

Efficient Linear Optics Quantum Computation: Supplementary Information

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Properties of teleportation with $|t_n\rangle$

In the paper we claimed that teleportation with $|t_n\rangle$ satisfies that if $0 < k < n + 1$, then the teleported state appears in mode $n + k$ and only needs to be corrected by applying a phase shift. The modes $2n - l$ are in state 1 for $0 \leq l < (n - k)$ and can be reused in future preparations requiring single photons. The modes $2n - l$ are in state 0 for $n - k < l < n$. If $k = 0$ we learn that the input state was $|0\rangle_0$ and if $k = n + 1$, that it was $|1\rangle_0$. The probability of these two events is $1/(n + 1)$, regardless of the input. Both the necessary correction and which mode we teleported to are unknown until after the measurement.

To prove the claim, consider a superposition $\alpha_0|0\rangle_0 + \alpha_1|1\rangle_0$ in the mode to be teleported. Suppose that k ($k \neq 0, k \neq n + 1$) photons are detected, more specifically, that the measurement detected r_j ($\sum_j r_j = k$) photons in mode j . The effect of the measurement can be seen as a projection onto $\alpha_0|x_0\rangle|y_0\rangle + \alpha_1|x_1\rangle|y_1\rangle$ followed by a measurement of the first $n + 1$ modes, where $|x_0\rangle = \hat{F}_{n+1}|0\rangle|1\rangle^k|0\rangle^{n-k}$, $|y_0\rangle = |0\rangle^k|1\rangle^{n-k}$, $|x_1\rangle = \hat{F}_{n+1}|1\rangle^k|0\rangle^{n-k+1}$ and $|y_1\rangle = |0\rangle^{k-1}|1\rangle^{n-k+1}$. Observe that applying $P_{2\pi l/(n+1)}$ to mode l for $0 \leq l \leq n$ after applying $\hat{F}_{n+1}$ is equivalent to shifting modes $0 \dots n$ circularly right before applying $\hat{F}_{n+1}$. This means that the states $|x_0\rangle$ and $|x_1\rangle$ differ only by phases in the number basis. Thus the measurement cannot distinguish between the two states. The relative phase of the detected number state in $|x_1\rangle$ with respect to $|x_0\rangle$ is given by $\prod_j \omega^{r_j j}$. Thus the output state is $\alpha_0|y_0\rangle + \alpha_1 \prod_j \omega^{r_j j} |y_1\rangle$ which has the desired properties after correcting this relative phase by applying a phase shift to mode $n + k$. The cases $k = 0$ and $k = n + 1$ can be analyzed similarly.

Some applications

A near-deterministic non-destructive parity measurement is possible using the state

$$|p_n\rangle = \sum_{i,j:i+j=\text{even}} |o\rangle^i |1\rangle^{n-i} |o\rangle^j |1\rangle^{n-j}$$

(bosonic qubit encoding as in the paper) for teleporting two modes using two instances of the protocol used with $|t_n\rangle$. The parity is determined by the total number of photons in the teleportation measurements. The method used to obtain the non-destructive parity measurement is similar to the one used to implement the controlled sign operation by teleportation.

It has been shown¹ that deterministic Bell basis measurements are impossible for bosonic qubits with linear optics. Fortunately, LOQC enables near-deterministic Bell basis measurements. Consider two qubits encoded in modes 1, 2 and 3, 4, respectively. The measurement is accomplished by first applying the non-destructive parity measurement to modes 1, 3. If the outcome is odd, apply $\text{BM}_1^{(12)}$ and $\text{BM}_1^{(34)}$. If it is even, apply $\text{BM}_1^{(13)}$ and $\text{BM}_1^{(24)}$. The sign in the Bell superposition can be inferred from the outcomes. The Bell basis measurement can be used to quantum teleport the state of a bosonic qubit given the state $|rt_1\rangle_{1234}$, where modes 1, 2 and 3, 4 belong to the sender and receiver, respectively. It is noteworthy that this state can be constructed by first sharing two unentangled photons using beam splitters, giving $(|10\rangle_{13} + |01\rangle_{13})(|10\rangle_{24} + |01\rangle_{24})$. The sender then uses $\text{RT}_1^{(12)}$, accepts the state if exactly two photons are detected, and informs the receiver via a classical channel when this happens.

