Supplementary material

Part A: Generation of problem instances

Since the number and type of fortification levels are assumed to be the same at all candidate locations for opening unreliable facilities, we denote this set by F in the remainder. Moreover, let $\mathcal{U}[a, b]$ indicate the generation of random numbers over the range $[a, b]$ according to a continuous uniform distribution.

Customer demand
In the first time period, we consider $d_{k1s} = \lceil \mathcal{U}[100, 300] \rceil$ for every $k \in K$ and $s \in S$, with $\lceil x \rceil$ denoting the smallest integer greater than or equal to x. In each of the following periods $t \in T \setminus \{1\}$, we set $d_{kts} = \lceil \beta_{ks}\, d_{k(t-1)s} \rceil$ with $k \in K$, $s \in S$, and β_{ks} is a random parameter that defines the demand pattern for each customer and scenario. Accordingly, we divide the set of scenarios S into three subsets, where the demand for all customers in each subset follows a specific pattern. For $s = 1, \ldots, |S|/3$, we define $\beta_{ks} \in \mathcal{U}[0.9, 1]$, which indicates that demand is monotone decreasing over the planning horizon. For the subset of scenarios $s = |S|/3+1, \ldots, 2|S|/3$, demand is monotone increasing since $\beta_{ks} \in \mathcal{U}[1, 1.1]$. In the last subset of scenarios, the demand pattern is irregular by taking $\beta_{ks} \in \mathcal{U}[0.9, 1.1]$ for $s = 2|S|/3 + 1, \ldots, |S|$. In this case, customer demand may fall by a maximum of 10% or rise by up to 10% from one period to the next. Recall that in our testbed, $|S|$ is a multiple of three (see Table 2 in Section 5.1 of the accompanying paper).

Capacity of suppliers and facilities
The regular capacity of each primary supplier $i \in I^1$ is defined as follows:

$$\overline{q}_i = \left\lceil \overline{\delta}_i \, \frac{\overline{d}}{|I^1|} \right\rceil \quad \text{with} \quad \overline{\delta}_i \in \mathcal{U}[1.5, 2.0]$$

and $\overline{d}$ provides an estimate of the total customer demand per period, i.e.

$$\overline{d} = \frac{\sum\limits_{k \in K} \sum\limits_{t \in T} \max\limits_{s \in S}\{d_{kts}\}}{|T|}.$$

The calculation of $\overline{q}_i$ for each backup supplier $i \in I^2$ is performed in a similar way as above but we take $|I^2|$ in the denominator instead of $|I^1|$.

A slightly different scheme is adopted for generating the capacity of the reliable facilities, namely $q^r_j = \lceil \delta_j \, \overline{d}/|J| \rceil$, with $\delta_j \in \mathcal{U}[3, 4]$, for every $j \in J^r$. Regarding the capacities of the unreliable facilities $j \in J^u$, these are generated in the same way, but the originally determined quantities are additionally multiplied by 0.5. Moreover, the amount of capacity is the same for each level of fortification. These settings are in line with those used by other authors, e.g. Nazemi and Parragh (2022).

Capacity loss due to disruptions
Each scenario specifies an individual disruption profile for every unreliable facility and primary supplier, defined by its timing, duration and capacity loss. We first select the earliest time period t^0_s in which a disaster may occur in scenario $s \in S$, with t^0_s chosen at random from $\{2, \ldots, n_T\}$.

For a given unreliable location $j \in J^u$ and scenario $s \in S$, we randomly determine the first period t^0_{js} in which this facility gets disrupted, with $t^0_{js} \in \{t^0_s, \ldots, n_T\}$. This choice applies to all fortification levels $f \in F$. On the other hand, the duration of the disturbance depends on f in such a way that the lower the level of fortification, the longer it takes for the facility to recover. Hence, the last period in which facility j with fortification level f is disturbed in scenario s is $t^1_{jfs} = \min\left\{t^0_{js} + g \cdot (|F| + 1 - f) - 1, n_T\right\}$, with g denoting a pre-specified parameter. We set $g = 2$, indicating that a facility with a high level of fortification (i.e. $f = 3$) will be disturbed in two consecutive periods, while a facility with a low level of fortification (i.e. $f = 1$) will be disturbed three times as long. The relative capacity loss faced by the unreliable facility $j \in J^u$ in the first period with a disturbance is determined by

