Living on the Edge of the Catastrophe

Supplementary Material

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1 Uncertain Decay Rate

As another particular variation of the base model, an uncertain decay rate, denoted $\widehat{\beta}$, is assumed to be revealed between the two time periods. During the first time period, the regulator does not know if the actual decay rate is high β_H or low β_L, with $\beta_H > \beta_L$. However, before the second time period, the uncertainty is resolved and the specific value (β_H or β_L) is revealed.

According to this particular case, since the actual decay rate β is unknown during the first time period, the regulator must form an expected decay rate $\widehat{\beta}$ to analyze the overall expected welfare across both periods. This expected value is calculated as the probability of the two possible outcomes: $\widehat{\beta} = p_H\beta_H + (1 - p_H)\beta_L$, where p_H is the probability of the high decay rate (β_H) occurring.

While the completely myopic regulator's first-period emission e_1^M is independent of β, this expectation $\widehat{\beta}$ becomes essential for the non myopic regulator e_1^{NM}, whose decision is entirely conditional on the expected degradation of the environmental stock.

Indeed, the analysis shows that the myopic regulator is not directly affected by the expected decay rate $\widehat{\beta}$ in the first period. This is because the myopic regulator chooses e_1^M solely to maximize the instantaneous welfare, which is independent of the decay rate β. In fact, $\frac{\partial}{\partial\beta}e_1^M = 0$.

However, the decision in the second time period is strongly influenced by the revealed decay rate. The emission amount e_2^M is positively related to the decay rate: $\frac{\partial}{\partial\beta}e_2^M > 0$. Specifically, if the standard linear solution holds, we have $\frac{\partial}{\partial\beta}e_2^M = \frac{1-X}{4} > 0$. Thus, during the first time period, the regulator emits e_1^M. After observing the revealed information (β_H or β_L) between the two periods, the regulator emits the suitable amount e_2^M.

If β_H is revealed, the second-period emission will be $e_2^M(\beta_H) = \frac{1-X}{4}(1 + \beta_H)$. Conversely, if β_L is revealed, $e_2^M(\beta_L) = \frac{1-X}{4}(1 + \beta_L)$ holds. Since $\beta_H > \beta_L$, we have $e_2^M(\beta_H) > e_2^M(\beta_L)$. A higher decay rate ($\beta_H$) reduces the environmental risk caused by the residual stock from the first time period. Consequently, the marginal social cost of emissions during the second time period is lower, allowing the myopic regulator to emit more (higher e_2^M) in the "lucky" scenario.

Under the Non Myopic Regulator case ($\gamma > 0$), the results are fundamentally different from the myopic case. The regulator chooses the sequence $\{e_1, e_2\}$ to maximize the expected discounted welfare based on the available information at the time of the decision.

At $t = 1$, the true decay rate is unknown. Therefore, the regulator forms the expectation $\widehat{\beta} = p_H\beta_H + (1 - p_H)\beta_L$ and chooses the optimal initial emission $\widehat{e}_1^{NM}$ based on this belief. The uncertainty plays a crucial role in the initial decision, as the regulator must anticipate the expected future costs determined by $\widehat{\beta}$.

After e_1 is emitted, the uncertainty is resolved. The regulator then exploits the revelation of the real absorption capacity ($\beta_i \in \{\beta_H, \beta_L\}$) to re-optimize e_2. The optimal emissions are therefore:

First Time-Period ($\widehat{e}_1^{NM}$): Chosen under uncertainty, dependent on $\widehat{\beta}$. The emission level is defined by the solution of the dynamic optimization problem, where the coefficient β is substituted by $\widehat{\beta}$: $\widehat{e}_1^{NM} = \frac{2(1-X)-(1+X)(1-\widehat{\beta})\delta}{4-(1-\widehat{\beta})^2\delta}$.

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Second Time-Period (e_2^{NM}): Chosen after the information is revealed. The optimal e_2^{NM} is determined by the reaction function linking e_2 to e_1 and the actual revealed decay rate β_i: $e_2^{NM}(\beta_i) = \frac{1-X}{2} - \frac{(1-\beta_i)}{2}\widehat{e}_1^{NM}$.

Thus, the non myopic regulator's second-period choice is state-contingent on the realized decay rate β_i.

