

Supplementary information

S1 Multiplex measures on randomized hypergraphs

In this section, we investigate how our proposed multiplex measures for higher-order networks behave under re-wiring. This analysis helps in understanding the robustness of the observed network properties and the significance of the original structure.

We propose two different randomization methods. They are available as part of Hypergraphx [59]. Results have been averaged over ten realizations of the random models.

For this analysis we employ:

- A *configuration model* for hypergraphs [66] which preserves the degree sequences and the hyperedge order distributions of the original layers of the multiplex hypergraphs while randomizing the actual nodes involved in the hyperedges.
- A *random model* for hypergraphs which preserves only the hyperedge order distributions of the original layers of the multiplex hypergraphs while randomizing the actual nodes involved in the hyperedges.

By preserving the degree sequence in each layer, the configuration model maintains the node activity and the degree correlations across layers. Additionally, it preserves the distribution of overlapping degrees across different layers, and consequently, the node participation coefficient. However, by re-wiring interactions and thus randomizing the neighbors in each interaction, we disrupt the micro and mesoscale structure of the hypergraphs. To assess this, in Fig. S1 we show the impact of the configuration model randomization process on the measure of hyperedge overlap in our empirical datasets. We notice a consistent drop in the number of overlapping hyperedges with respect to the original hypergraphs. In particular, after randomization, there are no hyperedges that appear in more than one layer in the case of movie collaborations.

Additionally, in Fig. S2 we show that the correlations between community structures are not preserved, as nodes tend to lose their preferential patterns of interactions under randomization. In particular, in HIGH SCHOOL the algorithm collapses every node into a single community in every layer since the networks lack clear separations.

Our second random model is more disruptive, as it disintegrates structures to a greater extent. While this model preserves the hyperedge size distributions for each layer of a multiplex hypergraph, it randomizes the nodes involved in the interactions. This process is sufficient to dismantle node activity patterns and disrupt node degree correlations. Since node degrees are not preserved, node and hyperedge participation coefficients are also not maintained. It is important to notice that since all layers of a multiplex hypergraph share the same node set, the only property of a layer that is preserved is layer density. The rewiring of interactions allows more nodes to become active in new layers. This is shown in Fig. S3, in which we analyze nodes' simultaneous activity across multiple layers and see an increase in such statistics with respect to original data.

The disruption of activity patterns is evident when computing the similarity among activity matrices between layers (Fig. S4). In fact, the cosine similarity of the activity

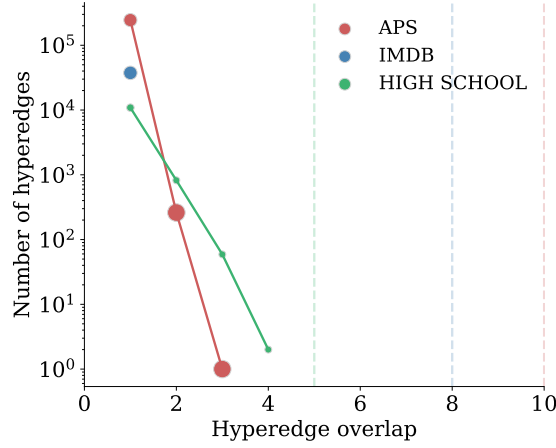


Fig. S1 Impact of configuration model randomizations on hyperedge overlap. We notice a consistent decrease in overlapping hyperedges compared to the original data, with no hyperedges appearing in more than one layer for movie collaborations after randomization.

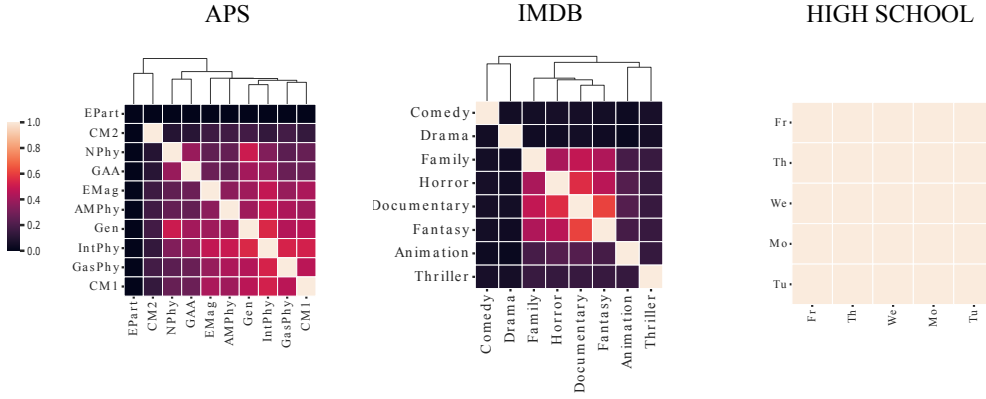


Fig. S2 Configuration model randomization leads to the loss of preferential interaction patterns, with nodes losing correlated community affiliations across layers.

matrices drops to zero for both APS and IMDB, while HIGH SCHOOL maintains a generally high level of similarity (explained by the abundance of interactions of size two and the high level of activity across layers). The disruption of activity patterns in APS and IMDB is caused by the rewiring of hyperedges with large cardinality, especially when these large hyperedges overlap in real-world data.

