

Supplementary Information A: GICS Classification System

The Global Industry Classification Standard (GICS), introduced in 1999 by MSCI in collaboration with S&P Dow Jones Indices, provides a comprehensive and globally consistent framework for classifying publicly traded companies by industry. Its goal is to improve transparency, comparability, and analytical rigor in the investment process.

The GICS system is designed to be:

1. Universal: applicable to companies worldwide,
2. Reliable: reflecting the current structure of the global equity market,
3. Flexible: offering four hierarchical levels of classification,
4. Evolving: updated regularly to remain aligned with changes in market structure.

The GICS hierarchy comprises four levels:

1. 11 Sectors,
2. 24 Industry Groups,
3. 69 Industries,
4. 158 Sub-Industries.

Recent revisions introduced a separate Real Estate sector (previously part of Financials) and redefined the former Telecommunication Services sector as Communication Services.

Each company is assigned a GICS classification based on its primary business activity. Classifications may be represented in text or in an 8-digit hierarchical coding system that ensures consistency and smooth transitions across tiers ([MSCI Inc. 2025](#)).

The 11 GICS Sectors

- Energy
- Materials
- Industrials
- Consumer Discretionary
- Consumer Staples
- Health Care
- Financials
- Information Technology
- Communication Services
- Utilities
- Real Estate

The list of stocks used in our empirical analysis is reported below. For each market, we select eight stocks from each GICS sector, yielding a balanced sample of 88 stocks. The table also reports the corresponding sector classification, which is used to ensure broad sectoral coverage across the S&P 500 and STOXX Europe 600 samples.

Panel A: Selected S&P 500 constituents

GICS Sector	Consumer Discretionary	Industrials	Financials	Utilities	Materials	Communication Services	Real Estate	Energy	Information Technology	Consumer Staples	Health Care
1	AMZN.OQ	GE.N	JPM.N	NEE.N	LIN.OQ	GOOGL.OQ	WELL.N	XOM.N	NVDA.OQ	WMT.N	LLY.N
2	HD.N	RTX.N	MA.N	SO.N	SHW.N	NFLX.OQ	PLD.N	CVX.N	MSFT.OQ	COST.OQ	JNJ.N
3	MCD.N	CAT.N	BAC.N	DUK.N	NEM.N	DIS.N	AMT.N	COF.N	AAPL.OQ	PG.N	UNH.N
4	BKNG.OQ	BA.N	WFC.N	AEP.OQ	ECL.N	T.N	EQIX.OQ	WAB.N	ORCL.N	KO.N	ABT.N
5	TJX.N	HON.OQ	MS.N	SRE.N	FCX.N	VZ.N	SPG.N	EOG.N	CSCO.OQ	PEP.OQ	MRK.N
6	LOW.N	ETN.N	AXP.N	D.N	APD.N	CMCSA.OQ	DLR.N	SLB.N	AMD.OQ	MO.N	TMO.N
7	NKE.N	UNP.N	GS.N	EXC.OQ	VMC.N	TTWO.OQ	O.N	VLO.N	CRM.N	MDLZ.OQ	BSX.N
8	SBUX.OQ	DE.N	C.N	XEL.OQ	MLM.N	EA.OQ	PSA.N	OKEN	IBM.N	CL.N	BSG.OQ

Panel B: Selected STOXX Europe 600 constituents

GICS Sector	Consumer Discretionary	Industrials	Financials	Utilities	Materials	Communication Services	Real Estate	Energy	Information Technology	Consumer Staples	Health Care
1	IYMH.PA	AIR.PA	BSBA.L	IBE.MC	AIRP.PA	ORAN.PA	URW.PA	SHEL.AS	SAPG.DE	OREPPA	ROG.S
2	HRMS.PA	SCHN.PA	ALVG.DE	ENEL.MI	RIOL.L	SCMN.S	SGRO.L	TTEF.PA	ASML.AS	NESN.S	NOVN.S
3	ITX.MC	SAF.PA	SAN.MC	NG.L	HOLN.S	TEF.MC	SPSN.S	BPL	DAST.PA	ULVR.L	AZN.L
4	DIOR.PA	ABBN.S	UBSG.S	ENGIE.PA	HEIG.DE	VOD.L	LOIM.PA	EQNR.OL	NOKIA.HE	BATS.L	ESLX.PA
5	CPL.S	RRL	CRDM.MI	ELE.MC	GIVN.S	BT.L	PSPN.S	ENLM.I	CAPP.PA	ARLB.R	SASV.PA
6	BMWG.DE	RHMG.DE	ISP.MI	RWEG.DE	SIKA.S	PUBP.PA	GFCP.PA	PKN.WA	ASML.AS	DGE.L	GSK.L
7	CPGL	BELL	BBVA.MC	NTGY.MC	AAL.L	TEL.OL	CVO.PA	REP.MC	STMML.MI	DANO.PA	MRCG.DE
8	CCLL	SGEF.PA	ZURN.S	VIE.PA	ANTO.L	KPN.AS	WDFF.BR	TENR.MI	LOGN.S	RKT.L	LONN.S

Table 1: Selected S&P 500 and STOXX Europe 600 constituents by GICS sector. The table reports the Refinitiv Instrument Codes (RICs) of the selected S&P 500 and STOXX Europe 600 constituents used in the empirical analysis. For each market, eight stocks are selected from each GICS sector. Within each sector, stocks are ordered by trading volume in the last observed year of the sample, with rank 1 denoting the highest trading volume and rank 8 the lowest among the selected stocks.

Supplementary Information B: Robustness of Factor Quantification to Window-Length Selection

This section examines whether RMT-based factor-count results are sensitive to the choice of rolling-window length. While Supplementary Information C focuses on the robustness of the continuous GMR, this section considers the number of eigenvalues

that exceed the MP upper bound. We compare factor-count estimates based on rolling windows of 23, 44, 66, 88, and 132 trading days for the U.S. and European markets.

Throughout this paper, the EWMA decay factor is kept fixed at $\delta = 0.94$ value. With $\delta = 0.94$, $\epsilon = 1 - \delta = 0.06$, and $p = 88$, this gives

$$q_{\text{EWMA}} = p\epsilon = 88 \times 0.06 = 5.28.$$

The largest solution of

$$\lambda_+ - \log(\lambda_+) = q_{\text{EWMA}} + 1$$

is

$$\lambda_+ \approx 8.41.$$

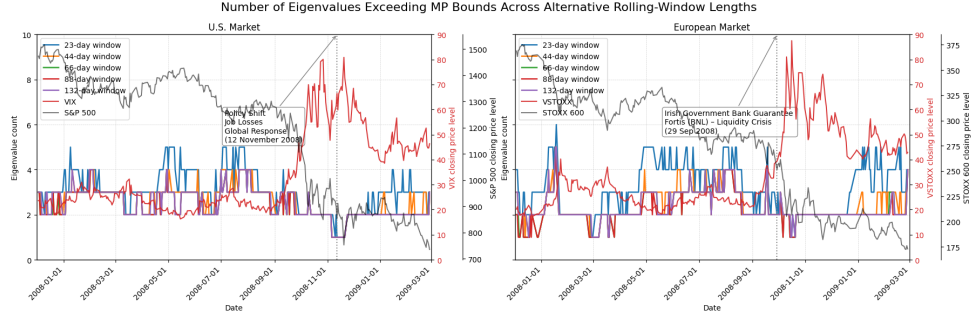
Thus, the exponentially weighted MP upper bound used in the baseline EWMA specification is approximately