Obviously, the uncertain decay rate $\widehat{\beta}$ does not alter the fundamental qualitative conclusions regarding the comparison between the myopic regulator and the non myopic regulator. In fact, the myopic procrastination is also preserved even in this setting when the decay rate is unknown in the first period.

Specifically, in the initial emission levels, the myopic regulator continues to emit more, and its decision e_1^M is independent with respect to the decay rate β. Conversely, the non myopic regulator's choice $\widehat{e}_1^{NM}$ is dependent on the expected decay rate $\widehat{\beta}$, reflecting an anticipation of the future cost. Consequently, first-period emission remains higher in myopic case: $e_1^M > \widehat{e}_1^{NM}$.

Moreover, as in the base model the myopic regulator begins with a higher initial emission, it leaves a larger residual stock for the second period. This forces the myopic regulator to emit less in the second period compared to the non myopic regulator, given the same realized decay rate $\beta_i \in \{\beta_H, \beta_L\}$, resulting in $e_2^M(\beta_i) < e_2^{NM}(\beta_i)$.

Indeed, the results regarding the terminal stock are the same: the myopic regulator's decision path always leads to a higher terminal stock at the end of the second period. The non myopic regulator carefully manages the intertemporal stock accumulation, ensuring that even if the environmental decay is stochastic, the final stock is lower than that resulting from myopic behavior.

Comparison of emission accumulated at the end of the second period remains valid for any realized state β_i: $e_2^M(\beta_i) + (1-\beta_i)e_1^M > e_2^{NM}(\beta_i) + (1-\beta_i)\widehat{e}_1^{NM}$. The stochastic decay rate simply affects the second-period decisions (e_2) that are contingent on the realized decay rate β_i, but does not fundamentally alter the comparative statics between the two regulators.

2 Alternative Specification with Concave Benefits

Up to this point, the analysis has utilized a linear benefit function, $B(e_i) = e_i$. This choice was primarily driven by the need for analytical simplicity and clarity in interpreting the resulting closed-form solutions for the optimal emission paths. However, a concave benefit function is undeniably a more accurate representation of reality in welfare economics, where the marginal benefit of emissions is typically diminishing.

Thus, in order to demonstrate the robustness of our core findings regarding the influence of myopia and dangerous learning β on the emissions path, we add to the analysis of a brief exploration of this alternative specification. We so replace the linear benefit function $B(e_i) = e_i$ with a general concave function $B(e_i)$ such that $B'(e_i) > 0$ and $B''(e_i) < 0$.

Crucially, while this change affects the quantitative levels of optimal emissions, the underlying philosophical and qualitative findings remain unchanged.

Specifically, we treat the problem employing a quadratic benefit function which maintains analytical tractability: $B(e_i) = e_i - \frac{1}{2}ce_i^2$, where $c > 0$ ensures concavity. The instantaneous utility functions for the three cases, incorporate the original probabilities $F(e_1) = e_1$ and $F_2(e_1, e_2) = \frac{e_2 - \beta e_1}{1 - e_1}$. Hence, we have an objective function for the myopic regulator during the first time period given by:

$$\max_{e_1} \quad \left\{ [1 - F(e_1)]\, B(e_1) + [F(e_1)]\, [-X] \right\} \tag{1}$$

Substituting the explicit functions:

$$\max_{e_1} \quad \left\{ (1 - e_1) \left(e_1 - \frac{1}{2}ce_1^2 \right) - Xe_1 \right\}$$

The First-Order Condition (FOC) $\frac{\partial WB_1^M}{\partial e_1} = 0$ is a quadratic equation in e_1:

$$\frac{\partial}{\partial e_1} \left[e_1 - \frac{1}{2}ce_1^2 - e_1^2 + \frac{1}{2}ce_1^3 - Xe_1 \right] = 0$$

$$1 - ce_1 - 2e_1 + \frac{3}{2}ce_1^2 - X = 0$$

$$\frac{3}{2}c(e_1^M)^2 - (2+c)e_1^M + (1-X) = 0$$

The optimal solution e_1^M is determined by the positive and relevant root of this quadratic equation.
Hence,$e_1^M = \frac{-[-(2+c)]\pm\sqrt{[-(2+c)]^2-4\left(\frac{3}{2}c\right)(1-X)}}{2\left(\frac{3}{2}c\right)}$,
Where, $\Delta = (2+c)^2 - 4\left(\frac{3}{2}c\right)(1-X)$
$\Rightarrow \Delta = (4+4c+c^2) - 6c(1-X)$
$\Rightarrow \Delta = 4+4c+c^2-6c+6cX$
$\Rightarrow \Delta = c^2-2c+4+6cX$

