Minimum Quality Standards and Benchmarking in Differentiated Duopoly

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Complete Appendix

In the following calculations, we frequently use the abbreviation $\eta := [405 - \lambda^2(18 - \lambda^2)]^{1/2}$ to simplify long expressions. Note that the term η is a strictly decreasing function of λ with $\eta = 2\sqrt{97} \approx 19.698$ for $\lambda = 1$ and $\eta = 18$ for $\lambda = 3$.

A.1 Check for Leapfrogging

The equilibria calculated in our paper require that leapfrogging can be ruled out. I.e., firm H has no incentive to choose a position $q_{Ht} < q_{Lt}$ when firm L sticks to its equilibrium position, and firm L has no incentive to choose a position $q_{Lt} > q_{Ht}$ when firm H sticks to its equilibrium position. As a pre-requisite, we note that under both regulations leapfrogging by firm H in the second period can be excluded a priori since it would violate the minimum quality standard.

We start with the *strict regulation*-equilibrium in the first period. Since the regulated and the unregulated equilibrium in $t=1$ coincide for $\lambda=3$, the following also proves that leapfrogging can be ruled out in the case without regulation. If firm H chooses a position $q_{H1} < q_{L1}$, its reduced profit function becomes $\hat{\pi}_{H1}(q_{H1}, q_{L1}) = (q_{L1} - q_{H1})[a - z - \gamma(q_{H1} + q_{L1})]^2 / 9z$. Replacing q_{L1} by firm $L's$ equilibrium position q_{L1}^s yields the profits of firm H under leapfrogging as a function of q_{H1}:

(A1) $$\hat{\pi}_{H1}(q_{H1}) = \frac{[a(1+\lambda) - z - 2\gamma(1+\lambda)q_{H1}][a(1+\lambda) - z(1+2\lambda) - 2\gamma(1+\lambda)q_{H1}]^2}{72z\gamma(1+\lambda)^3}.$$

Maximization of $\hat{\pi}_{H1}(q_{H1})$ leads to profits of $\hat{\pi}_{H1}^s = 4z^2\lambda^3 / 243\gamma(1+\lambda)^3$. Next, we define the ratio between profits without and profits with leapfrogging as $\sigma_{H1}^s := \pi_{H1}^s / \hat{\pi}_{H1}^s$. Inserting π_{H1}^s and $\hat{\pi}_{H1}^s$ yields $\sigma_{H1}^s = 27(3+\lambda)^2 / 4\lambda^2$. This ratio is a strictly decreasing function of λ with $\sigma_{H1}^s > 1$ for $\lambda = 3$. Hence, we obtain $\pi_{H1}^s > \hat{\pi}_{H1}^s$ for $\lambda \in [1,3)$ and leapfrogging by firm H in $t=1$ can be ruled out. In the same way, we obtain $\sigma_{L1}^s = \lambda^3$ in case of leapfrogging by firm L in $t=1$. For $\lambda \in (1,3)$ this implies $\pi_{L1}^s > \hat{\pi}_{L1}^s$, and leapfrogging by L can also be ruled out. Only the special case of $\lambda = 1$ leads to $\pi_{L1}^s = \hat{\pi}_{L1}^s$. However, if a cheap low-quality producer like firm L tries to penetrate the high-quality segment of the market, it will most likely be forced to change marketing strategies or distribution channels. Therefore, it seems to be reasonable to assume that leapfrogging will lead to additional costs. Although we are not able to quantify the

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latter, from $\pi_{L1}^s = \hat{\pi}_{L1}^s$ it is obvious that even assuming arbitrarily small costs suffices to prevent leapfrogging by L in $t=1$. Moreover, leapfrogging by L in $t=2$ can also be ruled out since we arrive at a ratio of $\sigma_{L2}^s = 27(1+3\lambda)^2/4$ implying $\pi_{L2}^s > \hat{\pi}_{L2}^s$ for $\lambda \in [1,3)$.

Calculating along the same lines for the equilibrium under *average regulation* yields the following expressions for the ratios $\sigma_{jt}^a = \pi_{jt}^a / \hat{\pi}_{jt}^a$:

(A2) $\sigma_{L1}^a = 8748(9+\lambda^2)(9+\lambda^2+\eta)^2/(27-\lambda^2)^3(63-\lambda^2-\eta)^2$,

(A3) $\sigma_{H1}^a = 8748(9+\lambda^2)(63-\lambda^2-\eta)^3/[(9+\lambda^2)(\lambda^2-27)+(45+\lambda^2)\eta]^3$,

(A4) $\sigma_{L2}^a = 27(45+\lambda^2+\eta)^2/(63-\lambda^2-\eta)^2$.

Due to $\eta = [405-\lambda^2(18-\lambda^2)]^{1/2}$, these ratios only depend on the magnitude of λ. Since the latter is restricted to $\lambda \in [1,3)$, it is easy to show by a simple plot that $\sigma_{jt}^a > 1$ holds within the relevant domain such that leapfrogging can be ruled out.

A.2 Proof of Proposition 1

We start with proving that firm H will not try to achieve a non-binding standard $\bar{q}^s \le q_{Lt}^o$ such that $\{q_{H1}^s, q_{L1}^s\}$ is indeed a Nash-equilibrium . In the first step, we assume that L sticks to q_{L1}^s and H chooses a quality $\tilde{q}_{H1} = q_{Lt}^o - \varepsilon$ with $\varepsilon \ge 0$ that results in a non-binding standard. Inserting $q_{Lt}^o = (4a-z)/8\gamma$ into $\tilde{q}_{H1}$ yields $\tilde{q}_{H1} = [4(a-2\gamma\varepsilon)-z]/8\gamma$. Next, inserting $\tilde{q}_{H1}$ as well as q_{L1}^s into $H's$ reduced profit function (6) and calculating the first derivative of $\pi_{H1}(\tilde{q}_{H1}, q_{L1}^s)$ with respect to ε yields:

(A5) $\partial\pi_{H1}(\tilde{q}_{H1}, q_{L1}^s)/\partial\varepsilon = -[8\gamma\varepsilon(1+\lambda)+z(21+17\lambda)][24\gamma\varepsilon(1+\lambda)+z(15+19\lambda)]/576z(1+\lambda)^2$.

Due to $\partial\pi_{H1}(\tilde{q}_{H1}, q_{L1}^s)/\partial\varepsilon < 0$, the profit of firm H in period $t=1$ is the larger, the smaller is the deviation ε. Consequently, $H's$ optimal strategy to achieve a non-binding standard, denoted by q_{H1}^n, requires $\varepsilon = 0$. This, in turn, implies $q_{H1}^n = q_{Lt}^o$.

