‘*What is Important is Seldom Urgent and What is Urgent is Seldom Important*’: A Study of the Strategic Implications of the Urgency Effect in a Competitive Setting

Supplement

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1 Proofs

Proof of Proposition 1

I prove the result in a sequence of lemmas.

Let $\sigma^H = \langle p, F_0, F_1 \rangle$ denote a symmetric equilibrium mixed strategy and $\pi((x,u),\sigma^H)$ a supplier's expected payoff of offering (x,u) when the opponent chooses the mixed strategy σ^H.

Lemma 1. *Suppose that the urgency frame is fixed at $u \in \{0,1\}$. Then, in symmetric equilibrium tasks are chosen probabilistically according to an atomless and strictly increasing cdf.*

Proof. Suppose that the urgency frame is fixed. When this is the case, the resulting game has the structure of an all-pay auction. Hence, it does not have pure-strategy equilibria. Denote by $F_u(x)$ the symmetric equilibrium mixed strategy.

Suppose, by contradiction, that a task y is assigned probability $\sigma(y) > 0$. Then, by offering y, each supplier makes $\frac{\sigma(y)}{2} + Prob[y > x] - c(y) - d(u)$ payoff. Assume that a supplier deviates to action $y+\epsilon$, for some $\epsilon > 0$. The deviant supplier makes $\sigma(y) + Prob[y > x] - c(y+\epsilon) - d(u)$. For a sufficiently small ϵ, the deviation is profitable. Hence, $F_u(x)$ is atomless.

Next, suppose that there exists an interval $[\underline{x}, \overline{x}]$ over which $F_u(x)$ is flat, with $\underline{x} < \overline{x}$. Observe that each supplier's payoff is $F_u(\underline{x}) - c(\underline{x}) - d(u)$ and $F_u(\underline{x}) - c(\overline{x}) - d(u)$ at $\underline{x}$ and $\overline{x}$, respectively. Observe that, since $c(\cdot)$ is strictly increasing, the latter is strictly greater than the former. However, this contradicts the assumption that both tasks are part of the support of the mixed-strategy equilibrium. Hence, $F_u(x)$ is strictly increasing. □

Lemma 2. *Suppose that the urgency frame is fixed at u. Then, suppliers make $-d(u)$ payoff in equilibrium and the least important task that is part of the support of $F_u(x)$ is a zero-importance task.*

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Proof. Suppose not. In particular, assume that the lower bound of the support of $F_u(x)$ is larger than zero, and equal to $\hat{x} > 0$. Then, task suppliers make payoff $-c(\hat{x}) - d(u)$ in equilibrium. However, if this is the case, they can deviate $(0, u)$ by making payoff $-d(u)$, a contradiction. □

Lemma 3. *The urgency frame u is chosen probabilistically if and only if $0 < c_f < 1 - \beta$.*

Proof. **Necessity**. I prove the statement by contrapositive. Assume first that $c_f \geq 1 - \beta$. Then, the candidate symmetric equilibrium strategy is for each supplier to choose $u = 0$ and probabilistically choose task importance according the following cdf.

$$F_0(x) = \begin{cases} c(x), & x \in [0, c^{-1}(1)] \\ 1, & x > c^{-1}(1) \end{cases} \tag{1}$$

At this profile, each supplier makes zero payoff. Assume that a supplier deviates to $(y, 1)$ with y arbitrarily close to zero. At the deviation, the deviant supplier obtains (as y approaches zero) $1 - \beta - c_f$, which constitutes a profitable deviation if and only if $1 - \beta > c_f$. Since, by assumption, $c_f \geq 1 - \beta$, the deviation is not profitable, and u is chosen deterministically.

Next, suppose that $c_f = 0$. The candidate symmetric equilibrium strategy is for each supplier to choose $u = 1$ and probabilistically set actions according the cdf $F_1 = F_0$. At this profile, each supplier payoff $-d(1) = -c_f = 0$. The only potentially profitable deviation is that to $(0, 0)$. Such deviation yields zero payoff. Hence, it is not profitable, and the urgency frame is again chosen deterministically, as desired.

Sufficiency. Suppose that $0 < c_f < 1 - \beta$. I distinguish two cases.

Case (i): both suppliers choose $u = 0$. The candidate symmetric equilibrium strategy is the same as that of the necessity part of the proof when $c_f \geq 1 - \beta$, with a resulting payoff of 0. However, in this case each supplier has an incentive to deviate to $(y, 1)$ with y arbitrarily close to zero, because the resulting payoff is $1 - \beta - c_f > 0$.

Case (ii): both suppliers choose $u = 1$. The candidate symmetric equilibrium strategy is the same as that of the necessity part of the proof when $c_f = 0$, with a resulting payoff of $-c_f < 0$. Observe that deviating to $(0, 0)$ is profitable, as the resulting payoff is $0 > -c_f$. In both cases suppliers have an incentive to deviate. Hence, the equilibrium - if it exists - is such that the urgency frame is chosen probabilistically. □

Lemma 4. *Suppose that $0 < c_f < 1 - \beta$. The support of F_u has the form $[0, \bar{x}_u]$, with $u \in \{0, 1\}$.*

Proof. Suppose not. By lemma 3, suppliers choose $u = 0$ with probability p and $u = 1$ with probability $1 - p$, with $p \in (0, 1)$. By the arguments of lemma 1, each F_u is atomless and strictly increasing. Hence, the support of F_u is given by an interval $[\underline{x}_u, \bar{x}_u]$ with $\underline{x}_u < \bar{x}_u$ for $u = 0, 1$. I distinguish three cases.

