Frequentist conditional variance for nonlinear mixed-effects models

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Supplementary Information

Stock Assessment Example

This example is based on the data and assessment model used for the American plaice (*Hippoglossoides platessoides*) fish stock in the Northwest Atlantic Fisheries Organization (NAFO; www.nafo.int) Subdivision 3Ps off the south coast of Newfoundland and Labrador, Canada. The data and the most recent assessment results are available in Morgan et al. (2020). A Bayesian state-space surplus production model (SPM) is used (e.g. Meyer and Millar, 1999); however, we replicate the assessment using only one prior which we treat as a known RE distribution (see below).

The Schaefer form of the American plaice SPM gives latent total stock biomass in year t (i.e. $B_t \geq 0$) as a function of the biomass in the previous year plus production (births+growth−natural deaths) minus the fishery catch (C_t, tonnes), with production modelled as a quadratic function of

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biomass; that is,

$$B_t = B_{t-1} + rB_{t-1}(1 - B_{t-1}/K) - C_{t-1}. \tag{1}$$

This is the latent process in the state-space model. If the catch is a constant harvest rate H times the biomass then $C_t = HB_t$ and in this case it is well know that the value of H that maximizes equilibrium catch (i.e. maximum sustainable yield, MSY) is $H_{msy} = r/2 \geq 0$ and the resulting equilibrium biomass from harvesting at this level is $B_{msy} = K/2 \geq 0$.

Similar to Morgan et al. (2020), our state-space SPM includes multiplicative lognormal process errors; however, different from Morgan et al. (2020), we assume that there is measurement error in catches and include a catch model, $C_t = H_t B_t$, where $H_t \geq 0$ is also based on a random walk stochastic process:

$$\begin{aligned} \ln(B_t) &= \ln\{B_{t-1} + rB_{t-1}(1 - B_{t-1}/K) - H_{t-1}B_{t-1}\} + \delta_{Bt}, \\ \ln(H_t) &= \ln(H_{t-1}) + \delta_{Ht}, \end{aligned} \tag{2}$$

where $t = 1, ..., T$, $\delta_{H1}, ..., \delta_{HT} \overset{iid}{\sim} N(0, \sigma^2_{\delta H})$, and $\delta_{B1}, ..., \delta_{BT} \overset{iid}{\sim} N(0, \sigma^2_{\delta B})$. We also assume that the initial biomass is random,

$$\ln(B_0/K) \sim N\left\{\ln(0.75), 0.125^2\right\},$$

which is broadly similar to the prior for B_0 in Morgan et al. (2020). SPM parameters are often poorly identified unless there is good contrast in the data, with periods of high catches and stock declines, and periods of low catches and stock increases. The latter condition is not common. This is why informative prior information is often used when fitting SPM's (e.g. McAllister et al., 2001). However, unlike Morgan et al. (2020), we do not use any other prior distributions. The parameters to estimate for the latent process part of the state-space model are $H_0, r, K, \sigma_{\delta H}$, and $\sigma_{\delta B}$.

The data available are annual estimates of total fishery catches of American plaice in Subdiv. 3Ps during 1960-2019, derived from various monitoring and surveillance programs (see Table 2 and Fig. 1 in Morgan et al., 2020). These authors treat the catch estimates as covariates in their model similar to Meyer and Millar (1999) and Winker et al. (2018), which implicitly assumes that measurement error in catches are negligible. This is a common assumption in stock assessment models; however, similar to Pedersen and Berg (2017), we assume that there are measurement errors in catches which are described below. Another common data source is a time-series of average catch from research surveys. These average catches are commonly referred to as stock size indices because trends in average survey catches should reflect trends in the stock size. The American plaice assessment uses indices derived from stratified random surveys conducted in the stock area since 1980. The state-space model observation equations for the time-series of survey indices

(I) and the catch observations (C_{ot}) are:

$$\begin{aligned} \ln(C_{ot}) &= \ln(H_t) + \ln(B_t) + \epsilon_{Ct}, \\ \ln(I_t) &= \begin{cases} \ln(q_E) + \ln(B_t) + \epsilon_{It} & \text{if year } \leq 1995 \\ \ln(q_C) + \ln(B_t) + \epsilon_{It} & \text{if year } > 1995. \end{cases} \end{aligned} \tag{3}$$

SPM times $t = 0, ..., T$ correspond to years 1960-2019. Both q_E and q_C are survey catchability parameters to estimate. These are different because there was a major change in survey gears and stratification in Subdiv. 3Ps since the 1996 survey. The $\epsilon_{It} \overset{iid}{\sim} N(0, \sigma^2_{\epsilon I})$ and $\epsilon_{Ct} \overset{iid}{\sim} N(0, \sigma^2_{\epsilon C})$; however, we assume that $\sigma_{\epsilon C} = 0.1$ and do not estimate this parameter. The total set of parameters to estimate for the state-space model are $H_0, r, K, \sigma_{\delta H}, \sigma_{\delta B}, q_E, q_C$, and $\sigma_{\epsilon I}$, along with RE predictions for B_t and $H_t, t = 1961, ..., 2019$ and B_0 for 1960.