$$\gamma_+^{\text{EWMA}} \approx 8.41 \sigma_{\text{res}}^2.$$

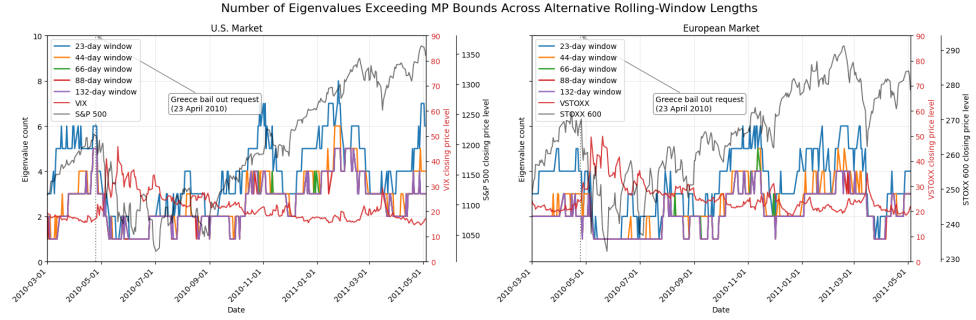
Figure 5 shows that the broad crisis-related pattern is robust across alternative window lengths. The 23-day window is the most reactive and produces the most volatile factor-count series, while longer windows are smoother and generally yield lower factor counts. This is expected because short windows respond more strongly to local changes in the correlation structure, but they are also more sensitive to transitory eigenvalue movements around the MP boundary. Longer windows average over a broader set of observations and therefore identify only more persistent eigenvalue separations from the random-matrix benchmark.

During the Global Financial Crisis, the COVID-19 crisis, and the European sovereign debt crisis, the factor count tends to decline around the most severe stress periods. This decline should not be interpreted as weaker market dependence. Rather, it indicates that the eigenvalue spectrum becomes more concentrated in the dominant market mode. In periods of severe stress, market-wide co-movement increases, the largest eigenvalue absorbs more variance, and fewer residual non-market eigenvalues remain clearly separated from the MP noise region.

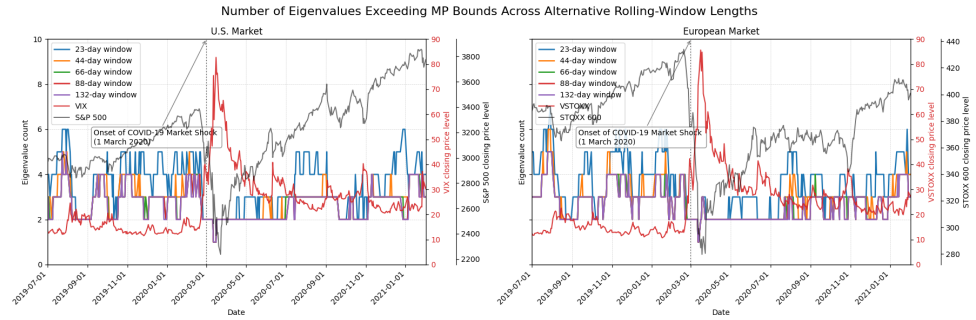
The fact that longer rolling windows often produce smaller factor counts than the 23-day window is therefore informative. It suggests that some of the additional factors detected by the shortest window reflect short-lived or locally unstable eigenvalue exceedances, whereas the longer-window specifications capture a smaller set of more persistent factor structures. The robustness check therefore supports the interpretation that the baseline 23-day window is useful for detecting rapid changes in market structure, while the longer windows confirm that the main crisis-related patterns are not an artefact of the short-window design.



(a) 2008 Financial Crisis



(b) Debt Crisis



(c) COVID-19 Crisis

Fig. 5: Robustness of RMT-based factor counts across alternative rolling-window lengths during the three major crisis episodes. The first row shows the Global Financial Crisis, the second row the European sovereign debt crisis, and the third row the COVID-19 crisis. Each panel compares the factor-count dynamics for rolling windows of 23, 44, 66, 88, and 132 trading days for the U.S. and European markets, together with the corresponding volatility and equity indices. The vertical dotted lines mark the crisis reference dates used throughout the article. Across specifications, the main crisis-related patterns remain qualitatively similar, while longer windows produce smoother dynamics and tend to retain fewer factors.

To quantify the similarity across window-length specifications, Figure 6 reports Spearman rank correlations between the factor-count series. Spearman correlations are used because the factor count is a discrete variable and because the main question is whether different window lengths preserve the same ordering of high- and low-factor-count periods.

The heatmaps show that correlations are generally positive and high, especially among the longer-window specifications. The strongest agreement is observed between the 66-, 88-, and 132-day windows, which often exhibit rank correlations close to one. The 23-day window shows weaker correlations with the longer windows, particularly during the Global Financial Crisis and the COVID-19 crisis. This is consistent with the interpretation that the shortest window captures more short-run variation, while longer windows identify more persistent factor-count dynamics.

Overall, the Spearman correlation evidence confirms that the alternative window-length specifications are not producing unrelated signals. Instead, they capture broadly similar crisis-related movements, with differences mainly reflecting the trade-off between short-run responsiveness and persistence.

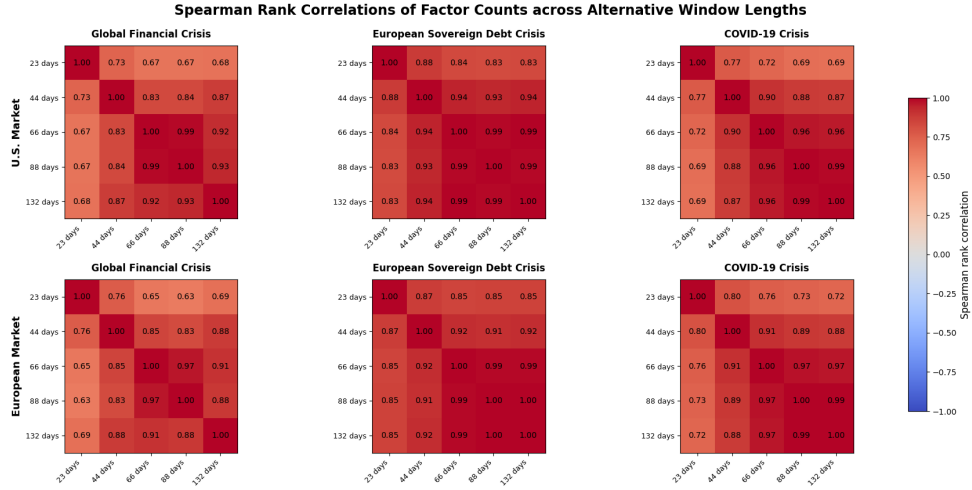


Fig. 6: Spearman rank correlations of MP-based factor counts across alternative rolling-window lengths. The first row reports results for the U.S. market and the second row reports results for the European market. Columns correspond to the Global Financial Crisis, the European sovereign debt crisis, and the COVID-19 crisis.

Supplementary Information C: Robustness of the GMR to Window-Length Selection

This subsection assesses the robustness of the proposed GMR with respect to the rolling-window length n . Since the GMR is constructed from rolling weighted correlation matrices, the choice of window length may affect the sensitivity and smoothness

of the resulting time series. Shorter windows are expected to react more strongly to short-term market fluctuations, while longer windows should produce smoother and more persistent signals.