In the second step, we obtain $\pi_{H1}(q_{H1}^n, q_{L1}^s)$ by inserting q_{H1}^n and q_{L1}^s into (6). Together with π_{H2}^o this leads to the present value of $H's$ profit in case of a non-binding standard, $\Pi_H^n = \pi_{H1}(q_{H1}^n, q_{L1}^s) + \delta \cdot \pi_{H2}^o$. The corresponding present value in case of the binding standard follows from $\Pi_H^s = \pi_{H1}^s + \delta \cdot \pi_{H2}^s$. Calculating the difference $\Pi_H^s - \Pi_H^n$ leads to:

(A6) $\Pi_H^s - \Pi_H^n = z^2(3+2\lambda)(3\lambda-1)^2[3+\lambda(13+6\lambda)]/4608\gamma(1+\lambda)^3 > 0$.

Due to $\Pi_H^s > \Pi_H^n$, the strategy $\{q_{H1}^n, q_{L1}^s\}$ does not pay for H and $\{q_{H1}^s, q_{L1}^s\}$ is indeed a Nash-equilibrium.

Now, we turn to the properties of the equilibrium as stated in Proposition 1. Calculating the corresponding differences in qualities, denoted by $\Delta q_{jt}^s := q_{jt}^s - q_{jt}^o$, yields:

(A7) $\Delta q_{L1}^s = z(\lambda-3)/8(1+\lambda)\gamma$,

(A8) $\Delta q_{H1}^s = 3z(\lambda-3)/8(1+\lambda)\gamma$, (A9) $\Delta q_{L2}^{s} = 3z(3\lambda - 1)/8(1+\lambda)\gamma$,

(A10) $\Delta q_{H2}^{s} = z(3\lambda - 1)/8(1+\lambda)\gamma$.

Due to $\lambda \in [1,3)$ these results imply $\Delta q_{j1}^{s} < 0$ and $\Delta q_{j2}^{s} > 0$ for $j = H, L$. Next, by rearranging terms the differences in product differentiation, denoted by $\Delta q_t^s := (q_{Ht}^s - q_{Lt}^s) - (q_{Ht}^o - q_{Lt}^o)$, can be written as $\Delta q_t^s = \Delta q_{Ht}^s - \Delta q_{Lt}^s$. Inserting (A7) to (A10) yields $\Delta q_1^s = z(\lambda - 3)/4(1+\lambda)\gamma < 0$ and $\Delta q_2^s = z(1 - 3\lambda)/4(1+\lambda)\gamma < 0$.

Moreover, due to $\Delta q_{j1}^{s} < 0$ and $\Delta q_1^s < 0$ we obtain $p_{j1}^s < p_{j1}^o$ for $j = H, L$ directly from (1) and (2). Calculating the differences in prices for the second period, $\Delta p_{j2}^s := p_{j2}^s - p_{j2}^o$, yields:

(A11) $\Delta p_{L2}^{s} = z(3\lambda - 1)[72a(1+\lambda) + z(39\lambda - 37)]/192\gamma(1+\lambda)^2$,

(A12) $\Delta p_{H2}^{s} = z(3\lambda - 1)[24a(1+\lambda) + z(15\lambda - 29)]/192\gamma(1+\lambda)^2$.

Due to $\lambda \in [1,3)$ and $z \le 2a$ for an interior solution, these results imply $\Delta p_{j2}^s > 0$ for $j = H, L$. Finally, calculating the differences in profits, denoted by $\Delta \pi_{jt}^s := \pi_{jt}^s - \pi_{jt}^o$, yields:

(A13) $\Delta \pi_{L1}^{s} = z^2(\lambda - 3)(9 + 30\lambda + 37\lambda^2)/144\gamma(1+\lambda)^3$,

(A14) $\Delta \pi_{H1}^{s} = z^2(\lambda - 3)(9 - 18\lambda - 11\lambda^2)/144\gamma(1+\lambda)^3$,

(A15) $\Delta \pi_{L2}^{s} = z^2(1 - 3\lambda)(9\lambda^2 - 18\lambda - 11)/144\gamma(1+\lambda)^3$,

(A16) $\Delta \pi_{H2}^{s} = z^2(1 - 3\lambda)(9\lambda^2 + 30\lambda + 37)/144\gamma(1+\lambda)^3$.

Due to $\lambda \in [1,3)$ these results imply $\Delta \pi_{L1}^{s} < 0$, $\Delta \pi_{H1}^{s} > 0$ and $\Delta \pi_{H2}^{s} < 0$. In contrast, the sign of the difference $\Delta \pi_{L2}^{s}$ depends on λ. Solving $\Delta \pi_{L2}^{s} = 0$ for λ and resubstituting $\delta = (9 - \lambda^2)/8$ we obtain $\Delta \pi_{L2}^{s} < 0$ if $\delta < \hat{\delta}$ and $\Delta \pi_{L2}^{s} > 0$ if $\delta > \hat{\delta}$ with $\hat{\delta} \approx 0.350$.

A.3 Proof of Proposition 2

First, we prove that none of the two firms has an incentive to strive for a non-binding standard such that $\{q_{H1}^a, q_{L1}^a\}$ is indeed a Nash-equilibrium. The calculations follow essentially the same lines as in Appendix A.2. We start with firm H: For given q_{L1}^a, a non-binding standard requires $(q_{L1}^a + q_{H1})/2 \le q_{Lt}^o$. Inserting q_{L1}^a according to (25) as well as $q_{Lt}^o = (4a - z)/8\gamma$ this inequality can be solved for $q_{H1} \le \hat{q}_{H1}$ with:

(A17) $\hat{q}_{H1} := [864a + z[45(15 - \eta) + (18 - \eta)\lambda^2 - \lambda^4]]/1728\gamma$.

