

Supplemental Material

Supplemental Material A: Methods

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84 Supplemental Material A: Methods

85

86 Tables

87 Supplementary table A1: All-Cause Mortality for males collected from the Federal Health Report 2017
88 [1] (Sourced from the “Federal Government's Online Database of Health Reporting”)

Age	Probability
0-0	0.003518
1-4	0.000152
5-9	0.000074
10-14	0.000083
15-19	0.000282
20-24	0.000419
25-29	0.000462
30-34	0.000679
35-39	0.000965
40-44	0.001416
45-49	0.002426
50-54	0.004187
55-59	0.007237
60-64	0.011822
65-69	0.017948
70-74	0.027003
75-79	0.04199
80-84	0.073153
85-89	0.138834
90+	0.256934

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91 Supplementary table A2: Age-specific HIV acquisition probabilities from reports for 2017 and 2019 [2,
92 3] (Robert Koch Institute’s Epidemiological Bulletin) .

Age	Modeled Estimated Probability
10-14	0
15-19	2.75E-09
20-24	0.005273
25-29	0.004056
30-34	0.003037
35-39	0.00347
40-44	2.33E-10
45-49	0.002796
50-54	0.027954

55-59	3.54E-07
60-64	4.24E-07
65-69	6.59E-09
70-74	0.007996
75-79	1.1E-07
80+	6.86E-08

Supplementary table A3: Distribution of number of male sexual partners by age from a report by Bochow et al [4] (A publication of the German AIDS Association)

Age	14-19 years	20-29 years	30-44 years	>44 years
0 Partners	18%	8%	6%	6%
1 Partner	26%	27%	22%	18%
2 to 10 Partners	46%	48%	44%	48%
11 to 50 Partners	9%	15%	24%	24%
>50 Partners	2%	2%	5%	4%

Supplementary table A4: Various estimates of age-dependent number of male sexual partners. The parentheses after the estimate type indicate the average number of partners assumed for each of the partner bands in Table A3

Estimate type	Low (0,1,2,11,50)	Middle (0,1,6,15,50)	High (0,1,11,50,50)
14-19 years	3.17	5.37	10.36
20-29 years	3.88	6.4	13.57
30-44 years	6.24	8.96	19.12
>44 years	5.78	8.66	18.98

Supplementary table A5: Annual sexual partners by age used in the model

Age	Annual Partners
14-19	3.1700
20-29	3.8800
30-44	6.2400
45-49	5.7800
50-54	2.8900
55-59	1.4450
60-64	0.7225
65+	0

108 Supplementary table A6: Data on Distribution of Sexual Activity from a 2014 study on the prevalence
109 of other STIs among MSM [5]

Range of Number of Partners in Last 6 Months	Not Known if HIV+	HIV+	Average No. of Partners over 6 months assumed in model.
≤1	15.9%	27.9%	0.5
2-5	47.2%	33.8%	3.5
6-10	19.5%	17.3%	8
≥11	17.4%	21%	11

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112 Supplementary table A7: Serostatus mixing matrix based on Hotton et al [6]

	HIV-	HIV+
HIV-	0.9693	0.0307
HIV+	0.7309	0.2691

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115 Supplementary table A8: Age mixing matrix, reproduced from Lin et al [7]

		Partner							
		12-13	14-15	16-19	20-24	25-34	35-44	45-54	55-74
Respondent	12-13	0.492	0.000	0.000	0.000	0.508	0.000	0.000	0.000
	14-15	0.000	0.321	0.679	0.000	0.000	0.000	0.000	0.000
	16-19	0.000	0.089	0.535	0.123	0.126	0.033	0.065	0.028
	20-24	0.000	0.000	0.092	0.459	0.373	0.042	0.034	0.000
	25-34	0.009	0.000	0.064	0.253	0.536	0.061	0.040	0.037
	35-44	0.000	0.000	0.027	0.046	0.098	0.538	0.234	0.057
	45-54	0.000	0.000	0.085	0.059	0.102	0.376	0.310	0.067
	55-74	0.000	0.000	0.055	0.000	0.141	0.136	0.099	0.569

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118 Supplementary table A9: Anal HPV infection parameters

Parameter Name	Parameter Description	Prior Ranges and Constraints	Reference and Comments	Point Estimates
Clear	Clearance rate of HPV infection for HIV- MSM.	HPV16: (0.56, 1.41), HPV18: (0.68, 1.47), HPV31+: Clearhiv<Clear	[8], Probabilities of clearance per 1000 person months are converted to annual rates. We assume that HIV infection does not	HPV16: 1.033 HPV18: 0.947 HPV31+: 1.029

			increase the chances of clearance.	
Wnim	Rate of waning immunity from natural infection for HIV- MSM.	All Types:(0,12), Wnim<Wnimihiv	Assumption. Other models [7] assume ranges from lifelong protection, to no protection. Here we assume that at least 1 month of protection is achieved, if any is going to be gained at all (see Serocon). We also assume that HIV infection can only increase the rate of waning.	HPV16: 0.018 HPV18: 0.832 HPV31+: 0.291
Progdys	Rate of progression from infection to precancer for HIV- MSM.	All Types: (0,1), Progdys<Progdysshiv	[9]. We assume that progression among HIV- MSM is no faster than HIV+ MSM. No type-specific data was available, so this range is used for all types.	HPV16: 0.056 HPV18: 0.094 HPV31+: 0.654
Progdis	Rate of progression from precancer to invasive cancer for HIV- MSM.	All types: (0,0.01), Progdis<Progdishiv	[10, 11].	HPV16: 0.003 HPV18: 0.002 HPV31+: 0.003
Rinfdis	Rate of regression of precancer to infectious state for HIV- MSM.	All Types: (0, 0.5)	[11]. Since the precancer state in our model encapsulates multiple precancerous states, we provide a smaller lower bound than in provided in the source to account for cases that progress from LSIL to HSIL, and then regress. In a similar way, we increase the upper bounds to account for	HPV16: 0.00001 HPV18: 0.00781 HPV31+: 0.00520

			LSIL that could clear before progression.	
Serocon	Percent of HIV-MSM that seroconvert and gain immunity after clearance of HPV infection.	(0,0.15), Serconhiv<Serocon	[12]. We assume that HIV infection does not increase the percentage of serconversions.	HPV16: 0.069 HPV18: 0.125 HPV31+: 0.119
TProb	Probability of Transmission for HPV infected HIV-MSM.	All types: (0,1)	Assumption	HPV16: 0.101 HPV18: 0.070 HPV31+: 0.027
Clearhiv	Clearance rate of HPV infection for HIV+ MSM.	HPV16: (0.10,0.7425), HPV18: (0.17,1.477) HPV31+: Clearhiv<Clear	[8, 13, 14], Probabilities of clearance per 1000 person months are converted to annual rates, and the union of the confidence intervals across the three sources is used. We assume that HIV infection does not increase the rate of clearance.	HPV16: 0.613 HPV18: 0.494 HPV31+: 0.008
Wnimhiv	Rate of waning immunity from natural infection for HIV- MSM.	All types: (0,12), Wnim<Wnimihiv	Assumption.	HPV16: 4.747 HPV18: 4.411 HPV31+: 3.011
Progdysshiv	Rate of progression from infection to precancer for HIV- MSM.	All types: (0,1), Progdys<Progdysshiv	[9]. We assume that progression among HIV- MSM is no faster than HIV+ MSM. No Type specific data was available, so this estimate is used for all types.	HPV16: 0.099 HPV18: 0.204 HPV31+: 0.937

Progdishiv	Rate of progression from precancer to invasive cancer for HIV- MSM.	All types: (0,0.01), Progdishiv<Progdishiv	[10, 11]	HPV16: 0.005 HPV18: 0.056 HPV31+: 0.036
Rinfdyshiv	Rate of regression of precancer to infectious state for HIV- MSM.	All types: (0, 0.5)	[11, 15]. See comments on Rinfdys. We do not assume anything about relative clearance rates based on HIV status.	HPV16: 0.000 HPV18: 0.008 HPV31+: 0.005
Seroconhiv	Percent of HIV- MSM that seroconvert and gain immunity after clearance of HPV infection.	All types: (0,0.15), Seroconhiv<Serocon	[12]	HPV16: 0.064 HPV18: 0.074 HPV31+: 0.073
TProbhiv	Probability of Transmission for HPV infected HIV- MSM.	All types: (0,1)	Assumption	HPV16: 0.151 HPV18: 0.001 HPV31+: 0.005

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120 There was little data to establish prior ranges for penile and oropharyngeal disease specifically. For
 121 the prior ranges, then, the same assumptions and sources as with anal disease were applied.

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123 Supplementary table A10: Penile and Oropharyngeal HPV Infection parameters

Parameter Name	Parameter Description	Penile Point Estimates	Oropharyngeal Point Estimates
Clear	Clearance rate of HPV infection for HIV- MSM.	HPV16: 1.673 HPV18: 0.164 HPV31+: 1.317	HPV16: 1.967 HPV18: 1.426 HPV31+: 1.778
Wnim	Rate of waning immunity from natural infection for HIV- MSM.	HPV16: 0.018 HPV18: 0.909 HPV31+: 0.846	HPV16: 1.522 HPV18: 0.874 HPV31+: 0.017
Progdys	Rate of progression from infection to precancer for HIV- MSM.	HPV16: 0.026 HPV18: 0.703 HPV31+: 0.381	HPV16: 0.012 HPV18: 0.292 HPV31+: 0.510
Progdish	Rate of progression from precancer to invasive cancer for HIV- MSM.	HPV16: 0.0027171 HPV18: 0.0000003 HPV31+: 0.0000009	HPV16: 0.00079 HPV18: 0.02713 HPV31+: 0.03721
Rinfdys	Rate of regression of precancer to infectious state for HIV- MSM.	HPV16: 0.00544 HPV18: 0.00455 HPV31+: 0.00084	HPV16: 0.011 HPV18: 0.058 HPV31+: 0.008

Serocon	Percent of HIV- MSM that seroconvert after clearance of HPV infection.	HPV16: 0.074 HPV18: 0.091 HPV31+: 0.046	HPV16: 0.147 HPV18: 0.041 HPV31+: 0.075
TProb	Probability of Transmission for HPV infected HIV- MSM.	HPV16: 0.185 HPV18: 0.051 HPV31+: 0.337	HPV16: 0.411 HPV18: 0.067 HPV31+: 0.048
Clearhiv	Clearance rate of HPV infection for HIV+ MSM.	HPV16: 1.643 HPV18: 0.002 HPV31+: 0.859	HPV16: 0.129 HPV18: 0.200 HPV31+: 1.351
Wnimhiv	Rate of waning immunity from natural infection for HIV- MSM.	HPV16: 3.099 HPV18: 1.840 HPV31+: 3.401	HPV16: 4.763 HPV18: 3.347 HPV31+: 3.551
Progdysshiv	Rate of progression from infection to precancer for HIV- MSM.	HPV16: 0.059 HPV18: 0.703 HPV31+: 0.039	HPV16: 0.041 HPV18: 0.513 HPV31+: 0.637
Progdishiv	Rate of progression from precancer to invasive cancer for HIV- MSM.	HPV16: 0.0049910 HPV18: 0.0000003 HPV31+: 0.0000009	HPV16: 0.00088 HPV18: 0.04174 HPV31+: 0.03739
Rinfdyshiv	Rate of regression of precancer to infectious state for HIV- MSM.	HPV16: 0.000 HPV18: 0.002 HPV31+: 0.008	HPV16: 0.169 HPV18: 0.037 HPV31+: 0.008
Seroconhiv	Percent of HIV- MSM that seroconvert after clearance of HPV infection.	HPV16: 0.034 HPV18: 0.091 HPV31+: 0.024	HPV16: 0.009 HPV18: 0.029 HPV31+: 0.042
TProbhiv	Probability of Transmission for HPV infected HIV- MSM.	HPV16: 0.001 HPV18: 0.957 HPV31+: 0.975	HPV16: 0.1678 HPV18: 0.0006 HPV31+: 0.0032

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126 Supplementary table A11: HPV6/11 Infection and Anogenital Warts parameters

Parameter Name	Parameter Description	Prior Ranges and Constraints	Reference and Comments	Point Estimates
Gwtreat	Rate of treatment for anogenital warts for HIV- MSM.	gwclean<gwtreat<4	Assumption	2.344626717013334
Gwclear	Rate of clearance of anogenital warts for HIV- MSM in the absence of treatment.	0.1<gwclear<1	Assumption	0.5834373196078692
Gwprog	Rate of progression from HPV6/11 infection to anogenital warts for HIV- MSM.	0<gwprog<1.6	[16]	0.060661483346827416

Hpvclear	Rate of clearance of HPV6/11 infection for HIV- MSM.	$0.05 < \text{hpvclear} < 2$	[17, 18]. Converted duration in months to annual rate.	0.05380707795151091
Hpvwnim	Rate of waning for immunity to HPV6/11 for HIV- MSM.	$0 < \text{hpvwnim} < 2$	Assumption. See above for Wnim for anal infection.	0.00026955614452616 97
Sympfrac	Fraction of anogenital warts cases that are treated for HIV- MSM.	$0 < \text{sympfrac} < 1$	Assumption	0.8820468231246689
Serocon	Percent of HIV- MSM that seroconvert after clearance of HPV infection for HIV- MSM.	$0 < \text{serocon} < 1$	Assumption	0.9879700080460507
Tprob	Probability of HPV transmission for infected HIV- MSM.	$0 < \text{Tprob} < 1$	Assumption	0.09553927472942719
Gwtreatshiv	Rate of treatment for anogenital warts for HIV+ MSM.	$\text{Gwtreat} < \text{gwtreat hiv} < 12$	Assumption	4.879284759141555
Gwclearshiv	Rate of clearance of anogenital warts for HIV+ MSM.	$0 < \text{gwclearshiv} < \text{gw clear}$	Assumption	0.07402274918405617
Gwproghiv	Rate of progression from HPV6/11 infection to anogenital warts for HIV+ MSM.	$\text{Gwprog} < \text{gwprog hiv} < 10$	Assumption	6.518566101278571
Hpvclearshiv	Rate of clearance of HPV6/11 infection for HIV+ MSM.	$0 < \text{hpvclearshiv} < \text{hpvclear}$	Assumption	0.0014297951001279
Hpvwnimshiv	Rate of waning for immunity to HPV6/11 for HIV+ MSM.	$0 < \text{hpvwnimshiv} < \text{hpvwnim}$	Assumption	0.00010340335867138 107
Sympfracshiv	Fraction of anogenital warts cases that are treated for HIV+ MSM.	$\text{Sympfrac} < \text{sympfrac hiv} < 1$	Assumption	0.8821170905117549
Seroconshiv	Percent of HIV- MSM that seroconvert after clearance of HPV infection for HIV+ MSM.	$0 < \text{seroconshiv} < \text{serocon}$	Assumption	0.02500902095173833 4
Tprobshiv	Probability of HPV transmission for infected HIV+ MSM.	$0 < \text{Tprobshiv} < 1$	Assumption	0.02649331267627953

129 Supplementary table A12: Historical uptakes used in the model

Age	2018	2019	2020
9	0	0.019	0.054
10	0	0.021	0.044852
11	0.001	0.08	0.127681
12	0	0.061061	0.128261
13	0	0.051	0.106603
14	0	0.034	0.084299
15	0	0.04	0.076605
16	0	0.034	0.061458
17	0	0.008	0.009317

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Supplementary table A13: Costs for the various diseases. The cost of palliative care is only incurred in patient populations that die from cancer

Disease	Diagnostic, Treatment, and Follow Up Costs (€)	Cost (€) of Inpatient Palliative Care	Source
Anal Cancer	9994.53	6482.87	[19] (A German-language paper of the cost-effectiveness of gender neutral vaccination)
Penile Cancer	7292.64	4956.05	[19]
Oropharyngeal Cancer	12863.33	8344.44	[19]
Anogenital Warts	430.82	0.00	[19]

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136 Supplementary table A14: Duration of treatment and associated utilities

Disease	Treatment Duration	Treatment Utility	Source
Anal Cancer	1 year	0.735	[7],[12]
Penile Cancer	1 year	0.845	[7],[12]
Oropharyngeal Cancer	1 year	0.74	[7],[12]
Anogenital Warts	1 year	0.9	[7],[12]

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139 Supplementary table A15: Duration of the survival period, in which there is a decrement to quality-of-
140 life following cancer treatment

Disease	Survivor Duration	Survivor Utility	Source
Anal Cancer	4.5 years	0.9	[19],[7]
Penile Cancer	4.5 years	0.9	[19],[7]
Oropharyngeal Cancer	4.5 years	0.9	[19],[7]

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143 Supplementary table A16: CHEERS 2022 Checklist

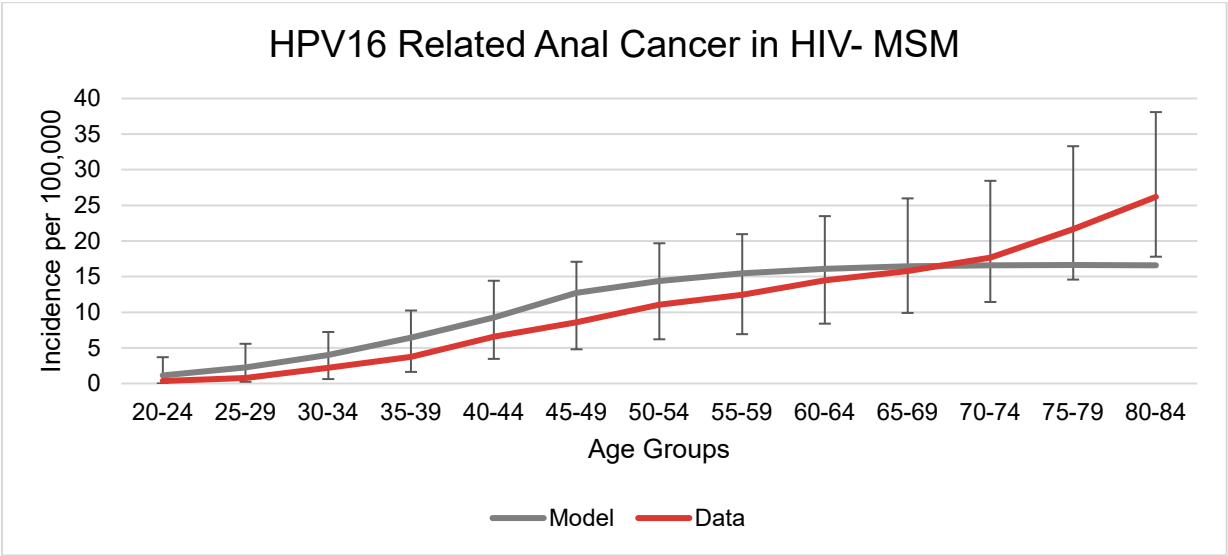
Topic	No.	Item	Location where item is reported
Title			
	1	Identify the study as an economic evaluation and specify the interventions being compared.	Manuscript, Title page
Abstract			
	2	Provide a structured summary that highlights context, key methods, results, and alternative analyses.	Manuscript, Section: Abstract
Introduction			
Background and objectives	3	Give the context for the study, the study question, and its practical relevance for decision making in policy or practice.	Manuscript, Section: Introduction
Methods			
Health economic analysis plan	4	Indicate whether a health economic analysis plan was developed and where available.	Not reported
Study population	5	Describe characteristics of the study population (such as age range, demographics, socioeconomic, or clinical characteristics).	Manuscript, Section: General model description and calibration
Setting and location	6	Provide relevant contextual information that may influence findings.	Manuscript, Section: General model description and calibration
Comparators	7	Describe the interventions or strategies being compared and why chosen.	Manuscript, Section: Vaccination strategies and analyses
Perspective	8	State the perspective(s) adopted by the study and why chosen.	Not reported