$$b_{jft^*s} = \begin{cases} 0.80 + 0.05\,k_j & \text{if } f = 1 \\ 0.45 + 0.05\,k_j & \text{if } f = 2 \\ 0.10 + 0.05\,k_j & \text{if } f = 3 \end{cases} \qquad \text{for} \quad t^* = t^0_{js}$$

and k_j selected at random from the set $\{0, 1, 2, 3\}$. Therefore, a facility with a low fortification level $(f = 1)$ may lose between 80% $(k_j = 0)$ and 95% $(k_j = 3)$ of its capacity, whereas the capacity loss faced by a facility with a high fortification level $(f = 3)$ varies between 10% and 25%. For the medium fortification level $f = 2$, the capacity loss lies between 45% and 60%. Moreover, we assume that there will be a gradual capacity recovery until period t^1_{jfs}. Without loss of generality, the latter is represented by a fixed proportion $\widetilde{q}$ applied to all periods up to t^1_{jfs} such that

$$b_{jft's} = b_{jf(t'-1)s} - \widetilde{q} \quad \text{for} \quad t' = t^0_{js} + 1, \ldots, t^1_{jfs}$$

and $\widetilde{q} = \dfrac{b_{jft^*s}}{t^1_{jfs} - t^0_{js} + 1}$ with $t^* = t^0_{js}$. In all time periods before t^0_{js} and after t^1_{jfs}, the unreliable facility can be operated at its full capacity, i.e. $b_{jfts} = 0$ for $t \in \{1, \ldots, t^0_{js} - 1\} \cup \{t^1_{jfs} + 1, \ldots, n_T\}$.

For the sake of illustration, let us assume that $n_T = 12$, $t^0_{js} = 5$ in some scenario s, $f = 2$ and $k_j = 1$. In this case, the unreliable facility j with a medium level of fortification loses half of its original capacity in period 5 of scenario s. In the following periods and until the end of period 8 $(t^1_{j2s} = 5 + 2 \cdot (3 + 1 - 2) - 1)$, the capacity recovery amounts to 12.5% per period $(\widetilde{q} = 0.5/(8 - 5 + 1))$. Hence, the capacity loss in periods 6, 7 and 8 is 37.5%, 25% and 12.5%, respectively.

Disruptions faced by primary suppliers $i \in I^1$ in scenario $s \in S$ start in period t^0_{is}, with t^0_{is} selected at random from $\{t^0_s, \ldots, n_T\}$, and end in period t^1_{is}. The latter parameter is given by $\min\{t^0_{is} + g\,k_i - 1, n_T\}$, with $g = 2$ and k_i is chosen at random from the set $\{0, 1, 2, 3\}$. In all other periods $t \in \{1, \ldots, t^0_{is} - 1\} \cup \{t^1_{is} + 1, \ldots, n_T\}$, no disturbances occur, so that $a_{its} = 0$. A similar scheme as for unreliable facilities is used to specify the capacity losses from period t^0_{is} to period t^1_{is}:

$$\begin{aligned} a_{it^*s} &= \mathcal{U}[0.3, 0.5] && \text{for } t^* = t^0_{is}, \\ a_{it's} &= a_{i(t'-1)s} - \frac{a_{it^*s}}{t^1_{is} - t^0_{is} + 1} && \text{for } t' = t^0_{is} + 1, \ldots, t^1_{is}. \end{aligned}$$

Fixed facility costs

The fixed costs of opening reliable facilities at candidate sites $j \in J^r$ reflect economies of scale as follows:

$$fc^r_{j1} \;=\; \lambda_j \,+\, \gamma_j \sqrt{q^r_j}, \qquad \lambda_j \in \mathcal{U}[5000, 10000], \quad \gamma_j \in \mathcal{U}[10000, 15000].$$