Now we assume that firm H chooses a quality $\tilde{q}_{H1} = \hat{q}_{H1} - \varepsilon$ with $\varepsilon \ge 0$ resulting in a non-binding standard. Inserting $\tilde{q}_{H1}$ as well as q_{L1}^a into $H's$ reduced profit function (6) yields $\pi_{H1}(\tilde{q}_{H1}, q_{L1}^a)$. Next, from calculating the first derivative of $\pi_{H1}(\tilde{q}_{H1}, q_{L1}^a)$ with respect to ε we obtain:

(A18) $$\frac{\partial \pi_{H1}(\tilde{q}_{H1}, q_{L1}^a)}{\partial \varepsilon} = -\frac{(9z + 4\gamma\varepsilon)[1296\gamma\varepsilon + z[(9 - \lambda^2)^2 + \eta(45 + \lambda^2)]]}{15552z} < 0.$$

Consequently, $H's$ optimal strategy to achieve a non-binding standard, denoted by q_{H1}^{n}, requires $\varepsilon=0$. This in turn implies $q_{H1}^{n}=\hat{q}_{H1}$. Moreover, calculating the difference between $\Pi_{H}^{n}=\pi_{H1}(q_{H1}^{n},q_{L1}^{a})+\delta\cdot\pi_{H2}^{o}$ and $\Pi_{H}^{a}=\pi_{H1}^{a}+\delta\cdot\pi_{H2}^{a}$ yields $\Pi_{H}^{a}-\Pi_{H}^{n}=\varpi_{1}\cdot z^{2}/3359232\gamma$ with:

(A19) $\varpi_{1}:=3645(279-11\eta)+81(44\eta-801)\lambda^{2}+54(54+\eta)\lambda^{4}+(63-\eta)\lambda^{6}-\lambda^{8}$.

Due to $\lambda\in[1,3)$ we obtain $\varpi_{1}>0$ such that $\Pi_{H}^{a}>\Pi_{H}^{n}$. Hence, the strategy $\{q_{H1}^{n},q_{L1}^{a}\}$ does not pay for firm H.

Next, we turn to firm L: For a given q_{H1}^{a} a non-binding standard requires $\left(q_{L1}+q_{H1}^{a}\right)/2\leq q_{Lt}^{o}$. Inserting q_{H1}^{a} according to (26) as well as $q_{Lt}^{o}=(4a-z)/8\gamma$ this inequality can be solved for $q_{L1}\leq\hat{q}_{L1}$ with:

(A20) $\hat{q}_{L1}:=[864a-z[27(17+\eta)-(\eta-90)\lambda^{2}-\lambda^{4}]]/1728\gamma$.[2]

Now we assume that firm L chooses a quality $\tilde{q}_{L1}=\hat{q}_{L1}-\varepsilon$ with $\varepsilon\geq0$ resulting in a non-binding standard. Contrary to the argumentation above, we need two separate steps to prove the optimality of $\varepsilon=0$. In the first step, inserting q_{H1}^{a} and $\tilde{q}_{L1}$ into (3) yields the market share of firm L as a function of ε: $s_{L1}=(3z-4\gamma\varepsilon)/12z$. Hence, increasing ε reduces $L's$ market share. In the second step, we consider firm $L's$ profit *per unit of output*, denoted by $\breve{\pi}_{L1}(q_{H1},q_{L1}):=p_{L1}(q_{H1},q_{L1})-\gamma q_{L1}^{2}$. From inserting q_{H1}^{a} as well as $\tilde{q}_{L1}$ and calculating the first derivative with respect to ε we obtain:

(A21) $$\frac{\partial\breve{\pi}_{L1}(q_{H1}^{a},\tilde{q}_{L1})}{\partial\varepsilon}=-\frac{1728\gamma\varepsilon+z[27(\eta-15)+(90-\eta)\lambda^{2}-\lambda^{4}]}{2592}<0.$$

Hence, increasing ε reduces not only the market share of firm L but also its profit per unit of output. Therefore $L's$ optimal strategy to achieve a non-binding standard, q_{L1}^{n}, requires $\varepsilon=0$ implying $q_{L1}^{n}=\hat{q}_{L1}$. Finally, calculating the difference between $\Pi_{L}^{n}=\pi_{L1}(q_{H1}^{a},q_{L1}^{n})+\delta\cdot\pi_{L2}^{o}$ and $\Pi_{L}^{a}=\pi_{L1}^{a}+\delta\cdot\pi_{L2}^{a}$ yields $\Pi_{L}^{a}-\Pi_{L}^{n}=\varpi_{2}\cdot z^{2}/3359232\gamma$ with:

(A22) $\varpi_{2}:=729(17\eta-117)+81(279-4\eta)\lambda^{2}+54(\eta-18)\lambda^{4}+(63-\eta)\lambda^{6}-\lambda^{8}$.

Due to $\lambda\in[1,3)$ we obtain $\varpi_{3}>0$ such that $\Pi_{L}^{a}>\Pi_{L}^{n}$. Hence, the strategy $\{q_{H1}^{a},q_{L1}^{n}\}$ does not pay for firm L and $\{q_{H1}^{a},q_{L1}^{a}\}$ is indeed a Nash-equilibrium.

Now, we turn to the properties of the equilibrium as stated in Proposition 2. Calculating the corresponding differences in qualities, denoted by $\Delta q_{jt}^{a}:=q_{jt}^{a}-q_{jt}^{o}$, yields:

(A23) $\Delta q_{L1}^{a}=z[9(5\eta-99)+(\eta-18)\lambda^{2}+\lambda^{4}]/1728\gamma$,

(A24) $\Delta q_{H1}^{a}=z[27(\eta-39)-(\eta-90)\lambda^{2}-\lambda^{4}]/1728\gamma$,

(A25) $\Delta q_{L2}^{a}=z(\eta-9+\lambda^{2})/48\gamma$,

(A26) $\Delta q_{H2}^{a}=z(\eta-9+\lambda^{2})/144\gamma$.

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[2] Note that if the ratio z/a becomes too large we obtain $\hat{q}_{L1}<0$ such that L has no possibility at all to achieve a non-binding standard. In contrast, with respect to H such a situation can be ruled out as long as the condition for an interior solution (i.e., $z\leq2a$) is satisfied.

Due to $\lambda \in [1,3)$ we obtain $\Delta q_{j1}^{a} < 0$ and $\Delta q_{j2}^{a} > 0$ for $j = H, L$. Moreover, similar to Appendix A.2, the differences in product differentiation are given by $\Delta q_{t}^{a} := \Delta q_{Ht}^{a} - \Delta q_{Lt}^{a}$. Inserting (A23) to (A26) yields $\Delta q_{1}^{a} = z[(54-\eta)\lambda^2 - 9(9+\eta) - \lambda^4]/864\gamma < 0$ and $\Delta q_{2}^{a} = z(9-\eta-\lambda^2)/72\gamma < 0$.

Next, due to $\Delta q_{j1}^{a} < 0$ and $\Delta q_{1}^{a} < 0$ we obtain $p_{j1}^{a} < p_{j1}^{o}$ for $j = H, L$ directly from (1) and (2). Moreover, calculating the differences in prices for the second period, $\Delta p_{j2}^{a} := p_{j2}^{a} - p_{j2}^{o}$, yields:

(A27) $\Delta p_{L2}^{a} = z(\lambda^2 + \eta - 9)[1296a + z(19\lambda^2 + 19\eta - 495)]/62208\gamma$,

(A28) $\Delta p_{H2}^{a} = z(\lambda^2 + \eta - 9)[432a + z(11\lambda^2 + 11\eta - 423)]/62208\gamma$.