Case (i): $\underline{x}_u > 0$ for all $u \in \{0, 1\}$. Assume WLOG that $\underline{x}_1 \leq \underline{x}_0$. Then, $\pi((\underline{x}_1, 1), \sigma^H) = (1 - \beta)p - c(\underline{x}_1) - c_f$. Observe that $\pi((\underline{x}_1 - \epsilon, 1), \sigma^H) = (1 - \beta)p - c(\underline{x}_1 - \epsilon) - c_f$. Hence, for a sufficiently small $\epsilon > 0$, the deviation is profitable. A similar argument can be applied to the case $\underline{x}_0 < \underline{x}_1$.

Case (ii): $\underline{x}_0 = 0$ and $\underline{x}_1 > 0$. If $\bar{x}_0 \leq \underline{x}_1$, then a contradiction is immediately obtained, as $\pi((\bar{x}_0, 0), \sigma^H) = p - c(\bar{x}_0)$ and $\pi((\underline{x}_1, 1), \sigma^H) = p - c(\underline{x}_1) - c_f$, and the former is strictly greater than the latter. Hence, suppose that $\bar{x}_0 > \underline{x}_1 > 0$. Observe that, for all $x \in [0, \underline{x}_1]$, there holds $\pi((x, 0), \sigma^H) = pF_0(x) - c(x)$. Given that $\pi((0, 0), \sigma^H) = 0$, by equating these two, it follows that $F_0(x) = \frac{c(x)}{p}$. Next, observe that:

$$\begin{aligned} \pi((\underline{x}_1, 1), \sigma^H) &= \beta p F_0(\underline{x}_1) + (1 - \beta)p - c(\underline{x}_1) - c_f \\ &= \beta c(\underline{x}_1) + (1 - \beta)p - c(\underline{x}_1) - c_f \end{aligned}$$

where in the second equation $F_0(\underline{x}_1)$ is replaced by $F_0(\underline{x}_1) = \frac{c(\underline{x}_1)}{p}$. Suppose that a supplier deviates to $(x, 1)$ where x is arbitrarily close to zero. At the deviation, the supplier makes $\lim_{x\to+0} \pi((x,1),\sigma^H) = (1-\beta)p - c_f$. Given that, by assumption, $0 < c_f < 1-\beta$, it follows that $\beta < 1$, and, as a result, $\lim_{x\to+0} \pi((x,1),\sigma^H) > \pi((\underline{x}_1,\sigma^H),1)$, i.e., the deviation is profitable.

Case (iii): $\underline{x}_0 > 0$ and $\underline{x}_1 = 0$. Assume first that $\underline{x}_0 \geq \bar{x}_1$. Observe that $\pi((\underline{x}_0,0),\sigma^H) = \beta(1-p) - c(\underline{x}_0)$ and $\pi((\bar{x}_1,1),\sigma^H) = \beta(1-p)+1-\beta-c(\underline{x}_0)-c_f$. Since, by assumption, $c_f < 1-\beta$, then $\pi((\underline{x}_0,0),\sigma^H) < \pi((\bar{x}_1,1),\sigma^H)$, a contradiction. Next, suppose that $\underline{x}_0 < \bar{x}_1$. Observe that, for all $x \in [0,\underline{x}_0]$, there holds $\pi((x,1),\sigma^H) = \beta(1-p)F_1(x)+(1-\beta)(p+(1-p)F_1(x))-c(x)-c_f$. Given that $\lim_{x\to+0} \pi((x,1),\sigma^H) = (1-\beta)p - c_f$, by equating these two, it follows that $F_1(x) = \frac{c(x)}{1-p}$. Next, observe that:

$$
\begin{aligned}
\pi((\underline{x}_0,0),\sigma^H) &= \beta(1-p)F_1(\underline{x}_0) - c(\underline{x}_0) \\
&= \beta c(\underline{x}_0) - c(\underline{x}_0)
\end{aligned}
$$

where in the second equation $F_1(\underline{x}_0)$ is replaced by $F_1(\underline{x}_0) = \frac{c(\underline{x}_0)}{1-p}$. Assume that a supplier deviates to $(0,0)$. At the deviation, the supplier makes $\pi((0,0),\sigma^H) = 0$. Given that, by assumption, $0 < c_f < 1-\beta$, it follows that $\beta < 1$, and, as a result, $\pi((0,0),\sigma^H) > \pi((\underline{x}_0,\sigma^H),1)$, i.e., the deviation is profitable.

Therefore, the only possibility that is left is that the support of F_u is $[0,\bar{x}_u]$ with $u \in \{0,1\}$, as desired. $\square$

Lemma 5. *Suppose that* $0 < c_f < 1-\beta$. *Then,* $\bar{x}_0 \geq \bar{x}_1$ *if and only if* $c_f \geq \frac{1-\beta}{2}$.

Proof. By lemma 4, the support of F_u has the form $[0,\bar{x}_u]$, with $u \in \{0,1\}$. Since $\pi(0,0),\sigma^H) = 0$ and $\lim_{x\to+0}((x,1),\sigma^H) = (1-\beta)p - c_f$. By equating these two, $p = \frac{c_f}{1-\beta}$.