In the remaining strategic periods $t \in T_L \setminus \{1\}$, we take $fc^r_{jt} = \overline{\lambda}_{jt}\, fc^r_{j(t-1)}$ with $\overline{\lambda}_{jt} \in \mathcal{U}[1.01, 1.03]$. This scheme is also used for defining the fixed opening costs of unreliable facilities $j \in J^u$ with high fortification level ($f = 3$). A medium ($f = 2$), resp. low ($f = 1$), level of fortification incurs a fixed cost that is 80%, resp. 70%, of the cost of the high level. Furthermore, the fixed cost of operating a facility in a strategic period $t \in T_L$ corresponds to 5% of the associated opening cost, i.e., $oc^r_{jt} = 0.05\, fc^r_{jt}$ ($j \in J^r$) and $oc^u_{jft} = 0.05\, fc^u_{jft}$ ($j \in J^u$, $f \in F$). In the remaining time periods $t \in T \setminus T_L$, we consider $oc^r_{jt} = \overline{\lambda}_{jt}\, oc^r_{j(t-1)}$ ($j \in J^r$) and $oc^u_{jft} = \overline{\lambda}_{jt}\, oc^u_{jf(t-1)}$ ($j \in J^u$, $f \in F$), with $\overline{\lambda}_{jt} \in \mathcal{U}[1.01, 1.03]$.

The fixed cost arising from the total damage of an unreliable facility is assumed to be 150% of its opening cost, i.e., $ac^u_{jft} = 1.5\, fc^u_{jft}$ ($j \in J^u, f \in F, t \in T_L$). In non-strategic periods, these fixed costs are defined in a similar way to the fixed operating costs, by multiplying the cost associated to period $t-1$ by $\overline{\lambda}_{jt} \in \mathcal{U}[1.01, 1.03]$.

Distribution costs

In the first time period, the unit distribution cost from a reliable facility $j \in J^r$ to a customer $k \in K$ in scenario $s \in S$ is defined as $dc_{jk1s} = 1.5\, \widetilde{d}_{jk}$, with $\widetilde{d}_{jk}$ denoting the Euclidean distance between the two locations j and k. In the remaining periods $t \in T\setminus\{1\}$, we take $dc_{jkts} = \mu_{jkts}\, dc_{jk(t-1)s}$ with $\mu_{jkts} \in \mathcal{U}[1.01, 1.03]$. The costs of distributing one unit of the product from unreliable facilities $j \in J^u$ to customers are determined in the same way in periods $t \in \{1, \ldots, t^0_{js}-1\} \cup \{t^1_{jfs}+1, \ldots, n_T\}$, i.e. periods without disturbances ($f \in F$). In the remaining periods $t = t^0_{js}, \ldots, t^1_{jfs}$, the distribution cost is more expensive, i.e., $dc_{jkts} = 1.1\, \mu_{jkts}\, dc_{jk(t-1)s}$.

Distribution costs from suppliers to facilities reflect the fact that transport from backup suppliers is more expensive than from primary suppliers, i.e., $sc_{\imath' jts} > sc_{ijts}$ for $\imath' \in I^2$ and $i \in I^1$. To ensure this, we first define for $i \in I^1$, $j \in J$ and $s \in S$:

$$sc_{ij1s} \;=\; \widetilde{d}_{ij}, \qquad sc_{ijts} = \overline{\overline{\mu}}_{ijts}\, sc_{ij(t-1)s}, \qquad \overline{\overline{\mu}}_{ijts} \in \mathcal{U}[1.01, 1.03], \quad t \in T \setminus \{1\}.$$

Next, we determine the smallest Euclidean distance from all backup suppliers to a given facility $j \in J$: $d^{min}_j = \min\limits_{\imath' \in I^2}\{\widetilde{d}_{\imath' j}\}$. In addition, we also identify the largest unit distribution cost to facility $j \in J$ among the primary suppliers, for period $t = 1$ and under scenario s: $\overline{sc}_{j1s} = \max\limits_{i \in I^1}\{sc_{ij1s}\}$. We then set

$$sc_{\imath' j1s} \;=\; \frac{\overline{sc}_{j1s} + 1}{d^{min}_j} \cdot \widetilde{d}_{\imath' j}, \qquad \imath' \in I^2,\ j \in J,\ s \in S.$$

A monotone increasing cost sequence is generated in the following periods as follows:

$$sc_{\imath' jts} = \overline{\overline{\mu}}_{\imath' jts}\, sc_{\imath' j(t-1)s} \qquad \imath' \in I^2,\ j \in J,\ t \in \{2, \ldots, n_T\},\ s \in S.$$