Topic	No.	Item	Location where item is reported
Time horizon	9	State the time horizon for the study and why appropriate.	Manuscript, Section: Vaccination strategies and analyses
Discount rate	10	Report the discount rate(s) and reason chosen.	Manuscript, Section: Vaccination strategies and analyses
Selection of outcomes	11	Describe what outcomes were used as the measure(s) of benefit(s) and harm(s).	Manuscript, Section: Vaccination strategies and analyses
Measurement of outcomes	12	Describe how outcomes used to capture benefit(s) and harm(s) were measured.	Not relevant
Valuation of outcomes	13	Describe the population and methods used to measure and value outcomes.	Not reported
Measurement and valuation of resources and costs	14	Describe how costs were valued.	Manuscript, Section: Vaccination strategies and analyses
Currency, price date, and conversion	15	Report the dates of the estimated resource quantities and unit costs, plus the currency and year of conversion.	Not reported
Rationale and description of model	16	If modelling is used, describe in detail and why used. Report if the model is publicly available and where it can be accessed.	Manuscript, Section: General model description and calibration
Analytics and assumptions	17	Describe any methods for analysing or statistically transforming data, any extrapolation methods, and approaches for validating any model used.	Manuscript, Section: General model description and calibration
Characterising heterogeneity	18	Describe any methods used for estimating how the results of the study vary for subgroups.	Manuscript, Section: General model description and calibration
Characterising distributional effects	19	Describe how impacts are distributed across different individuals or adjustments made to reflect priority populations.	Not reported
Characterising uncertainty	20	Describe methods to characterise any sources of uncertainty in the analysis.	Manuscript, Section: Sensitivity Analyses
Approach to engagement with patients and others affected by the study	21	Describe any approaches to engage patients or service recipients, the general public, communities, or stakeholders (such as clinicians or payers) in the design of the study.	Not reported

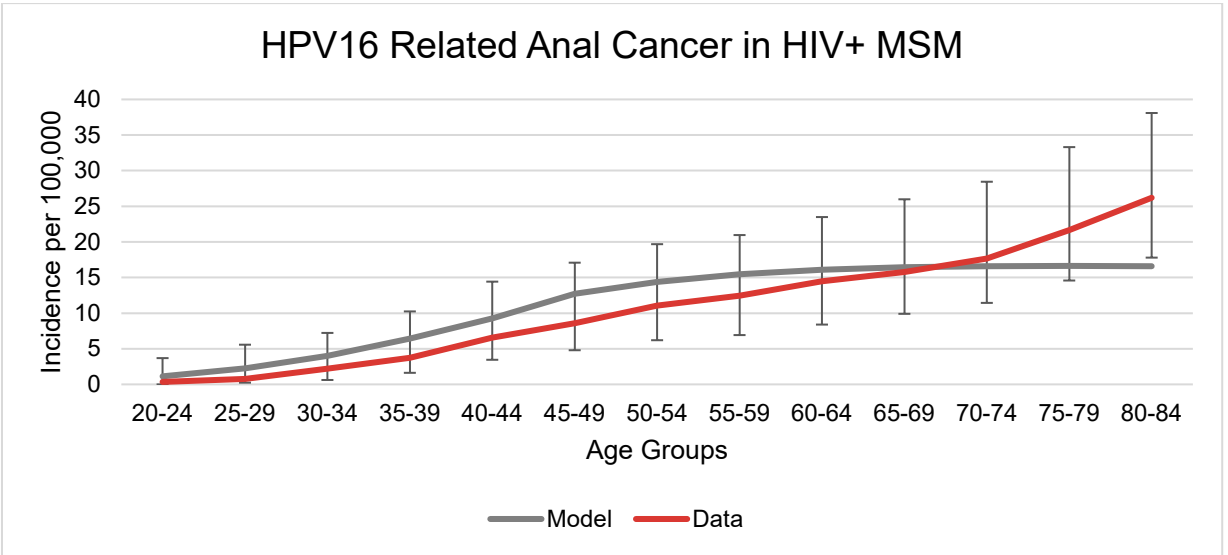
Topic	No.	Item	Location where item is reported
Results			
Study parameters	22	Report all analytic inputs (such as values, ranges, references) including uncertainty or distributional assumptions.	Manuscript, Section: Vaccination strategies and analyses; Supplementary Material
Summary of main results	23	Report the mean values for the main categories of costs and outcomes of interest and summarise them in the most appropriate overall measure.	Manuscript, Section: Results, Table 3
Effect of uncertainty	24	Describe how uncertainty about analytic judgments, inputs, or projections affect findings. Report the effect of choice of discount rate and time horizon, if applicable.	Manuscript, Section: Scenario and sensitivity analyses
Effect of engagement with patients and others affected by the study	25	Report on any difference patient/service recipient, general public, community, or stakeholder involvement made to the approach or findings of the study	Not reported
Discussion			
Study findings, limitations, generalisability, and current knowledge	26	Report key findings, limitations, ethical or equity considerations not captured, and how these could affect patients, policy, or practice.	Manuscript, Section: Discussion
Other relevant information			
Source of funding	27	Describe how the study was funded and any role of the funder in the identification, design, conduct, and reporting of the analysis	Manuscript, Section: Acknowledgements
Conflicts of interest	28	Report authors conflicts of interest according to journal or International Committee of Medical Journal Editors requirements.	Manuscript, Section: Acknowledgements

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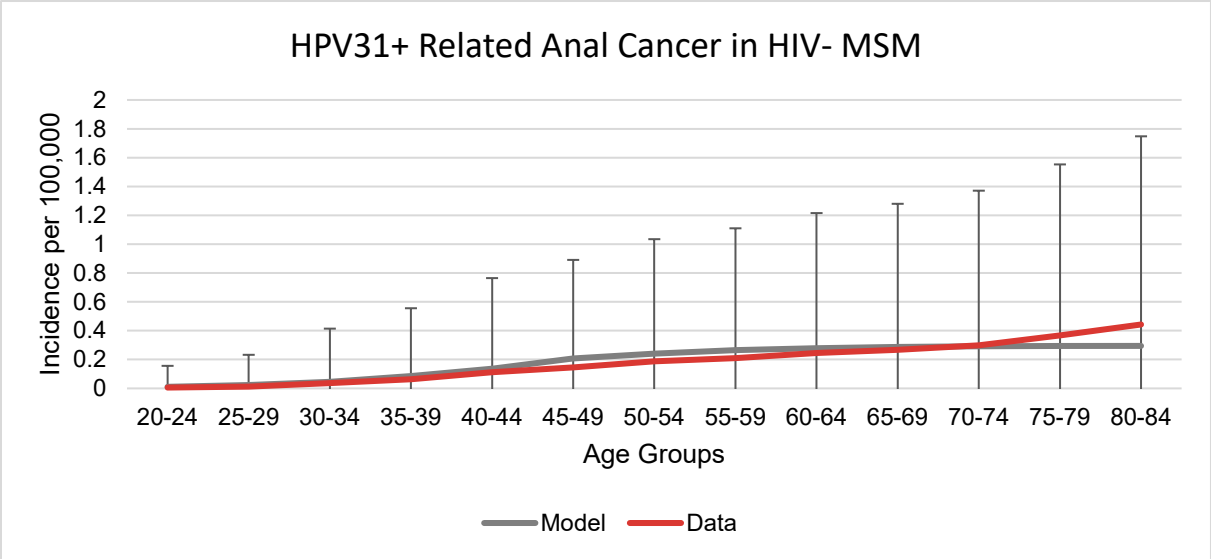
151 Figures



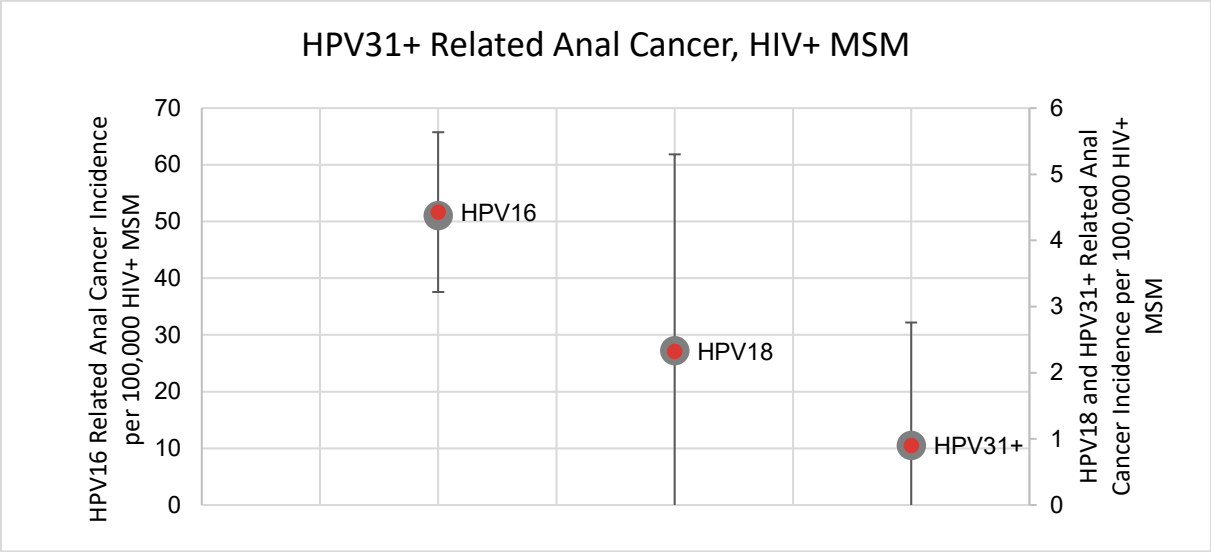
Supplementary Figure A1: Calibration results using a maximum likelihood estimation for hPV16 Related Anal Cancer in HIV- MSM



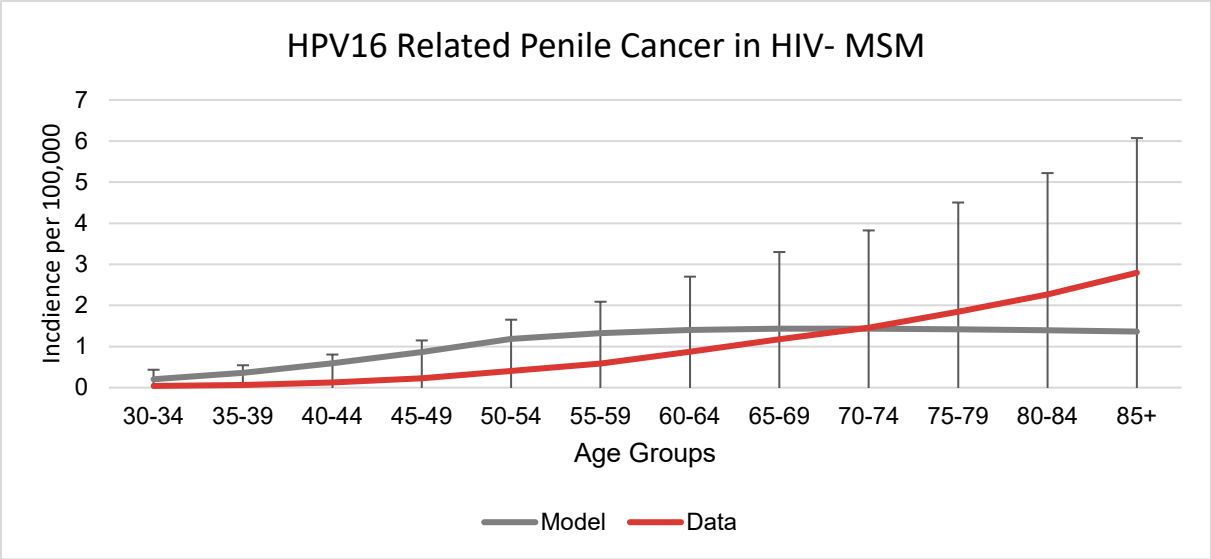
Supplementary Figure A2: Calibration results using a maximum likelihood estimation for HPV16 Related Anal Cancer in HIV+ MSM



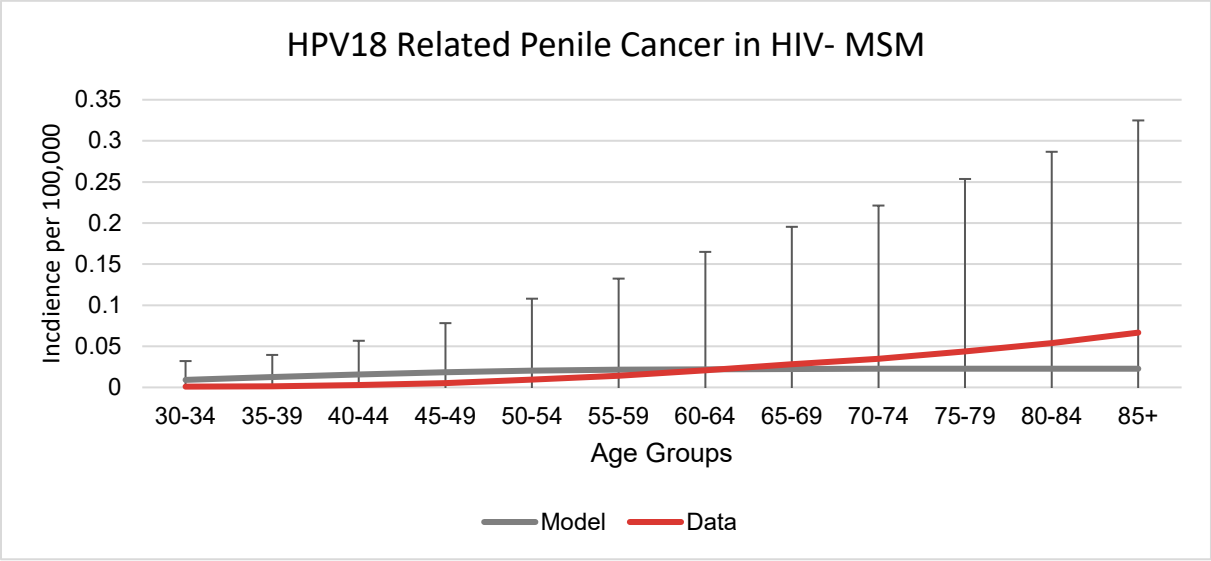
Supplementary Figure A3: Calibration results using a maximum likelihood estimation for HPV31+ Related Anal Cancer in HIV- MSM



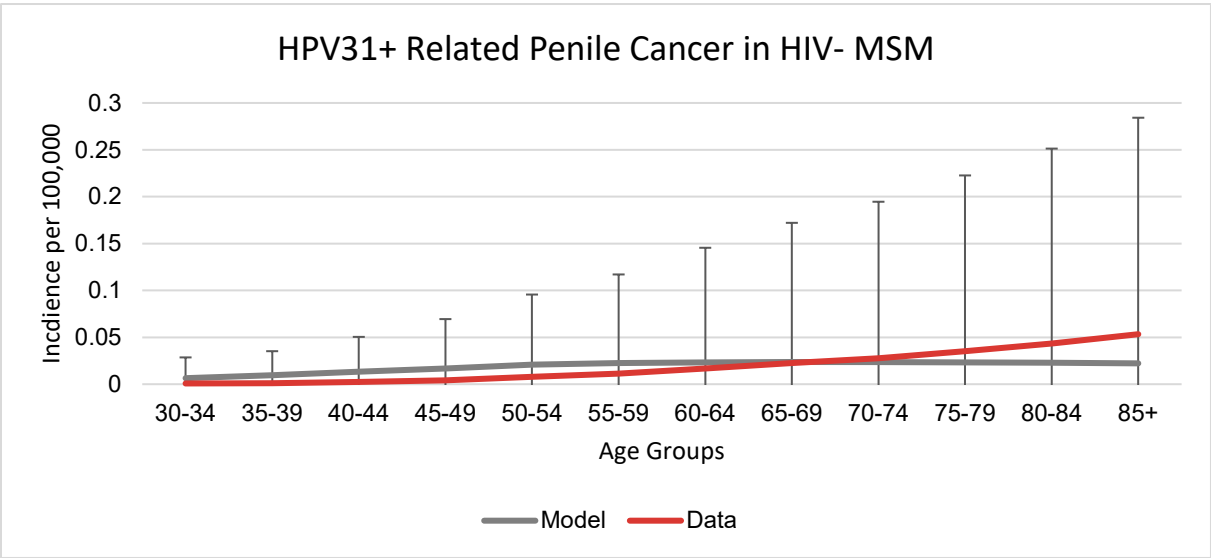
Supplementary Figure A4: Calibration results using a maximum likelihood estimation for HPV31+ Related Anal Cancer in HIV+ MSM



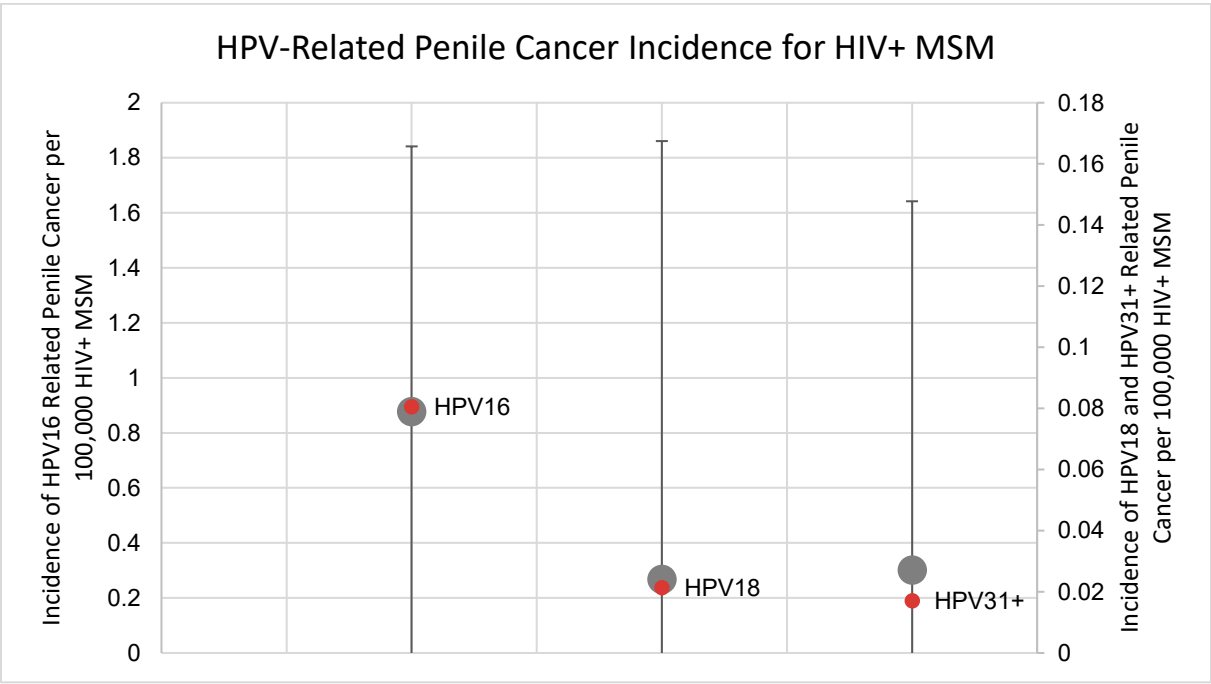
Supplementary Figure A5: Calibration results using a maximum likelihood estimation for HPV16 Related Penile Cancer in HIV- MSM