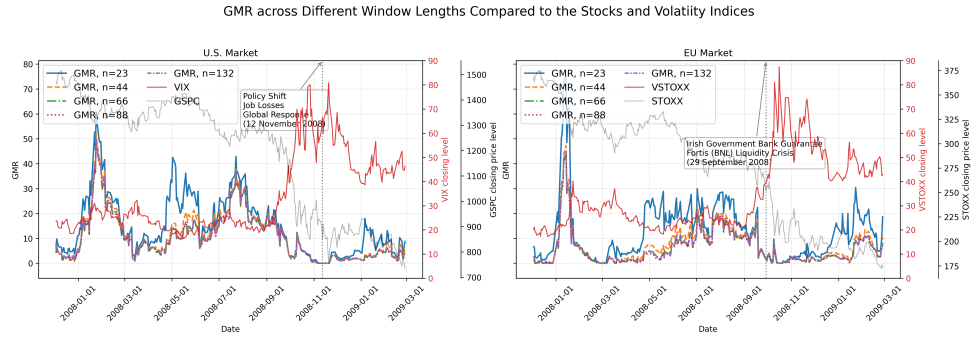
To examine whether the main findings are driven by the baseline choice of $n = 23$, the GMR is recalculated using several alternative rolling-window lengths including $\{44, 66, 88, 132\}$. The resulting series are compared using Spearman rank correlations. This allows us to evaluate whether the different window-length specifications capture the same underlying market-wide correlation structure and whether periods of elevated market connectedness are consistently identified across alternative window lengths.

The robustness results in Figures 7 indicates that the GMR is highly stable across alternative rolling-window lengths in both the European and U.S. markets. Spearman correlations, as shown in Figure 8, are consistently high, and correlations among the longer-window specifications are particularly strong. This suggests that the different window-length specifications capture the same underlying market-wide correlation structure rather than producing unrelated signals.

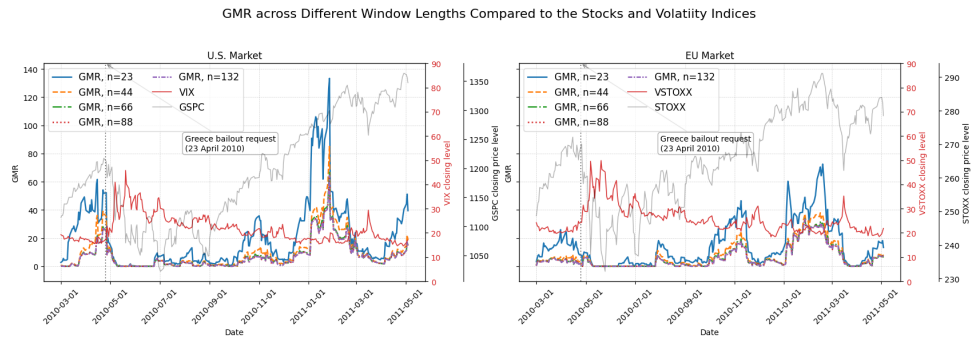
The weakest, but still strong, agreement is observed when the shortest window, $n = 23$ days, is compared with the longer windows. This is expected, since shorter rolling windows react more strongly to short-term fluctuations and crisis-related spikes, whereas longer windows smooth the signal more heavily. The higher responsiveness of the 23-day window makes it useful for detecting abrupt changes in the local correlation structure, while the longer-window specifications identify more persistent factor structures.

In addition, Table 2 reports the timing of volatility-index peaks and GMR minima across the alternative rolling-window lengths. The table shows that, in both markets and across the three crisis episodes, the GMR minimum generally occurs before the corresponding VIX or VSTOXX peak. The date gaps are negative in almost all cases, indicating that the decline in the GMR tends to precede the maximum of the volatility index. Importantly, this timing pattern is not limited to the baseline 23-day window, but is also visible for the longer-window specifications. This provides further evidence that the observed crisis-related decline in the GMR is not driven by a single window-length choice. Rather, the GMR consistently captures the weakening of non-market group modes and the increasing dominance of market-wide co-movement around periods of financial crisis.

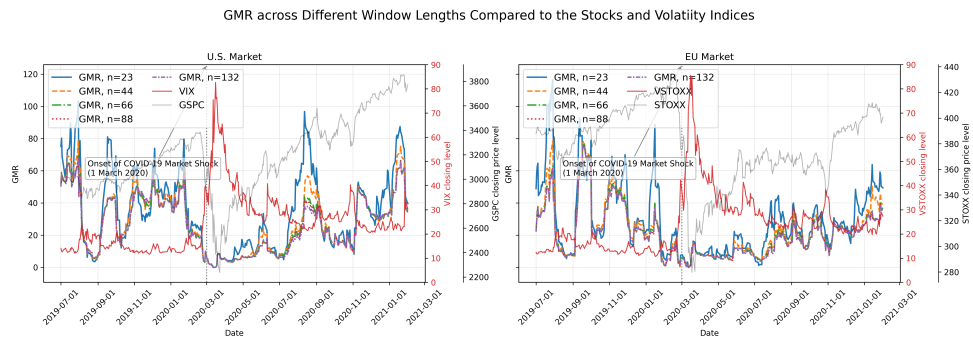
Overall, the results support the robustness of the GMR methodology with respect to the choice of rolling-window length. The main crisis-related patterns are not driven by one arbitrary window length. The 23-day window produces the most reactive and volatile signal, while longer windows produce smoother and more persistent estimates. The consistency of the main findings across these alternative specifications strengthens the interpretation of the GMR as a robust market-stress indicator based on deviations from the random-matrix benchmark.



(a) 2008 Financial Crisis



(b) Debt Crisis



(c) COVID-19 Crisis

Fig. 7: Robustness of the GM Ratio across alternative rolling-window lengths. The figures compare GMR dynamics with the corresponding volatility index during major crisis episodes. The first row shows the Global Financial Crisis, the second row the European sovereign debt crisis, and the third row the COVID-19 crisis.

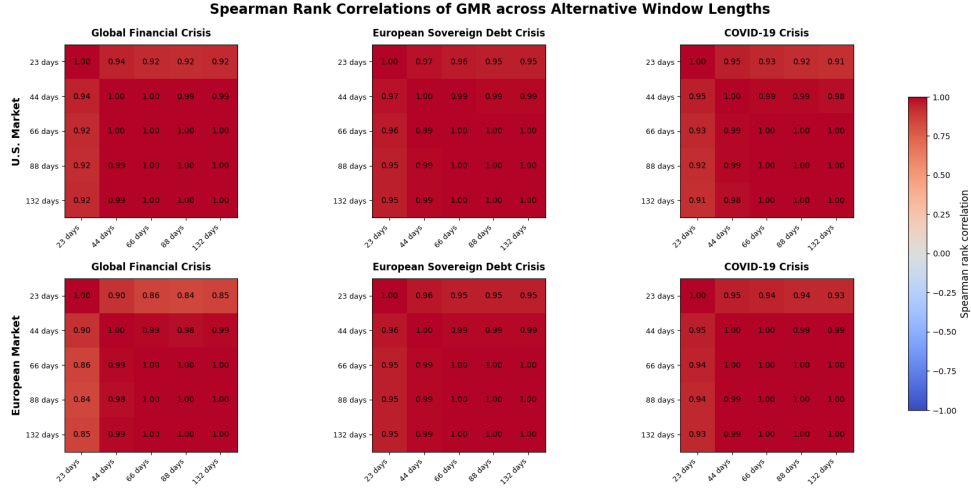


Fig. 8: Spearman rank correlations of the GMR across alternative rolling-window lengths during major crisis episodes. The first row reports results for the U.S. market and the second row reports results for the European market. Columns correspond to the Global Financial Crisis, the European sovereign debt crisis, and the COVID-19 crisis.