Better success boosting with quantum codes

The methods in the paper based on $\mathcal{X}_2$ can be improved in several ways. First, there is a great deal of flexibility in how to implement the state preparations and responses to failures. This can be exploited to show that with $\mathcal{X}_2$ it is possible to achieve $f_z < f$ for $f < 1/2$.²

Second, the techniques introduced for $\mathcal{X}_2$ readily generalize to stabilizer codes involving more than two qubits. Stabilizer codes are eigenspaces of commutative groups (the codes' *stabilizer groups*) consisting of products of Pauli operators. In this context it is convenient to use the abbreviation $U = \sigma_u$ for $U = X, Y, Z$. For $\mathcal{X}_2$, the stabilizer group is generated by XX (omitting labels when no ambiguity results). This implies that the projection operator onto the code is given by $I + XX$ (up to an omitted scale). For purposes of recovering from Z -measurement failures, one can use the code $\mathcal{X}_n$ on n qubits whose stabilizer is generated by operators of the form $X^{(i)}X^{(j)}$

for $i \neq j$. If any one qubit (say s) of the code is not measured, all the measured ones can be recovered by adapting the recovery procedure given in the paper so that the surviving qubit is teleported together with the measured ones using an entanglement modified by projecting with the operators $I + X^{(i)}X^{(s)}$ for i ranging over the labels of the measured qubits. Failed recovery attempts can be retried until one runs out of surviving qubits. As a result, it is possible in principle to achieve gains of arbitrarily high order with $f < 1/2$.

The code $\mathcal{X}_n$ is based on the classical repetition code. As a result, it can be used to correct phase errors in up to half the qubits. It is sufficient to periodically measure the stabilizer operators to extract a syndrome from which the phase errors can be deduced and corrected. This works also if the codes are concatenated, as, for example, the concatenation of $\mathcal{X}_2$ with itself is $\mathcal{X}_4$.

Third, to reduce the qubit overhead, it is possible to encode multiple logical qubits in one block using any other good classical linear code where the parity checks of the code are translated to products of X operators to obtain the stabilizer. Note that phase-error correction works with any quantum codes obtained from a good classical code in this way.

An erasure code for dealing with photon loss

The smallest erasure code that can be used to reduce the impact of photon loss involves four qubits. Examples are $\mathcal{E}_1$ with stabilizer $\langle XXXI, IYYY, ZIZZ \rangle$ for encoding one qubit and $\mathcal{E}_2$ with stabilizer $\langle XXXX, YYYY \rangle$ for encoding two qubits. The techniques used for $\mathcal{X}_n$ can be generalized for this (or any other) stabilizer code. With the exception of the 180° rotations, it is no longer possible to implement rotations directly. Consequently, at least one 45° rotation must be implemented by the teleportation method. $\mathcal{E}_1$ can be shown to lead to a quadratic improvement in the probability of erasure for $f_l \lesssim .01$,² where f_l is the probability of erasure before encoding. We believe that improvement is possible for much larger probabilities.

Additional figures:

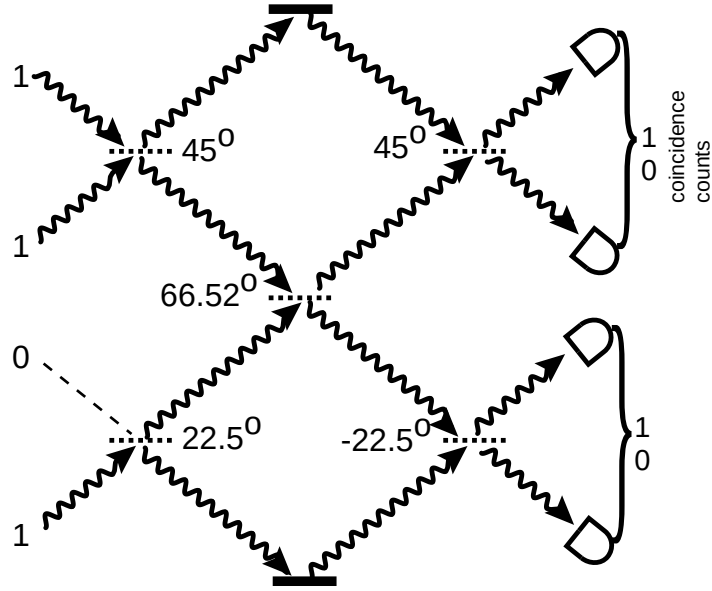


Figure 1. Experimental test of NS. The test may be performed by using two coupled Mach-Zehnder (MZ) interferometers. The upper MZ is adjusted so that single photons inserted in either or both arms are transmitted unchanged to the output. If two single photons are incident, this gives rise to a coincidence count probability of unity. If a single photon is present at the input to the lower MZ as indicated, the probability for coincidence counts at the upper MZ drops to zero conditional on the given single photon detection at the lower MZ. If one photon is inserted in the upper MZ, the conditional effect is that it appears in the opposite arm at the output. It is necessary to distinguish between 0, 1 and 2 photons at the lower detector of the lower MZ for proper conditioning. The experiment requires that up to three photons are incident to the device simultaneously. To achieve this a variant of the GHZ experiment of Bouwmeester et al.³ may be used. Alternatively, proposed devices for single photon sources^{4,5} can be used in conjunction with optical delay lines to bring the required single photon states to the interferometers with sufficient wavefunction overlap.

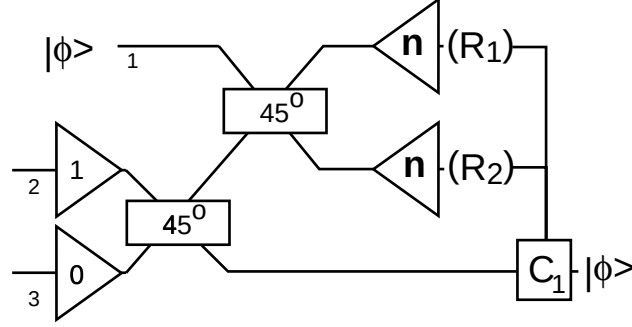


Figure 2. Non-deterministic quantum teleportation protocol. The input state $|\phi\rangle_1$ in mode 1 is teleported to mode 3, provided that the parity measurement's outcome is "odd" (see the paper). The parity in modes 1 and 2 is odd exactly when $R_1 + R_2 = 1$, which happens with probability $1/2$. The sign of the partial Bell measurement (see the paper) is '-' if $R_1 = 1$. $C_1^{(3)}$ is the correction operation, which consists of applying phase shifter $\mathbf{P}_{180^\circ}$ if $R_1 = 1$. If $R_1 + R_2 = 0$ or $R_1 + R_2 = 2$, the measurement fails. In the case of failure, the input state is effectively measured, with outcome $|0\rangle$ if $R_1 + R_2 = 0$ and $|1\rangle$ if $R_1 + R_2 = 2$.