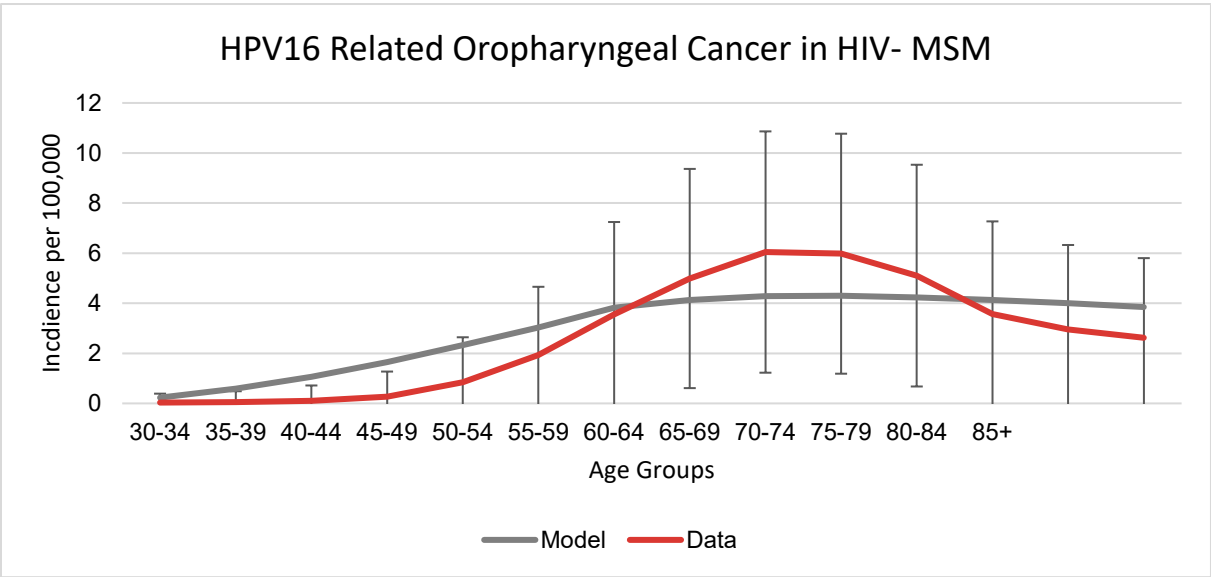
Supplementary Figure A6: Calibration results using a maximum likelihood estimation for HPV18 Related Penile Cancer in HIV- MSM



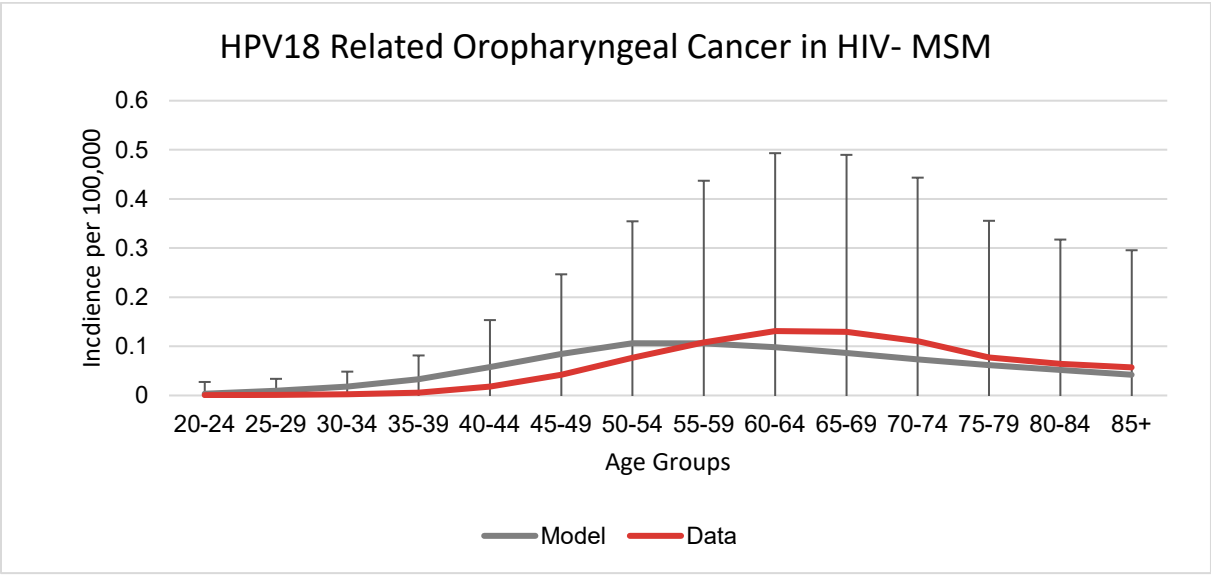
Supplementary Figure A7: Calibration results using a maximum likelihood estimation for HPV31+ Related Penile Cancer in HIV- MSM



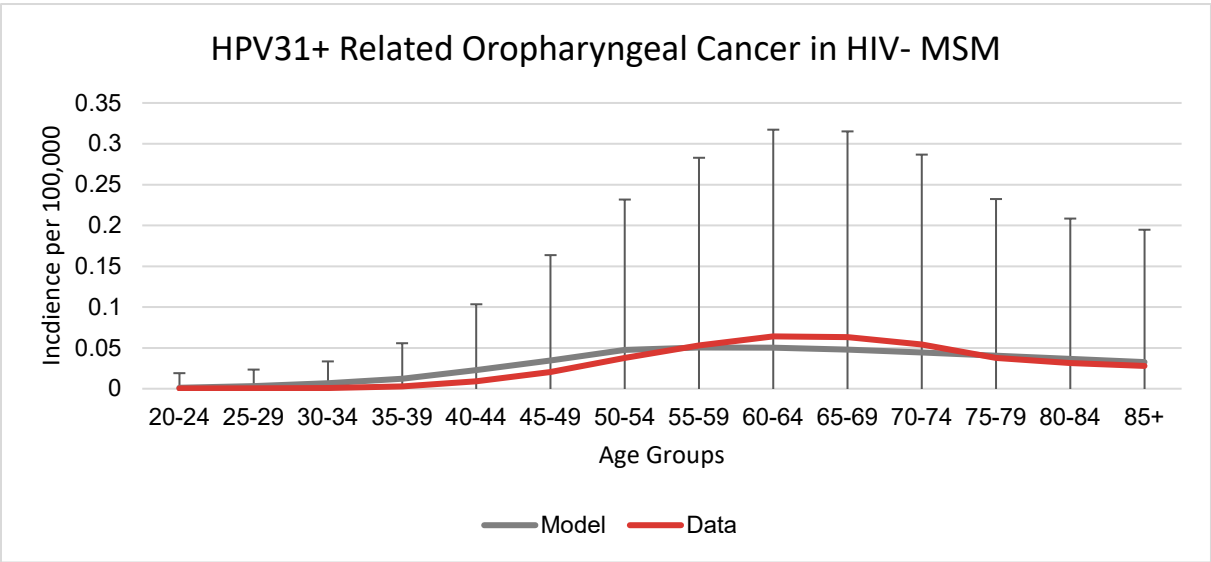
Supplementary Figure A8: Calibration results using a maximum likelihood estimation for HPV16 Related Penile Cancer Incidence for HIV+ MSM



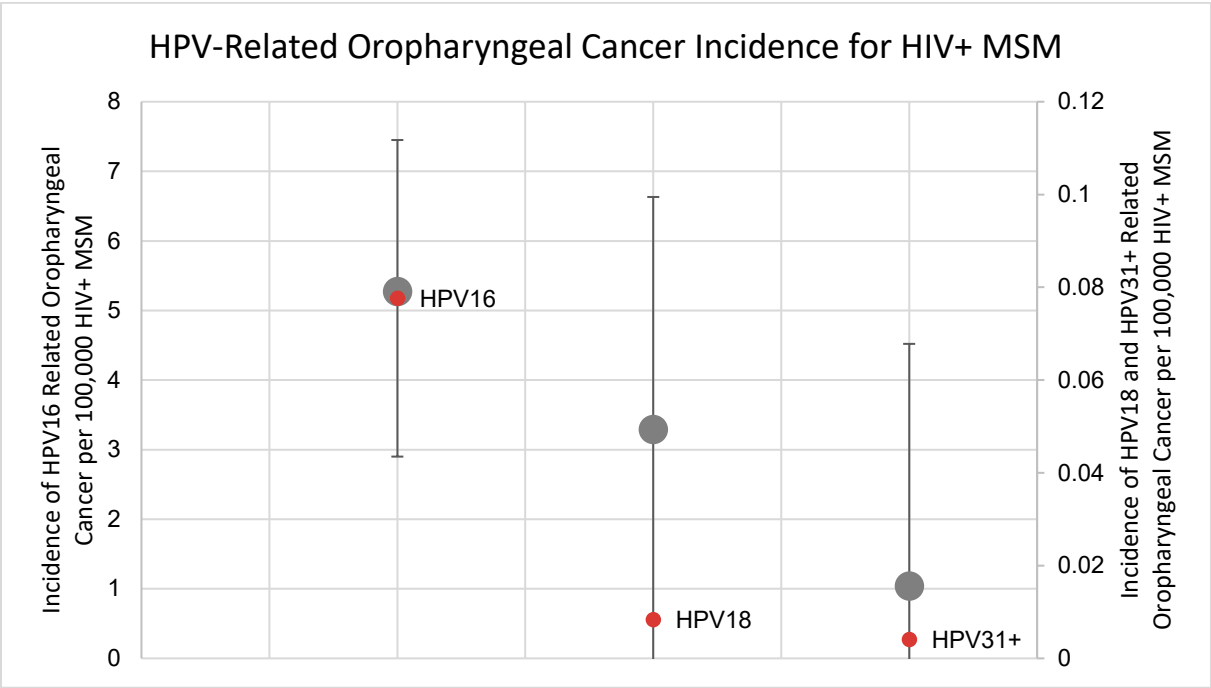
Supplementary Figure A9: Calibration results using a maximum likelihood estimation for HPV16 Related Oropharyngeal Cancer in HIV- MSM



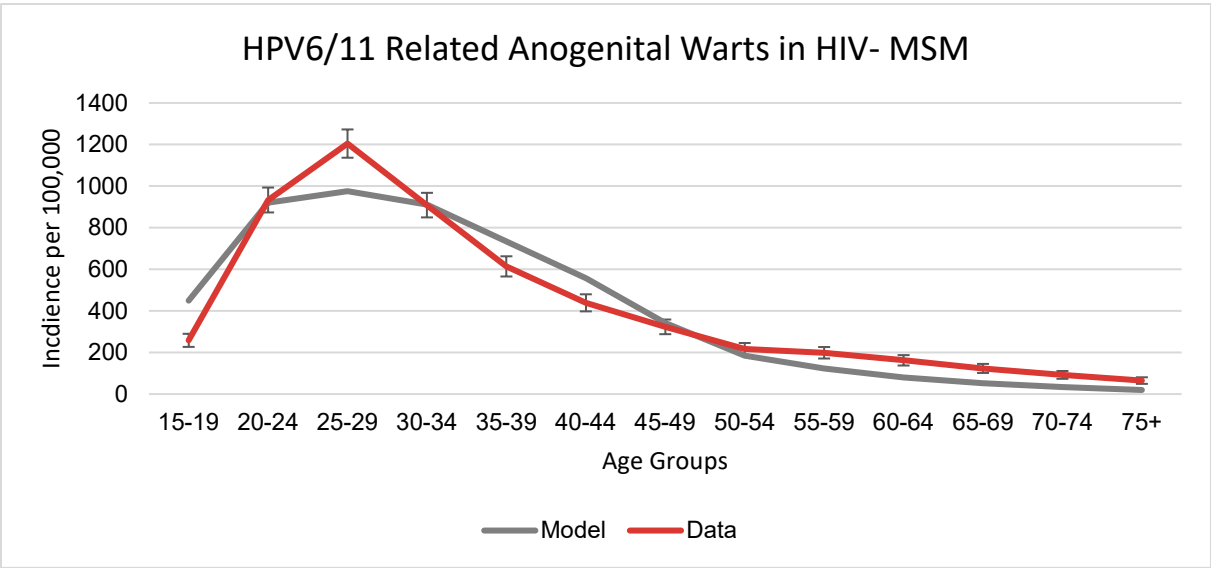
Supplementary Figure A10: Calibration results using a maximum likelihood estimation for HPV18 Related Oropharyngeal Cancer in HIV- MSM



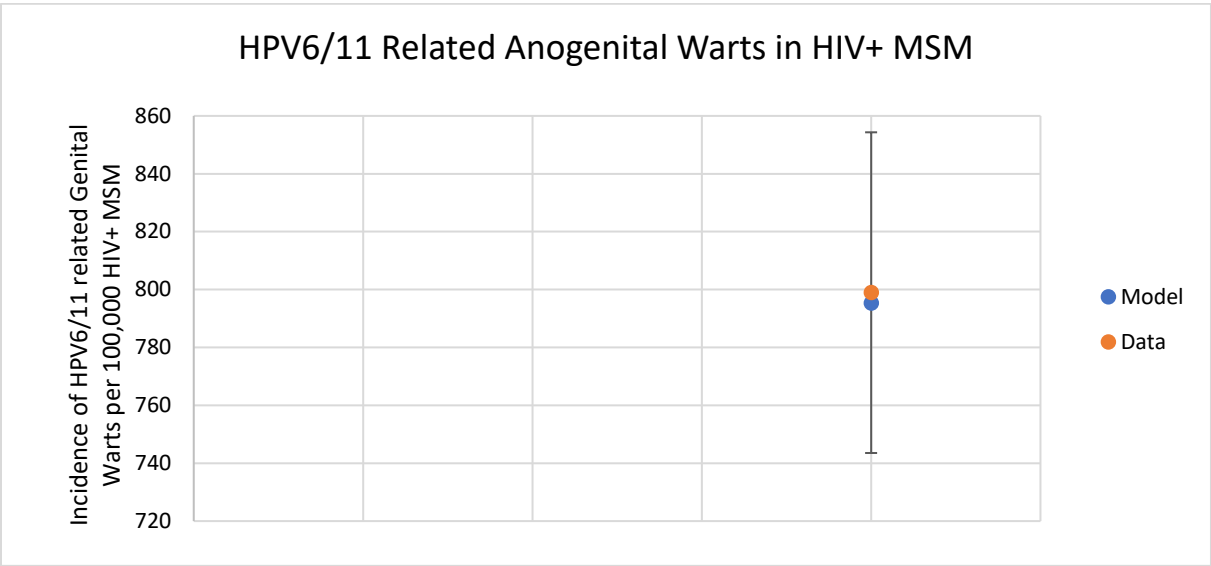
Supplementary Figure A11: Calibration results using a maximum likelihood estimation for HPV31+ Related Oropharyngeal Cancer in HIV- MSM



Supplementary Figure A12: Calibration results using a maximum likelihood estimation for HPV Related Oropharyngeal Cancer in HIV+ MSM



Supplementary Figure A13: Calibration results using a maximum likelihood estimation for HPV6/11 Related Anogenital Warts in HIV- MSM



Supplementary Figure A14: Calibration results using a maximum likelihood estimation for HPV6/11 Related Anogenital Warts in HIV+ MSM

213 Supplemental Material B: Results

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215 Tables

216 Supplementary table B1: Minimum NNV for the various disease endpoints and strategy, classified by
217 HIV Status and Uptake Scenarios

		HIV Negative		HIV Positive	
Disease	Strategy	Minimum NNV	Year minimum NNV is attained	Minimum NNV	Year minimum NNV is attained
Anal Cancer	Baseline	9562	2041	1353	2042
	18-26 MSM, 5% Uptake	9100	2039	1250	2028
	18-45 MSM, 5% Uptake	8244	2036	1007	2028
	18-26 MSM, 10% Uptake	8830	2038	1243	2028
	18-45 MSM, 10% Uptake	7825	2035	999	2028
Penile Cancer	Baseline	946514	2042	393912	2058
	18-26 MSM, 5% Uptake	868165	2037	356931	2028
	18-45 MSM, 5% Uptake	745151	2036	288870	2028
	18-26 MSM, 10% Uptake	833720	2036	352610	2028
	18-45 MSM, 10% Uptake	703819	2034	286040	2028
Oropharyngeal Cancer	Baseline	38456	2038	17171	2026
	18-26 MSM, 5% Uptake	35776	2035	13707	2028
	18-45 MSM, 5% Uptake	31388	2034	10030	2028
	18-26 MSM, 10% Uptake	34531	2034	13538	2028
	18-45 MSM, 10% Uptake	29510	2033	9956	2028
Anogenital Warts	Baseline	182	2034	8	2025
	18-26 MSM, 5% Uptake	167	2031	8	2025
	18-45 MSM,	161	2030	8	2025

	5% Uptake				
	18-26 MSM, 10% Uptake	161	2030	8	2025
	18-45 MSM, 10% Uptake	154	2029	8	2025

Supplementary table B2: Additional cancer cases, deaths and anogenital warts averted compared to vaccination of adolescent boys only in a 10% uptake scenario, given by disease site, HIV status and age groups

Disease site	HIV Status	MSM 18-26, 10% Uptake	MSM 18-45, 10% Uptake
Averted Cases (n, %)			
Anal Cancer	HIV-	1773 (17.10)	2682 (25.87)
	HIV+	961 (17.95)	1481 (27.67)
Oropharyngeal Cancer	HIV-	769 (26.30)	990 (33.86)
	HIV+	176 (33.56)	237 (45.21)
Penile Cancer	HIV-	32 (24.56)	41 (31.01)
	HIV+	5 (22.17)	7 (29.20)
Averted Deaths (n, %)			
Anal Cancer	HIV-	441 (15.94)	678 (24.52)
	HIV+	249 (16.93)	388 (26.36)
Oropharyngeal Cancer	HIV-	266 (22.57)	356 (30.23)
	HIV+	61 (30.16)	85 (41.89)
Penile Cancer	HIV-	6 (22.79)	8 (29.43)
	HIV+	1 (20.90)	1 (27.93)
Averted Anogenital Warts (n, %)			
Anogenital Warts	HIV-	125,696 (55.73)	154,474 (68.49)
	HIV+	6526 (59.33)	7914 (71.95)

Supplementary table B3: Minimum Number needed to vaccinate (NNV), Costs per Capita, QALYs lost per Capita and Cost Effectiveness compared to vaccinating of adolescent boys only in a 10% uptake scenario, given by disease site, HIV status and age groups