Necessity. Suppose that $\bar{x}_0 \geq \bar{x}_1$. Observe that $\pi((\bar{x}_0,0),\sigma^H) = \beta + (1-\beta)p - c(\bar{x}_0) = \beta + c_f - c(\bar{x}_0)$. Assume first that $\bar{x}_0 > \bar{x}_1$. Suppose, by contradiction, that $c_f < \frac{1-\beta}{2}$. Then, it must be the case that a supplier cannot profitably deviate to $(\bar{x}_0,1)$. At the deviation, the supplier makes $\pi((\bar{x}_0,1),\sigma^H) = 1 - c(\bar{x}_0) - c_f$. The deviation is not profitable whenever $1 - c(\bar{x}_0) - c_f \leq \beta + c_f - c(\bar{x}_0)$, or, equivalently, $c_f \geq \frac{1-\beta}{2}$, a contradiction. Next, assume that $\bar{x}_0 = \bar{x}_1$. In this case, $\pi((\bar{x}_1,1),\sigma^H) = 1 - c(\bar{x}_1) - c_f$ and it must be the case that $\pi((\bar{x}_0,0),\sigma^H) = \pi((\bar{x}_1,1),\sigma^H)$. Given that $\bar{x}_0 = \bar{x}_1$, it follows that $c_f = \frac{1-\beta}{2}$.

Sufficiency. Assume first that $c_f = \frac{1-\beta}{2}$. In this case, $\bar{x}_0 = \bar{x}_1$, for the same argument as that of the necessity part of the proof. Next, assume that $c_f > \frac{1-\beta}{2}$. Suppose, by contradiction, that $\bar{x}_0 < \bar{x}_1$. Then, $\pi((\bar{x}_1,1),\sigma^H) = 1 - c(\bar{x}_1) - c_f$. In addition, it must be the case that a supplier cannot profitably deviate to $(\bar{x}_1,0)$. At the deviation, the supplier makes $\pi((\bar{x}_1,0),\sigma^H) = \beta + (1-\beta)p - c(\bar{x}_0) = \beta + c_f - c(\bar{x}_0)$. The deviation is not profitable whenever $\beta + c_f - c(\bar{x}_0) \leq 1 - c(\bar{x}_1) - c_f$ or, equivalently, $c_f \leq \frac{1-\beta}{2}$, a contradiction, which is the desired result. $\square$

Lemma 6. *The symmetric equilibrium mixed strategy* σ^H *is defined as in the statement of proposition 1.*

Proof. I distinguish three cases.

Case (i): $c_f \geq 1-\beta$ or $c_f = 0$. In this case, I have already shown that the equilibrium cdf is that of equation 1, which is included in the proof of lemma 3.

Case (ii): $1-\beta > c_f \geq \frac{1-\beta}{2}$. By lemma 5, $\bar{x}_0 \geq \bar{x}_1$. Observe the following.

$$\begin{aligned}
\pi((0,0),\sigma^H) &= 0\\
\forall x\in[0,\overline{x}_1],\ \pi((x,0),\sigma^H) &= \beta\,[p\,F_0(x)+(1-p)F_1(x)]+(1-\beta)[p\,F_0(x)]-c(x)\\
\pi((\overline{x}_1,0),\sigma^H) &= \beta\,[p\,F_0(\overline{x}_1)+(1-p)]+(1-\beta)[p\,F_0(\overline{x}_1)]-c(\overline{x}_1)\\
\forall x\in[\overline{x}_1,\overline{x}_0],\ \pi((x,0),\sigma^H) &= \beta\,[p\,F_0(x)+(1-p)]+(1-\beta)[p\,F_0(x)]-c(x)\\
\pi((\overline{x}_0,0),\sigma^H) &= \beta+(1-\beta)p-c(\overline{x}_0)\\
\pi((0,1),\sigma^H) &= (1-\beta)p-c_f\\
\forall x\in[0,\overline{x}_1],\ \pi((x,1),\sigma^H) &= \beta\,[p\,F_0(x)+(1-p)F_1(x)]+(1-\beta)\,[p+(1-p)F_1(x)]\\
&\quad -c(x)-c_f\\
\pi((\overline{x}_1,1),\sigma^H) &= \beta\,[p\,F_0(\overline{x}_1)+(1-p)]+(1-\beta)-c(\overline{x}_1)-c_f
\end{aligned}$$

By setting up the usual indifference conditions and solving them, I obtain the equations of proposition 1(ii.a). Observe also that all variables are well-defined, as long as $1-\beta > c_f \geq \frac{1-\beta}{2}$.

Case(iii): $c_f < \frac{1-\beta}{2}$. By lemma 5, $\bar{x}_0 < \bar{x}_1$. Observe the following.

$$\begin{aligned}
\pi((0,0),\sigma^H) &= 0\\
\forall x\in[0,\overline{x}_0],\ \pi((x,0),\sigma^H) &= \beta\,[p\,F_0(x)+(1-p)F_1(x)]+(1-\beta)[p\,F_0(x)]-c(x)\\
\pi((\overline{x}_0,0),\sigma^H) &= \beta\,[p+(1-p)F_1(\overline{x}_0)]+(1-\beta)p-c(\overline{x}_0)\\
\pi((0,1),\sigma^H) &= (1-\beta)p-c_f\\
\forall x\in[0,\overline{x}_0],\ \pi_\sigma((x,1),\sigma^H) &= \beta\,[p\,F_0(x)+(1-p)F_1(x)]+(1-\beta)\,[p+(1-p)F_1(x)]\\
&\quad -c(x)-c_f\\
\pi((\overline{x}_0,1),\sigma^H) &= p+(1-p)F_1(\overline{x}_0)-c(\overline{x}_0)-c_f\\
\forall x\in[\overline{x}_0,\overline{x}_1],\ \pi((x,1),\sigma^H) &= p+(1-p)F_1(x)-c(x)-c_f\\
\pi((\overline{x}_1,1),\sigma^H) &= 1-c(\overline{x}_1)-c_f
\end{aligned}$$

By setting up the usual indifference conditions and solving them, I obtain the equations of proposition 1(ii.a). Observe also that all variables are well-defined, as long as $c_f < \frac{1-\beta}{2}$.