Finally, only arcs whose length does not exceed a certain limit are considered. The latter corresponds to a pre-specified percentage p of the maximum distance generated, $d^{max} = \max\limits_{j \in J}\{\max\limits_{i \in I} \widetilde{d}_{ij}, \max\limits_{k \in K} \widetilde{d}_{jk}\}$. In other words, variables $x_{jktt's}$ are only defined if $\widetilde{d}_{jk} \le p\, d^{max}$. Likewise, variables v_{ijts} are only considered for the arcs (i, j) such that $\widetilde{d}_{ij} \le p\, d^{max}$. In all the instances generated, we have taken

$p = 0.6$.

Tardiness penalty costs
Inspired by Correia and Melo (2021), the unit penalty cost for customer orders delivered with a delay is defined by

$$tc_{ktt'} \; = 0.01\,\theta_k^t \; (t'-t)^2 , \qquad t \in T,\; t' = t, t+1, \ldots, \min\{t+\rho_k, n_T\}$$

with

$$\theta_k^t \; = \frac{\sum\limits_{j \in J^r} oc^r_{jt} + \sum\limits_{j \in J^u} \sum\limits_{f \in F} oc^u_{jft}}{td\,|J|} + \frac{\sum\limits_{j \in J} \sum\limits_{s \in S} dc_{jkts}}{|J|\,|S|}$$

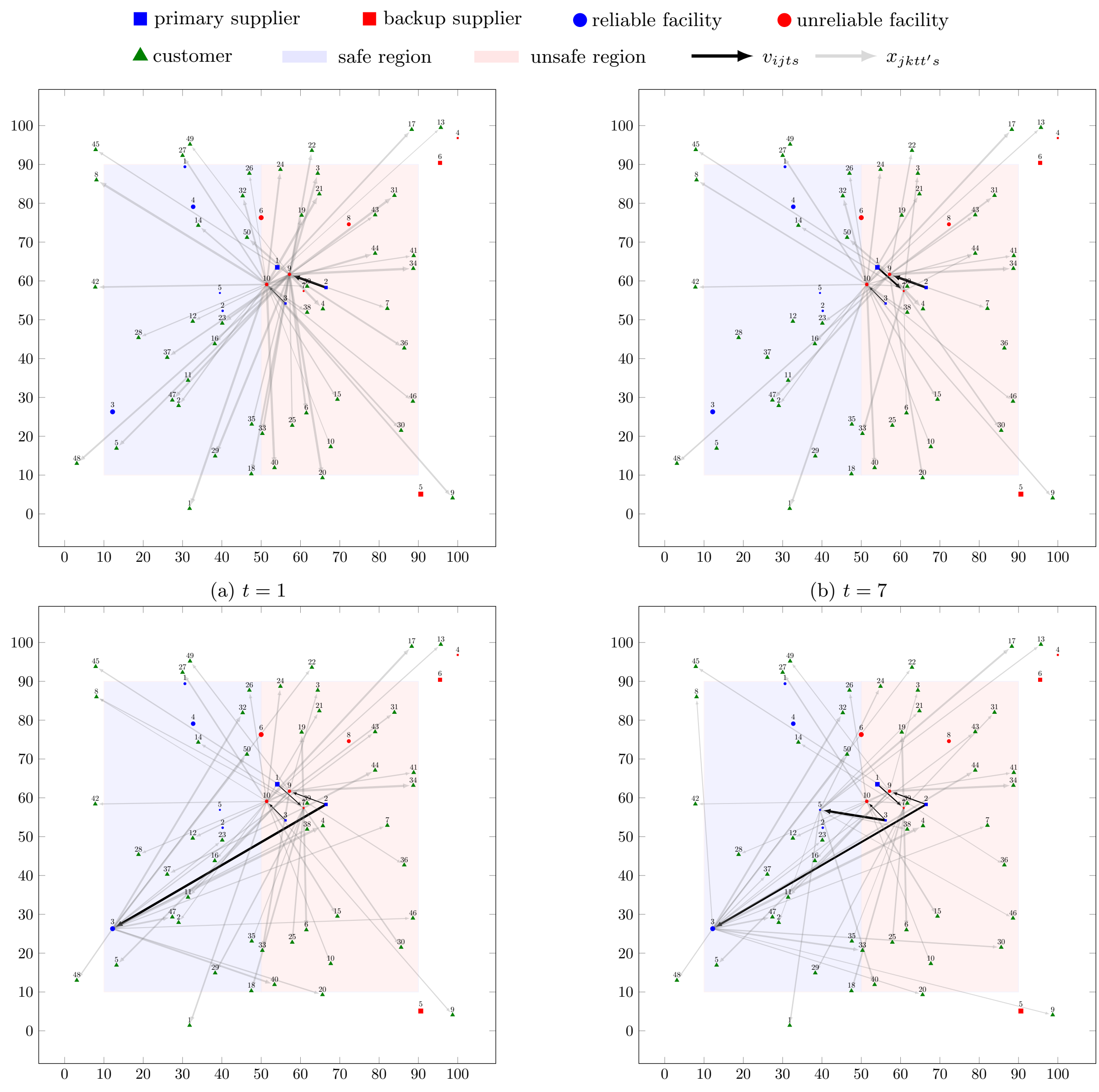
and td denoting the total maximum demand across all scenarios. Accordingly, $td = \sum\limits_{k \in K} \sum\limits_{t \in T} \max\limits_{s \in S} d_{kts}$. We note that the unit tardiness cost is a function of the average fixed facility operating costs and the average variable distribution costs to the customers.