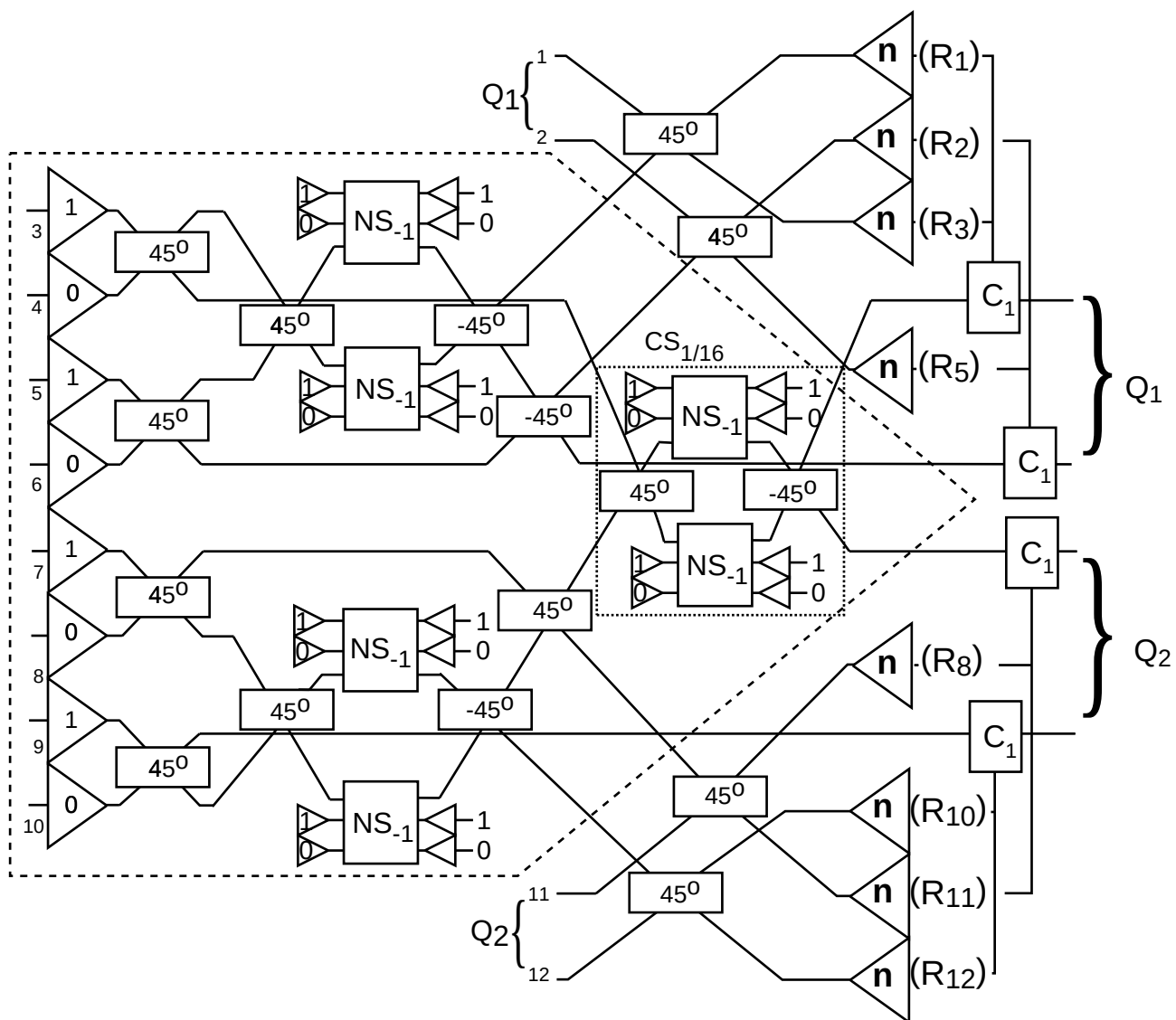


Figure 3. Conditional sign flip with loss detection. The outlined preparation consists of independently obtaining two copies of $|rt_1\rangle$ and one c - z operation, here implemented with c - $z_{1/16}$ (outlined). The failure behavior of the c - z operation can be exploited to avoid preparing both copies of $|rt_1\rangle$ in every attempt. Without exploiting this, the average number of times NS is attempted is $2 * 16 * (1 + 2 * 16) = 1056$. If the outlined c - $z_{1/16}$ is replaced by the network for c - $z_{1/4}$, the number of times NS is attempted becomes $2 * 4 * (16 + 2 * 16) = 384$. This illustrates how one can take advantage of the ability to combine independently generated states and the option to use any implementation of c - z . After the state preparation, the two qubits are independently teleported. The conditional sign flip succeeds with probability $1/4$. Loss is detected independently in each robust teleportation as described in the paper.