Rearranging, we obtain: $e_1^M = \frac{(2+c)\pm\sqrt{c^2-2c+4+6cX}}{3c}$

Where, the relevant root is $e_1^M = \frac{(2+c)-\sqrt{c^2-2c+4+6cX}}{3c}$, because the one with the positive sign does not met the second order condition.

Obviously, as in the base mode, the regulator maximizes the objective function during the second time period entailing the choice of the previous period as a stock:

$$\max_{e_2} \quad \left\{ [1-F_2(e_1,e_2)]\,B(e_2) + [F_2(e_1,e_2)]\,[-X] \right\} \tag{2}$$

Thus: $\max_{e_2} \quad \left\{ \left(1-\frac{e_2-\beta e_1}{1-e_1}\right)\left(e_2-\frac{1}{2}ce_2^2\right) - X\left(\frac{e_2-\beta e_1}{1-e_1}\right) \right\}$

The FOC $\frac{\partial WB_2^M}{\partial e_2} = 0$ is a cubic equation in e_2:

$$\frac{\partial}{\partial e_2}\left[\frac{(1-e_1)-(e_2-\beta e_1)}{1-e_1}B(e_2) - X\frac{e_2-\beta e_1}{1-e_1}\right] = 0$$

$$\frac{1-\beta e_1-e_2}{1-e_1}B'(e_2) - \frac{1}{1-e_1}B(e_2) - \frac{X}{1-e_1} = 0$$

Multiplying by $(1-e_1)$ and substituting $B(e_2)$ and $B'(e_2)$, we obtain:

$$(1-\beta e_1-e_2)(1-ce_2) - (e_2-\frac{1}{2}ce_2^2) - X = 0$$

$\Rightarrow \left(e_2-\frac{1}{2}ce_2^2\right) + X = \left[1-e_2-(1-\beta)e_1^M\right](1-ce_2)$
$\Rightarrow e_2-\frac{1}{2}ce_2^2+X = 1-(1+c)e_2+ce_2^2-(1-\beta)e_1^M+c(1-\beta)e_1^Me_2$
Moving all terms to the right-hand side and organizing by powers of e_2, we obtain:

$$\frac{3}{2}c(e_2^M)^2 - \left[2+c-c(1-\beta)e_1^M\right]e_2^M + \left[1-X-(1-\beta)e_1^M\right] = 0$$

After some tedious but straightforward calculations, we get to:

$$e_2^M = \frac{\left[2+c-c(1-\beta)e_1^M\right] - \sqrt{\left[2+c-c(1-\beta)e_1^M\right]^2 - 6c\left[1-X-(1-\beta)e_1^M\right]}}{3c}$$

The non myopic regulator simultaneously chooses e_1 and e_2 at $t=1$ to maximize the following objective function:

$$\max_{e_1,e_2} \quad \left\{[1-F(e_1)]B(e_1) + [1-F(e_1)][1-F_2(e_1,e_2)]B(e_2)\delta - F(e_1)X(1+\delta) - [1-F(e_1)]F_2(e_1,e_2)X\delta\right\} \tag{3}$$

The optimization problem is solved by setting up the Lagrangian:

$$\mathcal{L} = WB^{NM}(e_1,e_2) + \mu_1 e_1 + \mu_2 e_2$$

Where μ_1 and μ_2 are the multipliers associated with the non negativity constraints $e_1 \geq 0$ and $e_2 \geq 0$. Focusing on the interior solutions e_1^{NM} and e_2^{NM} are found by solving the system of two simultaneous FOCs ($\frac{\partial \mathcal{L}}{\partial e_1} = 0$ and $\frac{\partial \mathcal{L}}{\partial e_2} = 0$ with $\mu_1 = \mu_2 = 0$):