Finally, it is important to examine the degree correlations across layers and the participation coefficient of nodes. The rewiring process significantly affects degree correlations, resulting in all pairs of layers being uncorrelated in terms of node degrees (Fig. S5a). In Fig. S5b we show that hubs tend to disappear (lower overlapping degrees

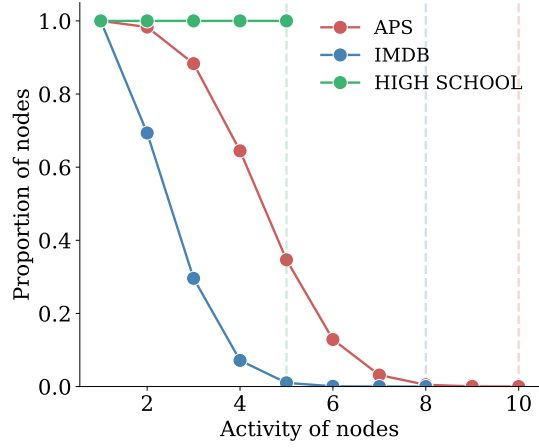


Fig. S3 Rewiring preserves layer density but increases node activity across multiple layers. This is shown by the increase in nodes' simultaneous activity across layers compared to the original data.

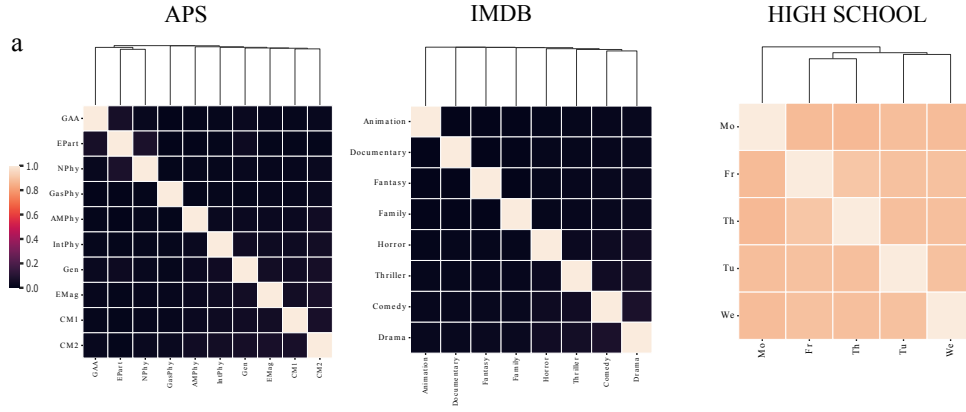


Fig. S4 Similarity among activity matrices between layers drops to zero for APS and IMDB but remains high for HIGH SCHOOL. This is due to the rewiring of large cardinality hyperedges, particularly when these hyperedges overlap in real-world data.

than original data), average interaction size tends to decrease (due to overabundance of low order interaction in the data) and participation coefficient tends to increase (given by the increase in activity of nodes).

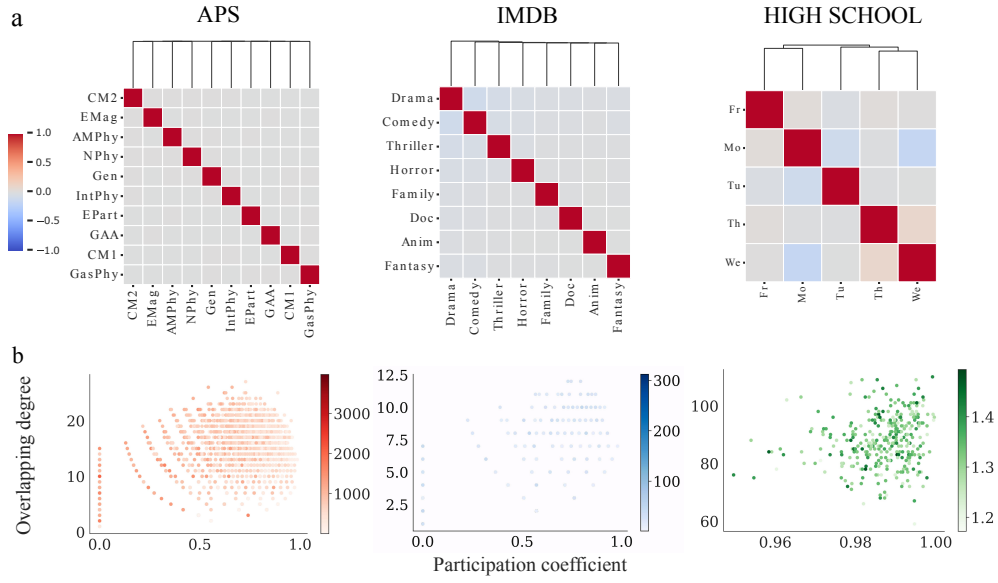


Fig. S5 Effects of rewiring on multiplex hypergraphs properties. (a) Rewiring results in uncorrelated node degrees across all layer pairs. (b) Hubs disappear (lower overlapping degrees), average interaction size decreases, and participation coefficient increases due to higher node activity.