Due to $\lambda \in [1,3)$ and $z \le 2a$ for an interior solution these results imply $\Delta p_{j2}^{a} > 0$ for $j = H, L$. Finally, calculating the differences in profits, denoted by $\Delta \pi_{jt}^{a} := \pi_{jt}^{a} - \pi_{jt}^{o}$, yields:

(A29) $\Delta \pi_{L1}^{a} = z^2[(9+\lambda^2)(9+\eta+\lambda^2)^2(63-\eta-\lambda^2) - 839808]/4478976\gamma$,

(A30) $\Delta \pi_{H1}^{a} = z^2[(9+\lambda^2)(63-\eta-\lambda^2)^3 - 839808]/4478976\gamma$,

(A31) $\Delta \pi_{L2}^{a} = z^2[(45+\eta+\lambda^2)^2(63-\eta-\lambda^2) - 157464]/839808\gamma$,

(A32) $\Delta \pi_{H2}^{a} = z^2[(63-\eta-\lambda^2)^3 - 157464]/839808\gamma$.

Due to $\lambda \in [1,3)$ we obtain $\Delta \pi_{L1}^{a} < 0$, $\Delta \pi_{L2}^{a} > 0$ and $\Delta \pi_{H2}^{a} < 0$. In contrast, the sign of $\Delta \pi_{H1}^{a}$ depends on λ. Solving $\Delta \pi_{H1}^{a} = 0$ for λ and resubstituting $\delta = (9-\lambda^2)/8$ we obtain $\Delta \pi_{H1}^{a} < 0$ if $\delta > \bar{\delta}$ and $\Delta \pi_{H1}^{a} > 0$ if $\delta < \bar{\delta}$ with $\bar{\delta} \approx 0.696$.

A.4 Comparison of Standards and Proof of Proposition 3

Starting with the comparison of standards, we calculate $\Delta \bar{q}^{s} := \bar{q}^{*} - \bar{q}^{s}$ and $\Delta \bar{q}^{a} := \bar{q}^{*} - \bar{q}^{a}$. This yields $\Delta \bar{q}^{s} = 3z[\sqrt{145} - 35 + 40(1+\lambda)^{-1}]/80\gamma$ and $\Delta \bar{q}^{a} = z[9\sqrt{145} - 5(\lambda^2 + \eta)]/240\gamma$. Due to $\lambda \in [1,3)$ we obtain $\Delta \bar{q}^{s} < 0$. In contrast, the sign of $\Delta \bar{q}^{a}$ depends on λ. Solving $\Delta \bar{q}^{a} = 0$ for λ and resubstituting $\delta = (9-\lambda^2)/8$ yields $\Delta \bar{q}^{a} > 0$ if $\delta > \breve{\delta}$ and $\Delta \bar{q}^{a} < 0$ if $\delta < \breve{\delta}$ with $\breve{\delta} \approx 0.806$.

We now turn to the differences in quality as stated in Proposition 3. In doing so, we use the notation $\Delta q_{jt}^{sa} := q_{jt}^{s} - q_{jt}^{a}$, $\Delta q_{jt}^{*s} := q_{jt}^{*} - q_{jt}^{s}$ and $\Delta q_{jt}^{*a} := q_{jt}^{*} - q_{jt}^{a}$. Concerning the comparison between *strict* and *average regulation* we obtain:

(A33) $\Delta q_{L1}^{sa} = z[27(41\lambda + 9) - (1+\lambda)[45\eta + (\eta - 18)\lambda^2 + \lambda^4]]/1728(1+\lambda)\gamma$,

(A34) $\Delta q_{H1}^{sa} = z[81(21\lambda - 11) - (1+\lambda)[27\eta - (\eta - 90)\lambda^2 - \lambda^4]]/1728(1+\lambda)\gamma$,

(A35) $\Delta q_{L2}^{sa} = z[63\lambda - 9 - (1+\lambda)(\eta + \lambda^2)]/48(1+\lambda)\gamma$,

(A36) $\Delta q_{H2}^{sa} = z[63\lambda - 9 - (1+\lambda)(\eta + \lambda^2)]/144(1+\lambda)\gamma$.

Due to $\lambda \in [1,3)$ we obtain $\Delta q_{j2}^{sa} > 0$ for $j = H, L$. In contrast, the signs of Δq_{j1}^{sa} depend on λ. Solving $\Delta q_{j1}^{sa} = 0$ for λ and resubstituting $\delta = (9-\lambda^2)/8$ yields:

– $\Delta q_{L1}^{sa} < 0$ if $\delta > \tilde{\delta}_L$ and $\Delta q_{L1}^{sa} > 0$ if $\delta < \tilde{\delta}_L$ with $\tilde{\delta}_L \approx 0.349$,

– $\Delta q_{H1}^{sa} < 0$ if $\delta > \tilde{\delta}_H$ and $\Delta q_{H1}^{sa} > 0$ if $\delta < \tilde{\delta}_H$ with $\tilde{\delta}_H \approx 0.829$.

We now turn to the comparison between *strict* or *average regulation* on the one hand and the optimal standard on the other hand. For $t=1$, an explicit calculation of Δq_{j1}^{*s} and Δq_{j1}^{*a} is not necessary since the results are obvious: The optimal standard induces an increase in qualities, whereas *strict* or *average regulation* induces a decrease in qualities. This implies $\Delta q_{j1}^{*s}>0$ and $\Delta q_{j1}^{*a}>0$ for $j=H,L$. For $t=2$ we note that $q_{L2}^{s}=\overline{q}^{s}$, $q_{L2}^{a}=\overline{q}^{a}$ and $q_{L2}^{*}=\overline{q}^{*}$ since all three standards are binding for firm L. Hence, our above-derived results concerning the relation between the different standards are directly transferable: $\Delta q_{L2}^{*s}=\Delta\overline{q}^{s}$ and $\Delta q_{L2}^{*a}=\Delta\overline{q}^{a}$. Moreover, concerning firm H we obtain $\Delta q_{H2}^{*s}=\Delta q_{L2}^{*s}/3$ and $\Delta q_{H2}^{*a}=\Delta q_{L2}^{*a}/3$. These results imply for $j=H,L$: $\Delta q_{j2}^{*s}<0$ as well as $\Delta q_{j2}^{*a}>0$ if $\delta>\breve{\delta}$ and $\Delta q_{j2}^{*a}<0$ if $\delta<\breve{\delta}$ with $\breve{\delta}\approx 0.806$.