FOC with respect to e_2 ($\frac{\partial \mathcal{L}}{\partial e_2} = 0$): This FOC is structurally identical to the myopic FOC for Period 2, but multiplied by δ and the survival probability of the first time period, $[1 - F(e_1)]$.

$$\frac{\partial WB^{NM}}{\partial e_2} = [1 - e_1] \frac{\partial}{\partial e_2} \left([1 - F_2(\cdot)]B(e_2) - F_2(\cdot)X \right) \delta = 0$$

Since $[1 - e_1]\delta > 0$, the equation simplifies to the same FOC found for e_2^M:

$$(1 - \beta e_1 - e_2)(1 - ce_2) - (e_2 - \frac{1}{2}ce_2^2) - X = 0$$

FOC with respect to e_1 ($\frac{\partial \mathcal{L}}{\partial e_1} = 0$): This FOC is the most complex, as it involves the direct effect of e_1 on $B(e_1)$ and X, and the indirect (learning) effect of e_1 on $F_2(\cdot)$ and $B(e_2)$ in the second time period.

$$\frac{\partial WB^{NM}}{\partial e_1} = \underbrace{[1 - e_1]B'(e_1) - B(e_1) - X(1 + \delta)}_{\text{Period 1: Direct Effects}} + \underbrace{\delta \frac{\partial}{\partial e_1} \left([1 - e_1][1 - F_2(\cdot)]B(e_2) - [1 - e_1]F_2(\cdot)X \right)}_{\text{Period 2: Intertemporal Effects}} = 0$$

Substituting all derivatives, this results in a system of the two simultaneous equations, that are cubic in e_1 and e_2 whose solution provides the optimal pair (e_1^{NM}, e_2^{NM}).

Optimal e_2^{NM} comes from:

$$\frac{3}{2}c(e_2^{NM})^2 - \left[2 + c - c(1 - \beta)e_1^{NM}\right] e_2^{NM} + \left[1 - X - (1 - \beta)e_1^{NM}\right] = 0$$

While, optimal e_1^{NM} from:

$$1 - (2 + c)e_1^{NM} + \frac{3}{2}c(e_1^{NM})^2 - X(1 + \delta) - (1 - \beta)\left(e_2^{NM} - \frac{1}{2}c(e_2^{NM})^2\right)\delta + \beta X\delta = 0$$

So, we can state that, as in the linear case $e_1^M > e_1^{NM}$. Moreover, $e_1^{NM} > e_2^{NM}$, if the future is heavily discounted (low δ) or the learning effect β is sufficiently small, while, $e_1^{NM} < e_2^{NM}$ if the social loss of the catastrophe X is large enough. These results are equals to respect the linear benefits function setting.

Thus, we can observe that the introduction of concave benefits strengthens the incentive to limit emissions, as the gains from higher e_i are lower. Consequently, *ceteris paribus* , the optimal emission levels e_i^* will be lower compared to the linear case. However, the qualitative findings remain unchanged: the myopic regulator still exhibits procrastination, tending to emit more initially and then reducing emissions sharply, whereas the forward looking regulator smooths emissions over time.

Focusing on the interior optimal solutions, e_1^M, e_2^M, e_1^{NM}, and e_2^{NM}, are as follows:

- **Myopic Solutions (e_1^M, e_2^M):**

  $e_1^M = \frac{(2+c)-\sqrt{c^2-2c+4+6cX}}{3c}$

  $e_2^M = \frac{\left[2+c-c(1-\beta)e_1^M\right]-\sqrt{\left[2+c-c(1-\beta)e_1^M\right]^2-6c\left[1-X-(1-\beta)e_1^M\right]}}{3c}$

- **Non Myopic Solutions (e_1^{NM}, e_2^{NM}):**

  $e_1^{NM} = \frac{(2+c)-\sqrt{(2+c)^2-6c\left[1-X(1+\delta)-(1-\beta)\left(e_2^{NM}-\frac{1}{2}c(e_2^{NM})^2\right)\delta+\beta X\delta\right]}}{3c}$

  $e_2^{NM} = \frac{\left[2+c-c(1-\beta)e_1^{NM}\right]-\sqrt{\left[2+c-c(1-\beta)e_1^{NM}\right]^2-6c\left[1-X-(1-\beta)e_1^{NM}\right]}}{3c}$.