Part B: Optimal network configuration

To illustrate how the network configuration evolves over the strategic periods, we have selected the right lexicographic solution (minimising total expected unmet demand) for an instance with 50 customers, 24 time periods, 3 primary suppliers, 3 backup suppliers, 5 potential locations for reliable facilities and 5 potential locations for unreliable facilities. Recall that the corresponding optimal (first-stage) location decisions are shown in Figure 7 (a) in Section 5.3 of the main paper. Figure S1 displays the allocation of primary suppliers to the operating facilities and the flows across the network for a particular scenario featuring monotonically increasing demand across consecutive periods.

Initially ($t = 1$), two unreliable facilities are opened at sites 9 and 10, both with a medium fortification level. The largest product quantity is delivered by primary supplier 2 to facility 9, while primary supplier 3 serves facility 10. In the next strategic period ($t = 7$), a third unreliable facility is opened at location 7 with the highest fortification level. Primary supplier 1 is assigned to it, likely due to proximity. In strategic period $t = 13$, the deployment of the reliable facility 3 takes place. Primary supplier 2 has sufficient capacity to serve this facility (in addition to location 9), although it is farther away than the other primary suppliers. Note that supplier-facility assignments are fixed across all scenarios. Finally, in the last strategic period ($t = 19$), reliable facility 5 becomes operational and is served by primary supplier 3.

In this particular scenario, unreliable facilities 7, 9 and 10 experience capacity losses in periods 14–17, 17–20 and 17–18, respectively. Thus, it is not surprising that reliable facilities 3 and 5 start operating in the second half of the planning horizon ($t \geq 13$). Moreover, backup supplier 5 is activated in periods 23 and 24 to compensate for the partial unavailability of primary supplier 3 in these periods. Accordingly, reliable facility 3 is served by both suppliers 3 and 5. Since in this Pareto-optimal solution all demand is satisfied, deliveries occur for all customers not only in the strategic periods depicted in Figures (a)–(d), but also in all other (tactical) periods.



(a) $t = 1$ (b) $t = 7$ (c) $t = 13$ (d) $t = 19$

Figure S1: Optimal network configuration in the right lexicographic solution (sol. 1) and scenario $s = 15$ for an instance with $|K| = 50$, $|T| = 24$, $|S| = 30$, $|I^1| = |I^2| = 3$ and $|J^r| = |J^u| = 5$.

Part C: Supply chain network design with limited responsiveness

Table S1 summarises the characteristics of the efficient solutions returned by the two executions of Algorithm 1 when late deliveries to customers are not permitted, i.e. $\rho_k = 0$ for all $k \in K$. This case limits the network's responsiveness to disruptions and demand fluctuations. The structure of this table is the same as that of Table 3 in Section 5.2.1 of the accompanying paper.

