Supplementary Information for

"Quantum support vector machines for classification and regression on a trapped-ion quantum computer"

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Supplementary Note 1: Comparison between the IonQ Harmony and Aria quantum processors

Table S1: Quality of the kernel matrix obtained from the IonQ Harmony and Aria quantum processors (4 qubits)

Fig. S1: Comparison in the quantum kernel between the IonQ Harmony and the IonQ Aria (4 qubits).

Fig. S2: The comparison in the quantum kernel between the quantum experiment on the IonQ Aria using 8 qubits and the noiseless simulation.

The financial dataset used for QSVR

Table S2: Financial dataset used for QSVR (training data)

Table S3: Financial dataset used for QSVR (test data)

Supplementary Note 1: Comparison between the IonQ Harmony and Aria quantum processors

In this section, we report our quantum experiments obtained using two trapped-ion quantum processors released from IonQ, called Harmony and Aria. The Fashion-MNIST dataset was used for the QSVC task in our quantum experiments. There is a significant improvement in terms of qubit gate error rates for the IonQ Aria. In particular, the two-qubit gate error rate is 0.4%, which is a significant improvement from that of the IonQ Harmony (2.7%).

In the first step, we evaluated the quantum kernel estimation derived from the two quantum processors (Table S1). Upon transitioning from the IonQ Harmony to the IonQ Aria, we observed an improvement in the alignment, an increase from 0.993 to 0.998. Consistently, the Frobenius norm between the noiseless kernel and the noisy kernel $\|K - K^{\mathrm{noise}}\|_F$ was significantly reduced.

Table S1: Quality of the kernel matrix obtained from the IonQ Harmony and Aria quantum processors (4 qubits)

	Alignment $A(K, K^{\mathrm{noise}})$	$\|K - K^{\mathrm{noise}}\|_F$
IonQ Harmony	0.993	2.436
IonQ Aria	0.998	1.446

Next, we took a closer look at the distribution of the deviation in the kernel entry $\Delta K_{ij} = K_{ij}^{\mathrm{noise}} - K_{ij}$ (Fig. S1). About 80% of the kernel matrix elements obtained from the near-term devices were underestimated in comparison with those obtained from the noiseless kernel, owing to the loss of information caused by noise and errors. On the other hand, less than 20% of the kernel matrix elements were somewhat overestimated. The average deviation ΔK_{ij} in the case of the IonQ Harmony was -0.091 whereas that in the case of the IonQ Aria was -0.051, indicating that the use of the IonQ Aria leads to a better quality of the quantum kernel. The deviation ΔK_{ij} obtained from the IonQ Harmony was more broadly distributed. On the other hand, the extent of the underestimation of the kernel entry was significantly reduced when using the IonQ Aria (part c in Fig. S1). Thus, this reduced underestimation played an important role in improving the quality of the quantum kernel. All the results suggest that the IonQ Aria can provide better results in terms of the quality of the quantum kernel matrix.