The comparative statics regarding β (increasing e_2 relative to e_1) and δ (reducing e_1 due to future damage anticipation) are preserved, confirming that $e_1^M > e_1^{NM}$ and $e_2^M < e_2^{NM}$ for sufficiently large social loss X.

3 General Form of the Equilibrium

Generalizing the solution of our two-periods base model to a case where the degree of foresightedness of the regulator is continuous, we obtain the following objective function for the general form:

$$\max_{e_1}\left\{ [\ 1 - F(e_1)][B(e_1) + \gamma \max_{e_2}\left\{ [1 - F(e_1, e_2)]B(e_2) + F(e_1, e_2)[-X] \right\}] + F(e_1)[-(1 + \delta)X] \right\} \quad (4)$$

Where γ represents the regulator's degree of foresight. The benchmark cases are defined by the specific values of γ:

$$\begin{cases} \gamma = 0 & Fully \quad Myopic \quad Regulator \\ 0 < \gamma < 1 & Limited \quad Non-Myopic \quad Regulator \\ \gamma = 1 & Fully \quad Non-Myopic \quad Regulator \end{cases} \tag{5}$$

Thus, the parameter γ allows us to continuously transition from a policy maker who completely ignores the future ($\gamma = 0$) to one who weighs the future equally to the present ($\gamma = 1$). Notice that we focus our attention on the decision taken in the first period. Any optimum of $\max_{e_1} \{g(e_1, \gamma)\}$, where $g(.)$ is our objective function, has a solution $e_1 \in E_1(\gamma)$. Thus, it is possible to use the envelope theorem because it holds at any differentiable point of the objective function (see, among others, Milgrom and Segal, 2002, Milgrom and Shannon, 1994).

Hence, the objective function (4) is a general form of the benchmark cases, allowing us to derive the optimum in a generalized manner.

Following Theorem 1 of Milgrom and Segal (2002), we narrow our analysis to the relevant parameters. We know that $\Theta(\gamma) = \sup_{e_1 \in E_1} g(e_1, \gamma)$. Given $E_1^*(\gamma) = \{e_1 \in E_1 : g(e_1, \gamma) = \Theta(\gamma)\}$, the derivative of the objective function with respect to the parameter is the optimum, $g'(e_1^*, \gamma)$. This generalization of previous results is useful because, following Theorem 2 of Milgrom and Segal (2002)[1], we observe that an optimum exists for any selection $e_1^*(\gamma) \in E_1^*(\gamma)$.

Since $g(e_1, \cdot)$ is continuous $\forall\ e_1 \in E_1$, there exists an integrable function $b : [0, 1] \to \Re_+$ such that $|g_\gamma(e_1, \gamma)| < b(\gamma) \quad \forall\ e_1 \in E_1$ and almost all $\gamma \in [0, 1]$. Therefore, Θ is continuous, and $e_1^*(\gamma) \in E_1^*(\gamma)$.

$$\Theta(\gamma) = \Theta(0) + \int_0^\gamma g'(e_1^*(s), s) ds \tag{6}$$

Thus, we obtain that $e_1^*(\gamma) \in E_1^*(\gamma)$ is the choice rule influenced by the degree of myopia γ. In other words, $e_1^*(\gamma)$ is the generalized form of e_1^M and e_1^{NM}[2]. In detail, this is possible on account of the fact that the function $g(e_1, \gamma)$ has *increasing differences* in (e_1, γ), because $g(e_1^*, \gamma > 0) - g(e_1, \gamma > 0) \geq g(e_1^*, \gamma = 0) - g(e_1, \gamma = 0)$, where $e_1 \in E_1$ and the set of γ.

Since, $g(e_1, \gamma)$ is twice continuously differentiable in (e_1, γ), and convex, it has increasing differences in (e_1, γ), thanks to the following:

$$\frac{\partial^2 g(e_1, \gamma)}{\partial e_1 \partial \gamma} \geq 0 \tag{7}$$

for all $e_1 \in E_1$ and $\gamma \in (0, 1)$.