It is easy to see that there are no profitable deviations. Hence, σ^H is the unique symmetric equilibrium mixed strategy. $\square$

Proof of Corollary 1

Proof. Assume that $1-\beta > c_f$. I first show that F_0 first-order stochastically dominates (FOSD) F_1 if and only if $c_f \geq \frac{1-\beta}{2}$. Note that F_0 FOSD F_1 whenever $F_0(x) \leq F_1(x)$ for all x in the support, or, equivalently, for all $x \in \left[0, c^{-1}\left(\frac{1+\beta}{1-\beta}(1-\beta-c_f)\right)\right]$,

$$\begin{aligned}
\frac{(1-\beta)}{c_f(1+\beta)}c(x) &\leq \frac{1-\beta}{(1+\beta)(1-c_f-\beta)}c(x)\\
\frac{1-\beta}{2} &\leq c_f\text{, as desired.}
\end{aligned}$$

In the other direction, F_1 FOSD F_0 whenever $F_1(x) \leq F_0(x)$ for all x in the support, or, equivalently, for all $x \in \left[0, c^{-1}\left(c_f\frac{1+\beta}{1-\beta}\right)\right]$,

$$\begin{aligned}\frac{1-\beta}{(1+\beta)(1-\beta-c_f)}c(x) &\leq \frac{(1-\beta)}{c_f(1+\beta)}c(x)\\ c_f &\leq \frac{1-\beta}{2}, \text{ as desired.}\end{aligned}$$

Given that, first, when $c_f \geq 1-\beta$, only non-urgent tasks are offered and, second, when $c_f < 1-\beta$, F_0 FOSD F_1 if and only if $c_f \geq \frac{1-\beta}{2}$, then it follows that task importance and urgency are negatively correlated whenever $c_f \geq \frac{1-\beta}{2}$. $\square$

Proof of Corollary 2

Proof. Let $\mathbb{E}^*_{\sigma^H} := \mathbb{E}_{\sigma^H}[x;\beta = 1, c_f]$.

The proof that $\mathbb{E}_{\sigma^H}[x;\beta, c_f] < \mathbb{E}^*_{\sigma^H}$ if and only if $p \in (0,1)$ is omitted, as it is a special case of the proof of proposition 2. It is immediate to see that the probability that BR decision-makers experience the urgency effect is positive (equation 8 in the main body) if and only if $p \in (0,1)$, as by prop. 1 when $p \in (0,1)$ both tasks framed as urgent and tasks framed as non-urgent are offered with positive probability. $\square$

Proof of Corollary 3

Proof. Let $((x_i, u_i), (x_j, u_j))$ denote a realisation of the equilibrium of prop. 1. I distinguish two cases.

First, suppose that $u_i = u_j$. In this case, the frame is held constant, and, therefore, no urgency effect occurs. As such, both the FR and the BR decision-makers select the tasks in the same way, i.e., by picking the most important one. Therefore, the average level of task importance that is chosen is identical across decision-makers' types in these realisations.

Second, suppose that $u_i \neq u_j$. In particular, assume without loss of generality that $u_i > u_j$. If $x_i > x_j$, then, by the same arguments as those of the previous paragraph, no urgency effect occurs, implying that FR and BR decision-makers both select task supplier i, because the task offered by i is more important *and* more urgent than that offered by j. The relevant case is when $u_i > u_j$ and $x_i \leq x_j$. In this case, while the FR decision-maker chooses task supplier j with probability greater than or equal to $\frac{1}{2}$ (with certainty, if $x_j > x_i$), the BR decision-maker chooses task supplier i with certainty. Therefore, in these classes of realisations the average level of task importance chosen by the BR decision-maker is *strictly smaller* than that of the FR decision-maker.

The above observations along with the results of prop. 1 and coroll. 2 lead to the desired result. $\square$

Proof of Proposition 2

Proof. Let $\mathbb{E}^*_{\sigma^H} := \mathbb{E}_{\sigma^H}[x;\beta = 1, c_f]$.

I begin by deriving the unconditional equilibrium task importance cdf in the case in which $c_f \in \left[\frac{1-\beta}{2}, 1-\beta\right)$. By prop. 1, it follows that:

$$
\begin{aligned}
F(x;\beta,c_f) &= p\cdot F_0(x;\beta,c_f)+(1-p)\cdot F_1(x;\beta,c_f)\\
&= \begin{cases}
\frac{c_f}{1-\beta}\frac{(1-\beta)}{c_f(1+\beta)}c(x)+\frac{1-\beta-c_f}{1-\beta}\frac{1-\beta}{(1+\beta)(1-c_f-\beta)}c(x), & x\in\left[0,c^{-1}\left(\frac{1+\beta}{1-\beta}(1-\beta-c_f)\right)\right]\\
\frac{c_f}{1-\beta}\frac{c(x)(1-\beta)-\beta(1-\beta-c_f)}{c_f}+\frac{1-\beta-c_f}{1-\beta}, & x\in\left[c^{-1}\left(\frac{1+\beta}{1-\beta}(1-\beta-c_f)\right),c^{-1}\left(c_f+\beta\right)\right]\\
1, & x>c^{-1}\left(c_f+\beta\right)
\end{cases}\\
&= \begin{cases}
\frac{2}{1+\beta}c(x), & x\in\left[0,c^{-1}\left(\frac{1+\beta}{1-\beta}(1-\beta-c_f)\right)\right]\\
1+c(x)-(\beta+c_f), & x\in\left[c^{-1}\left(\frac{1+\beta}{1-\beta}(1-\beta-c_f)\right),c^{-1}\left(c_f+\beta\right)\right]\\
1, & x>c^{-1}\left(c_f+\beta\right)
\end{cases}
\end{aligned}
$$