Supplementary Information D: Lead-Lag Structure of the GMR and Volatility Indices

This supplementary section provides a quantitative assessment of the relationship between the GMR and market volatility indices. The analysis has three parts. First, we report association measures between the eigenvalue-based indicators and the corresponding volatility indices, including Pearson correlations, bivariate R^2 values, and Spearman rank correlations. These statistics are computed for the GMR, the largest eigenvalue λ_1 , and the corresponding volatility index, using the VIX for the U.S. market and the VSTOXX for the European market. Second, we examine the lead-lag structure between the GMR and volatility to assess whether the GMR behaves as a leading, coincident, or lagging indicator of market-wide stress. Third, we complement the correlation evidence with event-study plots around the main crisis dates. Together, these analyses provide a more formal assessment of the visual co-movement documented in the main text.

Following the standard event-study logic that the event window should be wide enough to capture the market response but narrow enough to limit contamination from unrelated developments (MacKinlay 1997; Brown and Warner 1985), we use an event window of approximately three months before and three months after each crisis event. In trading-day terms, this corresponds to an event window of roughly $[-60, +60]$ trading days around the crisis date. The annotated crisis reference dates are chosen to match the event dates used in the figures. For the Global Financial Crisis, we use

Panel A: European market

Episode	n	Vol index	Vol peak date	Vol peak value	GMR min date	GMR min value	Date gap
Global Financial Crisis	23 days	VSTOXX	10/16/2008	87.513	10/15/2008	0	-1
Global Financial Crisis	44 days	VSTOXX	10/16/2008	87.513	10/15/2008	0	-1
Global Financial Crisis	66 days	VSTOXX	10/16/2008	87.513	10/6/2008	0	-10
Global Financial Crisis	88 days	VSTOXX	10/16/2008	87.513	10/6/2008	0	-10
Global Financial Crisis	132 days	VSTOXX	10/16/2008	87.513	10/15/2008	0	-1
European Sovereign Debt Crisis	23 days	VSTOXX	5/20/2010	49.874	5/10/2010	0	-10
European Sovereign Debt Crisis	44 days	VSTOXX	5/20/2010	49.874	5/10/2010	0	-10
European Sovereign Debt Crisis	66 days	VSTOXX	5/20/2010	49.874	4/27/2010	0	-23
European Sovereign Debt Crisis	88 days	VSTOXX	5/20/2010	49.874	4/27/2010	0	-23
European Sovereign Debt Crisis	132 days	VSTOXX	5/20/2010	49.874	4/27/2010	0	-23
COVID-19 Crisis	23 days	VSTOXX	3/16/2020	85.621	3/12/2020	0	-4
COVID-19 Crisis	44 days	VSTOXX	3/16/2020	85.621	3/12/2020	0	-4
COVID-19 Crisis	66 days	VSTOXX	3/16/2020	85.621	3/12/2020	0	-4
COVID-19 Crisis	88 days	VSTOXX	3/16/2020	85.621	3/12/2020	0	-4
COVID-19 Crisis	132 days	VSTOXX	3/16/2020	85.621	3/12/2020	0	-4

Panel B: U.S. market

Episode	n	Vol index	Vol peak date	Vol peak value	GMR min date	GMR min value	Date gap
Global Financial Crisis	23 days	VIX	11/20/2008	80.86	11/10/2008	0	-10
Global Financial Crisis	44 days	VIX	11/20/2008	80.86	11/6/2008	0	-14
Global Financial Crisis	66 days	VIX	11/20/2008	80.86	11/6/2008	0	-14
Global Financial Crisis	88 days	VIX	11/20/2008	80.86	11/6/2008	0	-14
Global Financial Crisis	132 days	VIX	11/20/2008	80.86	11/6/2008	0	-14
European Sovereign Debt Crisis	23 days	VIX	5/20/2010	45.79	5/11/2010	0	-9
European Sovereign Debt Crisis	44 days	VIX	5/20/2010	45.79	5/10/2010	0	-10
European Sovereign Debt Crisis	66 days	VIX	5/20/2010	45.79	5/10/2010	0	-10
European Sovereign Debt Crisis	88 days	VIX	5/20/2010	45.79	5/10/2010	0	-10
European Sovereign Debt Crisis	132 days	VIX	5/20/2010	45.79	5/10/2010	0	-10
COVID-19 Crisis	23 days	VIX	3/16/2020	82.69	3/12/2020	0	-4
COVID-19 Crisis	44 days	VIX	3/16/2020	82.69	3/12/2020	0	-4
COVID-19 Crisis	66 days	VIX	3/16/2020	82.69	3/12/2020	0	-4
COVID-19 Crisis	88 days	VIX	3/16/2020	82.69	3/12/2020	0	-4
COVID-19 Crisis	132 days	VIX	3/16/2020	82.69	3/12/2020	0	-4

Table 2: Timing of volatility-index peaks and minimum GMR values. Panel A reports results for the European market using the VSTOXX, while Panel B reports results for the U.S. market using the VIX. The date gap is defined as the difference in calendar days between the GMR minimum date and the volatility-index peak date. Negative values indicate that the GMR minimum occurred before the volatility-index peak.

market-specific dates: 12 November 2008 for the U.S. market, corresponding to the policy shift, job losses, and global response episode ([Gorton 2010](#)), and 29 September 2008 for the European market, corresponding to the Irish government bank guarantee and the Fortis liquidity crisis ([Reinhart and Rogoff 2009](#)). For the European sovereign debt crisis, we use the Greek bailout request on 23 April 2010 ([Lane 2012](#)). For the COVID-19 crisis, we use the onset of the COVID-19 market shock on 1 March 2020 ([Baker et al. 2020](#)).

We first quantify the association between the eigenvalue-based indicators and the corresponding volatility indices. Table 3 reports Pearson correlations, bivariate R^2 values, and Spearman rank correlations between the GMR, the largest eigenvalue λ_1 , and the volatility index. For the U.S. market, the volatility index is the VIX, while for the European market it is the VSTOXX. The statistics are reported separately for the Global Financial Crisis, the European sovereign debt crisis, the COVID-19 crisis, and the pooled crisis sample.

The results show a consistent pattern across both markets. The GMR is negatively correlated with the volatility indices, while the largest eigenvalue λ_1 is positively correlated with them. This is consistent with the interpretation that, during periods of market stress, the eigenvalue spectrum becomes increasingly dominated by the leading market mode, while the residual non-market group-mode structure weakens. The Spearman rank correlations confirm that this relationship is not only a linear association, but also holds in terms of the ordering of high- and low-stress observations.

However, contemporaneous correlations do not reveal whether the GMR leads, coincides with, or lags volatility-index movements. We therefore complement the correlation table with a lead-lag analysis.