Thus, the structure of the random variable and the property of increasing differences allow us to rearrange the optimum of (4) as (8):

$$e_1^*(\gamma) = \frac{[1 - F(e_1)]}{f(e_1)} - (1 + \delta)X - \gamma \tag{8}$$

Hence, the structure of the random variable and the property of increasing differences allow us to rearrange the optimum of (4) as (8). Normalizing ($\bar{e} = 1$), it is possible to rewrite this as $e_1^*(\gamma) = [1 - F(e_1)] - (1 + \delta)X - \gamma$. Thus, this is a generalized form of e_1^{NM} which, if $\gamma = 0$ becomes e_1^M[3]. This form of equilibrium is useful for a better interpretation of results, observing that the first element $\frac{[1-F(e_1)]}{f(e_1)}$ is the hazard. In other words, the closer the regulator is to the edge, the higher her benefits are; on the other side of the coin, going so close to the edge entails a cost. This cost is given by the possible loss from falling down from the edge and has been taken into account as $-(1 + \delta)X$. Moreover, the possibility of learning something regarding the real value of the threshold remains, and the more forward looking she is, the less she emits at present, preserving benefits for the future, which represents the opportunity for *future hazard* (i.e., related to the second period) $\frac{[1-F(e_2)]}{f(e_2)}$. In other words, γ becomes, in equilibrium,

[1] Our model specifically meets the necessary conditions of the cited Theorem 2, page 586.

[2] This is so because $\Theta(\gamma)$ follows all necessary and sufficient assumptions for implementing the choice rule $e_1^*(\gamma)$ that satisfies (6). Milgrom and Segal (2002) refers to (6) as an *integral condition* (Corollary 1, page 589).

[3] This is already clear even without this generalization. If we observe $e_1^{NM} = \frac{2(1-X)+(1+X)(1-\beta)\delta}{4-(1-\beta)^2\delta}$, in the case of a myopic regulator, δ is an irrelevant parameter, so it is treated as zero. Thus $e_1^{NM} = \frac{2(1-X)}{4} = \frac{(1-X)}{2} = e_2^M$.

the present prudence and an investment in knowledge about the real locus of the edge.

Mathematically, this occurs considering (4) once the maximization problem of the second time period is solved. This problem is given by: $\max_{e_2} \{[1-F(e_1,e_2)]B(e_2)+F(e_1,e_2)[-X]\}$, hence: $e_2^* = \frac{[1-F(e_1,e_2)]}{f(e_1,e_2)} - X$. Thus, it is possible to provide the optimum level of emissions as these are affected by the myopia γ. So, from $\max_{e_1(\gamma)} \left\{ [1-F(e_1)][B(e_1)+\gamma\left[\frac{[1-F(e_1,e_2)]}{f(e_1,e_2)} - X\right]] + F(e_1)[-(1+\delta)X] \right\}$ we get $e_1^*(\gamma) = \frac{[1-F(e_1)]}{f(e_1)} - (1+\delta)X + \gamma\left[\frac{[1-F(e_1)]f'(e_1,e_2)-f(e_1,e_2)^2}{f(e_1,e_2)^2}\right]$, where $\gamma\left[\frac{[1-F(e_1)]f'(e_1,e_2)-f(e_1,e_2)^2}{f(e_1,e_2)^2}\right]$ is $-\gamma$.

In fact, $f'(e_1,e_2)$ is zero because the probability that an infinitesimal additional unit of emission causes a natural disaster is zero. So, we obtain: $-\frac{f(e_1,e_2)}{f(e_1,e_2)}$, that is -1.

Hence, $\frac{[1-F(e_1)]f'(e_1,e_2)-f(e_1,e_2)^2}{f(e_1,e_2)^2}$ is a random variable with memory, also entailing free lunch for the second time period, given by dangerous learning as follows: $\Gamma_t(e_t,\beta) = \frac{[1-F(e_1)]f'(e_1,e_2)-f(e_1,e_2)^2}{f(e_1,e_2)^2}$. At the optimum, this takes negative value connected to the grade of forward looking. Rearranging, the optimum of (4) is given by (8).