Observe that when $c_f = 1-\beta$, $F(x;\beta,c_f = 1-\beta)$ simplifies to $F_0(x;\beta = 1,c_f)$. Hence, $\mathbb{E}^*_{\sigma^H} = \mathbb{E}_{\sigma^H}[x;\beta,c_f = 1-\beta]$. Next, let $\epsilon > 0$ be an arbitrarily small positive real number. Observe that $F(x;\beta,c_f+\epsilon)$ FOSD $F(x;\beta,c_f)$ whenever $F(x;\beta,c_f+\epsilon)\le F(x;\beta,c_f)$ for all x in the support, and $F(x;\beta,c_f+\epsilon) < F(x;\beta,c_f)$ for some x. By inspecting the unconditional equilibrium task importance cdf derived above, it is clear that $F(x;\beta,c_f+\epsilon)$ FOSD $F(x;\beta,c_f)$. Because FOSD is a transitive relation, this implies that $\mathbb{E}_{\sigma^H}[x;\beta,c_f]$ is strictly increasing in c_f for all $c_f\in\left[\frac{1-\beta}{2},1-\beta\right)$.

Next, consider the case in which $c_f < \frac{1-\beta}{2}$. By prop. 1, it follows that:

$$
\begin{aligned}
F(x;\beta,c_f) &= p\cdot F_0(x;\beta,c_f)+(1-p)\cdot F_1(x;\beta,c_f)\\
&= \begin{cases}
\frac{c_f}{1-\beta}\frac{(1-\beta)}{c_f(1+\beta)}c(x)+\frac{1-\beta-c_f}{1-\beta}\frac{1-\beta}{(1+\beta)(1-c_f-\beta)}c(x), & x\in\left[0,c^{-1}\left(c_f\frac{1+\beta}{1-\beta}\right)\right]\\
\frac{c_f}{1-\beta}+\frac{1-\beta-c_f}{1-\beta}\frac{(1-\beta)c(x)-c_f\beta}{1-\beta-c_f}, & x\in\left[c^{-1}\left(c_f\frac{1+\beta}{1-\beta}\right),c^{-1}(1-c_f)\right]\\
1, & x>c^{-1}(1-c_f)
\end{cases}\\
&= \begin{cases}
\frac{2}{1+\beta}c(x), & x\in\left[0,c^{-1}\left(c_f\frac{1+\beta}{1-\beta}\right)\right]\\
c(x)+c_f, & x\in\left[c^{-1}\left(c_f\frac{1+\beta}{1-\beta}\right),c^{-1}(1-c_f)\right]\\
1, & x>c^{-1}(1-c_f)
\end{cases}
\end{aligned}
$$

Observe that when $c_f = 0$, $F(x;\beta,c_f = 0)$ simplifies to $F_0(x;\beta = 1,c_f)$. Hence, $\mathbb{E}^*_{\sigma^H} = \mathbb{E}_{\sigma^H}[x;\beta,c_f = 0]$. Observe that for an arbitrarily small $\epsilon > 0$, $F(x;\beta,c_f)$ FOSD $F(x;\beta,c_f+\epsilon)$. Therefore, by transitivity of the FOSD relation, $\mathbb{E}_{\sigma^H}[x;\beta,c_f]$ is strictly decreasing in c_f for all $c_f\in\left(0,\frac{1-\beta}{2}\right)$.

Hence, $\mathbb{E}_{\sigma^H}[x;\beta,c_f]$

- is equal to $\mathbb{E}^*_{\sigma^H}$, when $c_f = 0$;
- is strictly decreasing in c_f, when $c_f\in\left(0,\frac{1-\beta}{2}\right)$;
- attains its minimum at $c_f = \frac{1-\beta}{2}$;
- is strictly increasing in c_f, when $c_f\in\left[\frac{1-\beta}{2},1-\beta\right)$;
- is equal to $\mathbb{E}^*_{\sigma^H}$, when $c_f\ge 1-\beta$.

Hence, $\mathbb{E}_{\sigma^H}[x;\beta,c_f]$ is a U-shaped function of c_f, as desired. $\square$

Proof of Proposition 3

Below I provide a statement of the symmetric equilibrium mixed strategy σ^{Aud}.

Proposition 3 (statement of equilibrium). *Assume that task suppliers are endowed with the payoff function of equation 9 in the main body of the paper. Then, there exists a unique symmetric equilibrium mixed strategy σ^{Aud} with the following properties.*

i If either $c_f \geq 1-\beta-\rho\cdot\tau$ or $c_f < 1-\beta-\rho\cdot\tau$ and $1-\beta \leq \rho\cdot\tau$, then $p=1$ and the cdf that suppliers use to randomise over importance is that of equation 3 in the main body of the paper (benchmark case).

ii If $c_f < 1-\beta-\rho\cdot\tau$ and $1-\beta > \rho\cdot\tau$, then $p=\frac{c_f+\rho\cdot\tau}{1-\beta}$. Moreover,

a if $c_f \in \left[\frac{1-\beta-2\rho\cdot\tau}{2}, 1-\beta-\rho\cdot\tau\right)$, then

$$F_0(x;\beta,c_f,\rho,\tau)=\begin{cases}\frac{(1-\beta)}{(c_f+\rho\cdot\tau)(1+\beta)}c(x), & x\in\left[0,c^{-1}\left(\frac{1+\beta}{1-\beta}(1-\beta-c_f-\rho\cdot\tau)\right)\right]\\ \frac{c(x)(1-\beta)-\beta(1-\beta-c_f-\rho\cdot\tau)}{(c_f+\rho\cdot\tau)}, & x\in\left[c^{-1}\left(\frac{1+\beta}{1-\beta}(1-\beta-c_f-\rho\cdot\tau)\right),c^{-1}\left(c_f+\beta+\rho\cdot\tau\right)\right]\\ 1, & x>c^{-1}\left(c_f+\beta+\rho\cdot\tau\right)\end{cases}$$