Disease site	HIV Status	MSM 18-26, 10% Uptake	MSM 18-45, 10% Uptake
Minimum NNV per vaccination strategy (n)			
Anal Cancer	HIV-	8830	7825
	HIV+	1243	999
Oropharyngeal Cancer	HIV-	34531	29510
	HIV+	13538	9956
Penile Cancer	HIV-	833720	703819
	HIV+	352610	286040

Anogenital Warts	HIV-	161	154
	HIV+	8	8
Costs per Capita (€)			
Vaccination	Total	182.0729	287.5553
Anal Cancer	Total	29.67147	27.86411
Oropharyngeal Cancer	Total	8.095721	7.594411
Penile Cancer	Total	0.172819	0.16549
Genital Warts	Total	25.79566	22.04582
QALYs lost per Capita			
Anal Cancer	Total	0.009891	0.009364
Oropharyngeal Cancer	Total	0.002027	0.0019
Penile Cancer	Total	8.1E-05	7.82E-05
Anogenital Warts	Total	0.002628	0.002166
Cost Effectiveness			
Incremental Costs (€)	Total	104.502061	203.9186032
Incremental QALYs	Total	0.002836524	0.003955374
ICER (€/QALY)	Total	36841.59	51554.82

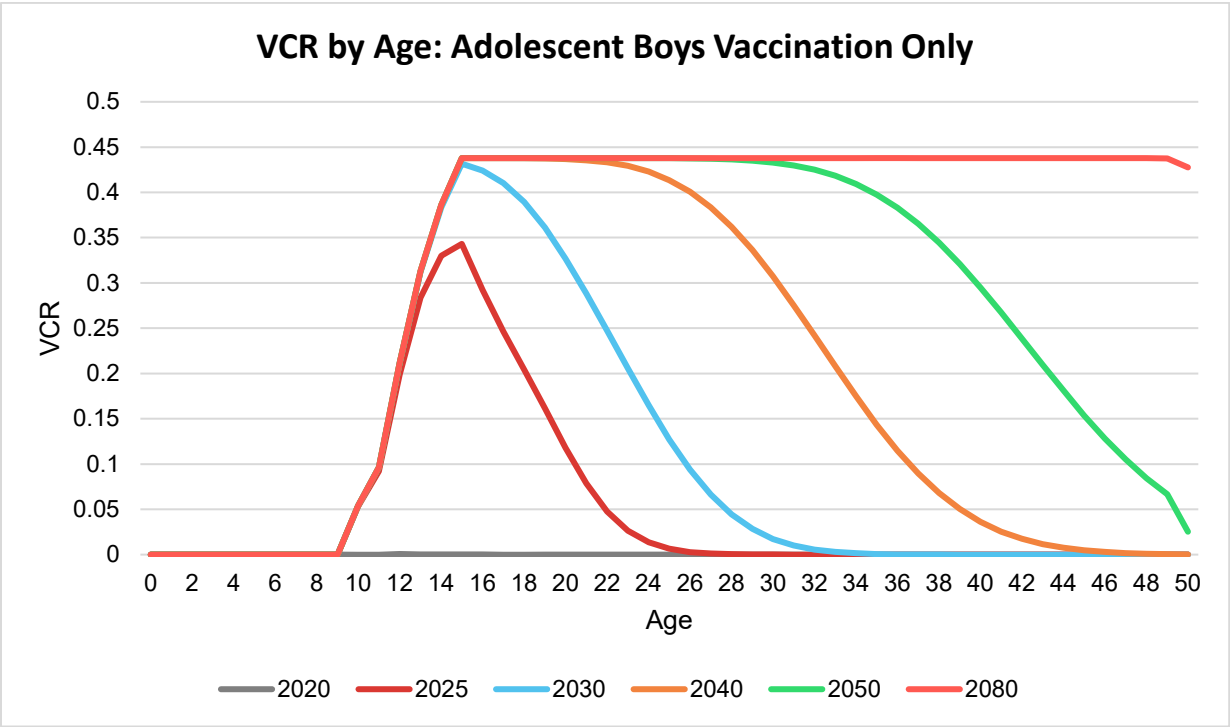
Supplementary table B4: Economic results for different uptake scenarios

Strategy	Cost Per Capita (€)	QALYs per Capita	ICER (€/QALY)	Notes
Adolescent Boys Only	141.306545	0.017464	--	
18-26 Targeted Vaccination of MSM, 5% Uptake	192.085727	0.016025	35,300.091623	Vs. Adolescent Boys Only
18-45 Targeted Vaccination of MSM, 5% Uptake	239.342273	0.015134	53,049.557589	Vs. 18-26 MSM, 5% Uptake
18-26 Targeted Vaccination of MSM, 10% Uptake	245.808606	0.014627	36,841.904107	Vs. Adolescent Boys Only
18-45 Targeted Vaccination of MSM, 10% Uptake	345.225148	0.013508	88,852.035124	Vs. 18-26 MSM, 10% Uptake

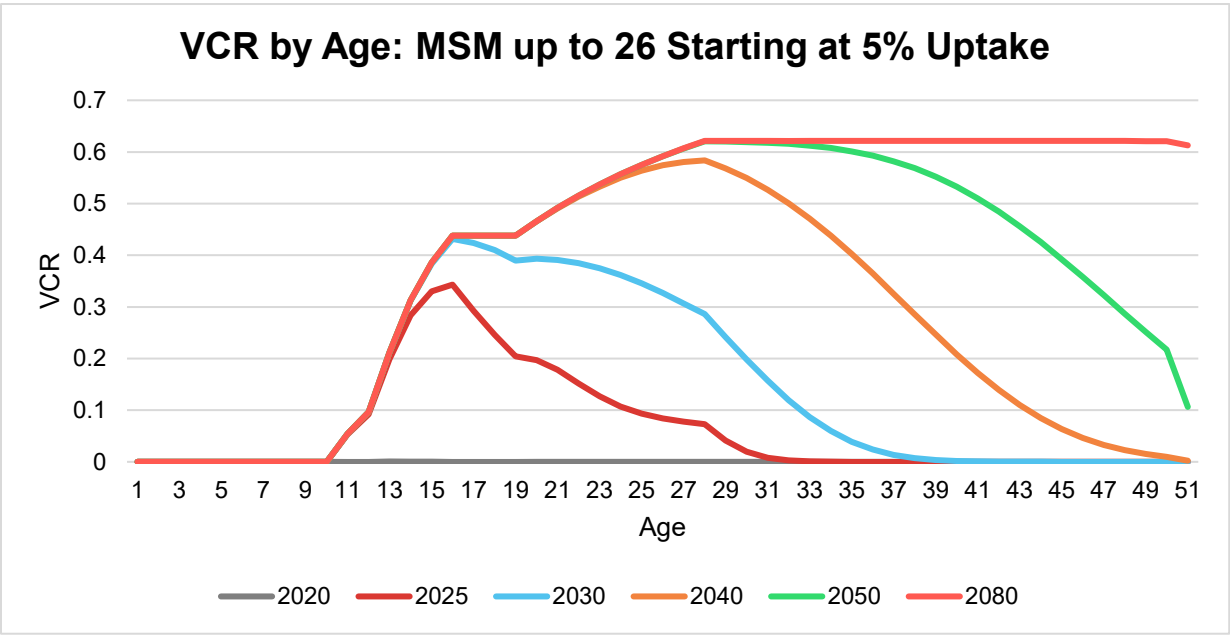
233 Supplementary table B5: Total cases of cancers with no vaccination compared to boys-only
234 vaccination.

Disease site	HIV Status	No Vaccination	Boys Only Vaccination
Total Cases over the 100 year time horizon (% compared to No vaccination).			
Anal Cancer	HIV-	18752 (100%)	10368 (55%)
	HIV+	9282 (100%)	5354 (58%)
		28034 (100%)	15722 (56%)
Oropharyngeal Cancer	HIV-	4881 (100%)	2923 (60%)
	HIV+	847 (100%)	525 (62%)
		5728 (100%)	3448 (60%)
Penile Cancer	HIV-	194 (100%)	131 (68%)
	HIV+	30 (100%)	23 (77%)
		224 (100%)	154 (69%)

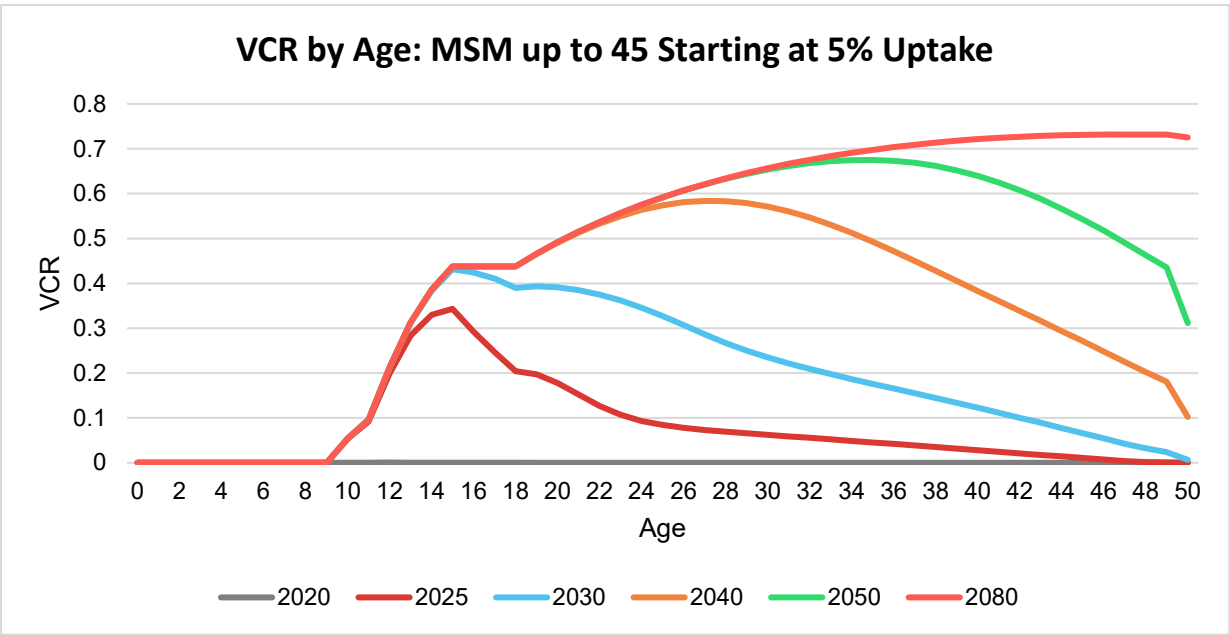
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237 **VCR Rates**
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240 Supplementary figure B1: Vaccination Coverage Rate (VCR) by Age: Adolescent Boys Vaccination Only
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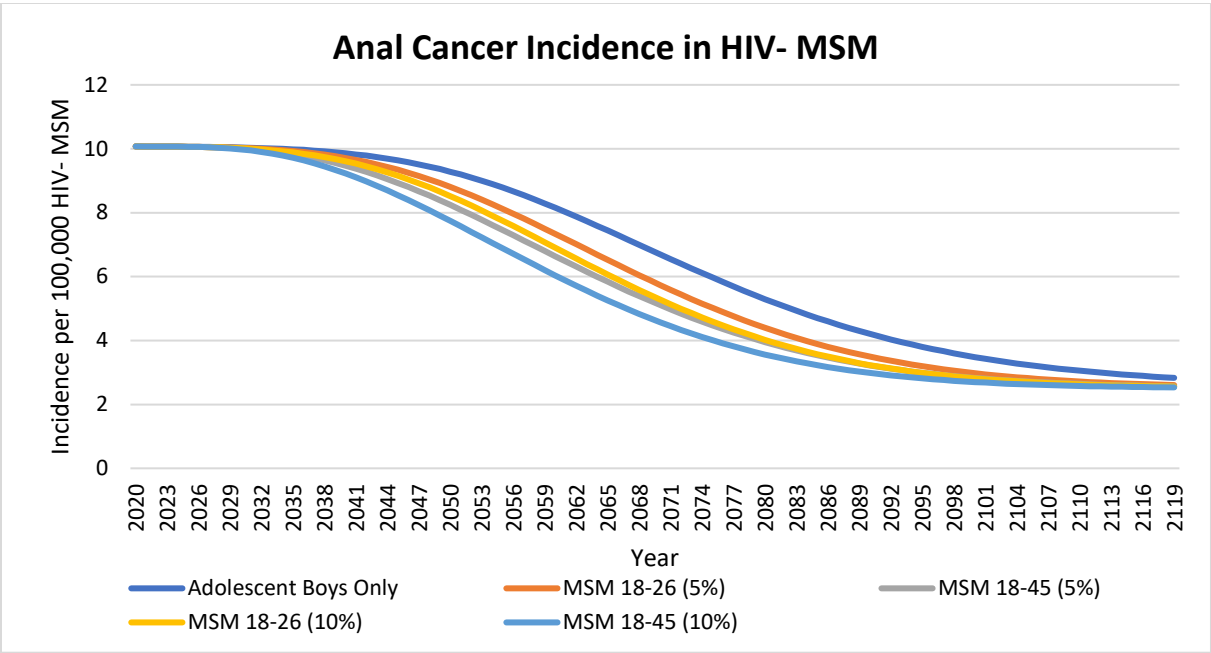


Supplementary figure B2: VCR by Age: MSM up to 26, Starting at 5% Uptake

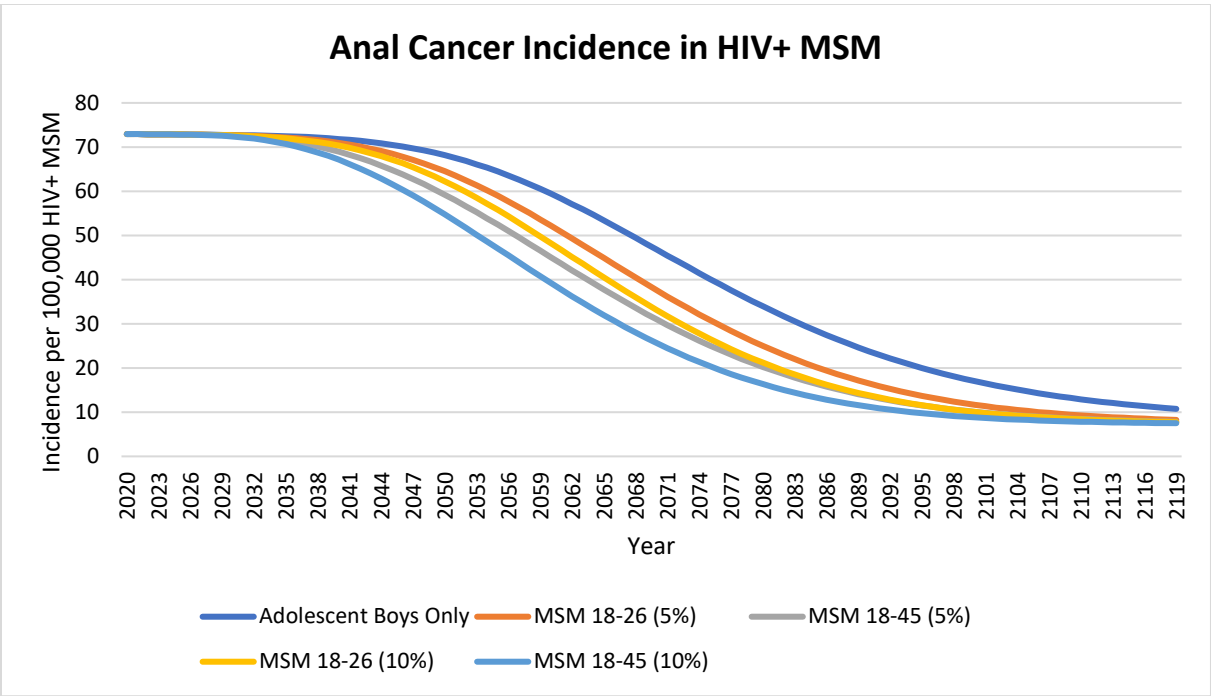


Supplementary figure B3: VCR by Age: MSM up to 45, Starting at 5% Uptake

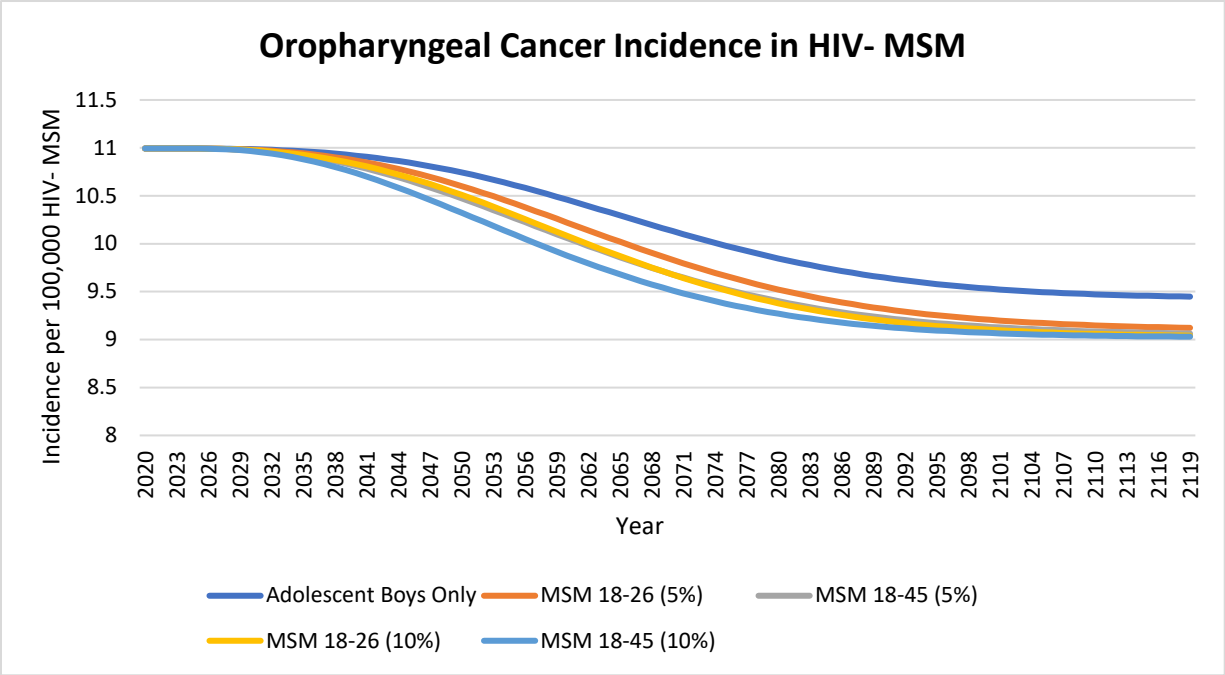
Disease Incidence Over Time



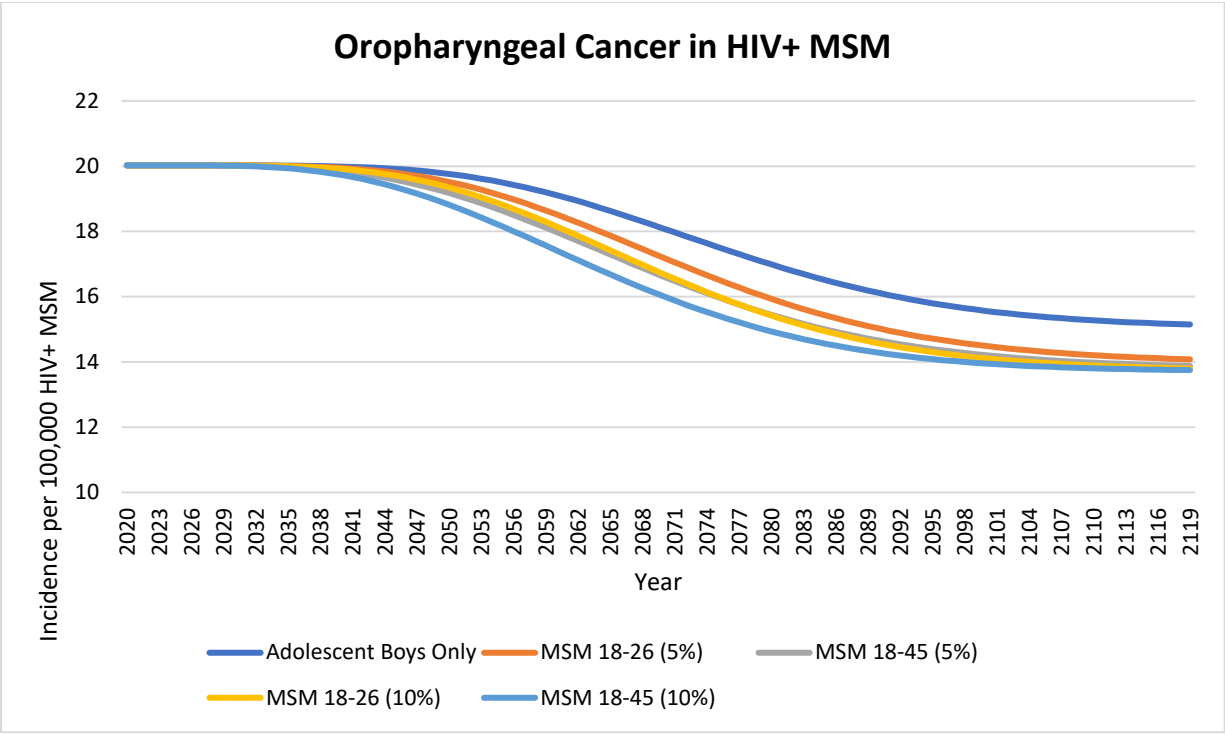
Supplementary figure B4: Disease Incidence Over Time: Anal Cancer Incidence in HIV- MSM