Finally, we turn to the degree of product differentiation. For comparing *strict* and *average regulation* we calculate the differences $\Delta q_t^{sa}:=(q_{Ht}^{s}-q_{Lt}^{s})-(q_{Ht}^{a}-q_{Lt}^{a})$:

(A37) $\Delta q_1^{sa}=z[(1+\lambda)[9\eta+(\eta-54)\lambda^2+\lambda^4]+297\lambda-567]/864(1+\lambda)\gamma$,

(A38) $\Delta q_2^{sa}=z[9+\eta+\lambda^2(1+\lambda)-\lambda(63-\eta)]/72(1+\lambda)\gamma$.

Due to $\lambda\in[1,3)$ we obtain $\Delta q_1^{sa}>0$ and $\Delta q_2^{sa}<0$. For comparing *strict* and *average regulation* with the optimal standard we calculate the differences $\Delta q_t^{*s}:=(q_{Ht}^{*}-q_{Lt}^{*})-(q_{Ht}^{s}-q_{Lt}^{s})$ and $\Delta q_t^{*a}:=(q_{Ht}^{*}-q_{Lt}^{*})-(q_{Ht}^{a}-q_{Lt}^{a})$:

(A39) $\Delta q_1^{*s}=z[40(1+\lambda)^{-1}-5-\sqrt{145}]/40\gamma$,

(A40) $\Delta q_2^{*s}=z[35-\sqrt{145}-40(1+\lambda)^{-1}]/40\gamma$,

(A41) $\Delta q_1^{*a}=z[945-108\sqrt{145}+45\eta+5\lambda^2(\lambda^2+\eta-54)]/4320\gamma$,

(A42) $\Delta q_2^{*a}=z[5(\lambda^2+\eta)-9\sqrt{145}]/360\gamma$.

From (A.40) we obtain $\Delta q_2^{*s}>0$ for $\lambda\in[1,3)$. The signs of the other three differences depend on the magnitude of λ. Solving $\Delta q_1^{*s}=0$, $\Delta q_1^{*a}=0$ and $\Delta q_2^{*a}=0$ for λ and re-substituting $\delta=(9-\lambda^2)/8$ we obtain:

– $\Delta q_1^{*s}>0$ if $\delta>\overline{\delta}_1$ and $\Delta q_1^{*s}<0$ if $\delta<\overline{\delta}_1$ with $\overline{\delta}_1\approx 0.898$,
– $\Delta q_1^{*a}>0$ if $\delta>\overline{\delta}_2$ and $\Delta q_1^{*a}<0$ if $\delta<\overline{\delta}_2$ with $\overline{\delta}_2\approx 0.736$,
– $\Delta q_2^{*a}>0$ if $\delta<\overline{\delta}_3$ and $\Delta q_2^{*a}<0$ if $\delta>\overline{\delta}_3$ with $\overline{\delta}_3\approx 0.806$.

A.5 Proof of Proposition 4

We first compare aggregated profits in the unregulated case with the results from the different standards. Calculating $\Delta\Pi_t^{so}:=\Pi_t(q_{Lt}^{s},q_{Ht}^{s})-\Pi_t(q_{Lt}^{o},q_{Ht}^{o})$, $\Delta\Pi_t^{ao}:=\Pi_t(q_{Lt}^{a},q_{Ht}^{a})-\Pi_t(q_{Lt}^{o},q_{Ht}^{o})$ and $\Delta\Pi_t^{*o}:=\Pi_t(q_{Lt}^{*},q_{Ht}^{*})-\Pi_t(q_{Lt}^{o},q_{Ht}^{o})$ we obtain:

(A43) $\Delta\Pi_1^{so}=z^2(\lambda-3)[9+\lambda(6+13\lambda)]/72\gamma(1+\lambda)^3$,

(A44) $\Delta\Pi_2^{so}=z^2(1-3\lambda)[13+\lambda(6+9\lambda)]/72\gamma(1+\lambda)^3$,

(A45) $\Delta\Pi_1^{ao}=z^2[13122(14-\eta)+891(27-\eta)\lambda^2+27(2\eta-57)\lambda^4+(63-\eta)\lambda^6-\lambda^8]/559872\gamma$, (A46) $\Delta\Pi_2^{ao} = z^2(9-\eta-\lambda^2)[3483-\eta(72-\eta)+2(\eta-36)\lambda^2+\lambda^4]/419904\gamma$,

(A47) $\Delta\Pi_t^{*o} = z^2(595-71\sqrt{145})/3600\gamma$ for $t=1,2$.

Due to $\lambda\in[1,3)$ these results imply $\Delta\Pi_t^{so}<0$, $\Delta\Pi_t^{ao}<0$ and $\Delta\Pi_t^{*o}<0$ for $t=1,2$.

Next, we calculate the differences $\Delta\Pi_t^{sa} := \Pi_t(q_{Lt}^s, q_{Ht}^s) - \Pi_t(q_{Lt}^a, q_{Ht}^a)$ for the comparison between *strict* and *average regulation*. This leads to $\Delta\Pi_t^{sa} = z^2\theta_t/839808\gamma(1+\lambda)^3$ with:

(A48) $\theta_1 := \frac{3}{16}(1+\lambda)^3(9+\lambda^2)(\eta+\lambda^2-63)[(\eta+\lambda^2-63)^2+(\eta+\lambda^2+9)^2]+93312\lambda[9+\lambda(6+5\lambda)]$,

(A49) $\theta_2 := 93312[9+\lambda(6+9\lambda)]+(1+\lambda)^3(\eta+\lambda^2-63)[(\eta+\lambda^2-63)^2+(\eta+\lambda^2+45)^2]$.

Due to $\lambda\in[1,3)$ we obtain $\theta_1>0\rightarrow\Delta\Pi_1^{sa}>0$ and $\theta_2<0\rightarrow\Delta\Pi_2^{sa}<0$.

Next, we calculate $\Delta\Pi_t^{*s} := \Pi_t(q_{Lt}^*, q_{Ht}^*) - \Pi_t(q_{Lt}^s, q_{Ht}^s)$ for comparing *strict regulation* with the optimal standard:

(A50) $\Delta\Pi_1^{*s} = z^2[1945-71\sqrt{145}-400\lambda(9+6\lambda+5\lambda^2)(1+\lambda)^{-3}]/3600\gamma$,

(A51) $\Delta\Pi_2^{*s} = z^2[1945-71\sqrt{145}-400(5+6\lambda+9\lambda^2)(1+\lambda)^{-3}]/3600\gamma$.