Our SPM parameter estimates and stock assessment results (Table 2) are broadly similar to Morgan et al. (2020). For example, we estimate that B_{msy} is 55 128 tonnes whereas Morgan et al. (2020) estimate 70 290 tonnes. This difference is not that large because the coefficients of variation (CV), which are commonly reported for stock assessments, are fairly high for both CVr and Morgan et al. (2020). Note that they only report 95% credible intervals and not CV's, but the value inferred from 25% of the log-difference of the percentiles is 0.43 which is similar to the CVr value in Table 2. The CVf's are usually substantially smaller than CVr's, but because of the conditional bias in parameter estimates and RE predictions, the CVr's provide more reliable confidence intervals. This is what we found from the simulations in Section 2.4 and this will also be the case for SPM simulation results we provide later in this section. Confidence intervals derived from the CVr's are shown in Fig. 1 for American plaice biomass and harvest rates. However, a primary focus of stock assessment is to estimate stock size and harvest rates relative to management reference points (i.e. stock status). There are limit reference points in place for 3Ps American plaice that management should avoid with high probability, which are H_{msy} for harvest rates and $0.4 \times B_{msy}$ for biomass. Estimation and inference for biomass and harvest rates relative to limit reference points are used as the basis to determine the status of the stock. Projections of stock status are used directly by managers to decide future harvest levels. Our SPM stock status estimates (Fig. 2) are very similar to those in Figs. 12 and 13 in Morgan et al. (2020), but our confidence intervals are somewhat narrower than their 95% credible intervals.

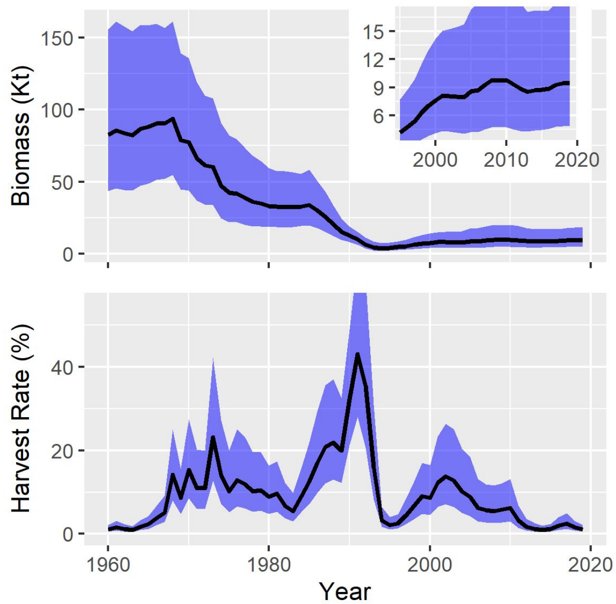

Figure 1: Surplus production model estimates of total stock biomass (top panel) and fishery harvest rates (bottom panel) for American plaice in NAFO Subdivision 3Ps. The blue shaded regions indicate 95% confidence intervals based on Vr (Eq. 4).

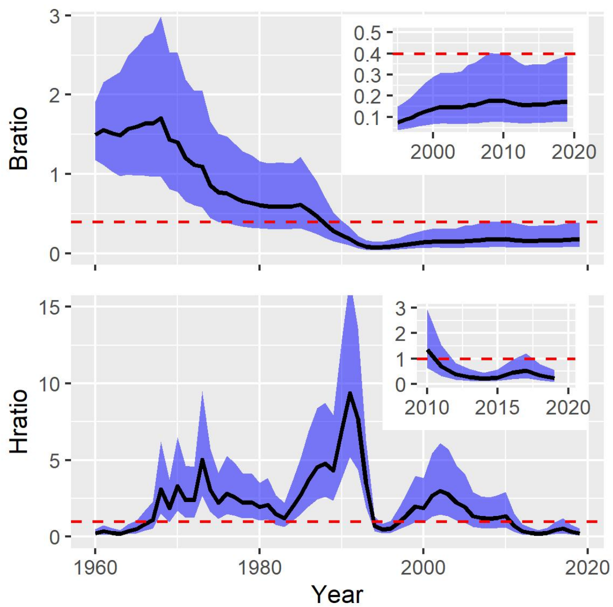

Figure 2: Surplus production model estimates of total stock biomass relative to B_{msy} (top panel) and fishery harvest rates (bottom panel) relative to H_{msy}. Red horizontal dashed lines are fisheries management limits (see text for explanation). See Fig. 1 for other details.

References

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