Market	Episode	Predictor	Vol index	Pearson	R squared	Spearman
US Market	Global Financial Crisis	GMR	VIX	-0.84	0.71	-0.83
US Market	Global Financial Crisis	MaxEigenvalue	VIX	0.91	0.83	0.85
US Market	Sovereign Debt Crisis	GMR	VIX	-0.66	0.44	-0.75
US Market	Sovereign Debt Crisis	MaxEigenvalue	VIX	0.81	0.65	0.87
US Market	COVID-19 Crisis	GMR	VIX	-0.76	0.58	-0.91
US Market	COVID-19 Crisis	MaxEigenvalue	VIX	0.93	0.87	0.97
US Market	Pooled	GMR	VIX	-0.61	0.37	-0.7
US Market	Pooled	MaxEigenvalue	VIX	0.8	0.65	0.89

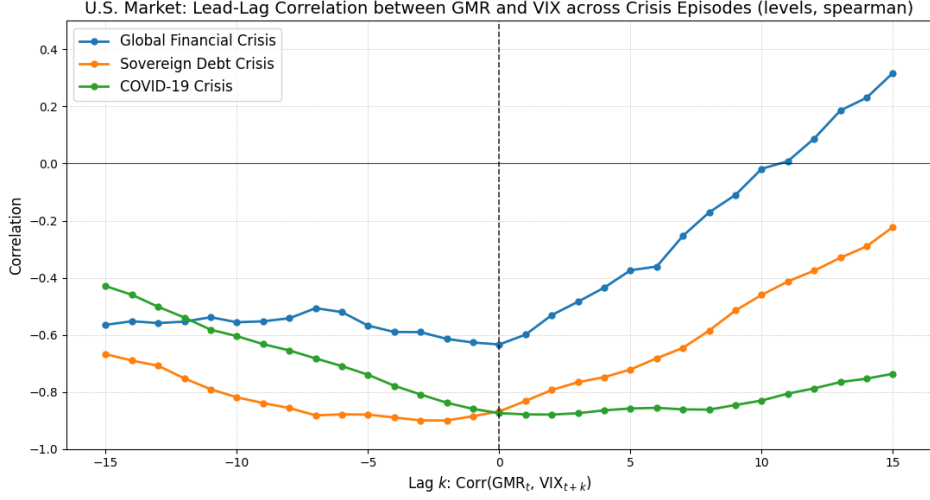
Market	Episode	Predictor	Vol index	Pearson	R squared	Spearman
EU Market	Global Financial Crisis	GMR	VSTOXX	-0.82	0.68	-0.82
EU Market	Global Financial Crisis	MaxEigenvalue	VSTOXX	0.84	0.71	0.85
EU Market	Sovereign Debt Crisis	GMR	VSTOXX	-0.71	0.51	-0.8
EU Market	Sovereign Debt Crisis	MaxEigenvalue	VSTOXX	0.83	0.68	0.85
EU Market	COVID-19 Crisis	GMR	VSTOXX	-0.49	0.24	-0.54
EU Market	COVID-19 Crisis	MaxEigenvalue	VSTOXX	0.88	0.78	0.92
EU Market	Pooled	GMR	VSTOXX	-0.53	0.28	-0.65
EU Market	Pooled	MaxEigenvalue	VSTOXX	0.75	0.56	0.84

Table 3: Contemporaneous association between eigenvalue-based indicators and volatility indices. The table reports Pearson correlations, bivariate R^2 values, and Spearman rank correlations between the GMR, the largest eigenvalue λ_1 , and the corresponding volatility index. Panel A reports results for the U.S. market using the VIX, while Panel B reports results for the European market using the VSTOXX. Statistics are reported separately for each crisis episode and for the pooled crisis sample.

We compute lead-lag correlations between the GMR and the corresponding volatility index over lags $k = -15, \dots, +15$. Positive values of k correspond to $\text{Corr}(\text{GMR}_t, \text{Volatility}_{t+k})$, so that a strong association at positive lags would indicate that the GMR leads the volatility index. Negative values of k indicate that the volatility index leads the GMR.

The use of a $[-60, +60]$ trading-day event window is particularly appropriate in the present setting because the GMR is constructed from 23-day rolling windows. A substantially narrower window would leave too few observations for a stable lead-lag correlation analysis, especially when lags up to ± 15 trading days are considered. At

the same time, the three-month window remains focused on the crisis-onset period and avoids mixing the event dynamics with longer tranquil or post-crisis periods.



(a) U.S. market: Crisis lead-lag correlation between GMR and VIX.



(b) European market: Crisis lead-lag correlation between GMR and VSTOXX.

Fig. 9: The upper panel reports the U.S. market using VIX, while the lower panel reports the European market using VSTOXX. The lag convention is $\text{Corr}(\text{GMR}_t, \text{Volatility}_{t+k})$. Positive lags therefore indicate that the GMR is compared with future volatility, while negative lags indicate that the GMR is compared with past volatility.

Figure 9 reports the crisis-specific lead-lag profiles between the GMR and the corresponding volatility indices. The correlations are mostly negative, confirming that lower values of the GMR are associated with higher values of VIX or VSTOXX. This is consistent with the interpretation that a decline in the GMR reflects the weakening of non-market group modes and the increasing dominance of the common market factor during periods of stress.

The timing of the strongest association differs across markets and crisis episodes. In the U.S. market, the Global Financial Crisis profile is negative at negative and near-contemporaneous lags, with the strongest negative association around the event window close to $k = 0$. At longer positive lags, the correlation weakens and eventually turns positive, suggesting that the GMR does not provide a persistent leading signal for VIX throughout the full post-event horizon in this episode. During the sovereign debt crisis, the U.S. correlation is strongly negative across almost the entire lag range, with the strongest association at negative or near-contemporaneous lags. This indicates that VIX movements largely lead or coincide with changes in the GMR in this episode. During the COVID-19 crisis, the U.S. profile is negative throughout and reaches its strongest values around the contemporaneous window and shortly thereafter, indicating close co-movement between the collapse in GMR and the volatility spike.

For the European market, the Global Financial Crisis and sovereign debt crisis also show persistently negative correlations between the GMR and VSTOXX. In both cases, the strongest negative correlations occur around near-contemporaneous or slightly positive lags, suggesting that low GMR values are closely aligned with elevated volatility and, in some windows, precede higher VSTOXX values over the following trading days. The COVID-19 profile is more asymmetric: correlations are positive at longer negative lags, but become negative as the crisis window approaches and remain negative at contemporaneous and positive lags. This pattern suggests that the European GMR becomes informative mainly around the onset and continuation of the COVID-19 volatility shock, rather than far in advance of it. Overall, the lead-lag evidence supports interpreting the GMR primarily as a contemporaneous tracking measure of market-wide dependence and diversification collapse, with some episode-specific leading information rather than as a uniform early-warning indicator across all crises. This interpretation is economically sound, because the GMR is strongly associated with VIX and VSTOXX during crisis periods. Since the VIX is an option-implied measure of expected stock-market volatility for the next 30 days, and since previous studies show that VIX and related implied-volatility measures contain predictive information for future stock-market volatility and returns (Cboe Global Markets 2026; Banerjee et al. 2007; Bekaert and Hoerova 2014), the close association between the GMR and volatility indices suggests that our method may also provide useful information for predictive market-risk monitoring. In this sense, the GMR can be used to monitor the strengthening of market-wide interconnectedness, diversification collapse, and common-factor dominance. In addition, the GMR is available when no option implied measures exist.

As a complementary timing check, we also conduct an event-study analysis around the same crisis dates. For each event, the nearest available trading day is used as the

event date, denoted by $\tau = 0$. We align the transformed stress measure $100 - \text{GMR}$, the largest eigenvalue λ_1 , and the corresponding volatility index over the $[-60, +60]$ trading-day event window. Each series is standardized relative to a pre-event baseline window, so that the event-study plots show deviations from the pre-crisis baseline. This allows us to inspect whether the deterioration in the GMR occurs before, at, or after the volatility response.