Since the index t' can now be eliminated from the flow variables $x_{jktt's}$ defined in (6), the objective functions (7) and (8), as well as constraints (17), (22)–(25) and (31) are simplified. Therefore, compared to the case $\rho_k = 2$ ($k \in K$), the two-stage stochastic formulation has on average 64.6% fewer continuous variables, but about 1% more constraints due to inequalities (33).

Table S1: Characteristics of the solutions identified when delivery delays to customers are not permitted ($\rho_k = 0,\ k \in K$).

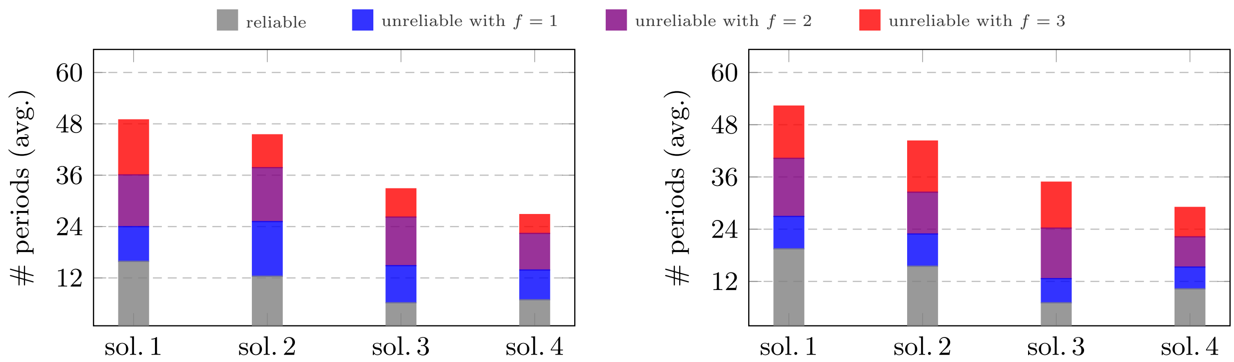
Inst.	Sol.	Algorithm 1 with CPLEX: Total cost (m.u.)	Demand met (%)	MIP gap (%)	CPU time (s)	Algorithm 1 with two-phase heuristic: % cost dev. to CPLEX	Demand met (%)	% CPU to CPLEX
G1	1	10,237,223.7	100.0	0.00	15.6	0.15	100.0	14.10
	2	6,625,043.1	75.0	0.00	28.1	0.41	75.0	8.47
	3	4,501,614.9	50.0	0.00	21.9	0.82	50.0	5.75
	4	2,827,742.4	25.0	0.00	6.3	0.00	25.0	4.13
G2	1	21,398,390.1	100.0	0.00	55.7	0.73	100.0	12.93
	2	13,443,138.6	75.0	0.00	172.6	0.85	75.0	4.73
	3	8,565,923.1	50.0	0.00	119.0	0.92	50.0	2.91
	4	4,832,311.2	25.0	0.00	13.8	0.85	25.0	6.38
G3	1	32,151,960.8	100.0	0.00	133.3	0.43	100.0	7.17
	2	20,651,823.6	75.0	0.00	404.7	0.97	75.0	5.87
	3	12,890,635.8	50.0	0.00	276.2	0.42	50.0	2.32
	4	7,192,008.6	25.0	0.00	25.2	0.79	25.0	7.14
G4	1	10,725,824.5	100.0	0.00	46.6	0.90	100.0	12.66
	2	6,590,065.7	75.0	0.00	122.4	1.14	75.0	7.83
	3	4,248,102.9	50.0	0.00	68.0	0.64	50.0	4.97
	4	2,756,980.7	25.0	0.00	15.1	0.00	25.0	5.30
G5	1	22,297,469.4	100.0	0.00	176.9	0.37	100.0	10.21
	2	14,203,377.6	75.0	0.00	588.2	0.63	75.0	4.44
	3	8,851,517.3	50.0	0.00	336.3	1.01	50.0	5.26
	4	5,346,748.9	25.0	0.00	40.8	0.77	25.0	8.09
G6	1	31,514,352.2	100.0	0.00	369.2	1.49	100.0	8.35
	2	19,433,285.9	75.0	0.00	1,877.5	1.26	75.0	3.14
	3	12,806,659.8	50.0	0.00	1,241.2	1.18	50.0	4.08
	4	7,331,246.6	25.0	0.00	72.8	1.05	25.0	6.95
G7	1	17,722,727.9	100.0	0.00	1,852.7	0.66	100.0	0.49
	2	10,326,481.0	75.0	0.00	237.1	0.53	75.0	2.87
	3	6,834,607.6	50.0	0.00	143.5	1.57	50.0	3.60
	4	4,042,580.3	25.0	0.00	17.1	0.91	25.0	7.02
G8	1	37,155,835.3	100.0	0.00	1,338.7	1.41	100.0	1.75
	2	22,979,329.4	75.0	0.00	2,101.6	0.32	75.0	2.45
	3	14,564,967.4	50.0	0.00	1,074.5	0.30	50.0	2.60
	4	8,212,232.5	25.0	0.00	57.4	0.42	25.0	4.67
G9	1	53,387,439.5	100.0	0.00	2,475.1	1.07	100.0	1.30
	2	33,562,143.7	75.0	0.85	5,093.9	1.61	75.0	2.21
	3	20,721,062.4	50.0	0.88	4,570.9	−0.64	50.0	0.69
	4	11,263,568.9	25.0	0.00	101.9	0.00	25.0	4.53
G10	1	18,441,486.9	100.0	0.00	997.7	0.68	100.0	1.32
	2	11,339,602.0	75.0	0.00	1,154.9	0.93	75.0	2.77
	3	7,182,100.6	50.0	0.00	567.6	1.44	50.0	2.58
	4	4,388,481.8	25.0	0.00	55.1	0.00	25.0	2.65
G11	1	36,548,618.2	100.0	0.29	3,933.9	1.57	100.0	1.56
	2	23,021,511.0	75.0	1.38	7,200.2	0.70	75.0	1.75
	3	14,214,956.7	50.0	0.60	4,188.1	0.62	50.0	1.81
	4	8,155,230.7	25.0	0.00	143.9	0.75	25.0	5.32
G12	1	56,791,349.4	100.0	0.65	6,493.4	1.74	100.0	2.31
	2	36,973,150.7	75.0	1.68	6,387.3	−0.43	75.0	6.37
	3	22,724,806.5	50.0	1.93	6,578.9	−1.65	50.0	3.55
	4	11,988,338.1	25.0	0.00	305.6	0.90	25.0	2.98
G13	1	63,694,523.4	100.0	3.63	2,641.4	−1.09	98.4	27.37
	2	-	-		7,200.0		74.8	18.76
	3	-	-		7,200.2		49.8	16.67
	4	12,801,782.0	25.0	2.99	1,508.5	−2.01	25.1	8.68