Focusing on welfare, the general form of the interior solution provides a clear understanding of the dynamic depending on the grade of myopia γ. In fact, $WB[e_1,e_2] = [1-F(e_1)]e_1 + [1-F(e_1)][1-F(e_2)]e_2\delta - F(e_1)[X(1+\delta)] - [1-F(e_1)]F(e_2)X\delta$, in general form, is given by:

$$WB[e_1^*(\gamma)] = \frac{1}{2}\gamma(1-\gamma) - \frac{\beta}{2}\gamma(e_2+X)\delta + \left[[1-e_2+\frac{\beta}{2}[1-(1+\delta)X]]e_2 - [e_2-\frac{\beta}{2}[1-(1+\delta)X]]X\right]\delta \quad (9)$$

In fact, $WB[e_1,e_2] = [1-F(e_1)]e_1 + [1-F(e_1)][1-F(e_2)]e_2\delta - F(e_1)[X(1+\delta)] - [1-F(e_1)]F(e_2)X\delta$ is generalizable starting from $e_1^*(\gamma) = \frac{[1-F(e_1)]}{f(e_1)} - (1+\delta)X - \gamma$. Hence, we have $e_1 = \frac{1}{2}[1-(1+\delta)X-\gamma]$. Thus, we can rearrange $WB[e_1,e_2]$ as: $\frac{1}{2}[1-F(e_1)][1-(1+\delta)X-\gamma] + [1-F(e_1)][1-F(e_2)]e_2\delta - F(e_1)(1+\delta)X - [1-F(e_1)]F(e_2)X\delta \quad \Rightarrow$
$\frac{1}{2}(1-e_1)[1-(1-\delta)X-\gamma] + (1-e_2-\beta e_1)e_2\delta - e_1(1+\delta)X - (e_2-\beta e_1)X\delta$
$\frac{1}{2}[1-1+(1+\delta)X+\gamma][1-(1+\delta)X-\gamma] + [1-e_2+\frac{\beta}{2}[1-(1+\delta)X-\gamma]]e_2\delta - \frac{1}{2}[1-(1+\delta)X-\gamma](1+\delta)X - [e_2-\frac{\beta}{2}[1-(1+\delta)X-\gamma]]X\delta$ leading to the (9) and, since $\frac{1-X}{2}$ is the optimal initial emission level for the myopic regulator e_1^M, we can rearrange (9) as $\frac{1}{2}\gamma(1-\gamma) - \frac{\beta}{2}\gamma(e_2+X)\delta + \left[[1-e_2+\beta e_1^M(1+\delta)]e_2 - [e_2-\beta e_1^M(1+\delta)]X\right]\delta$, this clearly shows null welfare with respect to the benefits and probabilities generated during the first time period. This feature is due to the fact that, under expected utility theory, emissions are set as close as possible to the edge, to obtain information on where the latter is, (element $\frac{\bar{e}-X}{2}$). Thus, in $WB[e_1^*(\gamma)]$ emissions are set according to the risk of experiencing the catastrophe, but are never equal to the threshold. The first time period benefit remains relative to the dangerous learning in a positive way, $\beta e_1^M(1+\delta)e_2$, as a prize for taking risk because it provides more information regarding the real wherebouts of the disaster threshold. It also appears with a negative term, $\beta e_1^M(1+\delta)X$, as a crude probability of suffering X for two time periods, i.e., $(1+\delta)X$.

$WB[e_1^*(\gamma)]$ is also negatively connected with the degree of myopia γ and, jointly, with the dangerous learning β. In fact, there is the negative term $\frac{\beta}{2}\gamma(e_2+X)$. Thus, in the case of high value of β and low level of disaster severity X, welfare is equal. Moreover, for any value of X, if $\beta = 1$ welfare for the myopic and non myopic regulator is equal, as it is clear, graphically, through Figure 4.

Further, the marginal effect of the degree of far-sightedness on welfare $\frac{\partial WB[e_1^*(\gamma)]}{\partial\gamma} : 1-\gamma-\beta(e_2+X)\delta$, is negative if $\gamma > 1-\beta(e_2+X)\delta$, or if the dangerous learning is sufficiently high: $\beta > \frac{1-\gamma}{(e_2+X)\delta}$.

These findings are interpreted here as an assessment of the over caution (or fear) regarding disaster probability, which decreases welfare. This can be likened to the difference between driving a car while respecting the speed limit versus not driving at all. Furthermore, the welfare analysis also highlights the role of dangerous learning β.

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