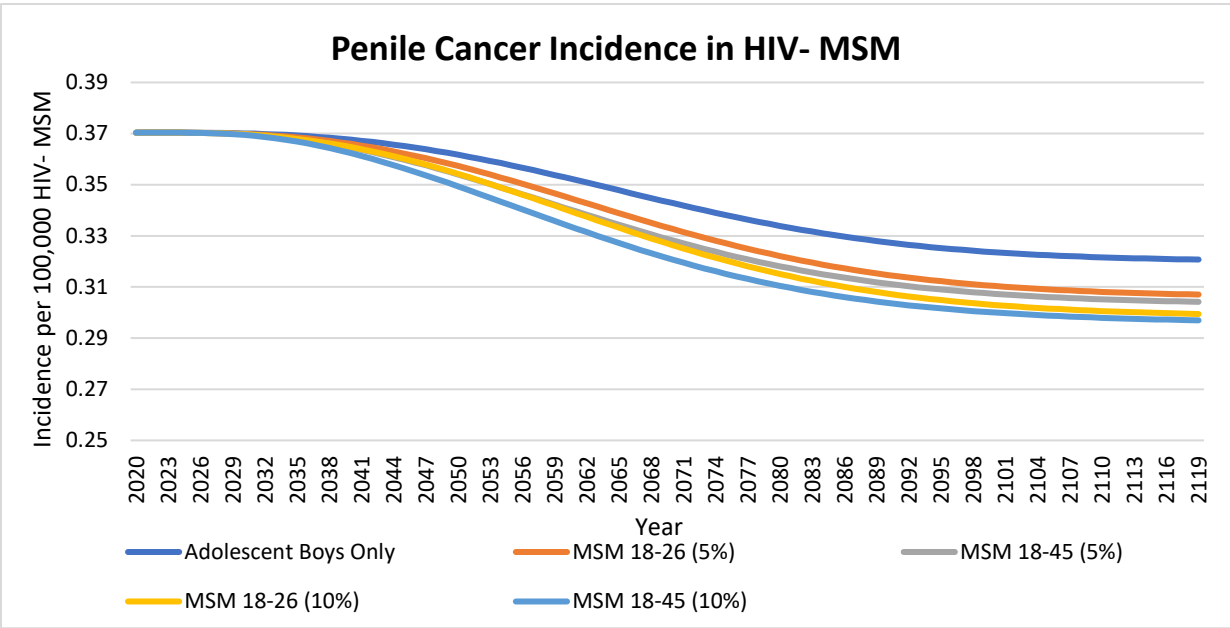
Supplementary figure B5: Anal Cancer Incidence in HIV+ MSM



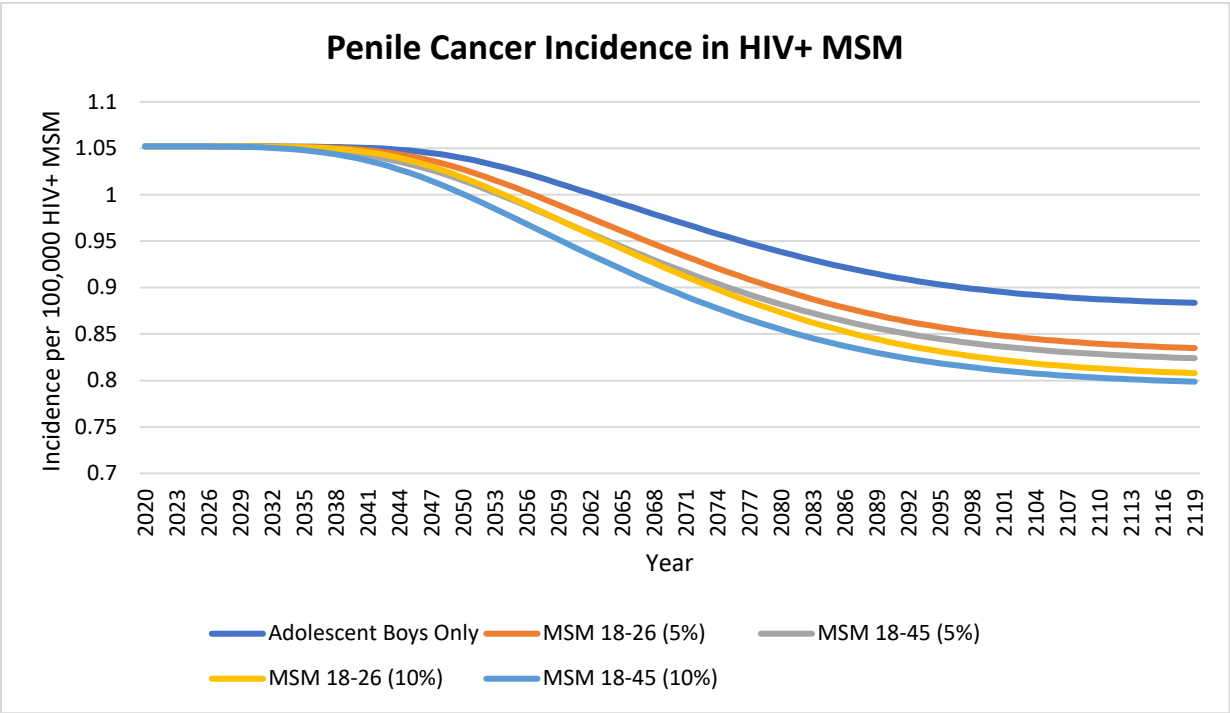
Supplementary figure B6: Oropharyngeal Cancer Incidence in HIV- MSM



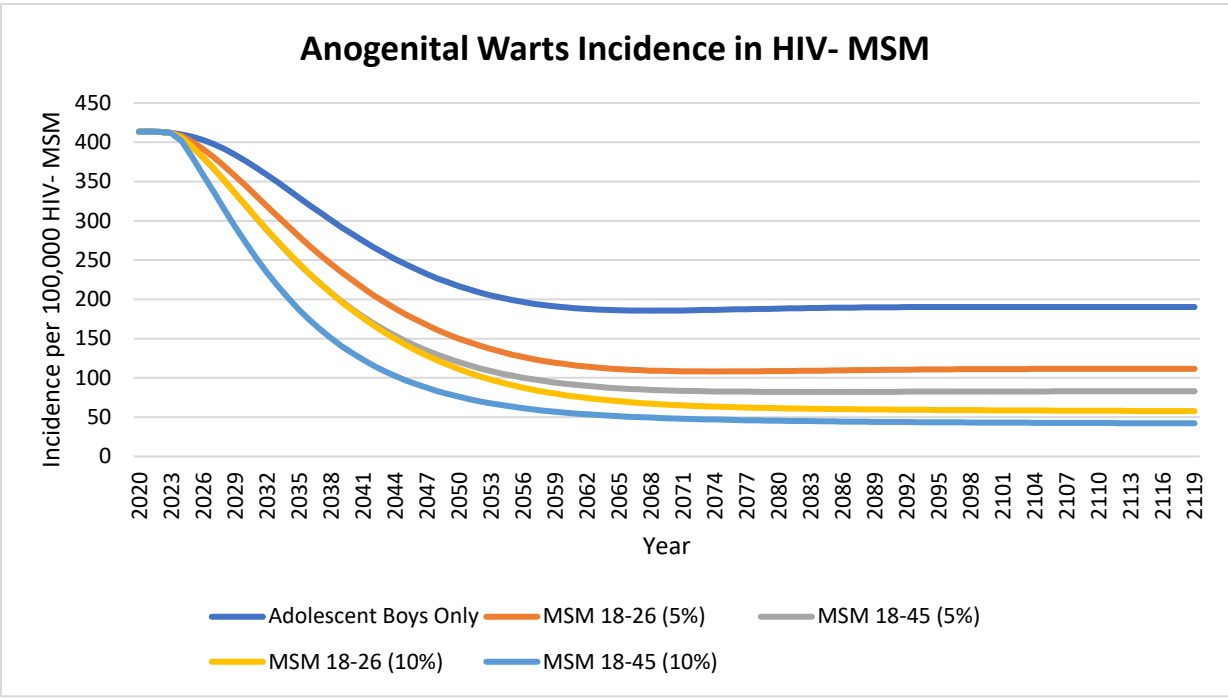
Supplementary figure B7: Oropharyngeal Cancer in HIV+ MSM



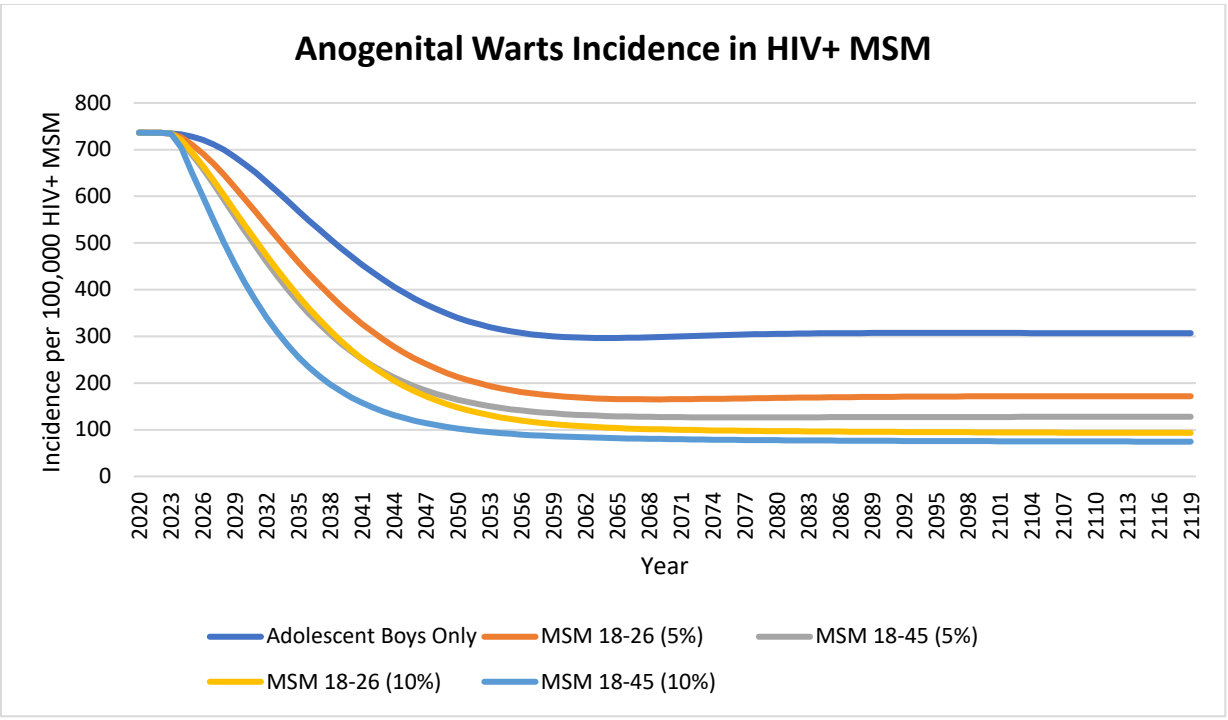
Supplementary figure B8: Penile Cancer Incidence in HIV- MSM



Supplementary figure B9: Penile Cancer Incidence in HIV+ MSM



Supplementary figure B10: Anogenital Warts Incidence in HIV- MSM



Supplementary figure B11: Anogenital Warts Incidence in HIV+ MSM

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- 346

Supplemental Information C

Ordinary Differential Equations for the Transmission of HPV among Men Who Have Sex With Men

Throughout, take $2 \leq i \leq age_{max}$, $1 \leq j \leq act_{max}$, and $u \in \{\text{pos}, \text{neg}\}$.

1 Demographic Model

1.1 MSM HIV- Demographics

age_{max} is the number of age groups in the model, and act_{max} is the number of sexual activity groups in the model. cmu is the death rate due to undetected cancer, and $dcmu$ is the death rate in detected cancer. $hivacqprob(i)$ is the probability of acquiring HIV at in the age group i , d_i is the rate of aging out of age group i , and is computed based on assumption of steady state demographics. μ_i is the background all cause mortality rate. $hivrisk(j)$ represents the increased chance of getting HIV in the sexual activity group j .

$$\begin{aligned} Na'_{1,j}(t) &= -cmu_1 dis_{1,j}(t) - (d_1 + \mu_1) Na_{1,j}(t) - dcmu_1 detdis_{1,j}(t) + B_j \\ Na'_{i,j}(t) &= -cmu_i dis_{i,j}(t) + d_{i-1}(1 - hivacqprob(i-1)hivrisk(j))Na_{i-1,j}(t) - (d_i + \mu_i) Na_{i,j}(t) - dcmu_i detdis_{i,j}(t) \end{aligned}$$

1.2 MSM HIV+ Demographics

$chivmu$ is the death rate due to undetected cancer in the HIV+ population, and $dchivmu$ is the death rate due to detected cancer in the HIV+ population. $dhiv_i$ is the aging rate out of the i th age group in the HIV+ population, and is generally equal to d_i (though they may be different if background all-cause mortality is different in the HIV+ group, in order to get steady state

demographics. hmu is the background all cause mortality for the HIV+ population.

$$\begin{aligned} Nhiv'_{1,j}(t) &= -chivmu_1 dishiv_{1,j}(t) - (d_1 + hmu_1) Nhiv_{1,j}(t) - dchivmu_1 detdishiv_{1,j}(t) + 0 \\ Nhiv'_{i,j}(t) &= -chivmu_i dishiv_{i,j}(t) + d_{i-1} hivacqprob(i-1) hivrisk(j) Na_{i-1,j}(t) - dchivmu_i detdishiv_{i,j}(t) \\ &\quad - (dhiv_i + hmu_i) Nhiv_{i,j}(t) + dhiv_{i-1} Nhiv_{i-1,j}(t) \end{aligned}$$

2 Cancer Model

2.1 Force of Infection

$\beta_{i,j,u,m,n,neg}$ is the number of *per capita* partnerships between the i, j th age and activity group and the m, n th HIV- age and activity groups, multiplied by the per partnership transmission probability.

$$\begin{aligned} \lambda_{i,j,u}(t) &= \\ &\sum_{m=1}^{age_{max}} \sum_n^{act_{max}} \frac{\beta_{i,j,u,m,n,neg} (detdis_{m,n}(t) + dis_{m,n}(t) + dys_{m,n}(t) + inf_{m,n}(t) + vdys_{m,n}(t) + vi_{m,n}(t) + vsdys_{m,n}(t) + vsi_{m,n}(t))}{Na_{m,n}(t)} \\ &\quad + \frac{\beta_{i,j,u,m,n,pos} (detdishiv_{m,n}(t) + dishiv_{m,n}(t) + dyshiv_{m,n}(t) + infhiv_{m,n}(t) + vdyshiv_{m,n}(t) + vihiv_{m,n}(t) + vsdyshiv_{m,n}(t) + vsihiv_{m,n}(t))}{Nhiv_{m,n}(t)} \end{aligned}$$

2.2 HIV-, Susceptible to HPV

$clear$ is the clearance rate of HPV. $serocon$ is the proportion that seroconvert upon clearance and result in protection against reinfection. $cleardys$ is the rate of clearance of dysplasia, $wnim$ is the rate of waning for natural protection against infection.

$$\begin{aligned} s'_{1,j}(t) &= clear_1(1 - serocon)inf_{1,j}(t) + cleardys_1(1 - serocon)dys_{1,j}(t) - s_{1,j}(t) (\lambda_{1,j,neg}(t) + d_1 + \mu_1) + wnim_1 r_{1,j}(t) + B_j \\ s'_{i,j}(t) &= (1 - serocon)clear_i inf_{i,j}(t) + (1 - serocon)cleardys_i dys_{i,j}(t) + d_{i-1} (1 - uptake_{i-1}(t)) s_{i-1,j}(t) (1 - hivrisk(j) hivacqprob(i-1)) \\ &\quad - s_{i,j}(t) (\lambda_{i,j,neg}(t) + d_i + \mu_i) + wnim_i r_{i,j}(t) \end{aligned}$$

2.3 HIV-, Infected with HPV

progdys is the rate of progression from infection to dysplasia. rinfdys is the rate of clearance from dysplasia to infection. uptake is the proportion of the unvaccinated population that is getting vaccinated at that time.

$$\begin{aligned}\text{inf}'_{1,j}(t) &= -\text{inf}_{1,j}(t) (\text{clear}_1 + d_1 + \mu_1 + \text{progdys}_1) + \text{rinfdys}_1 \text{dys}_{1,j}(t) + s_{1,j}(t) \lambda_{1,j,\text{neg}}(t) \\ \text{inf}'_{i,j}(t) &= -\text{inf}_{i,j}(t) (\text{clear}_i + d_i + \mu_i + \text{progdys}_i) + d_{i-1} (1 - \text{uptake}_{i-1}(t)) \text{inf}_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) \\ &\quad + \text{rinfdys}_i \text{dys}_{i,j}(t) + s_{i,j}(t) \lambda_{i,j,\text{neg}}(t)\end{aligned}$$

