Minimum Quality Standards and Benchmarking in Differentiated Duopoly

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Numerical example for the case of n=3 firms

In the following, the firms are labelled by $j=L,M,H$ with M indicating the "medium quality"-firm. Calculating along the same lines as in Section 3 of our paper, we obtain $q^o_{Lt}=(8a+z)/16\gamma$, $q^o_{Mt}=(8a+4z)/16\gamma$ and $q^o_{Ht}=(8a+7z)/16\gamma$ for the unregulated case. Assuming *strict* or *average regulation*, however, our model is for $n=3$ solvable only in numerical applications. In doing so, we assume $\gamma=1$ and $\alpha_i\in[1,2]$.[2] Moreover, to ensure market coverage in all scenarios considered, we fix the baseline utility at $\bar{u}=0.3$.[3] Since the discount factor is decisive for most of our results, we consider a "low discounting"-case with $\delta=0.9$ and a „high discounting"-case with $\delta=0.5$.[4] Finally, in order to guarantee an equilibrium under *average regulation* we have to assume that the standard introduced in $t=2$ relies on the average calculated over the highest and the lowest quality supplied in $t=1$.[5]

Table 1 displays the equilibria for $n=3$ and compares them to those obtained for $n=2$.[6] These figures suggest that our main results concerning the impact on qualities obtained for duopoly carry over to the case of three firms. In particular, under *strict* or *average regulation*, qualities decrease in $t=1$ and increase in $t=2$ compared to the unregulated equilibrium. Moreover, in the "high discounting"-case, the standards introduced in the second period are more severe than in the "low discounting"-case.

Calculating the corresponding profits for $n=3$ reveals that the commitment effect described in Section of our 4 still works in favor of firm L: Under *strict regulation*, the profits of firms M and H decrease compared to the unregulated equilibrium by up to 98 % after the standard has been introduced, whereas the profit of firm L decreases only by up to 31 %. Under *average regulation*, the profits of M and H decrease by up to 82 %, whereas the profit of L even increases by up to 28 %.

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2 In the literature, most numerical specified models on minimum quality standards assume $\alpha_i\in[0,1]$. Within our model, however, this assumption would lead to a corner solution in the case of $n=2$.

3 Note that the magnitude of $\bar{u}$ has no impact on the optimal triplet of qualities $\{q_L^{**},q_M^{**},q_H^{**}\}$ and on the standards introduced as long as the condition for market coverage is satisfied.

4 For example, if the useful life of the considered goods amounts to 5 years, the discount factor $\delta=0.9$ implies an annual discount rate of about 2.1 %, and $\delta=0.5$ implies about 14.4 %.

5 Using the average calculated over all three qualities supplied in $t=1$ would also allow firm M to directly influence the standard. In this case, however, there are too many strategical effects at the same time such that an equilibrium no longer exists.

6 In Tables 1 and 2, all uneven results are rounded to five decimal digits.

n = 2 firms		n = 3 firms		
Unregulated Equilibrium				
$q_{Lt}^{o}=0.37500$	$q_{Ht}^{o}=1.12500$	$q_{Lt}^{o}=0.56250$	$q_{Mt}^{o}=0.75000$	$q_{Ht}^{o}=0.93750$
Optimum				
$q_{Lt}^{**}=0.62500$	$q_{Ht}^{**}=0.87500$	$q_{Lt}^{**}=0.57393$	$q_{Mt}^{**}=0.75000$	$q_{Ht}^{**}=0.92607$
Optimal Standard				
$\overline{q}^{*}=0.63906$		$\overline{q}^{*}=0.57239$		
$q_{Lt}^{*}=0.63906$	$q_{Ht}^{*}=1.21302$	$q_{Lt}^{*}=0.57239$	$q_{Mt}^{*}=0.75565$	$q_{Ht}^{*}=0.93891$
Strict regulation – low discounting: $\delta=0.9$				
$\overline{q}^{s}=0.85942$		$\overline{q}^{s}=0.90533$		
$q_{L1}^{s}=0.28647$	$q_{H1}^{s}=0.85942$	$q_{L1}^{s}=0.55791$	$q_{M1}^{s}=0.73162$	$q_{H1}^{s}=0.90533$
$q_{L2}^{s}=0.85942$	$q_{H2}^{s}=1.28647$	$q_{L2}^{s}=0.90533$	$q_{M2}^{s}=0.94591$	$q_{H2}^{s}=0.98648$
Average regulation – low discounting: $\delta=0.9$				
$\overline{q}^{a}=0.62889$		$\overline{q}^{a}=0.67606$		
$q_{L1}^{a}=0.36755$	$q_{H1}^{a}=0.89022$	$q_{L1}^{a}=0.46204$	$q_{M1}^{a}=0.67606$	$q_{H1}^{a}=0.89009$
$q_{L2}^{a}=0.62889$	$q_{H2}^{a}=1.20963$	$q_{L2}^{a}=0.67606$	$q_{M2}^{a}=0.81493$	$q_{H2}^{a}=0.95372$
Strict regulation – high discounting: $\delta=0.5$				
$\overline{q}^{s}=1.03647$		$\overline{q}^{s}=0.92030$		
$q_{L1}^{s}=0.34549$	$q_{H1}^{s}=1.03647$	$q_{L1}^{s}=0.56004$	$q_{M1}^{s}=0.74017$	$q_{H1}^{s}=0.92030$
$q_{L2}^{s}=1.03647$	$q_{H2}^{s}=1.34549$	$q_{L2}^{s}=0.92030$	$q_{M2}^{s}=0.95446$	$q_{H2}^{s}=0.98862$
Average regulation – high discounting: $\delta=0.5$				
$\overline{q}^{a}=0.67581$		$\overline{q}^{a}=0.70490$		
$q_{L1}^{a}=0.35530$	$q_{H1}^{a}=0.99633$	$q_{L1}^{a}=0.49950$	$q_{M1}^{a}=0.70490$	$q_{H1}^{a}=0.91030$
$q_{L2}^{a}=0.67581$	$q_{H2}^{a}=1.22527$	$q_{L2}^{a}=0.70490$	$q_{M2}^{a}=0.83137$	$q_{H2}^{a}=0.95784$