$$F_1(x;\beta,c_f,\rho,\tau)=\begin{cases}\frac{1-\beta}{(1+\beta)(1-c_f-\beta-\rho\cdot\tau)}c(x), & x\in\left[0,c^{-1}\left(\frac{1+\beta}{1-\beta}(1-\beta-c_f-\rho\cdot\tau)\right)\right]\\ 1, & x>c^{-1}\left(\frac{1+\beta}{1-\beta}(1-\beta-c_f-\rho\cdot\tau)\right)\end{cases}$$

b if $c_f \in \left[0, \frac{1-\beta-2\rho\cdot\tau}{2}\right)$, then

$$F_0(x;\beta,c_f)=\begin{cases}\frac{(1-\beta)}{(c_f+\rho\cdot\tau)(1+\beta)}c(x), & x\in\left[0,c^{-1}\left((c_f+\rho\cdot\tau)\frac{1+\beta}{1-\beta}\right)\right]\\ 1, & x>c^{-1}\left((c_f+\rho\cdot\tau)\frac{1+\beta}{1-\beta}\right)\end{cases}$$

$$F_1(x;\beta,c_f)=\begin{cases}\frac{1-\beta}{(1+\beta)(1-\beta-c_f-\rho\cdot\tau)}c(x), & x\in\left[0,c^{-1}\left((c_f+\rho\cdot\tau)\frac{1+\beta}{1-\beta}\right)\right]\\ \frac{(1-\beta)c(x)-\beta(c_f+\rho\cdot\tau)}{1-\beta-c_f-\rho\cdot\tau}, & x\in\left[c^{-1}\left((c_f+\rho\cdot\tau)\frac{1+\beta}{1-\beta}\right),c^{-1}(1-c_f-\rho\cdot\tau)\right]\\ 1, & x>c^{-1}(1-c_f-\rho\cdot\tau)\end{cases}$$

iii Each task supplier makes zero payoff in equilibrium.

The existence and uniqueness proof of the equilibrium of proposition 3 is omitted, because it is similar to that of proposition 1. Hence, I turn to the proof of proposition 3. I do so through a sequence of lemmas.

Lemma 7. *Let σ^{Aud} denote the unique symmetric eq. strategy (defined above) resulting from suppliers being endowed with the payoff function of equation 9 in the main body of the paper. Moreover, let $\mathbb{E}^*_{\sigma^{Aud}} := \mathbb{E}_{\sigma^{Aud}}[x;\beta=1,c_f,\rho,\tau]$. Then, $\mathbb{E}_{\sigma^{Aud}}[x;\beta,c_f,\rho,\tau]$ is*

- *is strictly decreasing in c_f, when $c_f \in \left[0, \frac{1-\beta-2\rho\cdot\tau}{2}\right)$;*
- *attains its minimum at $c_f = \frac{1-\beta-2\rho\cdot\tau}{2}$;*
- *is strictly increasing in c_f, when $c_f \in \left[\frac{1-\beta-2\rho\cdot\tau}{2}, 1-\beta-\rho\cdot\tau\right)$;*
- *is equal to $\mathbb{E}^*_{\sigma^{Aud}}$, when $c_f \geq 1-\beta-\rho\cdot\tau$.*

Moreover, when $c_f \in \left[0, \frac{1-\beta-2\rho\cdot\tau}{2}\right)$, $\mathbb{E}_{\sigma^{Aud}}[x;\beta, c_f = 0, \rho > 0, \tau] < \mathbb{E}^*_{\sigma^{Aud}}$.

Proof. Let σ^{Aud} denote the unique symmetric eq. strategy (defined above) resulting from suppliers being endowed with the payoff function of equation 9 in the main body of the paper. Moreover, let $\mathbb{E}^*_{\sigma^{Aud}} := \mathbb{E}_{\sigma^{Aud}}[x;\beta = 1, c_f, \rho, \tau]$.

It is clear that the unconditional equilibrium task importance cdf $F(x;\beta, c_f \geq 1-\beta-\rho\cdot\tau, \rho, \tau)$ in the case in which $c_f \geq 1-\beta-\rho\cdot\tau$ is the same as that of the benchmark case, i.e., $F(x;\beta = 1, c_f, \rho, \tau)$.

Next, the unconditional equilibrium task importance cdf $F(\cdot;\cdot)$ when $c_f \in \left[\frac{1-\beta-2\rho\cdot\tau}{2}, 1-\beta-\rho\cdot\tau\right)$ is given by:

$$F(x;\beta,c_f) = \begin{cases} \frac{2}{1+\beta}c(x), & x \in \left[0, c^{-1}\left(\frac{1+\beta}{1-\beta}(1-\beta-c_f-\rho\cdot\tau)\right)\right] \\ 1 + c(x) - (\beta + c_f + \rho\cdot\tau), & x \in \left[c^{-1}\left(\frac{1+\beta}{1-\beta}(1-\beta-c_f-\rho\cdot\tau)\right), c^{-1}\left(c_f+\beta+\rho\cdot\tau\right)\right] \\ 1, & x > c^{-1}\left(c_f+\beta+\rho\cdot\tau\right) \end{cases}$$

Next, the unconditional equilibrium task importance cdf $F(\cdot;\cdot)$ when when $c_f \in \left[0, \frac{1-\beta-2\rho\cdot\tau}{2}\right)$ is given by:

$$F(x;\beta,c_f,\rho,\tau) = \begin{cases} \frac{2}{1+\beta}c(x), & x \in \left[0, c^{-1}\left((c_f+\rho\cdot\tau)\frac{1+\beta}{1-\beta}\right)\right] \\ c(x) + c_f + \rho\cdot\tau, & x \in \left[c^{-1}\left((c_f+\rho\cdot\tau)\frac{1+\beta}{1-\beta}\right), c^{-1}(1-c_f-\rho\cdot\tau)\right] \\ 1, & x > c^{-1}(1-c_f-\rho\cdot\tau) \end{cases}$$

By FOSD arguments similar to those of the proof of proposition 2, $\mathbb{E}_{\sigma^{Aud}}[x;\beta, c_f, \rho, \tau]$ is too an U-shaped function of c_f.