2.4 HIV+, Infected with HPV

Parameters are concatenated with an “hiv” and are correspondingly defined compared to the HIV- population.

$$\begin{aligned}\text{infhiv}'_{1,j}(t) &= -\text{infhiv}_{1,j}(t) (\text{clearhiv}_1 + \text{dhiv}_1 + \text{hmu}_1 + \text{progdys}_{\text{hiv}_1}) + \text{rinfdyshiv}_1 \text{dyshiv}_{1,j}(t) + \text{shiv}_{1,j}(t) \lambda_{1,j,\text{pos}}(t) \\ \text{infhiv}'_{i,j}(t) &= -\text{infhiv}_{i,j}(t) (\text{clearhiv}_i + \text{dhiv}_i + \text{hmu}_i + \text{progdys}_{\text{hiv}_i}) + d_{i-1} \text{hivrisk}(j) (1 - \text{uptake}_{i-1}(t)) \text{hivacqprob}(i-1) \text{inf}_{i-1,j}(t) \\ &\quad + \text{dhiv}_{i-1} (1 - \text{uptake}_{\text{hiv}_{i-1}}(t)) \text{infhiv}_{i-1,j}(t) + \text{rinfdyshiv}_i \text{dyshiv}_{i,j}(t) + \text{shiv}_{i,j}(t) \lambda_{i,j,\text{pos}}(t)\end{aligned}$$

2.5 HIV-, Dysplasia

progdys is progression from dysplasia to cancer.

$$\begin{aligned}\text{dys}'_{1,j}(t) &= \text{progdys}_1 \text{inf}_{1,j}(t) - \text{dys}_{1,j}(t) (\text{clear}_{\text{dys}_1} + d_1 + \mu_1 + \text{progdys}_1 + \text{rinfdys}_1) \\ \text{dys}'_{i,j}(t) &= -\text{dys}_{i,j}(t) (\text{clear}_{\text{dys}_i} + d_i + \mu_i + \text{progdys}_i + \text{rinfdys}_i) \\ &\quad + d_{i-1} (1 - \text{uptake}_{i-1}(t)) \text{dys}_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) + \text{progdys}_i \text{inf}_{i,j}(t)\end{aligned}$$

2.6 HIV+, Dysplasia

$$\begin{aligned}\text{dyshiv}'_{1,j}(t) &= \text{progdys}_{\text{hiv}_1} \text{infhiv}_{1,j}(t) - \text{dyshiv}_{1,j}(t) (\text{clear}_{\text{dyshiv}_1} + \text{dhiv}_1 + \text{hmu}_1 + \text{progdys}_{\text{hiv}_1} + \text{rinfdyshiv}_1) \\ \text{dyshiv}'_{i,j}(t) &= -\text{dyshiv}_{i,j}(t) (\text{clear}_{\text{dyshiv}_i} + \text{dhiv}_i + \text{hmu}_i + \text{progdys}_{\text{hiv}_i} + \text{rinfdyshiv}_i) \\ &\quad + d_{i-1} \text{hivrisk}(j) (1 - \text{uptake}_{i-1}(t)) \text{hivacqprob}(i-1) \text{dys}_{i-1,j}(t) + \text{dhiv}_{i-1} (1 - \text{uptake}_{\text{hiv}_{i-1}}(t)) \text{dyshiv}_{i-1,j}(t) + \text{progdys}_{\text{hiv}_i} \text{infhiv}_{i,j}(t)\end{aligned}$$

2.7 HIV-, Undetected Cancer

det is the rate of detection of undetected cancer. Treatment for the cancer begins upon detection.

$$\begin{aligned} \text{dis}'_{1,j}(t) &= \text{progdis}_1 (\text{dys}_{1,j}(t) + \text{vdys}_{1,j}(t) + \text{vsdys}_{1,j}(t)) - \text{dis}_{1,j}(t) (\text{cmu}_1 + d_1 + \text{det}_1 + \mu_1) \\ \text{dis}'_{i,j}(t) &= -\text{dis}_{i,j}(t) (\text{cmu}_i + d_i + \text{det}_i + \mu_i) + d_{i-1} \text{dis}_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) + \\ &\quad \text{progdis}_i (\text{dys}_{i,j}(t) + \text{vdys}_{i,j}(t) + \text{vsdys}_{i,j}(t)) \end{aligned}$$

2.8 HIV+, Undetected Cancer

$$\begin{aligned} \text{dishiv}'_{1,j}(t) &= \text{progdishiv}_1 (\text{dyshiv}_{1,j}(t) + \text{vdyshiv}_{1,j}(t) + \text{vsdyshiv}_{1,j}(t)) - \text{dishiv}_{1,j}(t) (\text{chivmu}_1 + \text{dethiv}_1 + \text{dhiv}_1 + \text{hmu}_1) \\ \text{dishiv}'_{i,j}(t) &= -\text{dishiv}_{i,j}(t) (\text{chivmu}_i + \text{dethiv}_i + \text{dhiv}_i + \text{hmu}_i) + d_{i-1} \text{hivrisk}(j) \text{hivacqprob}(i-1) \text{dis}_{i-1,j}(t) + \\ &\quad \text{dhiv}_{i-1} \text{dishiv}_{i-1,j}(t) + \text{progdishiv}_i (\text{dyshiv}_{i,j}(t) + \text{vdyshiv}_{i,j}(t) + \text{vsdyshiv}_{i,j}(t)) \end{aligned}$$

2.9 HIV-, Detected Cancer

treat is the rate at which the cancer is treated and the population moves into “survivor” status.

$$\begin{aligned} \text{detdis}'_{1,j}(t) &= \text{det}_1 \text{dis}_{1,j}(t) - \text{detdis}_{1,j}(t) (d_1 + \text{dcmu}_1 + \mu_1 + \text{treat}_1) \\ \text{detdis}'_{i,j}(t) &= -\text{detdis}_{i,j}(t) (d_i + \text{dcmu}_i + \mu_i + \text{treat}_i) + d_{i-1} \text{detdis}_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1) + \text{det}_i \text{dis}_{i,j}(t)) \end{aligned}$$

2.10 HIV+, Detected Cancer

$$\begin{aligned} \text{detdishiv}'_{1,j}(t) &= \text{dethiv}_1 \text{dishiv}_{1,j}(t) - \text{detdishiv}_{1,j}(t) (\text{dchivmu}_1 + \text{dhiv}_1 + \text{hmu}_1 + \text{treathiv}_1) \\ \text{detdishiv}'_{i,j}(t) &= d_{i-1} \text{hivrisk}(j) \text{hivacqprob}(i-1) \text{detdis}_{i-1,j}(t) - \text{detdishiv}_{i,j}(t) (\text{dchivmu}_i + \text{dhiv}_i + \text{hmu}_i + \text{treathiv}_i) \\ &\quad + \text{dhiv}_{i-1} \text{detdishiv}_{i-1,j}(t) + \text{dethiv}_i \text{dishiv}_{i,j}(t) \end{aligned}$$

2.11 HIV-, Cancer Survivor

qolend is the rate at which the population that has survived cancer moves into a state of “normalcy” in which there is no loss of quality of life due to having cancer e.g. followup tests and screens no longer impact quality of life.

$$\begin{aligned} \text{surv}'_{1,j}(t) &= \text{treat}_1 \text{detdis}_{1,j}(t) - (d_1 + \mu_1 + \text{qolend}_1) \text{surv}_{1,j}(t) \\ \text{surv}'_{i,j}(t) &= d_{i-1} \text{surv}_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) - (d_i + \mu_i + \text{qolend}_i) \text{surv}_{i,j}(t) + \text{treat}_i \text{detdis}_{i,j}(t) \end{aligned}$$

2.12 HIV+, Cancer Survivor

$$\text{survhiv}'_{1,j}(t) = \text{treathiv}_1 \text{detdishiv}_{1,j}(t) - (\text{dhiv}_1 + \text{hmu}_1 + \text{qolendhiv}_1) \text{survhiv}_{1,j}(t)$$

$$\text{survhiv}'_{i,j}(t) = d_{i-1} \text{hivrisk}(j) \text{hivacqprob}(i-1) \text{survhiv}_{i-1,j}(t) + \text{treathiv}_i \text{detdishiv}_{i,j}(t) - (\text{dhiv}_i + \text{hmu}_i + \text{qolendhiv}_i) \text{survhiv}_{i,j}(t) + \text{dhiv}_{i-1} \text{survhiv}_{i-1,j}(t)$$

2.13 HIV-, Cancer Survivor, No Disutility

$$\text{survtwo}'_{1,j}(t) = \text{qolend}_1 \text{surv}_{1,j}(t) - (d_1 + \mu_1) \text{survtwo}_{1,j}(t)$$

$$\text{survtwo}'_{i,j}(t) = d_{i-1} \text{survtwo}_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) - (d_i + \mu_i) \text{survtwo}_{i,j}(t) + \text{qolend}_i \text{surv}_{i,j}(t)$$

2.14 HIV+, Cancer Survivor, No Disutility

$$\text{survhivtwo}'_{1,j}(t) = \text{qolendhiv}_1 \text{survhiv}_{1,j}(t) - (\text{dhiv}_1 + \text{hmu}_1) \text{survhivtwo}_{1,j}(t)$$

$$\text{survhivtwo}'_{i,j}(t) = d_{i-1} \text{hivrisk}(j) \text{hivacqprob}(i-1) \text{survhivtwo}_{i-1,j}(t) - (\text{dhiv}_i + \text{hmu}_i) \text{survhivtwo}_{i,j}(t) + \text{dhiv}_{i-1} \text{survhivtwo}_{i-1,j}(t) + \text{qolendhiv}_i \text{survhiv}_{i,j}(t)$$

2.15 HIV-, Recovered from HPV

$$r'_{1,j}(t) = \text{serocon} (\text{clear}_1 \text{inf}_{1,j}(t) + \text{clear}_{\text{dys}_1} \text{dys}_{1,j}(t)) - (d_1 + \mu_1 + \text{wnim}_1) r_{1,j}(t)$$

$$r'_{i,j}(t) = \text{serocon} (\text{clear}_i \text{inf}_{i,j}(t) + \text{clear}_{\text{dys}_i} \text{dys}_{i,j}(t)) + d_{i-1} (1 - \text{uptake}_{i-1}(t)) r_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) - (d_i + \mu_i + \text{wnim}_i) r_{i,j}(t)$$

2.16 HIV+, Recovered from HPV

$$\text{rhiv}'_{1,j}(t) = \text{seroconhiv} (\text{clear}_{\text{dyshiv}_1} \text{dyshiv}_{1,j}(t) + \text{clearhiv}_1 \text{infhiv}_{1,j}(t)) - (\text{dhiv}_1 + \text{hmu}_1 + \text{wnimhiv}_1) \text{rhiv}_{1,j}(t)$$

$$\begin{aligned} \text{rhiv}'_{i,j}(t) = & \text{seroconhiv} (\text{clear}_{\text{dyshiv}_i} \text{dyshiv}_{i,j}(t) + \text{clearhiv}_i \text{infhiv}_{i,j}(t)) + d_{i-1} \text{hivrisk}(j) (1 - \text{uptake}_{i-1}(t)) \text{hivacqprob}(i-1) r_{i-1,j}(t) \\ & - (\text{dhiv}_i + \text{hmu}_i + \text{wnimhiv}_i) \text{rhiv}_{i,j}(t) + \text{dhiv}_{i-1} (1 - \text{uptake}_{i-1}(t)) \text{rhiv}_{i-1,j}(t) \end{aligned}$$

2.17 HIV-, Death from Cancer

These death-related states are for the computation of QALYs lost due to death. It is basically a counterfactual: we are computing the QALYs that would have been gained in this population if they had survived for their cancer to be cured. Thus, we have

post-death states for both the detected cancer states, as well as the survivor states.

$$\begin{aligned} \text{death}'_{1,j}(t) &= \text{dcmu}_1 \text{detdis}_{1,j}(t) - (d_1 + \mu_1) \text{death}_{1,j}(t) \\ \text{death}'_{i,j}(t) &= d_{i-1}(1 - \text{hivacqprob}(i-1)) \text{death}_{i-1,j}(t) - (d_i + \mu_i + \text{treat}_i) \text{death}_{i,j}(t) + \text{dcmu}_i \text{detdis}_{i,j}(t) \end{aligned}$$

2.18 HIV+, Death from Cancer

$$\begin{aligned} \text{deathhiv}'_{1,j}(t) &= \text{dhivmu}_1 \text{detdishiv}_{1,j}(t) - (\text{dhiv}_1 + \text{hmu}_1 + \text{treathiv}_1) \text{deathhiv}_{1,j}(t) \\ \text{deathhiv}'_{i,j}(t) &= d_{i-1} \text{hivacqprob}(i-1) \text{death}_{i-1,j}(t) + \text{dhivmu}_i \text{detdishiv}_{i,j}(t) - (\text{dhiv}_i + \text{hmu}_i + \text{treathiv}_i) \text{deathhiv}_{i,j}(t) + \text{dhiv}_{i-1} \text{deathhiv}_{i-1,j}(t) \end{aligned}$$

2.19 HIV-, Death, Survivor Counterfactual

$$\begin{aligned} \text{dsurv}'_{1,j}(t) &= \text{treat}_1 \text{death}_{1,j}(t) - (d_1 + \mu_1 + \text{qolend}_1) \text{dsurv}_{1,j}(t) \\ \text{dsurv}'_{i,j}(t) &= d_{i-1}(1 - \text{hivacqprob}(i-1)) \text{dsurv}_{i-1,j}(t) - (d_i + \mu_i + \text{qolend}_i) \text{dsurv}_{i,j}(t) + \text{treat}_i \text{death}_{i,j}(t) \end{aligned}$$

2.20 HIV+, Death, Survivor Counterfactual

$$\begin{aligned} \text{dsurvshiv}'_{1,j}(t) &= \text{treathiv}_1 \text{deathhiv}_{1,j}(t) - (\text{dhiv}_1 + \text{hmu}_1 + \text{qolendhiv}_1) \text{dsurvshiv}_{1,j}(t) \\ \text{dsurvshiv}'_{i,j}(t) &= d_{i-1} \text{hivacqprob}(i-1) \text{dsurv}_{i-1,j}(t) + \text{treathiv}_i \text{deathhiv}_{i,j}(t) - (\text{dhiv}_i + \text{hmu}_i + \text{qolendhiv}_i) \text{dsurvshiv}_{i,j}(t) \\ &\quad + \text{dhiv}_{i-1} \text{dsurvshiv}_{i-1,j}(t) \end{aligned}$$

2.21 HIV-, Death, Survivor No Disutility, Counterfactual

$$\begin{aligned} \text{dsurvtwo}'_{1,j}(t) &= \text{qolend}_1 \text{dsurv}_{1,j}(t) - (d_1 + \mu_1) \text{dsurvtwo}_{1,j}(t) \\ \text{dsurvtwo}'_{i,j}(t) &= d_{i-1}(1 - \text{hivacqprob}(i-1)) \text{dsurvtwo}_{i-1,j}(t) - (d_i + \mu_i) \text{dsurvtwo}_{i,j}(t) + \text{qolend}_i \text{dsurv}_{i,j}(t) \end{aligned}$$

2.22 HIV-, Vaccinated

$$\begin{aligned} v'_{1,j}(t) &= \text{clearvac}_1(1 - \text{serocon}) \text{vi}_{1,j}(t) - (v_{1,j}(t) ((1 - \text{efficacyinf}) \lambda_{1,j,\text{neg}}(t) + d_1 + \mu_1 + \text{wvim}_1)) + \text{wnim}_1 \text{vr}_{1,j}(t) \\ v'_{i,j}(t) &= (1 - \text{serocon}) \text{clearvac}_i \text{vi}_{i,j}(t) - v_{i,j}(t) ((1 - \text{efficacyinf}) \lambda_{i,j,\text{neg}}(t) + d_i + \mu_i + \text{wvim}_i) \\ &\quad + (1 - \text{failure}) d_{i-1} \text{uptake}_{i-1}(t) (1 - \text{hivacqprob}(i-1)) s_{i-1,j}(t) + d_{i-1}(1 - \text{hivacqprob}(i-1)) v_{i-1,j}(t) + \text{wnim}_i \text{vr}_{i,j}(t) \end{aligned}$$

2.23 HIV+, Vaccinated

$$\begin{aligned}
\text{vhiv}'_{1,j}(t) &= \text{clearhivvac}_1(1 - \text{seroconhiv})\text{vihiv}_{1,j}(t) - \text{vhiv}_{1,j}(t)((1 - \text{efficacyinhiv})\lambda_{1,j,\text{pos}}(t) + \text{dhiv}_1 + \text{hmu}_1 + \text{wvimhiv}_1)\text{vrhiv}_{1,j}(t) \\
\text{vhiv}'_{i,j}(t) &= (1 - \text{seroconhiv})\text{clearhivvac}_i\text{vihiv}_{i,j}(t) + (1 - \text{failure})d_{i-1}\text{uptake}_{i-1}(t)\text{hivacqprob}(i-1)s_{i-1,j}(t) + d_{i-1}\text{hivacqprob}(i-1)v_{i-1,j}(t) - \\
&\quad \text{vhiv}_{i,j}(t)((1 - \text{efficacyinhiv})\lambda_{i,j,\text{pos}}(t) + \text{dhiv}_i + \text{hmu}_i + \text{wvimhiv}_i) + (1 - \text{failurehiv})\text{dhiv}_{i-1}\text{uptakehiv}_{i-1}(t)\text{shiv}_{i-1,j}(t) - \\
&\quad + \text{dhiv}_{i-1}\text{vhiv}_{i-1,j}(t) + \text{wnimhiv}_i\text{vrhiv}_{i,j}(t)
\end{aligned}$$

2.24 HIV-, Breakthrough Infection

$$\begin{aligned}
\text{vi}'_{1,j}(t) &= -\text{vi}_{1,j}(t)(\text{clearvac}_1 + d_1 + (\text{efficacydys})\text{progdys}_1 + \mu_1 + \text{wvim}_1) + (1 - \text{efficacyinf})v_{1,j}(t)\lambda_{1,j,\text{neg}}(t) + \text{rinf dys}_1\text{vdys}_{1,j}(t) \\
\text{vi}'_{i,j}(t) &= -\text{vi}_{i,j}(t)(\text{clearvac}_i + d_i + (\text{efficacydys})\text{progdys}_i + \mu_i + \text{wvim}_i) \\
&\quad + (1 - \text{failure})d_{i-1}\text{uptake}_{i-1}(t)(1 - \text{hivacqprob}(i-1))\text{inf}_{i-1,j}(t) + d_{i-1}(1 - \text{hivacqprob}(i-1))\text{vi}_{i-1,j}(t) \\
&\quad + (1 - \text{efficacyinf})v_{i,j}(t)\lambda_{i,j,\text{neg}}(t) + \text{rinf dys}_i\text{vdys}_{i,j}(t)
\end{aligned}$$