Due to $\lambda\in[1,3)$, (A.51) implies $\Delta\Pi_2^{*s}>0$. In contrast, the sign of $\Delta\Pi_1^{*s}$ depends on λ. From solving $\Delta\Pi_1^{*s}=0$ and resubstituting $\delta=(9-\lambda^2)/8$ we obtain $\Delta\Pi_1^{*s}>0$ if $\delta>\tilde{\delta}_1$ and $\Delta\Pi_1^{*s}<0$ if $\delta<\tilde{\delta}_1$ with $\tilde{\delta}_1\approx0.898$.

Finally, we calculate $\Delta\Pi_t^{*a} := \Pi_t(q_{Lt}^*, q_{Ht}^*) - \Pi_t(q_{Lt}^a, q_{Ht}^a)$ for comparing *average regulation* with the optimal standard. This leads to $\Delta\Pi_t^{*a} = z^2\psi_t/5248800\gamma$ with:

(A52) $\psi_1 := \frac{3}{16}[7776(1945-71\sqrt{145})+25(9+\lambda^2)(\lambda^2+\eta-63)[27(45-\eta)+(\eta-36)\lambda^2+\lambda^4]]$,

(A53) $\psi_2 := 1458(1945-71\sqrt{145})+25(\lambda^2+\eta-63)[9(189-\eta)+(\eta-18)\lambda^2+\lambda^4]$.

From solving $\psi_t=0$ for λ and resubstituting $\delta=(9-\lambda^2)/8$ we obtain:

– $\psi_1>0\rightarrow\Delta\Pi_1^{*a}>0$ if $\delta>\tilde{\delta}_2$ and $\psi_1<0\rightarrow\Delta\Pi_1^{*a}<0$ if $\delta<\tilde{\delta}_2$ with $\tilde{\delta}_2\approx0.662$,
– $\psi_2<0\rightarrow\Delta\Pi_2^{*a}<0$ if $\delta>\tilde{\delta}_3$ and $\psi_2>0\rightarrow\Delta\Pi_2^{*a}>0$ if $\delta<\tilde{\delta}_3$ with $\tilde{\delta}_3\approx0.806$.

A.6 Proof of Proposition 5

We first compare consumers' aggregated net utility in the unregulated case with the results obtained from using the different standards. Calculating $\Delta U_t^{so} := U_t(q_{Lt}^s, q_{Ht}^s) - U_t(q_{Lt}^o, q_{Ht}^o)$, $\Delta U_t^{ao} := U_t(q_{Lt}^a, q_{Ht}^a) - U_t(q_{Lt}^o, q_{Ht}^o)$ and $\Delta U_t^{*o} := U_t(q_{Lt}^*, q_{Ht}^*) - U_t(q_{Lt}^o, q_{Ht}^o)$ yields:

(A54) $\Delta U_1^{so} = z^2(3-\lambda)[5\lambda(30+29\lambda)-27]/576\gamma(1+\lambda)^3$,

(A55) $\Delta U_2^{so} = z^2(3\lambda-1)[3\lambda(50-9\lambda)+145]/576\gamma(1+\lambda)^3$,

(A56) $\Delta U_1^{ao} = z^2[\lambda^4[243(\eta-30)+(288-5\eta)\lambda^2-5\lambda^4]+81[\eta(1449+\lambda^2)-81(233+8\lambda^2)]]/4478976\gamma$,

(A57) $\Delta U_2^{ao} = z^2[81(143\eta-1953)+9(1566-35\eta)\lambda^2-(297+2\eta+2\lambda^2)\lambda^4]/839808\gamma$, (A58) $\Delta U_t^{*o} = z^2(713\sqrt{145} - 5785)/28800\gamma$ for $t=1,2$.

Due to $\lambda \in [1,3)$ these results imply $\Delta U_t^{so} > 0$ and $\Delta U_t^{ao} > 0$ and $\Delta U_t^{*o} > 0$ for $t=1,2$.

Next, we calculate the differences $\Delta U_t^{sa} := U_t(q_{Lt}^s, q_{Ht}^s) - U_t(q_{Lt}^a, q_{Ht}^a)$ for comparing *strict* and *average regulation*. This leads to $\Delta U_t^{sa} = z^2(\varphi_t - \mu_t)/839808\gamma(1+\lambda)^3$ with:

(A59) $\varphi_1 := \frac{3}{16}(1+\lambda)^3[5103(615-23\eta)+81(648-\eta)\lambda^2+243(30-\eta)\lambda^4+(5\eta+5\lambda^2-288)\lambda^6]$,

(A60) $\mu_1 := 23328[18+9\lambda+(21+22\lambda)\lambda^2]$,

(A61) $\varphi_2 := (1+\lambda)^3[81(5679-143\eta)+9(35\eta-1566)\lambda^2+(297+2\eta)\lambda^4+2\lambda^6]$,

(A62) $\mu_2 := 23328[22+21\lambda+(9+18\lambda)\lambda^2]$.

Due to $\lambda \in [1,3)$ we obtain $\varphi_1 - \mu_1 < 0 \to \Delta U_1^{sa} < 0$. In contrast, the sign of $\varphi_2 - \mu_2$ depends on λ. Solving $\varphi_2 - \mu_2 = 0$ for λ and resubstituting $\delta = (9-\lambda^2)/8$ we obtain $\varphi_2 - \mu_2 > 0 \to \Delta U_2^{sa} > 0$ if $\delta > \hat{\delta}_1$ and $\varphi_2 - \mu_2 < 0 \to \Delta U_2^{sa} < 0$ if $\delta < \hat{\delta}_1$ with $\hat{\delta}_1 \approx 0.549$.

Next, we calculate $\Delta U_t^{*s} := U_t(q_{Lt}^*, q_{Ht}^*) - U_t(q_{Lt}^s, q_{Ht}^s)$ for comparing *strict regulation* with the optimal standard:

(A63) $\Delta U_1^{*s} = z^2[1465+713\sqrt{145}+800[8(1+\lambda)^{-3}+33(1+\lambda)^{-2}+45(1+\lambda)^{-1}]]/28800\gamma$,

(A64) $\Delta U_2^{*s} = z^2[713\sqrt{145}-1735-800[8(1+\lambda)^{-3}-57(1+\lambda)^{-2}+45(1+\lambda)^{-1}]]/28800\gamma$.