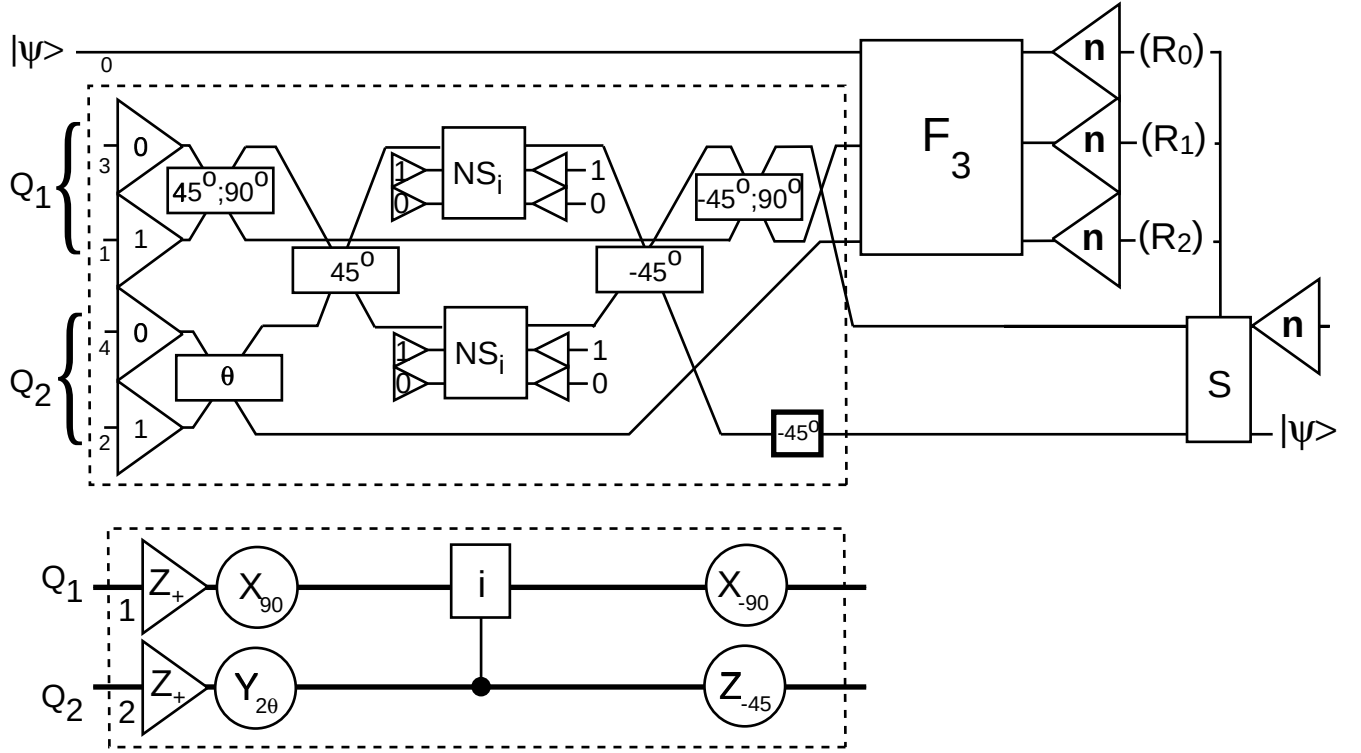


Figure 4. Teleportation with probability $2/3$. Top. The full protocol with the state preparation (outlined) in terms of a linear optics network. The angle θ satisfies $\tan(\theta) = \sqrt{2}$ and F_3 is the Fourier transform on three modes. Success occurs if $1 \leq R = R_0 + R_1 + R_2 \leq 2$ (probability $2/3$). The correction operation S swaps mode 3 with mode 4 if $R = 1$ and applies a phase change described in the text. Bottom. The state preparation using a qubit-based quantum network. The mode identifications for the bosonic qubits are shown and the qubit gates are aligned with the corresponding optical elements.