2.25 HIV+, Breakthrough Infection

$$\begin{aligned}
\text{vihiv}'_{1,j}(t) &= -\text{vihiv}_{1,j}(t)(\text{clearhivvac}_1 + d_1 + \text{efficacydyshiv}\text{progdys}\text{shiv}_1 + \mu_1 + \text{wvim}_1) + (1 - \text{efficacyinhiv})\text{vhiv}_{1,j}(t)\lambda_{1,j,\text{pos}}(t) + \text{rinf dyshiv}_1\text{vdyshiv}_{1,j}(t) \\
\text{vihiv}'_{i,j}(t) &= -\text{vihiv}_{i,j}(t)(\text{clearhivvac}_i + d_i + \text{efficacydyshiv}\text{progdys}\text{shiv}_i + \mu_i + \text{wvim}_i) + (1 - \text{failure})d_{i-1}\text{uptake}_{i-1}(t)\text{hivacqprob}(i-1)\text{inf}_{i-1,j}(t) \\
&\quad + d_{i-1}\text{hivacqprob}(i-1)\text{vi}_{i-1,j}(t) + (1 - \text{failurehiv})\text{dhiv}_{i-1}\text{uptakehiv}_{i-1}(t)\text{inhiv}_{i-1,j}(t) + \text{dhiv}_{i-1}\text{vihiv}_{i-1,j}(t) \\
&\quad + (1 - \text{efficacyinhiv})\text{vhiv}_{i,j}(t)\lambda_{i,j,\text{pos}}(t) + \text{rinf dyshiv}_i\text{vdyshiv}_{i,j}(t)
\end{aligned}$$

2.26 HIV-, Breakthrough Dysplasia

$$\begin{aligned}
\text{vdys}'_{1,j}(t) &= \text{efficacydys}\text{progdys}_1\text{vi}_{1,j}(t) - \text{vdys}_{1,j}(t)(d_1 + \mu_1 + \text{progdis}_1 + \text{rinf dys}_1 + \text{wvim}_1) \\
\text{vdys}'_{i,j}(t) &= (1 - \text{failure})d_{i-1}\text{uptake}_i(t)(1 - \text{hivacqprob}(i-1))\text{dys}_{i-1,j}(t) + d_{i-1}(1 - \text{hivacqprob}(i-1))\text{vdys}_{i-1,j}(t) \\
&\quad - \text{vdys}_{i,j}(t)(d_i + \mu_i + \text{progdis}_i + \text{rinf dys}_i + \text{wvim}_i) + \text{efficacydys}\text{progdys}_i\text{vi}_{i,j}(t)
\end{aligned}$$

2.27 HIV+, Breakthrough Dysplasia

$$\begin{aligned}
\text{vdyshiv}'_{1,j}(t) &= \text{efficacydyshivprogdyshiv}_1 \text{vihiv}_{1,j}(t) - \text{vdyshiv}_{1,j}(t) (\text{dhiv}_1 + \text{hmu}_1 + \text{progdishiv}_1 + \text{rinfdyshiv}_1 + \text{wvimhiv}_1) \\
\text{vdyshiv}'_{i,j}(t) &= (1 - \text{failure}) d_{i-1} \text{uptake}_{i-1}(t) \text{hivacqprob}(i-1) \text{dys}_{i-1,j}(t) + d_{i-1} \text{hivacqprob}(i-1) \text{vdys}_{i-1,j}(t) \\
&\quad + (1 - \text{failure}) \text{dhiv}_{i-1} \text{uptake}_{i-1}(t) \text{dyshiv}_{i-1,j}(t) - \text{vdyshiv}_{i,j}(t) (\text{dhiv}_i + \text{hmu}_i + \text{progdishiv}_i + \text{rinfdyshiv}_i + \text{wvimhiv}_i) \\
&\quad + \text{dhiv}_{i-1} \text{vdyshiv}_{i-1,j}(t) + \text{efficacydyshivprogdyshiv}_i \text{vihiv}_{i,j}(t)
\end{aligned}$$

2.28 HIV-, Recovered

$$\begin{aligned}
\text{vr}'_{1,j}(t) &= \text{clearvac}_1 \text{seroconvi}_{1,j}(t) - \text{vr}_{1,j}(t) (d_1 + \mu_1 + \text{wnim}_1 + \text{wvim}_1) \\
\text{vr}'_{i,j}(t) &= \text{seroconclearvac}_i \text{vi}_{i,j}(t) + (1 - \text{failure}) d_{i-1} \text{uptake}_{i-1}(t) (1 - \text{hivacqprob}(i-1)) r_{i-1,j}(t) \\
&\quad + d_{i-1} (1 - \text{hivacqprob}(i-1)) \text{vr}_{i-1,j}(t) - \text{vr}_{i,j}(t) (d_i + \mu_i + \text{wnim}_i + \text{wvim}_i)
\end{aligned}$$

2.29 HIV+, Recovered

$$\begin{aligned}
\text{vrhiv}'_{1,j}(t) &= \text{clearhivvac}_1 \text{seroconhivvihiv}_{1,j}(t) - \text{vrhiv}_{1,j}(t) (\text{dhiv}_1 + \text{hmu}_1 + \text{wnimhiv}_1 + \text{wvimhiv}_1) \\
\text{vrhiv}'_{i,j}(t) &= \text{seroconhivclearhivvac}_i \text{vihiv}_{i,j}(t) + (1 - \text{failure}) d_{i-1} \text{uptake}_{i-1}(t) \text{hivacqprob}(i-1) r_{i-1,j}(t) \\
&\quad + d_{i-1} \text{hivacqprob}(i-1) \text{vr}_{i-1,j}(t) + (1 - \text{failure}) \text{dhiv}_{i-1} \text{uptake}_{i-1}(t) \text{rhiv}_{i-1,j}(t) - \text{vrhiv}_{i,j}(t) (\text{dhiv}_i + \text{hmu}_i + \text{wnimhiv}_i + \text{wvimhiv}_i) \\
&\quad + \text{dhiv}_{i-1} \text{vrhiv}_{i-1,j}(t)
\end{aligned}$$

2.30 HIV-, Vaccinated with No Protection

$$\begin{aligned}
\text{vs}'_{1,j}(t) &= \text{clear}_1 (1 - \text{serocon}) \text{vsi}_{1,j}(t) - \text{vs}_{1,j}(t) (\lambda_{1,j,\text{neg}}(t) + d_1 + \mu_1) + \text{wvim}_1 v_{1,j}(t) + \text{wnim}_1 \text{vsr}_{1,j}(t) \\
\text{vs}'_{i,j}(t) &= (1 - \text{serocon}) \text{clear}_i \text{vsi}_{i,j}(t) + \text{failed}_{i-1} \text{uptake}_{i-1}(t) (1 - \text{hivacqprob}(i-1)) s_{i-1,j}(t) \\
&\quad + d_{i-1} (1 - \text{hivacqprob}(i-1)) \text{vs}_{i-1,j}(t) - \text{vs}_{i,j}(t) (\lambda_{i,j,\text{neg}}(t) + d_i + \mu_i) + \text{wvim}_i v_{i,j}(t) + \text{wnim}_i \text{vsr}_{i,j}(t)
\end{aligned}$$

2.31 HIV+, Vaccinated with No Protection

$$\begin{aligned}
vshiv'_{1,j}(t) &= clearhiv_1(1 - seroconhiv)vshiv_{1,j}(t) - vshiv_{1,j}(t)(\lambda_{1,j, \text{pos}}(t) + dhiv_1 + hmu_1) + wvimhiv_1vhiv_{1,j}(t) + wnimhiv_1vsrhiv_{1,j}(t) \\
vshiv'_{i,j}(t) &= (1 - seroconhiv)clearhiv_ivshiv_{i,j}(t) + failed_{i-1}uptake_{i-1}(t)hivacqprob(i-1)s_{i-1,j}(t) \\
&\quad + d_{i-1}hivacqprob(i-1)vs_{i-1,j}(t) + failurehivdhiv_{i-1}uptakehiv_{i-1}(t)shiv_{i-1,j}(t) - vshiv_{i,j}(t)(\lambda_{i,j, \text{pos}}(t) + dhiv_i + hmu_i) \\
&\quad + dhiv_{i-1}vshiv_{i-1,j}(t) + wvimhiv_ivhiv_{i,j}(t) + wnimhiv_ivsrhiv_{i,j}(t)
\end{aligned}$$

2.32 HIV-, Vaccinated with No Protection, Infected

$$\begin{aligned}
vsi'_{1,j}(t) &= -vsi_{1,j}(t)(clear_1 + d_1 + \mu_1 + progdyshiv_1) + vs_{1,j}(t)\lambda_{1,j, \text{neg}}(t) + rinfdyshiv_1vsdys_{1,j}(t) + wvim_1vi_{1,j}(t) \\
vsi'_{i,j}(t) &= -vsi_{i,j}(t)(clear_i + d_i + \mu_i + progdyshiv_i) + failed_{i-1}uptake_{i-1}(t)(1 - hivacqprob(i-1))inf_{i-1,j}(t) \\
&\quad + d_{i-1}(1 - hivacqprob(i-1))vsi_{i-1,j}(t) + vs_{i,j}(t)\lambda_{i,j, \text{neg}}(t) + rinfdyshiv_ivsdys_{i,j}(t) + wvim_ivi_{i,j}(t)
\end{aligned}$$

2.33 HIV+, Vaccinated with No Protection, Infected

$$\begin{aligned}
vshiv'_{1,j}(t) &= -vshiv_{1,j}(t)(clearhiv_1 + d_1 + \mu_1 + progdyshiv_1) + vshiv_{1,j}(t)\lambda_{1,j, \text{pos}}(t) + rinfdyshiv_1vsdyshiv_{1,j}(t) + wvim_1vihiv_{1,j}(t) \\
vshiv'_{i,j}(t) &= -vshiv_{i,j}(t)(clearhiv_i + d_i + \mu_i + progdyshiv_i) + failed_{i-1}uptake_{i-1}(t)hivacqprob(i-1)inf_{i-1,j}(t) \\
&\quad + d_{i-1}hivacqprob(i-1)vsi_{i-1,j}(t) + failurehivdhiv_{i-1}uptakehiv_{i-1}(t)infhiv_{i-1,j}(t) + dhiv_{i-1}vshiv_{i-1,j}(t) + vshiv_{i,j}(t)\lambda_{i,j, \text{pos}}(t) \\
&\quad + rinfdyshiv_ivsdyshiv_{i,j}(t) + wvim_1vihiv_{i,j}(t)
\end{aligned}$$

2.34 HIV-, Vaccinated with No Protection, Dysplasia

$$\begin{aligned}
vsdys'_{1,j}(t) &= -vsdys_{1,j}(t)(d_1 + \mu_1 + progdis_1 + rinfdyshiv_1) + progdyshiv_1vsi_{1,j}(t) + wvim_1vdys_{1,j}(t) \\
vsdys'_{i,j}(t) &= failed_{i-1}uptake_{i-1}(t)(1 - hivacqprob(i-1))dys_{i-1,j}(t) + d_{i-1}(1 - hivacqprob(i-1))vsdys_{i-1,j}(t) \\
&\quad - vsdys_{i,j}(t)(d_i + \mu_i + progdis_i + rinfdyshiv_i) + progdyshiv_ivsi_{i,j}(t) + wvim_ivdys_{i,j}(t)
\end{aligned}$$

2.35 HIV+, Vaccinated with No Protection, Dysplasia

$$\begin{aligned}
\text{vsdyshiv}'_{1,j}(t) &= -\text{vsdyshiv}_{1,j}(t) (\text{dhiv}_1 + \text{hmu}_1 + \text{progdishiv}_1 + \text{rinfdyshiv}_1) + \text{progdysshiv}_1 \text{vsihiv}_{1,j}(t) + \text{wvimhiv}_1 \text{vdyshiv}_{1,j}(t) \\
\text{vsdyshiv}'_{i,j}(t) &= \text{failed}_{i-1} \text{uptake}_{i-1}(t) \text{hivacqprob}(i-1) \text{dys}_{i-1,j}(t) + d_{i-1} \text{hivacqprob}(i-1) \text{vsdys}_{i-1,j}(t) + \\
&\quad \text{failed}_{i-1} \text{uptake}_{i-1}(t) \text{dyshiv}_{i-1,j}(t) - \text{vsdyshiv}_{i,j}(t) (\text{dhiv}_i + \text{hmu}_i + \text{progdishiv}_i + \text{rinfdyshiv}_i) + \text{dhiv}_{i-1} \text{vsdyshiv}_{i-1,j}(t) \\
&\quad + \text{progdysshiv}_i \text{vsihiv}_{i,j}(t) + \text{wvimhiv}_i \text{vdyshiv}_{i,j}(t)
\end{aligned}$$

2.36 HIV+, Vaccinated with No Protection, Recovered

$$\begin{aligned}
\text{vsr}'_{1,j}(t) &= \text{clear}_1 \text{seroconvsi}_{1,j}(t) - (d_1 + \mu_1 + \text{wnim}_1) \text{vsr}_{1,j}(t) + \text{wvim}_1 \text{vr}_{1,j}(t) \\
\text{vsr}'_{i,j}(t) &= \text{seroconclear}_i \text{vsi}_{i,j}(t) + \text{failed}_{i-1} \text{uptake}_{i-1}(t) (1 - \text{hivacqprob}(i-1)) r_{i-1,j}(t) + d_{i-1} (1 - \text{hivacqprob}(i-1)) \text{vsr}_{i-1,j}(t) \\
&\quad - (d_i + \mu_i + \text{wnim}_i) \text{vsr}_{i,j}(t) + \text{wvim}_i \text{vr}_{i,j}(t)
\end{aligned}$$

2.37 HIV-, Vaccinated with No Protection, Recovered

$$\begin{aligned}
\text{vsrhiv}'_{1,j}(t) &= \text{clearhiv}_1 \text{seroconhivvsihiv}_{1,j}(t) - (\text{dhiv}_1 + \text{hmu}_1 + \text{wnimhiv}_1) \text{vsrhiv}_{1,j}(t) + \text{wvimhiv}_1 \text{vrhiv}_{1,j}(t) \\
\text{vsrhiv}'_{i,j}(t) &= \text{seroconhivclearhiv}_i \text{vsihiv}_{i,j}(t) + \text{failed}_{i-1} \text{uptake}_{i-1}(t) \text{hivacqprob}(i-1) r_{i-1,j}(t) + d_{i-1} \text{hivacqprob}(i-1) \text{vsr}_{i-1,j}(t) \\
&\quad + \text{failed}_{i-1} \text{uptake}_{i-1}(t) \text{rhiv}_{i-1,j}(t) - (\text{dhiv}_i + \text{hmu}_i + \text{wnimhiv}_i) \text{vsrhiv}_{i,j}(t) + \text{dhiv}_{i-1} \text{vsrhiv}_{i-1,j}(t) + \text{wvimhiv}_i \text{vrhiv}_{i,j}(t)
\end{aligned}$$

3 Genital Warts Model

3.1 Force of Infection

$$\begin{aligned}
\lambda_{i,j,u}(t) &= \sum_{m=1}^{\text{age}_{max}} \sum_n^{\text{act}_{max}} \frac{(\text{agw}_{m,n}(t) + \text{gw}_{m,n}(t) + \text{inf}_{m,n}(t)) \text{beta}(i, j, u, m, n, \text{neg})}{\text{Na}_{m,n}(t)} \\
&\quad + \sum_m^{\text{age}_{max}} \sum_n^{\text{act}_{max}} \frac{(\text{agwhiv}_{m,n}(t) + \text{gwhiv}_{m,n}(t) + \text{infhiv}_{m,n}(t)) \text{beta}(i, j, u, m, n, \text{pos})}{\text{Nhiv}_{m,n}(t)};
\end{aligned}$$

3.2 HIV-, Susceptibles

cleargw is the clearance rate of genital warts.

$$\begin{aligned} s'_{1,j}(t) &= \text{cleargw}_1(1 - \text{serocon})\text{agw}_{1,j}(t) + \text{clear}_1(1 - \text{serocon})\text{inf}_{1,j}(t) - s_{1,j}(t)(\lambda_{1,j,\text{neg}}(t) + d_1 + \mu_1) + (1 - \text{serocon})\text{treatgw}_1\text{gw}_{1,j}(t) + \text{wnim}_1 r_{1,j}(t) + B_j \\ s'_{i,j}(t) &= (1 - \text{serocon})\text{cleargw}_i\text{agw}_{i,j}(t) + (1 - \text{serocon})\text{clear}_i\text{inf}_{i,j}(t) + d_{i-1}(1 - \text{uptake}_{i-1}(t))s_{i-1,j}(t)(1 - \text{hivrisk}(j)\text{hivacqprob}(i-1)) \\ &\quad - s_{i,j}(t)(\lambda_{i,j,\text{neg}}(t) + d_i + \mu_i) + (1 - \text{serocon})\text{treatgw}_i\text{gw}_{i,j}(t) + \text{wnim}_i r_{i,j}(t) \end{aligned}$$

3.3 HIV+, Susceptibles

$$\begin{aligned} \text{shiv}'_{1,j}(t) &= \text{cleargwhiv}_1(1 - \text{seroconhiv})\text{agwhiv}_{1,j}(t) + \text{clearhiv}_1(1 - \text{seroconhiv})\text{infhiv}_{1,j}(t) - \text{shiv}_{1,j}(t)(\lambda_{1,j,\text{pos}}(t) + \text{dhiv}_1 + \text{hmu}_1) \\ &\quad + (1 - \text{seroconhiv})\text{treatgwhiv}_1\text{gwhiv}_{1,j}(t) + \text{wnimhiv}_1 r_{1,j}(t) \\ \text{shiv}'_{i,j}(t) &= (1 - \text{seroconhiv})\text{cleargwhiv}_i\text{agwhiv}_{i,j}(t) + (1 - \text{seroconhiv})\text{clearhiv}_i\text{infhiv}_{i,j}(t) + d_{i-1}\text{hivrisk}(j)(1 - \text{uptake}_{i-1}(t))\text{hivacqprob}(i-1)s_{i-1,j}(t) \\ &\quad - \text{shiv}_{i,j}(t)(\lambda_{i,j,\text{pos}}(t) + \text{dhiv}_i + \text{hmu}_i) + \text{dhiv}_{i-1}(1 - \text{uptakehiv}_i(t))\text{shiv}_{i-1,j}(t) + (1 - \text{seroconhiv})\text{treatgwhiv}_i\text{gwhiv}_{i,j}(t) + \text{wnimhiv}_i r_{i,j}(t) \end{aligned}$$

3.4 HIV-, Infected

proggw is the rate of progression of infection to genital warts.

$$\begin{aligned} \text{inf}'_{1,j}(t) &= s_{1,j}(t)\lambda_{1,j,\text{neg}}(t) - \text{inf}_{1,j}(t)(\text{clear}_1 + d_1 + \mu_1 + \text{proggw}_1) \\ \text{inf}'_{i,j}(t) &= -\text{inf}_{i,j}(t)(\text{clear}_i + d_i + \mu_i + \text{proggw}_i) + d_{i-1}(1 - \text{uptake}_i(t))\text{inf}_{i-1,j}(t)(1 - \text{hivrisk}(j)\text{hivacqprob}(i-1)) + s_{i,j}(t)\lambda_{i,j,\text{neg}}(t) \end{aligned}$$