The signs of ΔU_1^{*s} and ΔU_2^{*s} depend on λ. From solving both terms for λ and resubstituting $\delta = (9-\lambda^2)/8$ we obtain:

– $\Delta U_1^{*s} < 0$ if $\delta > \hat{\delta}_2$ and $\Delta U_1^{*s} > 0$ if $\delta < \hat{\delta}_2$ with $\hat{\delta}_2 \approx 0.898$,

– $\Delta U_2^{*s} < 0$ if $\delta > \hat{\delta}_3$ and $\Delta U_2^{*s} > 0$ if $\delta < \hat{\delta}_3$ with $\hat{\delta}_3 \approx 0.433$.

Finally, we calculate the difference $\Delta U_t^{*a} := U_t(q_{Lt}^*, q_{Ht}^*) - U_t(q_{Lt}^a, q_{Ht}^a)$ for comparing *average regulation* with the optimal standard. This leads to $\Delta U_t^{*a} = z^2\kappa_t/111974400\gamma$ with:

(A65) $\kappa_1 := 243(64715+11408\sqrt{145}-12075\eta)+25\lambda^2[81(648-\eta)+243(30-\eta)\lambda^2+(5\eta-288)\lambda^4+5\lambda^6]$,

(A66) $\kappa_2 := \frac{16}{3}[81(6417\sqrt{145}-3240-3575\eta)+25\lambda^2[9(35\eta-1566)+(297+2\eta)\lambda^2+2\lambda^4]]$.

The signs of κ_1 and κ_2 depend on λ. From solving both terms for λ and resubstituting $\delta = (9-\lambda^2)/8$ we obtain:

– $\kappa_1 < 0 \to \Delta U_1^{*a} < 0$ if $\delta > \hat{\delta}_4$ and $\kappa_1 > 0 \to \Delta U_1^{*a} > 0$ if $\delta < \hat{\delta}_4$ with $\hat{\delta}_4 \approx 0.636$,

– $\kappa_2 > 0 \to \Delta U_2^{*a} > 0$ if $\delta > \hat{\delta}_5$ and $\kappa_2 < 0 \to \Delta U_2^{*a} < 0$ if $\delta < \hat{\delta}_5$ with $\hat{\delta}_5 \approx 0.806$.

A.7 Proof of Proposition 6

We first compare welfare in period *t* in the unregulated case with the results from the different standards. Calculating $\Delta W_t^{so} := W_t(q_{Lt}^s, q_{Ht}^s) - W_t(q_{Lt}^o, q_{Ht}^o)$, $\Delta W_t^{ao} := W_t(q_{Lt}^a, q_{Ht}^a) - W_t(q_{Lt}^o, q_{Ht}^o)$ and $\Delta W_t^{*o} := W_t(q_{Lt}^*, q_{Ht}^*) - W_t(q_{Lt}^o, q_{Ht}^o)$ yields:

(A67) $\Delta W_1^{so} = z^2(3-\lambda)[\lambda(102+41\lambda)-99]/576\gamma(1+\lambda)^3$,

(A68) $\Delta W_2^{so} = z^2(3\lambda-1)[\lambda(102-99\lambda)+41]/576\gamma(1+\lambda)^3$,

(A69) $\Delta W_1^{ao} = z^2[729(17\eta-81)+243(576-29\eta)\lambda^2+27(25\eta-726)\lambda^4+(792-13\eta-13\lambda^2)\lambda^6]/4478976\gamma$,

(A70) $\Delta W_2^{ao} = z^2[81(31\eta-369)+9(54+5\eta)\lambda^2-5(2\eta+2\lambda^2-27)\lambda^4]/839808\gamma$,

(A71) $\Delta W_t^{*o} = z^2(29\sqrt{145}-205)]/5760\gamma$ for $t=1,2$.

Due to $\lambda\in[1,3)$ we obtain $\Delta W_1^{so}>0$ as well as $\Delta W_t^{ao}>0$ and $\Delta W_t^{*o}>0$ for $t=1,2$. In contrast, the sign of ΔW_2^{so} depends on λ. From solving $\Delta W_2^{so}=0$ for λ and resubstituting $\delta=(9-\lambda^2)/8$ we obtain $\Delta W_2^{so}>0$ if $\delta>\bar{\delta}_1$ and $\Delta W_2^{so}<0$ if $\delta<\bar{\delta}_1$ with $\bar{\delta}_1\approx 0.901$.

Next, we calculate $\Delta W_t^{sa} := W_t(q_{Lt}^s, q_{Ht}^s) - W_t(q_{Lt}^a, q_{Ht}^a)$ for comparing *strict* and *average regulation*. This leads to $\Delta W_t^{sa} = z^2(\omega_t - \chi_t)/839808\gamma(1+\lambda)^3$ with:

(A72) $\omega_1 := \frac{3}{16}(1+\lambda)^3[729(81-17\eta)+243(29\eta-576)\lambda^2+27(726-25\eta)\lambda^4+(13\eta+13\lambda^2-792)\lambda^6]$,

(A73) $\chi_1 := 1458(\lambda-3)(102\lambda+41\lambda^2-99)$,

(A74) $\omega_2 := (1+\lambda)^3[81(369-31\eta)-9(54+5\eta)\lambda^2+5(2\eta+2\lambda^2-27)\lambda^4]$,

(A75) $\chi_2 := 1458(3\lambda-1)(99\lambda^2-102\lambda-41)$.

Due to $\lambda\in[1,3)$ we obtain $\omega_t-\chi_t<0 \rightarrow \Delta W_t^{sa}<0$ for $t=1,2$.

In the next step, we calculate $\Delta W_t^{*s} := W_t(q_{Lt}^*, q_{Ht}^*) - W_t(q_{Lt}^s, q_{Ht}^s)$ for comparing *strict regulation* with the optimal standard. This leads to $\Delta W_t^{*s} = z^2\phi_t/5760\gamma(1+\lambda)^3$ with:

(A76) $\phi_1 := 2765+29\sqrt{145}+(87\sqrt{145}-4665)\lambda+(87\sqrt{145}-825)\lambda^2+(205+29\sqrt{145})\lambda^3$,

(A77) $\phi_2 := 205+29\sqrt{145}+(87\sqrt{145}-825)\lambda+(87\sqrt{145}-4665)\lambda^2+(2765+29\sqrt{145})\lambda^3$.

Due to $\lambda\in[1,3)$, these results imply $\phi_t>0 \rightarrow \Delta W_t^{*s}>0$ for $t=1,2$. The only exception is the special case of $\delta\approx 0.898$ which leads to $\Delta W_1^{*s}=0$.