Finally, assume that $c_f \in \left[0, \frac{1-\beta-2\rho\cdot\tau}{2}\right)$, so that $\frac{1-\beta}{2} > \rho\cdot\tau$. Observe that when $c_f = 0$, $F(x;\beta = 1, c_f, \rho, \tau)$ FOSD $F(x;\beta, c_f = 0, \rho > 0, \tau)$ implying that $\mathbb{E}_{\sigma^{Aud}}[x;\beta, c_f = 0, \rho > 0, \tau] < \mathbb{E}^*_{\sigma^{Aud}}$. □

Lemma 8. *Let* σ^{Aud} *denote the unique symmetric eq. strategy (defined above) resulting from suppliers being endowed with the payoff function of equation 9 in the main body of the paper. Then, there exists a threshold* $\overline{c_f} := \max\left\{0, \frac{1-\beta-\rho\cdot\tau}{2}\right\}$ *such that:*

a for all $c_f \geq \overline{c_f}$*, there holds*

$$\mathbb{E}_{\sigma^{Aud}}[x;\beta, c_f, \tau, \rho > 0] \geq \mathbb{E}_{\sigma^{Aud}}[x;\beta, c_f, \tau, \rho = 0]$$

b for all $c_f < \overline{c_f}$*, there holds*

$$\mathbb{E}_{\sigma^{Aud}}[x;\beta, c_f, \tau, \rho > 0] < \mathbb{E}_{\sigma^{Aud}}[x;\beta, c_f, \tau, \rho = 0]$$

Proof. Let σ^{Aud} denote the unique symmetric eq. strategy (defined above) resulting from suppliers being endowed with the payoff function of equation 9 in the main body of the paper.

By proposition 2 and lemma 7, both $\mathbb{E}_{\sigma^{Aud}}[x;\beta, c_f, \rho = 0, \tau]$ and $\mathbb{E}_{\sigma^{Aud}}[x;\beta, c_f, \rho > 0, \tau]$ are U-shaped function of c_f, where the former attains its minimum at $\frac{1-\beta}{2}$ and the latter attains its minimum at $\frac{1-\beta-2\rho\cdot\tau}{2}$. Observe that $\frac{1-\beta}{2} > \frac{1-\beta-2\rho\cdot\tau}{2}$. Since both $\mathbb{E}_{\sigma^{Aud}}[x;\beta, c_f, \rho = 0, \tau]$ and $\mathbb{E}_{\sigma^{Aud}}[x;\beta, c_f, \rho > 0, \tau]$ are continuous in c_f, there exists one point at which $\mathbb{E}_{\sigma^{Aud}}[x;\beta, c_f, \rho =$

$0,\tau] = \mathbb{E}_{\sigma^{Aud}}[x;\beta,c_f,\rho>0,\tau]$. Observe that $\mathbb{E}_{\sigma^{Aud}}[x;\beta,c_f,\rho=0,\tau] = \mathbb{E}_{\sigma^{Aud}}[x;\beta,c_f,\rho>0,\tau]$ if and only if the corresponding unconditional cdfs - $F(x;\beta,c_f,\rho=0,\tau)$ and $F(x;\beta,c_f,\rho>0,\tau)$ - are identical. By proposition 2 and lemma 7, the relevant unconditional cdfs are:

$$F(x;\beta,c_f,\rho=0,\tau) = \begin{cases} \frac{2}{1+\beta}c(x), & x \in \left[0, c^{-1}\left(c_f\frac{1+\beta}{1-\beta}\right)\right] \\ c(x)+c_f, & x \in \left[c^{-1}\left(c_f\frac{1+\beta}{1-\beta}\right), c^{-1}(1-c_f)\right] \\ 1, & x > c^{-1}(1-c_f) \end{cases}$$

for $c_f \leq \frac{1-\beta}{2}$, and

$$F(x;\beta,c_f,\rho>0,\tau) = \begin{cases} \frac{2}{1+\beta}c(x), & x \in \left[0, c^{-1}\left(\frac{1+\beta}{1-\beta}(1-\beta-c_f-\rho\cdot\tau)\right)\right] \\ 1+c(x)-(\beta+c_f+\rho\cdot\tau), & x \in \left[c^{-1}\left(\frac{1+\beta}{1-\beta}(1-\beta-c_f-\rho\cdot\tau)\right), c^{-1}\left(c_f+\beta+\rho\cdot\tau\right)\right] \\ 1, & x > c^{-1}\left(c_f+\beta+\rho\cdot\tau\right) \end{cases}$$

$c_f \in \left[\frac{1-\beta-2\rho\cdot\tau}{2}, 1-\beta-\rho\cdot\tau\right)$.