3.5 HIV+, Infected

$$\begin{aligned} \text{infhiv}'_{1,j}(t) &= \text{shiv}_{1,j}(t)\lambda_{1,j,\text{pos}}(t) - \text{infhiv}_{1,j}(t)(\text{clearhiv}_1 + \text{dhiv}_1 + \text{hmu}_1 + \text{proggwhiv}_1) \\ \text{infhiv}'_{i,j}(t) &= -\text{infhiv}_{i,j}(t)(\text{clearhiv}_i + \text{dhiv}_i + \text{hmu}_i + \text{proggwhiv}_i) + d_{i-1}\text{hivrisk}(j)(1 - \text{uptake}_i(t))\text{hivacqprob}(i-1)\text{inf}_{i-1,j}(t) + \\ &\quad \text{dhiv}_{i-1}(1 - \text{uptakehiv}_i(t))\text{infhiv}_{i-1,j}(t) + \text{shiv}_{i,j}(t)\lambda_{i,j,\text{pos}}(t) \end{aligned}$$

3.6 HIV-, Symptomatic Genital Warts

sympfrac is the proportion of genital warts cases that are symptomatic, meaning that patients will seek treatment.

$$\begin{aligned} \text{gw}'_{1,j}(t) &= \text{proggw}_1 \text{sympfracinf}_{1,j}(t) - (d_1 + \mu_1 + \text{treatgw}_1) \text{gw}_{1,j}(t) \\ \text{gw}'_{i,j}(t) &= d_{i-1} (1 - \text{uptake}_i(t)) \text{gw}_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) - (d_i + \mu_i + \text{treatgw}_i) \text{gw}_{i,j}(t) + \text{sympfracproggw}_i \text{inf}_{i,j}(t) \end{aligned}$$

3.7 HIV+, Symptomatic Genital Warts

$$\begin{aligned} \text{gwhiv}'_{1,j}(t) &= \text{proggwhiv}_1 \text{sympfrachivinfhiv}_{1,j}(t) - (\text{dhiv}_1 + \text{hmu}_1 + \text{treatgwhiv}_1) \text{gwhiv}_{1,j}(t) \\ \text{gwhiv}'_{i,j}(t) &= d_{i-1} \text{hivrisk}(j) (1 - \text{uptake}_i(t)) \text{hivacqprob}(i-1) \text{gw}_{i-1,j}(t) - (\text{dhiv}_i + \text{hmu}_i + \text{treatgwhiv}_i) \text{gwhiv}_{i,j}(t) \\ &\quad + \text{dhiv}_{i-1} (1 - \text{uptakehiv}_i(t)) \text{gwhiv}_{i-1,j}(t) + \text{sympfrachivproggwhiv}_i \text{infhiv}_{i,j}(t) \end{aligned}$$

3.8 HIV-, Asymptomatic Genital Warts

$$\begin{aligned} \text{agw}'_{1,j}(t) &= \text{proggw}_1 (1 - \text{sympfrac}) \text{inf}_{1,j}(t) - (\text{cleargw}_1 + d_1 + \mu_1) \text{agw}_{1,j}(t) \\ \text{agw}'_{i,j}(t) &= -(\text{cleargw}_i + d_i + \mu_i) \text{agw}_{i,j}(t) + d_{i-1} (1 - \text{uptake}_i(t)) \text{agw}_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) + (1 - \text{sympfrac}) \text{proggw}_i \text{inf}_{i,j}(t) \end{aligned}$$

3.9 HIV+, Asymptomatic Genital Warts

$$\begin{aligned} \text{agwhiv}'_{1,j}(t) &= \text{proggwhiv}_1 (1 - \text{sympfrachiv}) \text{infhiv}_{1,j}(t) - (\text{cleargwhiv}_1 + \text{dhiv}_1 + \text{hmu}_1) \text{agwhiv}_{1,j}(t) \\ \text{agwhiv}'_{i,j}(t) &= d_{i-1} \text{hivrisk}(j) (1 - \text{uptake}_i(t)) \text{hivacqprob}(i-1) \text{agw}_{i-1,j}(t) - (\text{cleargwhiv}_i + \text{dhiv}_i + \text{hmu}_i) \text{agwhiv}_{i,j}(t) \\ &\quad + \text{dhiv}_{i-1} (1 - \text{uptakehiv}_i(t)) \text{agwhiv}_{i-1,j}(t) + (1 - \text{sympfrachiv}) \text{proggwhiv}_i \text{infhiv}_{i,j}(t) \end{aligned}$$

3.10 HIV-, Recovered

$$\begin{aligned}
r'_{1,j}(t) &= \text{cleargw}_1 \text{seroconagw}_{1,j}(t) + \text{clear}_1 \text{seroconinf}_{1,j}(t) - (d_1 + \mu_1 + \text{wnim}_1) r_{1,j}(t) + \text{serocontreatgw}_1 \text{gw}_{1,j}(t) \\
r'_{i,j}(t) &= \text{seroconcleargw}_i \text{agw}_{i,j}(t) + \text{seroconclear}_i \text{inf}_{i,j}(t) + d_{i-1} (1 - \text{uptake}_i(t)) r_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) \\
&\quad - (d_i + \mu_i + \text{wnim}_i) r_{i,j}(t) + \text{serocontreatgw}_i \text{gw}_{i,j}(t)
\end{aligned}$$

3.11 HIV+, Recovered

$$\begin{aligned}
\text{rhiv}'_{1,j}(t) &= \text{clearghiv}_1 (\text{seroconhiv}) \text{agwhiv}_{1,j}(t) + \text{clearhiv}_1 (\text{seroconhiv}) \text{infhiv}_{1,j}(t) - (\text{dhiv}_1 + \text{hmu}_1 + \text{wnimhiv}_1) \text{rhiv}_{1,j}(t) \\
&\quad + (\text{seroconhiv}) \text{treatghiv}_1 \text{gwhiv}_{1,j}(t) \\
\text{rhiv}'_{i,j}(t) &= \text{seroconhivclearghiv}_i \text{agwhiv}_{i,j}(t) + \text{seroconhivclearhiv}_i \text{infhiv}_{i,j}(t) + d_{i-1} \text{hivrisk}(j) (1 - \text{uptake}_i(t)) \text{hivacqprob}(i-1) r_{i-1,j}(t) \\
&\quad - (\text{dhiv}_i + \text{hmu}_i + \text{wnimhiv}_i) \text{rhiv}_{i,j}(t) + \text{dhiv}_{i-1} (1 - \text{uptakehiv}_i(t)) \text{rhiv}_{i-1,j}(t) + \text{seroconhivtreatghiv}_i \text{gwhiv}_{i,j}(t)
\end{aligned}$$

3.12 HIV-, Vaccinated

$$\begin{aligned}
v'_{1,j}(t) &= \text{clearvac}_1 (1 - \text{serocon}) \text{vinf}_{1,j}(t) + \text{clearvacgw}_1 (1 - \text{serocon}) \text{vagw}_{1,j}(t) + v_{1,j}(t) (-((1 - \text{efficacyinf}) \lambda_{1,j,\text{neg}}(t) + d_1 + \mu_1 + \text{wvim}_1)) \\
&\quad + (1 - \text{serocon}) \text{treatgw}_1 \text{vgw}_{1,j}(t) + \text{wnim}_1 \text{vr}_{1,j}(t) \\
v'_{i,j}(t) &= (1 - \text{serocon}) \text{clearvac}_i \text{vinf}_{i,j}(t) + (1 - \text{serocon}) \text{clearvacgw}_i \text{vagw}_{i,j}(t) - v_{i,j}(t) ((1 - \text{efficacyinf}) \lambda_{i,j,\text{neg}}(t) + d_i + \mu_i + \text{wvim}_i) \\
&\quad + (1 - \text{failure}) d_{i-1} \text{uptake}_{i-1}(t) s_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) + d_{i-1} v_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) \\
&\quad + (1 - \text{serocon}) \text{treatgw}_i \text{vgw}_{i,j}(t) + \text{wnim}_i \text{vr}_{i,j}(t)
\end{aligned}$$

3.13 HIV+, Vaccinated

$$\begin{aligned}
\text{vhiv}'_{1,j}(t) &= \text{clearhivvac}_1(1 - \text{seroconhiv})\text{vinfhiv}_{1,j}(t) + \text{clearvacgwhiv}_1(1 - \text{seroconhiv})\text{vagwhiv}_{1,j}(t) \\
&\quad - \text{vhiv}_{1,j}(t)((1 - \text{efficacyinf})\lambda_{1,j,\text{pos}}(t) + \text{dhiv}_1 + \text{hmu}_1) + (1 - \text{seroconhiv})\text{treatgwhiv}_1\text{vgwhiv}_{1,j}(t) + \text{wnimhiv}_1\text{vrhiv}_{1,j}(t) \\
\text{vhiv}'_{i,j}(t) &= (1 - \text{seroconhiv})\text{clearhivvac}_i\text{vinfhiv}_{i,j}(t) + (1 - \text{seroconhiv})\text{clearvacgwhiv}_i\text{vagwhiv}_{i,j}(t) \\
&\quad + (1 - \text{failurehiv})d_{i-1}\text{hivrisk}(j)\text{uptake}_{i-1}(t)\text{hivacqprob}(i-1)s_{i-1,j}(t) + (1 - \text{failurehiv})d_{i-1}\text{uptakehiv}_{i-1}(t)\text{shiv}_{i-1,j}(t) + \\
&\quad d_{i-1}\text{hivrisk}(j)\text{hivacqprob}(i-1)v_{i-1,j}(t) - \text{vhiv}_{i,j}(t)((1 - \text{efficacyinfhiv})\lambda_{i,j,\text{pos}}(t) + \text{dhiv}_i + \text{hmu}_i) + \text{dhiv}_{i-1}\text{vhiv}_{i-1,j}(t) \\
&\quad + (1 - \text{seroconhiv})\text{treatgwhiv}_i\text{vgwhiv}_{i,j}(t) + \text{wnimhiv}_i\text{vrhiv}_{i,j}(t)
\end{aligned}$$

3.14 HIV-, Breakthrough Infections

$$\begin{aligned}
\text{vinf}'_{1,j}(t) &= (1 - \text{efficacyinf})v_{1,j}(t)\lambda_{1,j,\text{neg}}(t) - \text{vinf}_{1,j}(t)(\text{clearvac}_1 + d_1 + \mu_1 + \text{proggwvac}_1) \\
\text{vinf}'_{i,j}(t) &= -\text{vinf}_{i,j}(t)(\text{clearvac}_i + d_i + \mu_i + \text{proggwvac}_i) + (1 - \text{failure})d_{i-1}\text{uptake}_{i-1}(t)\text{inf}_{i-1,j}(t)(1 - \text{hivrisk}(j)\text{hivacqprob}(i-1)) \\
&\quad + d_{i-1}\text{vinf}_{i-1,j}(t)(1 - \text{hivrisk}(j)\text{hivacqprob}(i-1)) + (1 - \text{efficacyinf})v_{i,j}(t)\lambda_{i,j,\text{neg}}(t)
\end{aligned}$$

3.15 HIV+, Breakthrough Infections

$$\begin{aligned}
\text{vinfhiv}'_{1,j}(t) &= (1 - \text{efficacyinfhiv})\text{vhiv}_{1,j}(t)\lambda_{1,j,\text{pos}}(t) - \text{vinfhiv}_{1,j}(t)(\text{clearhivvac}_1 + \text{dhiv}_1 + \text{hmu}_1 + \text{proggwvachiv}_1) \\
\text{vinfhiv}'_{i,j}(t) &= -\text{vinfhiv}_{i,j}(t)(\text{clearhivvac}_i + \text{dhiv}_i + \text{hmu}_i + \text{proggwvachiv}_i) + (1 - \text{failurehiv})d_{i-1}\text{hivrisk}(j)\text{uptake}_{i-1}(t)\text{hivacqprob}(i-1)\text{inf}_{i-1,j}(t) \\
&\quad + d_{i-1}\text{hivrisk}(j)\text{hivacqprob}(i-1)\text{vinf}_{i-1,j}(t) + (1 - \text{failurehiv})\text{dhiv}_{i-1}\text{uptakehiv}_{i-1}(t)\text{infhiv}_{i-1,j}(t) + \text{dhiv}_{i-1}\text{vinfhiv}_{i-1,j}(t) \\
&\quad + (1 - \text{efficacyinfhiv})\text{vhiv}_{i,j}(t)\lambda_{i,j,\text{pos}}(t)
\end{aligned}$$

3.16 HIV-, Breakthrough Symptomatic Genital Warts

$$\begin{aligned} \text{vgw}'_{1,j}(t) &= \text{proggwvac}_1 \text{sympfracvinf}_{1,j}(t) - (d_1 + \mu_1 + \text{treatgw}_1) \text{vgw}_{1,j}(t) \\ \text{vgw}'_{i,j}(t) &= (1 - \text{failure}) d_{i-1} \text{uptake}_{i-1}(t) \text{gw}_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) + d_{i-1} \text{vgw}_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) \\ &\quad - (d_i + \mu_i + \text{treatgw}_i) \text{vgw}_{i,j}(t) + \text{sympfracproggwvac}_i \text{vinf}_{i,j}(t) \end{aligned}$$

3.17 HIV+, Breakthrough Symptomatic Genital Warts

$$\begin{aligned} \text{vgwhiv}'_{1,j}(t) &= \text{proggwvachiv}_1 \text{sympfrachivvinfhiv}_{1,j}(t) - (\text{dhiv}_1 + \text{hmu}_1 + \text{treatgwhiv}_1) \text{vgwhiv}_{1,j}(t) \\ \text{vgwhiv}'_{i,j}(t) &= (1 - \text{failurehiv}) d_{i-1} \text{hivrisk}(j) \text{uptake}_{i-1}(t) \text{hivacqprob}(i-1) \text{gw}_{i-1,j}(t) + d_{i-1} \text{hivrisk}(j) \text{hivacqprob}(i-1) \text{vgw}_{i-1,j}(t) \\ &\quad + (1 - \text{failurehiv}) \text{dhiv}_{i-1} \text{uptakehiv}_{i-1}(t) \text{gwhiv}_{i-1,j}(t) - (\text{dhiv}_i + \text{hmu}_i + \text{treatgwhiv}_i) \text{vgwhiv}_{i,j}(t) + \text{dhiv}_{i-1} \text{vgwhiv}_{i-1,j}(t) \\ &\quad + \text{sympfrachivproggwvachiv}_i \text{vinfhiv}_{i,j}(t) \end{aligned}$$

3.18 HIV-, Breakthrough Asymptomatic Genital Warts

$$\begin{aligned} \text{vagw}'_{1,j}(t) &= \text{proggwvac}_1 (1 - \text{sympfrac}) \text{vinf}_{1,j}(t) - (\text{cleargw}_1 + d_1 + \mu_1) \text{vagw}_{1,j}(t) \\ \text{vagw}'_{i,j}(t) &= (1 - \text{failure}) d_{i-1} \text{uptake}_{i-1}(t) \text{agw}_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) - (\text{cleargw}_i + d_i + \mu_i) \text{vagw}_{i,j}(t) \\ &\quad + d_{i-1} \text{vagw}_{i-1,j}(t) (1 - \text{hivrisk}(j) \text{hivacqprob}(i-1)) + (1 - \text{sympfrac}) \text{proggwvac}_i \text{vinf}_{i,j}(t) \end{aligned}$$

3.19 HIV+, Breakthrough Asymptomatic Genital Warts

$$\begin{aligned} \text{vagwhiv}'_{1,j}(t) &= \text{proggwvachiv}_1 (1 - \text{sympfrachiv}) \text{vinfhiv}_{1,j}(t) - (\text{cleargwhiv}_1 + \text{dhiv}_1 + \text{hmu}_1) \text{vagwhiv}_{1,j}(t) \\ \text{vagwhiv}'_{i,j}(t) &= (1 - \text{failurehiv}) d_{i-1} \text{hivrisk}(j) \text{uptake}_{i-1}(t) \text{hivacqprob}(i-1) \text{agw}_{i-1,j}(t) + (1 - \text{failurehiv}) \text{dhiv}_{i-1} \text{uptakehiv}_{i-1}(t) \text{agwhiv}_{i-1,j}(t) \\ &\quad - (\text{cleargwhiv}_i + \text{dhiv}_i + \text{hmu}_i) \text{vagwhiv}_{i,j}(t) + d_{i-1} \text{hivrisk}(j) \text{hivacqprob}(i-1) \text{vagw}_{i-1,j}(t) + \text{dhiv}_{i-1} \text{vagwhiv}_{i-1,j}(t) \\ &\quad + (1 - \text{sympfrachiv}) \text{proggwvachiv}_i \text{vinfhiv}_{i,j}(t) \end{aligned}$$

3.20 HIV-, Vaccinated Recovered

$$\begin{aligned}
vr'_{1,j}(t) &= clear_1 seroconvinf_{1,j}(t) + cleargw_1 seroconvagw_{1,j}(t) - (d_1 + \mu_1 + wnim_1) vr_{1,j}(t) + serocontreatgw_1 vgw_{1,j}(t) \\
vr'_{i,j}(t) &= seroconcleargw_i vagw_{i,j}(t) + seroconclearvac_i vinf_{i,j}(t) + (1 - failure) d_{i-1} uptake_{i-1}(t) r_{i,j}(t) (1 - hivrisk(j) hivacqprob(i-1)) \\
&\quad + d_{i-1} vr_{i-1,j}(t) (1 - hivrisk(j) hivacqprob(i-1)) - (d_i + \mu_i + wnim_i) vr_{i,j}(t) + serocontreatgw_i vgw_{i,j}(t)
\end{aligned}$$

3.21 HIV+, Vaccinated Recovered

$$\begin{aligned}
vrhiv'_{1,j}(t) &= clearghiv_1 seroconhivvagwhiv_{1,j}(t) + clearhiv_1 seroconhivvinfhiv_{1,j}(t) - (dhiv_1 + hmu_1 + wnimhiv_1) vrhiv_{1,j}(t) \\
&\quad + seroconhivtreatghiv_1 vgwhiv_{1,j}(t) \\
vrhiv'_{i,j}(t) &= seroconhivclearghiv_i vagwhiv_{i,j}(t) + seroconhivclearhiv_i vinfhiv_{i,j}(t) + (1 - failurehiv) d_{i-1} hivrisk(j) uptake_i(t) hivacqprob(i-1) r_{i,j}(t) \\
&\quad + (1 - failurehiv) d_{i-1} uptakehiv_i(t) rhiv_{i,j}(t) + d_{i-1} hivrisk(j) hivacqprob(i-1) vr_{i-1,j}(t) - (dhiv_i + hmu_i + wnimhiv_i) vrhiv_{i,j}(t) \\
&\quad + dhiv_{i-1} vrhiv_{i-1,j}(t) + seroconhivtreatghiv_i vgwhiv_{i,j}(t)
\end{aligned}$$

3.22 Failed Vaccination

All the vaccinated compartments above have a corresponding “failed” vaccination state, where either the vaccine protection waned, or that, when vaccinated, the vaccine failed to provide any protection. We do not reproduce these equations here, but see the compartments in the cancer model for similar expressions.