

“The formulas for the synthetic periodic regression analysis with 24-hour and 12-hour cycles”

1. In periodic regression analysis, the following periodic regression model is assumed:

$$\text{周期回帰モデル: } y = A_0 + \sum_{k=1}^m A_k \cos(\omega_k t - \theta_k) + \sum_{k=1}^m A_k \cos(\omega_k t - \theta_k)$$

A_0 : 定数、メサール(MESOR) A_k : 振幅(amplitude) $\omega_k = \frac{360}{T} k$: 角周波数(angular frequency)
 t : 時間 T : 基本周期(fundamental period) $\frac{T}{k}$: 周期成分 k の周期(period)
 θ_k : 位相(acrophase) $\frac{\theta_k}{\omega_k}$: 位相時間 $m \leq (\text{データ数})/2$: 周期成分数 ϵ : 回帰誤差

※周期回帰モデル: Periodic regression model、定数: constant term、時間: time、位相時間: phase time、周期成分数: number of periodic components、回帰誤差: regression error

2. The actual calculations

Regarding the assumed periodic regression model, it is calculated as a multiple regression model as follows.

$$\beta_0 = \frac{a_0}{2} \quad \beta_1 = a_1 \quad \beta_2 = b_1 \quad \cdots \quad \beta_{2k-1} = a_k \quad \beta_{2k} = b_k \quad \cdots \quad \beta_{2m-1} = a_m \quad \beta_{2m} = b_m$$

$$x_0 = 1$$

$$x_1 = \cos(\omega_1 t) = \cos\left(\frac{2\pi}{T} t\right) \quad x_2 = \sin(\omega_1 t) = \sin\left(\frac{2\pi}{T} t\right)$$

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$$x_{2k-1} = \cos(\omega_k t) = \cos\left(\frac{2\pi k}{T} t\right) \quad x_{2k} = \sin(\omega_k t) = \sin\left(\frac{2\pi k}{T} t\right)$$

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$$x_{2m-1} = \cos(\omega_m t) = \cos\left(\frac{2\pi m}{T} t\right) \quad x_{2m} = \sin(\omega_m t) = \sin\left(\frac{2\pi m}{T} t\right)$$

$$y = X\beta + \epsilon = \hat{y} + \epsilon$$

$$y = \begin{bmatrix} y_1 \\ \vdots \\ y_j \\ \vdots \\ y_n \end{bmatrix} \quad X = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1(2k-1)} & x_{1(2k)} & \cdots & x_{1(2m)} \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 1 & x_{j1} & \cdots & x_{j(2k-1)} & x_{j(2k)} & \cdots & x_{j(2m)} \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 1 & x_{n1} & \cdots & x_{n(2k-1)} & x_{n(2k)} & \cdots & x_{n(2m)} \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{2k-1} \\ \beta_{2k} \\ \vdots \\ \beta_{2m} \end{bmatrix} \quad \epsilon = \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_j \\ \vdots \\ \epsilon_n \end{bmatrix}$$

The least squares solution and various parameters are calculated using the least squares method as follows.

$$\hat{\beta} [X'X]^{-1} X'y$$

$$A_0 = \frac{a_0}{2} = \hat{\beta}_0 \quad a_k = \hat{\beta}_{2k-1} \quad b_k = \hat{\beta}_{2k} \quad A_k = \sqrt{a_k^2 + b_k^2} \quad \theta_k^* = \tan^{-1} \left(\frac{b_k}{a_k} \right)$$