– no feasible solution found.

As expected, a significantly larger number of instances can be solved to optimality within the time limit by Algorithm 1 run exclusively with CPLEX, namely groups G1–G8 and G10. For the remaining instances (G9, G11 and G12), high-quality solutions are obtained, with optimality gaps smaller than

2%. However, the G13 instances with 180 scenarios remain very challenging, and CPLEX fails again to find feasible solutions to all of them. Specifically, only the lexicographic solutions (sol. 1 and sol. 4) could be identified for the five instances in this group. In contrast, Algorithm 1 with the two-phase heuristic requires, on average, 95% less CPU time than CPLEX and finds feasible solutions to all instances, including G13. Furthermore, the quality of the heuristic solutions is consistently high, with optimality gaps not exceeding 1.74%. In some of the larger instances (G9, G12 and G13), the heuristic even outperforms CPLEX.

Figure S2 (a) reports the average number of time periods a facility is operated when Algorithm 1 is executed with CPLEX and delivery delays to customers are allowed. In Figure S2 (b) the setting without delivery delays is depicted. Recall that a similar pattern is also observed for the heuristic solutions (see Figure 10 in Section 5.3 of the main paper).



(a) Delivery delays ($\rho_k = 2$, $k \in K$).

(b) Timely deliveries ($\rho_k = 0$, $k \in K$).

Figure S2: Average number of periods that a facility is in operation for solutions returned by Algorithm 1 with CPLEX (instances G1–G12).

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