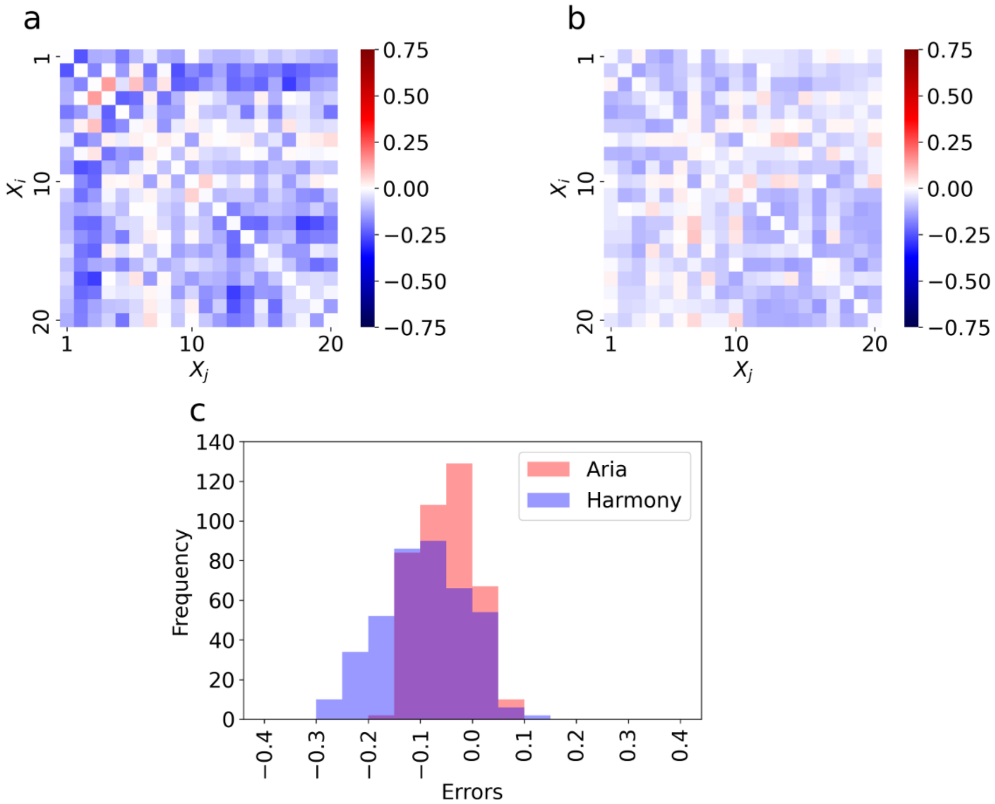

Fig. S1: Comparison in the quantum kernel between the IonQ Harmony and the IonQ Aria (4 qubits). (a) Deviation ΔK_{ij} obtained using the IonQ Harmony. (b) Deviation ΔK_{ij} obtained using the IonQ Aria. (c) The histogram for the deviation ΔK_{ij} for the IonQ Harmony (indicated by blue) and for the IonQ Aria (indicated by red). The Fashion-MNIST dataset was used; 4 qubits were used for both quantum processors.

Motivated by our preliminary results, we further performed our quantum experiment on the IonQ Aria. In this case, we constructed our QSVC models using 8 qubits. The Fashion-MNIST dataset was used for the QSVC task. In our quantum experiment, the alignment between the noiseless kernel and the noisy kernel was 0.977 for the training data, suggesting that the noisy quantum kernel maintained the essential information of the Gram matrix. Our QSVM model achieved 100% accuracy for the test data, without the use of any error mitigation technique (the number of test data was 20). The alignment between the noiseless kernel and the noisy kernel was 0.996, and the average deviation ΔK_{ij} was -0.092 (Fig. S2). The results indicate the usefulness of our quantum circuit on the real device with 8 qubits.