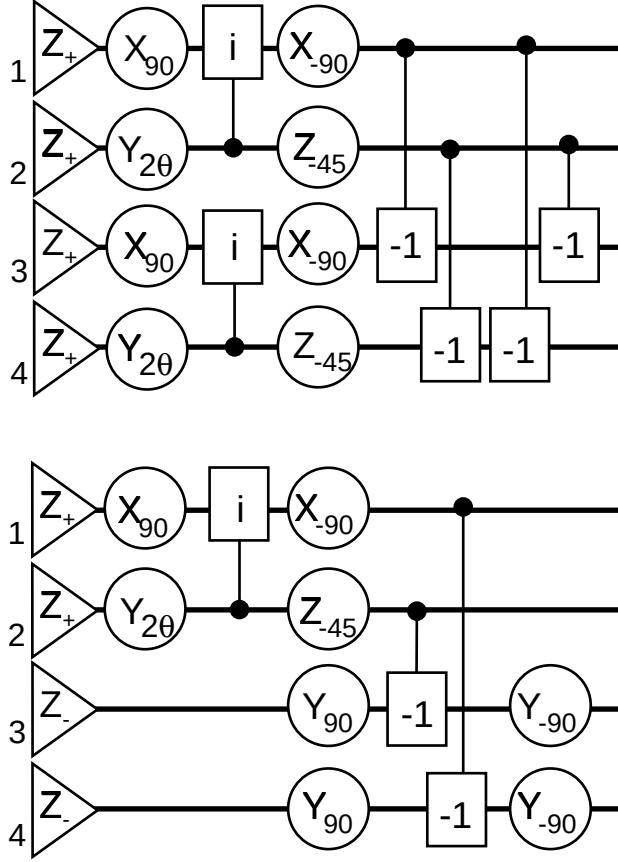


Figure 5. State preparation networks. Top. A qubit network for preparing the state $|cs_2\rangle$ needed for $c\text{-}z_{4/9}$. A corresponding optical network can be obtained as in Fig. 4. Bottom. A network for preparing the state $|rt_2\rangle$. Linear size networks to obtain states such as $|cs_n\rangle$ exist but may be difficult to implement with good probabilities of success. However, together with quantum coding methods, $n \leq 3$ suffices.

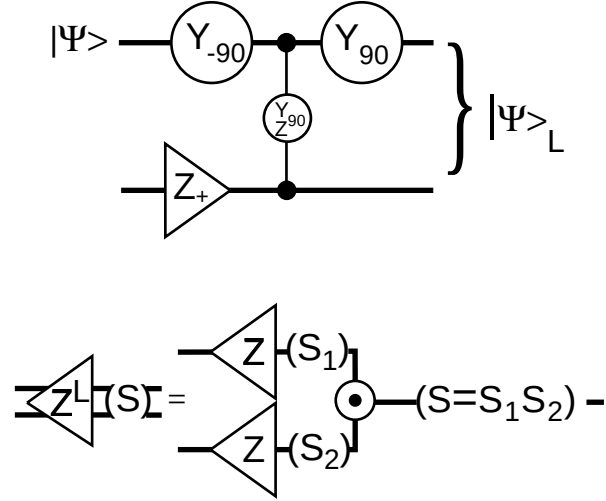


Figure 6. State preparation and measurement for code $\mathcal{X}_2$. Top. Encoding an arbitrary state $|\Psi\rangle$. The output is accepted if the coupling gate succeeds. Eigenstates of one of the standard operators are prepared by using the output of a standard preparation instead of $|\Psi\rangle$. Bottom. The logical Z measurement. S is the eigenvalue of the logical operator and computed as the product of the measurement outcomes S_1 and S_2 .

References

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