1 Relaxation time estimate for Lévy flights in a confined two dimensional region

One can estimate the time to reach equilibrium for a two-dimensional Lévy flight in a confined region of linear extent L as follows. Assume that the single step radial distribution of the random walk can be approximated by

$$p_{\Delta t}(\mathbf{x}) = (1-Q)\delta(\mathbf{x}) + Q\, f_{\delta L}(\mathbf{x}). \tag{1.1}$$

Here Δt denotes the typical time between single steps, Q the fraction of walkers which jump a distance $d > \delta L$ and $(1-Q)$ the fraction which remains in a disk defined by $|\mathbf{x}| \leq \delta L$. The function $f_{\delta L}(\mathbf{x})$ comprises the power-law in the single steps, characteristic for Lévy flights:

$$f_{\delta L}(\mathbf{x}) = C\, \delta L^{\beta} |\mathbf{x}|^{-(1+\beta)} \qquad |\mathbf{x}| \geq \delta L.$$

Inserting this into Eq. (1.1) one obtains that $f_{\delta L}(\mathbf{x})$ is normalized to unity and that the normalization constant C is independent of the microscopic length δL. The Fourier-transform of $p(\mathbf{x})$ is given by

$$\tilde{p}(\mathbf{k}) = (1-Q) + Q \tilde{f}_{\delta L}(\mathbf{k}).$$

The Fourier-transform of the probability density function $W_N(\mathbf{x})$ of the walker being located at a position $\mathbf{x}$ after N steps can be computed in terms of $\tilde{p}(\mathbf{k})$ according to

$$\tilde{W}_N(\mathbf{k}) = \tilde{p}(\mathbf{k})^N \approx \left(1 - Q\delta L^{\beta} |\mathbf{k}|^{\beta}\right)^N \approx e^{-QN\,|\delta L\,\mathbf{k}|^{\beta}}. \tag{1.2}$$

The relaxation time in a confined region is provided by the lowest mode

$$k_{\text{min}} = \frac{L}{2\pi}.$$

Inserted into (1.2) with $N = t/\Delta t$ on obtains

$$T_{\text{eq}} \approx \delta T/Q\, (L/2\pi\delta L)^{\beta}.$$