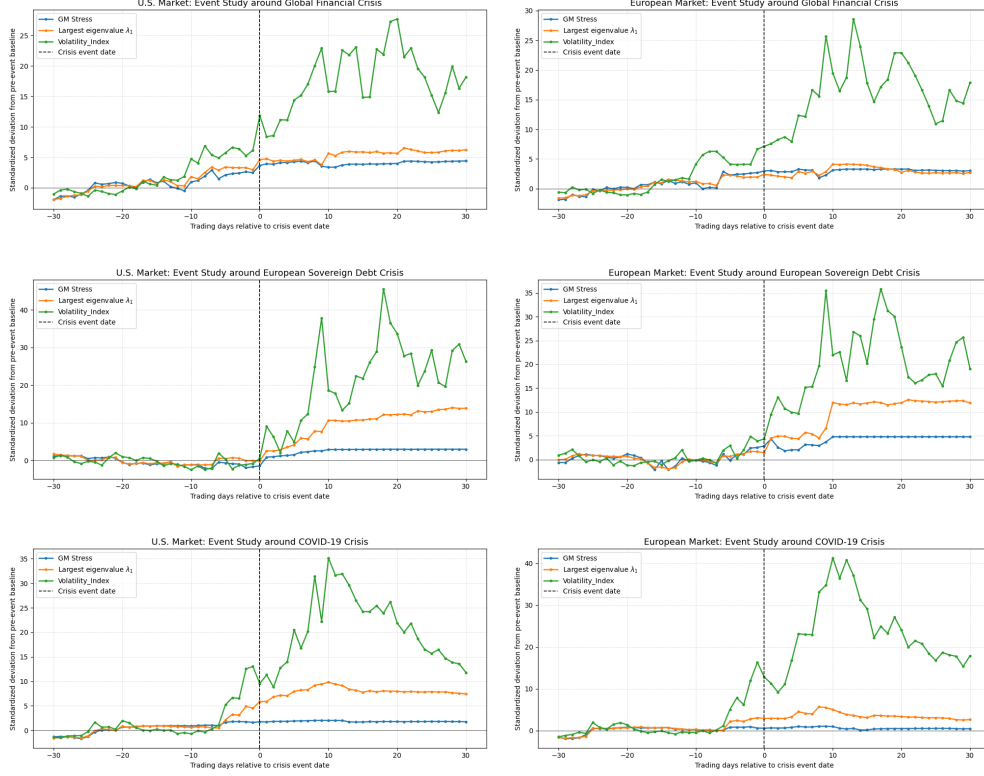


Fig. 10: Event-study dynamics around the three crisis episodes. Each row corresponds to one crisis episode: the Global Financial Crisis, the European sovereign debt crisis, and the COVID-19 crisis. The left column reports the U.S. market and the right column reports the European market. The figures show standardized deviations of GM Stress, the largest eigenvalue λ_1 , and the corresponding volatility index from the pre-event baseline. The vertical dashed line denotes the crisis-event date.

The event-study plots provide additional evidence that the GMR-based stress measure reacts strongly around major crisis episodes. Across the six panels, GM Stress, the largest eigenvalue λ_1 , and the corresponding volatility index are standardized relative to the pre-event baseline. This allows the dynamics of the spectral indicators and volatility indices to be compared within the same event window. In most cases,

the volatility index exhibits the largest standardized response, while GM Stress and λ_1 move in the same direction, indicating that volatility spikes are accompanied by a stronger dominant market mode and a weakening of diversified group structure.

During the Global Financial Crisis, both the U.S. and European markets show a clear post-event increase in the volatility index, GM Stress, and the largest eigenvalue. The European market response is particularly pronounced, with VSTOXX rising sharply after the crisis reference date and the spectral indicators also moving upward. This suggests that the crisis was associated with a broad concentration of market risk, in which the leading eigenvalue became more dominant and the GMR-based stress measure increased. The strongest movements occur mainly around or after the event date, indicating that the GMR captures the realized build-up of market-wide dependence rather than providing a uniformly early signal before the event.

The European sovereign debt crisis shows a more heterogeneous pattern across markets, but the regional character of the shock is still visible. In the European market, GM Stress and λ_1 increase strongly after the event date and remain elevated, while VSTOXX also displays large post-event spikes. The U.S. market also reacts, but the response is less directly tied to the event date and appears more gradual. This supports the interpretation that the GMR does not mechanically generate a crisis signal around every event date. Instead, it reacts most clearly when the event is associated with genuine market-wide correlation stress in the relevant market.

For the COVID-19 crisis, both markets show a strong and persistent increase in the volatility index, GM Stress, and the largest eigenvalue around the event window. Importantly, some movement is already visible before the event date, especially in the volatility indices and the spectral indicators. This is consistent with the fact that financial markets began pricing pandemic-related risks before the formal crisis reference date. After the event date, the indicators remain elevated for several trading days, suggesting that the GMR-based stress measure captures not only the initial shock but also the persistence of market-wide spectral concentration during the COVID-19 episode.

Overall, both the lead-lag correlations and the event-study plots support a cautious interpretation. The GMR is highly informative about contemporaneous market stress and, in some cases, reacts early within the crisis-onset window. However, the evidence is not sufficiently systematic to support a strong predictive or forecasting interpretation. We therefore interpret the GMR as a concurrent, and in some episodes early-reacting, indicator of market-wide spectral stress.

Supplementary Information E : Sensitivity of the GMR to Alternative MP-Threshold

Table 4 summarizes the sensitivity of the GMR to alternative Marčenko–Pastur threshold calibrations, while Figure 11 visualizes the corresponding time-series dynamics across the three crisis episodes. Together, the table and figure distinguish between level sensitivity and dynamic robustness: the table reports how the average magnitude of the GMR changes across threshold choices, whereas the figure shows whether the main crisis-related movements remain visible over time.

The sensitivity analysis shows that the absolute level of the GMR depends on the calibration of the MP threshold, as expected. Increasing σ_{res}^2 raises the MP upper bound and therefore reduces the amount of spectral variance classified as lying above the noise threshold. In the U.S. market, this is reflected in the declining mean GMR, from 31.50 under the dynamic specification and 30.00 under fixed $\sigma_{\text{res}}^2 = 0.6$, to 17.68 and 10.53 under fixed $\sigma_{\text{res}}^2 = 0.8$ and $\sigma_{\text{res}}^2 = 1.0$, respectively. In the European market, the dynamic specification yields a mean GMR of 19.83. The fixed $\sigma_{\text{res}}^2 = 0.6$ and $\sigma_{\text{res}}^2 = 0.8$ specifications produce higher mean values of 22.94, while the more conservative fixed $\sigma_{\text{res}}^2 = 1.0$ specification reduces the mean GMR substantially to 5.77.

However, the time-series pattern of the GMR remains robust across the alternative calibrations, especially for moderate threshold choices. The dynamic specification calibrates σ_{res}^2 using the average of the residual-variance estimates obtained from previous rolling windows only. In the U.S. market, it is almost perfectly correlated with the fixed $\sigma_{\text{res}}^2 = 0.6$ specification, with a Pearson correlation of 0.99 and a Spearman correlation of 0.99. Even under the more conservative thresholds $\sigma_{\text{res}}^2 = 0.8$ and $\sigma_{\text{res}}^2 = 1.0$, the correlations with the dynamic specification remain high, with Pearson correlations of 0.96 and 0.88 and Spearman correlations of 0.96 and 0.89, respectively.

The European market shows a similar, although somewhat more threshold-sensitive, pattern. The fixed $\sigma_{\text{res}}^2 = 0.6$ and $\sigma_{\text{res}}^2 = 0.8$ specifications remain strongly associated with the dynamic specification, with Pearson correlations of 0.88 and 0.87 and Spearman correlations of 0.94 in both cases. Under the most conservative calibration, $\sigma_{\text{res}}^2 = 1.0$, the correlation declines to 0.65 for Pearson and 0.75 for Spearman. This indicates that very high threshold choices affect not only the level but also, to some extent, the relative time-series dynamics of the GMR in the European market. Nevertheless, the positive and economically meaningful correlations show that the main stress-related movements remain visible.