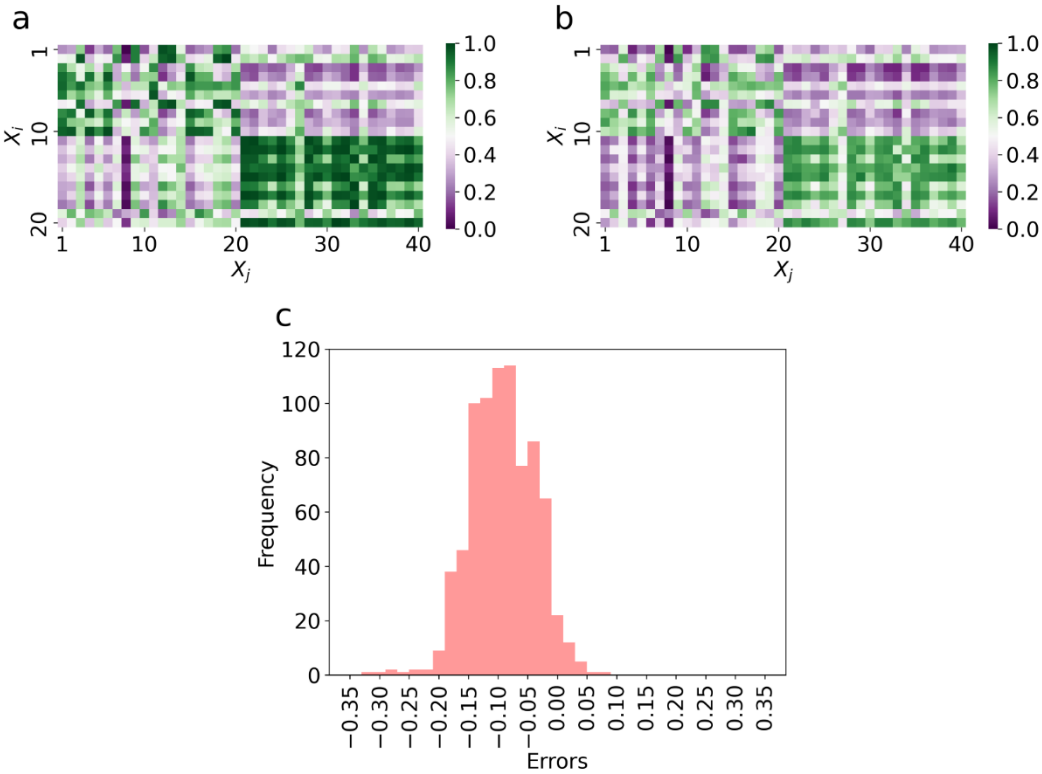

Fig. S2: The comparison in the quantum kernel between the quantum experiment on the IonQ Aria using 8 qubits and the noiseless simulation. (a) Train-test quantum kernel for the noiseless simulation. (b) Train-test quantum kernel for the quantum experiment. (c) Histogram for the deviation ΔK_{ij}. We used the Fashion-MNIST dataset (40 data points for training and 20 data points for testing). The alignment of the noisy quantum kernel with the noiseless quantum kernel was 0.996, and the average deviation ΔK_{ij} was -0.092. Using the noisy quantum kernel, our QSVM model achieved 100% accuracy for the test data.

The financial dataset used for QSVR

Table S2 Financial dataset used for QSVR (training data)

Number	SSE Index	WTI Crude Oil	US Dollar Index	UK Nickel
1	3676.59	68.35	92.514	19506
2	3536.29	74.83	94.353	18342
3	3581.73	69.29	92.033	19789
4	3272.00	96.35	106.804	21613
5	3547.34	83.57	94.121	19448
6	3228.06	97.59	107.911	19802
7	3607.43	66.26	96.114	20030
8	3387.64	108.43	104.909	21824
9	3067.76	108.26	103.802	30114
10	3253.69	112.12	98.499	31583
11	3662.60	70.46	92.614	19623
12	3489.15	92.10	96.188	24396
13	3047.06	104.69	103.230	31771
14	3567.10	68.59	92.449	19332
15	3693.13	68.14	92.48	20190
16	3579.54	78.90	95.722	20734
17	3562.70	69.95	96.338	20141
18	3186.27	94.42	106.104	22492
19	3429.58	91.32	95.392	23398
20	3607.09	72.61	92.919	19401
21	3518.42	82.81	93.326	19574
22	3613.97	71.97	93.176	19358
23	3609.86	83.76	93.809	20306
24	3593.15	82.96	93.728	20050
25	3275.76	97.26	106.331	21700
26	3409.21	111.76	104.261	23158
27	3555.26	82.12	94.775	22176
28	3597.43	82.64	94.901	22064
29	3284.83	120.67	104.151	27264
30	3613.07	73.98	93.335	19387
31	3625.13	71.12	96.489	19615
32	3558.28	81.31	93.961	19294
33	3189.04	88.54	105.566	22210
34	3167.13	94.29	99.924	32483
35	3516.30	68.44	92.512	19656
36	3582.08	76.75	96.544	20336
37	3524.74	69.09	93.038	19678
38	3522.16	68.74	92.694	19011
39	3576.89	65.57	96.024	19946
40	3637.57	72.36	95.881	20230

Table S3 Financial dataset used for QSVR (test data)

Number	SSE Index	WTI Crude Oil	US Dollar Index	UK Nickel
1	3561.76	80.44	94.080	18918
2	3582.60	83.76	93.625	19739
3	3569.91	85.43	95.723	22073
4	3477.13	65.64	92.971	18893
5	2928.51	98.54	101.769	32636
6	3675.19	69.30	92.650	19713
7	3186.43	114.67	101.766	28392
8	3498.54	80.86	93.853	19162
9	3642.22	73.30	93.03	19351
10	3146.86	110.29	102.097	27732
11	3054.99	106.13	104.897	27810
12	3398.62	105.76	104.464	22698
13	3238.95	121.51	103.218	28023
14	3546.94	80.64	94.519	18978
15	3225.64	106.95	100.326	33175
16	3675.02	72.38	96.015	19624
17	3582.83	75.45	93.381	18946
18	3462.31	95.72	96.694	24282
19	3130.24	115.07	101.698	28284
20	3320.15	104.27	104.192	24038
21	3004.14	103.09	103.689	28185
22	3271.03	114.93	98.615	32380
23	3446.98	66.59	93.145	19193
24	3282.72	99.27	98.627	33223
25	3186.82	104.25	99.913	32981
26	3715.37	70.45	92.652	19726
27	3267.2	106.19	103.981	24449
28	3573.84	66.50	96.158	19953
29	3451.41	91.59	96.619	24361
30	3465.83	93.66	95.697	23406
31	3628.49	72.23	93.460	19221
32	3586.08	79.46	96.326	20383
33	3544.48	84.05	93.879	19702
34	3263.79	122.11	102.546	28855
35	3266.60	107.82	97.875	32893
36	3656.22	72.61	92.534	20016
37	3567.44	81.22	95.625	21794
38	3595.09	72.05	96.370	20189
39	3313.58	104.09	107.829	21880
40	3214.50	105.96	99.055	32725