Table 1: Numerical example with $n=3$ firms – standards and qualities in equilibrium.

Moreover, in line with Schmidt (2009) we observe that the differences between the unregulated and the optimal qualities drastically shrink down at the transition from $n=2$ to $n=3$. This effect is reflected in the results of our welfare analysis as shown in Table 2: Applying the optimal standard $\overline{q}^{*}$ leads only to a marginal increase in welfare, whereas *strict* or *average* regulation reduces welfare regardless of the discount factor applied (although the households' utility partially increases). Hence, concerning the benefits of an MQS in the case of more than two firms, our model essentially confirms the basic results already known from the literature.

	n = 2 firms		n = 3 firms	
	Unregulated Equilibrium			
Profits Π_t^o	Π_1^o=0.37500	Π_2^o=0.37500	Π_1^o=0.04720	Π_2^o=0.04720
Utility U_t^o	U_1^o=0.44063	U_2^o=0.44063	U_1^o=0.83329	U_2^o=0.83329
Welfare W_t^o	W_1^o=0.81563	W_2^o=0.81563	W_1^o=0.88049	W_2^o=0.88049
	Optimum			
Profits Π_t^{**}	Π_1^{**}=0.12500	Π_2^{**}=0.12500	Π_1^{**}=0.04445	Π_2^{**}=0.04445
Utility U_t^{**}	U_1^{**}=0.75313	U_2^{**}=0.75313	U_1^{**}=0.83610	U_2^{**}=0.83610
Welfare W_t^{**}	W_1^{**}=0.87813	W_2^{**}=0.87813	W_1^{**}=0.88055	W_2^{**}=0.88055
	Optimal Standard			
Profits Π_t^*	Π_1^*=0.30279	Π_2^*=0.30279	Π_1^*=0.04619	Π_2^*=0.04619
Utility U_t^*	U_1^*=0.53787	U_2^*=0.53787	U_1^*=0.83432	U_2^*=0.83432
Welfare W_t^*	W_1^*=0.84066	W_2^*=0.84066	W_1^*=0.88051	W_2^*=0.88051
	Strict regulation – low discounting: δ=0.9			
Profits Π_t^s	Π_1^s=0.30244	Π_2^s=0.25312	Π_1^s=0.04399	Π_2^s=0.01405
Utility U_t^s	U_1^s=0.53822	U_2^s=0.56236	U_1^s=0.83639	U_2^s=0.82252
Welfare W_t^s	W_1^s=0.84066	W_2^s=0.81548	W_1^s=0.88038	W_2^s=0.83657
	Average regulation – low discounting: δ=0.9			
Profits Π_t^a	Π_1^a=0.26815	Π_2^a=0.30516	Π_1^a=0.05597	Π_2^a=0.03664
Utility U_t^a	U_1^a=0.58524	U_2^a=0.53546	U_1^a=0.82195	U_2^a=0.84075
Welfare W_t^a	W_1^a=0.85339	W_2^a=0.84062	W_1^a=0.87792	W_2^a=0.87739
	Strict regulation – high discounting: δ=0.5			
Profits Π_t^s	Π_1^s=0.34763	Π_2^s=0.20793	Π_1^s=0.04545	Π_2^s=0.03664
Utility U_t^s	U_1^s=0.48107	U_2^s=0.54163	U_1^s=0.83503	U_2^s=0.81958
Welfare W_t^s	W_1^s=0.82870	W_2^s=0.74956	W_1^s=0.88048	W_2^s=0.83168
	Average regulation – high discounting: δ=0.5			
Profits Π_t^a	Π_1^a=0.32365	Π_2^a=0.29437	Π_1^a=0.05237	Π_2^a=0.03415
Utility U_t^a	U_1^a=0.51466	U_2^a=0.54567	U_1^a=0.82700	U_2^a=0.84103
Welfare W_t^a	W_1^a=0.83831	W_2^a=0.84004	W_1^a=0.87937	W_2^a=0.87518

Table 2: Numerical example with n=3 firms – welfare analysis.