Observe that the former is equal to the latter if and only if $c(x)+c_f = 1+c(x)-(\beta+c_f+\rho\cdot\tau)$, or, equivalently, $c_f = \frac{1-\beta-\rho\cdot\tau}{2}$. Therefore, by setting $\overline{c_f} := \max\left\{0, \frac{1-\beta-\rho\cdot\tau}{2}\right\}$, the result follows. □

Lemma 9. *Let σ^{Aud} denote the unique symmetric eq. strategy (defined above) resulting from suppliers being endowed with the payoff function of equation 9 in the main body of the paper. There exists a threshold $\overline{\rho\cdot\tau} := \frac{1-\beta}{2}$ such that, for all $\rho\cdot\tau \geq \overline{\rho\cdot\tau}$, $\mathbb{E}_{\sigma^{Aud}}[x;\beta,c_f,\tau,\rho]$ is non-decreasing in c_f.*

Proof. Let σ^{Aud} denote the unique symmetric eq. strategy (defined above) resulting from suppliers being endowed with the payoff function of equation 9 in the main body of the paper.

By setting an expected fine $\rho\cdot\tau$ such that $\rho\cdot\tau = \frac{1-\beta}{2}$, it follows - by Lemma 7 - that $\mathbb{E}_{\sigma^{Aud}}[x;\beta,c_f,\rho,\tau]$ attains its minimum at $c_f = 0$. Therefore, by Lemma 7 and the fact that c_f cannot be negative, $\mathbb{E}_{\sigma^{Aud}}[x;\beta,c_f,\rho,\tau]$ is non-decreasing in c_f for all feasible values of c_f. □

2 Contest Success Function

As discussed in remark 3 in the main body of the paper, the contest success function (CSF) is the function that maps the players's efforts into the individual probabilities of winning the contest (Corchón and Serena, 2018). The purpose of this section is to explicitly define the CSF induced by the proposed model, and link it with the static contest-theory literature with multi-dimensional effort.

Let $((x_i,u_i),(x_j,u_j)) \in (X\times\{0,1\})^2$ denote a strategy profile. Letting $i,j \in \{1,2\}$ with $i \neq j$, I begin by defining task supplier i's CSF for the FR decision-makers - denoted CSF_i^{FR} - as follows.

$$CSF_i^{FR}((x_i,u_i),(x_j,u_j)) := \begin{cases} 1 & \text{, if } x_i > x_j \\ \frac{1}{2} & \text{, if } x_i = x_j \\ 0 & \text{, if } x_i < x_j \end{cases} \tag{2}$$

Next, I define task supplier i's CSF for the BR decision-makers - denoted CSF_i^{BR} - as follows.

$$CSF_i^{BR}((x_i,u_i),(x_j,u_j)) := \begin{cases} 1 & \text{, if } u_i > u_j \\ 1 & \text{, if } u_i = u_j \text{ and } x_i > x_j \\ \frac{1}{2} & \text{, if } x_i = x_j \text{ and } u_i = u_j \\ 0 & \text{, if } u_i = u_j \text{ and } x_i < x_j \\ 0 & \text{, if } u_i < u_j \end{cases} \tag{3}$$

Finally, letting $\beta \in [0,1]$ denote the fraction of decision-makers that are FR, I define the overall task supplier i's CSF - denoted CSF_i - as follows.

$$CSF_i((x_i,u_i),(x_j,u_j)) := \beta \cdot CSF_i^{FR}(\cdot) + (1-\beta)\cdot CSF_i^{BR}(\cdot) \tag{4}$$

Observe that in the proposed model the CSF (equation 4) is given by the convex combination of two components, where the weights are given by the fractions of FR and BR decision-makers in the population, respectively. The first component (equation 2) is a standard all-pay auction CSF that captures the behaviour of FR decision-makers. The second component (equation 3), instead, is a 'lexicographic' CSF, in which one effort dimension (the task-urgency dimension) is more important than the other effort dimension (the task-importance dimension) in determining the winning probability.

The literature that investigates static contests that have the property that effort is a multi-dimensional decision variable include Clark and Kai (2007), Arbatskaya and Mialon (2010), and Lagerlöf (2020). Clark and Kai (2007) study a two-player Tullock contest, in which effort is n-dimensional and the winner is the contestant that wins on at least a certain number k of such dimensions, with the property that the probability of winning on one dimension is independent of the effort exerted in the other dimensions. Arbatskaya and Mialon (2010), on the other hand, axiomatise a multi-dimensional two-player Tullock contest success function by studying the equilibrium of the resulting (possibly asymmetric) contest and assuming that the total cost of effort is linearly additive over effort dimensions. More recently, Lagerlöf (2020) examines a (possibly asymmetric) multi-player hybrid contest in which the probability of winning is affected by both an all-pay effort dimension and a winner-pay effort dimension. In that paper, the two effort dimensions are aggregated via a production function to determine a player's score and the total cost of effort is assumed to be linearly additive over effort dimensions.

In contrast, the proposed model studies a symmetric two-player contest in which effort is two-dimensional, and differs from these papers in at least two ways. First, as illustrated above, the proposed contest success function is of neither a generalised Tullock nor a standard all-pay auction nor an hybrid all-pay and winner-pay form. Instead, this paper studies a *novel contest success function* that captures the urgency effect, and that can be expressed as the convex combination of two contest success functions: an *all-pay auction contest success function* that captures the behaviour of FR decision-makers, and a *'lexicographic' contest success function*, in which one effort dimension (task urgency) is more important than the other effort dimension (task importance) in determining the winning probabilities, that captures the behaviour of BR decision-makers. Nonetheless, as discussed above, the proposed model *does* include a standard all-pay auction as a special case, when all decision-makers are FR. Second, in the proposed model the total cost of effort is additively separable across dimension, but - unlike in the above papers - not necessarily linear within dimensions. In fact, the only relevant assumption that is imposed on the task-importance cost function is that of it being strictly increasing.

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