The mean absolute difference (MAD) provides an additional measure of the level sensitivity of the GMR. In the U.S. market, the fixed $\sigma_{\text{res}}^2 = 0.6$ specification differs only slightly from the dynamic specification, with a MAD of 2.01, while the deviations increase to 13.82 for $\sigma_{\text{res}}^2 = 0.8$ and to 20.97 for $\sigma_{\text{res}}^2 = 1.0$. In the European market, the MAD is 8.54 for both $\sigma_{\text{res}}^2 = 0.6$ and $\sigma_{\text{res}}^2 = 0.8$, and rises to 14.20 under the most conservative $\sigma_{\text{res}}^2 = 1.0$ calibration. This confirms that the absolute level of the GMR is affected by the threshold calibration. However, the correlation results show that, despite these level differences, the timing and relative dynamics of the GMR remain broadly similar across reasonable specifications.

Overall, the results confirm that the level of the GMR is threshold-sensitive, while the main crisis-related dynamics remain stable across alternative MP-threshold calibrations. The dynamic specification therefore provides a useful compromise: it allows the MP threshold to adapt over time while preserving the main variation in the GMR across crisis episodes.

Figure 11 complements the summary statistics in Table 4 by showing the time-series behaviour of the GMR under each threshold calibration. The visual evidence confirms that higher fixed values of σ_{res}^2 lower the GMR mechanically by raising the MP upper edge. However, the main crisis-related movements are preserved across calibrations: periods of severe market stress are still associated with sharp declines in the

Panel A: U.S. market							
Specification	Mean	Median	SD	Max.	Pearson	Spearman	MAD
Dynamic $\sigma_{\text{res},t}^2$	31.50	25.11	25.83	145.47	–	–	–
Fixed $\sigma^2 = 0.6$	30.00	23.29	25.75	142.16	0.99	0.99	2.01
Fixed $\sigma^2 = 0.8$	17.68	11.72	17.87	90.93	0.96	0.96	13.82
Fixed $\sigma^2 = 1.0$	10.53	5.25	12.95	69.20	0.88	0.89	20.97

Panel B: European market							
Specification	Mean	Median	SD	Max.	Pearson	Spearman	MAD
Dynamic $\sigma_{\text{res},t}^2$	19.83	20.10	10.34	48.44	–	–	–
Fixed $\sigma^2 = 0.6$	22.94	17.10	20.50	103.79	0.88	0.94	8.54
Fixed $\sigma^2 = 0.8$	22.94	17.10	20.50	103.78	0.87	0.94	8.54
Fixed $\sigma^2 = 1.0$	5.77	1.65	8.11	51.23	0.65	0.75	14.20

Table 4: Sensitivity of the GMR to alternative Marčenko–Pastur threshold calibrations in the U.S. and European markets. Panel A reports results for the U.S. market and Panel B reports results for the European market. Pearson, Spearman, and MAD are computed relative to the dynamic specification.

GMR, indicating a weakening of non-market group modes and a stronger concentration of spectral mass in the dominant market component. The figure therefore supports the interpretation that the dynamic specification affects the scale of the indicator, but does not generate the crisis-related pattern artificially.

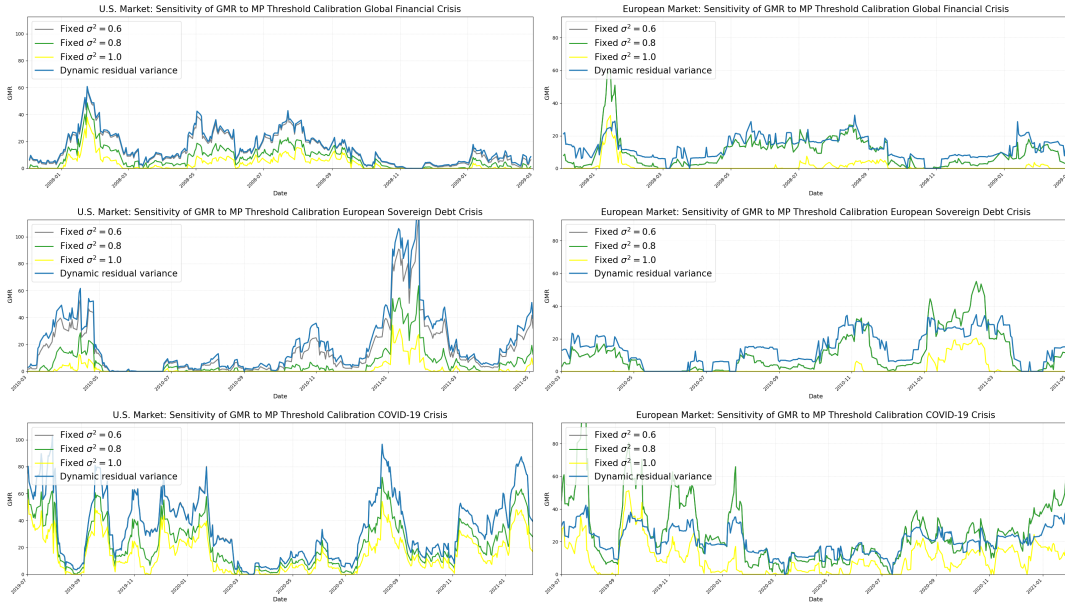


Fig. 11: Sensitivity of the GM Ratio to alternative Marčenko–Pastur threshold calibrations across crisis episodes. The panels compare the dynamic residual-variance specification with fixed variance calibrations for the U.S. and European markets during the Global Financial Crisis, the European sovereign debt crisis, and the COVID–19 crisis.

Supplementary Information F: Zoomed Eigenvalue Spectra

Figure 12 reports zoomed versions of the empirical eigenvalue spectra shown in Figure 1. The full spectra contain very large eigenvalues, especially market-mode eigenvalues. These large eigenvalues are important for identifying common market structure, but they compress the lower part of the spectrum visually. The zoomed plots therefore focus on the lower eigenvalue range, making the comparison between empirical eigenvalues, Wishart simulations, and MP bounds easier to inspect.

Supplementary Information G: Quantitative Comparison of Factor-Selection Methods

This section provides a detailed quantitative comparison of the factor-selection methods used in the main text. We compare the RMT-based MP count, the standard PCA 90% cumulative-variance rule, Horn’s parallel analysis, and the Onatski-type eigenvalue-spacing criterion. The comparison is reported separately for the U.S. and European markets and for the three crisis episodes.