Finally, we calculate $\Delta W_t^{*a} := W_t(q_{Lt}^*, q_{Ht}^*) - W_t(q_{Lt}^a, q_{Ht}^a)$ for comparing *average regulation* with the optimal standard. This leads to $\Delta W_t^{*a} = z^2\nu_t/22394880\gamma$ with:

(A78) $\nu_1 := 243(464\sqrt{145}-2065-255\eta)+5\lambda^2[243(29\eta-576)+27(726-25\eta)\lambda^2+(13\eta-792)\lambda^4+13\lambda^6]$,

(A79) $\nu_2 := \frac{16}{3}[81(261\sqrt{145}-155\eta)-5\lambda^2[9(54+5\eta)-5(2\eta-27)\lambda^2-10\lambda^4]]$.

Due to $\lambda\in[1,3)$, (A79) implies $\nu_2>0 \rightarrow \Delta W_2^{*a}>0$ except for the special case of $\delta\approx 0.806$ which leads to $\Delta W_2^{*a}=0$. In contrast, the sign of ν_1 depends on λ. From solving $\nu_1=0$ for λ and resubstituting $\delta=(9-\lambda^2)/8$ we obtain $\nu_1<0 \rightarrow \Delta W_1^{*a}<0$ if $\delta>\bar{\delta}_2$ and $\nu_1>0 \rightarrow \Delta W_1^{*a}>0$ if $\delta<\bar{\delta}_2$ with $\bar{\delta}_2\approx 0.520$.

A.8 Proof of Proposition 7

The relations $W^* > \max\{W^o, W^s\}$ and $W^a > \max\{W^o, W^s\}$ follow directly from Proposition 6. To check the remaining relations, we first calculate $\Delta W^{so} := W(q^s_{Lt}, q^s_{Ht}) - W(q^o_{Lt}, q^o_{Ht})$:

(A80) $\Delta W^{so} = z^2(\lambda-3)[915+\lambda[27\lambda(7+11\lambda)-1753]]/4608\gamma(1+\lambda)^2$.

The sign of ΔW^{so} depends on λ. Solving $\Delta W^{so} = 0$ for λ and resubstituting $\delta = (9-\lambda^2)/8$ we obtain $\Delta W^{so} > 0$ if $\delta > \breve{\delta}_1$ and $\Delta W^{so} < 0$ if $\delta < \breve{\delta}_1$ with $\breve{\delta}_1 \approx 0.747$. Next, we calculate the difference $\Delta W^{*a} := W(q^*_{Lt}, q^*_{Ht}) - W(q^a_{Lt}, q^a_{Ht})$. This yields $\Delta W^{*a} = z^2(\upsilon_1 + \upsilon_2)/67184640\gamma$ with:

(A81) $\upsilon_1 := 729(986\sqrt{145} - 565\eta - 2065) - 81(522\sqrt{145} - 1565\eta + 26460)\lambda^2$,

(A82) $\upsilon_2 := 135(2124 - 65\eta)\lambda^4 + (95\eta - 9630)\lambda^6 + 95\lambda^8$.

The sign of $\upsilon_1 + \upsilon_2$ depends on λ. Solving $\upsilon_1 + \upsilon_2$ for λ and resubstituting $\delta = (9-\lambda^2)/8$ we obtain $\upsilon_1 + \upsilon_2 < 0 \rightarrow \Delta W^{*a} < 0$ if $\delta > \breve{\delta}_2$ and $\upsilon_1 + \upsilon_2 > 0 \rightarrow \Delta W^{*a} > 0$ if $\delta < \breve{\delta}_2$ with $\breve{\delta}_2 \approx 0.558$.

Moreover, calculating the difference $\Delta W^{ao} := W(q^a_{Lt}, q^a_{Ht}) - W(q^o_{Lt}, q^o_{Ht})$ and differentiating the resulting expression with respect to λ yields $\partial \Delta W^{ao}/\partial\lambda = \lambda z^2 \upsilon_3 / 3359232\gamma\eta$ with:

(A83) $\upsilon_3 := 729(335\eta - 7751) + 243(4503 - 236\eta)\lambda^2 + 27(107\eta - 2829)\lambda^4 + (3231 - 38\eta - 38\lambda^2)\lambda^6$.

For the difference in normalized welfare, $\Delta\tilde{W}^{ao} := \tilde{W}(q^a_{Lt}, q^a_{Ht}) - \tilde{W}(q^o_{Lt}, q^o_{Ht})$, an analogous calculation yields $\partial\Delta\tilde{W}^{ao}/\partial\lambda = \lambda z^2(\upsilon_4 + \upsilon_5)/839808\gamma(\lambda^2 - 17)^2\eta$ with:

(A84) $\upsilon_4 := 729(10409\eta - 210969) + 243(150051 - 8024\eta)\lambda^2 + 54(2881\eta - 65481)\lambda^4$,

(A85) $\upsilon_5 := (197982 - 5144\eta)\lambda^6 + (57\eta - 5657)\lambda^8 + 57\lambda^{10}$.

Due to $\lambda \in [1,3)$ we obtain $\upsilon_3 < 0 \rightarrow \partial\Delta W^{ao}/\partial\lambda < 0$ and $\upsilon_4 + \upsilon_5 < 0 \rightarrow \partial\Delta\tilde{W}^{ao}/\partial\lambda < 0$. Due to $\delta = (9-\lambda^2)/8$ this implies $\partial\Delta W^{ao}/\partial\delta > 0$ as well as $\partial\Delta\tilde{W}^{ao}/\partial\delta > 0$.

A.9 Conditions for market coverage

To show that the thresholds $\hat{u}^a_t$ $(t = 1,2)$ are strictly increasing in the parameter λ, we calculate $\hat{u}^a_t = p_{Lt}(q^a_{Lt}, q^a_{Ht}) - a q_{Lt}(q^a_{Lt}, q^a_{Ht})$ and differentiate the resulting expressions with respect to λ. This yields:

(A86) $$\frac{\partial \hat{u}^a_1}{\partial\lambda} = \frac{\lambda z^2[2187(9\eta - 19) + 27(28\eta - 495)\lambda^2 + 9(3 - 5\eta)\lambda^4 + (2\eta + 2\lambda^2 - 63)\lambda^6]}{373248\gamma\eta},$$

(A87) $$\frac{\partial \hat{u}^a_2}{\partial\lambda} = \frac{\lambda z^2[5346 - 252\eta + (19\eta + 19\lambda^2 - 423)\lambda^2]}{373248\gamma\eta}.$$

Due to $\lambda \in [1,3)$ we obtain $\partial\hat{u}^a_t/\partial\lambda > 0$ for $t = 1,2$. Hence, within the relevant domain the thresholds $\hat{u}^a_t$ are strictly increasing in λ.