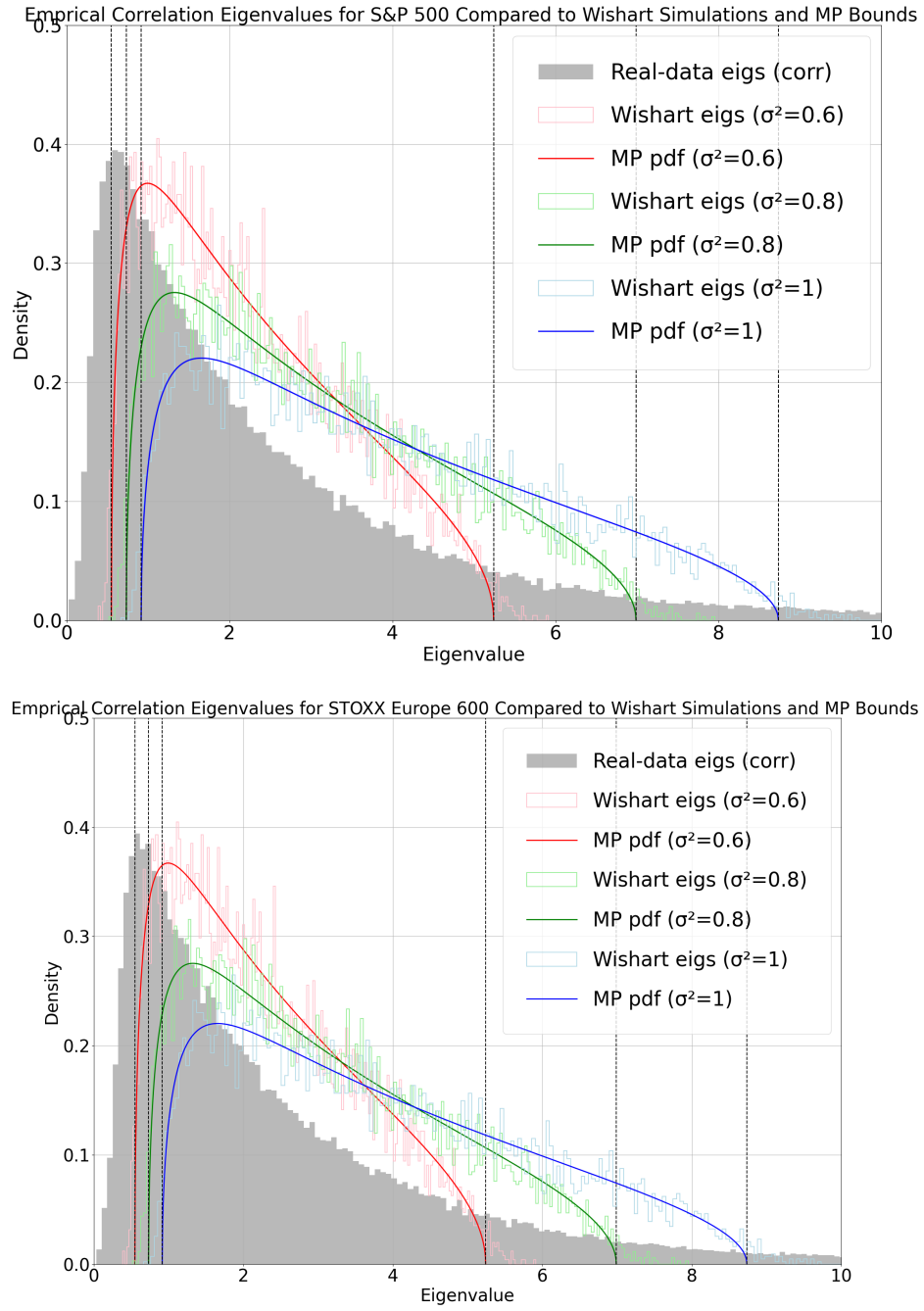


Fig. 12: Zoomed empirical weighted correlation eigenvalue spectra compared with Wishart simulations and Marčenko–Pastur bounds. The upper panel reports the S&P 500 universe, while the lower panel reports the STOXX Europe 600 universe. The zoomed spectra focus on the lower part of the eigenvalue distribution, where the simulated Wishart eigenvalues and MP bounds are more visible. The corresponding full spectra are reported in Figure 1.

Table 5 reports descriptive statistics for the number of retained factors. The results show a clear ranking across methods. The PCA 90% rule retains by far the largest number of components in all crisis episodes, with mean factor counts generally around 11 to 13 in both markets. By contrast, the RMT-based MP count is more conservative, typically selecting around three to four factors. Horn’s parallel analysis and the Onatski-type spacing criterion are even more restrictive, usually retaining only one to two factors.

This pattern confirms that the PCA 90% cumulative-variance rule tends to retain many components mechanically, because it continues adding principal components until the chosen variance threshold is reached. The noise-based and eigenvalue-separation methods are more selective. Horn’s parallel analysis compares empirical eigenvalues with simulated random-eigenvalue benchmarks, while the Onatski-type criterion focuses on large gaps in the ordered eigenvalue spectrum. The RMT-based MP count provides a complementary high-dimensional noise benchmark based on the MP upper edge.

The crisis-specific results also show that the factor structure becomes more concentrated during periods of market stress. In both markets, the RMT count remains substantially below the PCA count, while Horn and Onatski retain only a small number of factors. This is consistent with the interpretation that crisis periods are characterized by a stronger dominant market mode and fewer clearly distinguishable non-market factors. The reported correlations with VIX and VSTOXX are mostly negative, especially for the RMT and PCA counts, indicating that higher volatility is associated with a lower number of retained non-market factors.

Taken together, the supplementary evidence reinforces the main-text conclusion. The PCA 90% rule provides a useful descriptive benchmark, but it tends to retain many components that may not be clearly distinguishable from finite-sample noise. The RMT, Horn, and Onatski criteria provide more conservative factor-selection benchmarks and support a more parsimonious interpretation of the eigenvalue spectrum during crisis episodes.

Panel A: U.S. market

Period	Method	Mean	Median	Std. dev.	Min.	Max.	Corr. with VIX
Global Financial Crisis	RMT	3.05	3	0.89	1	5	-0.66
Global Financial Crisis	PCA	11.33	12	2.09	6	15	-0.76
Global Financial Crisis	Horn	1.81	2	0.73	1	4	-0.53
Global Financial Crisis	Onatski	1.78	2	0.63	1	3	-0.29
European Sovereign Debt Crisis	RMT	3.84	4	1.59	1	8	-0.75
European Sovereign Debt Crisis	PCA	12.8	13	2.03	8	16	-0.82
European Sovereign Debt Crisis	Horn	1.64	1	0.99	1	6	-0.43
European Sovereign Debt Crisis	Onatski	1.29	1	0.56	1	4	-0.16
COVID-19 Crisis	RMT	3.54	4	1.1	1	6	-0.61
COVID-19 Crisis	PCA	11.63	12	2.5	3	15	-0.86
COVID-19 Crisis	Horn	2.38	2	0.78	1	5	-0.42
COVID-19 Crisis	Onatski	2.19	2	0.61	1	4	-0.06

Panel B: European market

Period	Method	Mean	Median	Std. dev.	Min.	Max.	Corr. with VSTOXX
Global Financial Crisis	RMT	3.16	3	1.1	1	6	-0.43
Global Financial Crisis	PCA	12.35	13	1.75	7	15	-0.53
Global Financial Crisis	Horn	1.55	1	0.65	1	4	-0.34
Global Financial Crisis	Onatski	1.43	1	0.57	1	4	-0.14
European Sovereign Debt Crisis	RMT	3.35	3	1.37	1	6	-0.6
European Sovereign Debt Crisis	PCA	12.64	13	1.74	7	15	-0.72
European Sovereign Debt Crisis	Horn	1.46	1	0.68	1	4	-0.37
European Sovereign Debt Crisis	Onatski	1.29	1	0.47	1	3	-0.28
COVID-19 Crisis	RMT	3.53	3	1.13	1	7	-0.58
COVID-19 Crisis	PCA	12.1	12	1.89	4	15	-0.81
COVID-19 Crisis	Horn	2.31	2	0.83	1	6	-0.36
COVID-19 Crisis	Onatski	2.02	2	0.56	1	4	-0.05

Table 5: Factor-selection method comparison by period in the U.S. and European markets. The table reports descriptive statistics for the number of retained factors across the RMT-based MP count, the PCA 90% cumulative-variance rule, Horn’s parallel analysis, and the Onatski-type eigenvalue-spacing criterion.