Supplementary Information

Some History

Vega is probably the most observed star in the sky after the Sun. Its prominent hydrogen spectrum places it as a critical spectral type standard at A0 Va[1,2]. Its apparent brightness and high declination made it convenient for the northern hemisphere observatories for calibration as the absolute photometric standard in the visible[3], which was extended somewhat into the UV with rockets flights[4]. Angular diameter measurements with the Intensity Interferometer[5] allowed an accurate determination of its effective temperature[4] and radius. These and the sharp lined nature of the spectrum lent confidence to the extrapolation of the calibrations over the whole UV[6] and near IR[7] using model atmospheres. Numerous abundance analyses[8–10] have established Vega as mildly metal deficient.

But the star does not comfortably fit preconceptions. Reports that Vega was photometrically variable, fairly widespread over the years[11], have persisted[12,13] to the point where the Bright Star Catalogue[14] continues to show it as variable. Reports of emission lines appear[15] and are refuted[16,17].

Although it is typed as a main sequence star, Petrie[18] early on noted it was about 0.7 mag too bright. This led Gray[19] to suggest that perhaps rapid rotation was involved, predicting a fractional angular velocity range of $\omega = 0.9 - 1.0$, if that were the explanation. Unusual line shapes seen at high dispersion persuaded Gulliver, Hill and Adelman[20–22] of this last suggestion, who concluded that Vega was indeed a rapid rotator seen $5° - 8°$ from pole-on.

In the meantime, Vega has taken on a whole new life as the brightest member of the eponymous group of objects which host so-called “debris disks”[23,24]. In the case of Vega, the disk was not detected until observations at $20\,\mu$m showed an excess[23,25]. The early “pointed” IRAS measurements and recent Spitzer measurements[26] show the disk is essentially circular. The view emerges of a pole-on star surrounded by a pole-on disk.

The near IR has also presented problems. In a number of papers Blackwell and coworkers[27–31] have reported that models for Vega consistently underestimate the observed flux at longer than $1\,\mu m$ by a few hundredths of a magnitude. Recently, near IR interferometric observations with the Palomar Testbed Interferometer[32] have suggested that Vega’s disk may make a measurable contribution at $2\,\mu m$.

The Observations

Interferometric observations have been obtained during late May and early June 2001 in the visible with the Navy Prototype Optical Interferometer[33] (NPOI) at substantially improved resolution and with multiple apertures and closure phase techniques that allow us to coarsely resolve the visible surface of Vega. A number of innovative observing and reduction techniques were used. The interested reader should refer to our Altair[34] paper, where data are reported from the same observing run, for details.

As we have described[34], these observations were substantially compromised by detector deadtimes which were flux dependent (through the effects of detector heating) and varied from channel to channel. As for Altair, we believe the phases survived those problems basically unaffected. However, since Vega is twice as bright and was observed essentially directly overhead, no attempt will be made to use the visibility amplitudes until we have secured a stable calibration of the deadtime/heating effects. Fortunately the analysis for Vega is not so dependent on the visibilities, unlike the case of Altair.

ζ Aql, a rapidly rotating A star, was used as a calibrator. As described[34], other observations have allowed us to put small, $\sim \pm 1^\circ$ limits on any triple phases that might be result from rotation induced asymmetry in the calibrator. The configuration of the back end in use during these runs had each of the baselines detected on separate spectrometers. As a result, small spectrometer to spectrometer differences required that calibrators be observed for the phase measurements. It is important that ζ Aql presented no measurable offsets in these circumstances.

Model Fitting

As described in the main text, we focus here on the triple phase data taken by NPOI on May 25, 2001. We augment these with the V magnitude[6], $V = 0.026 \pm 0.008$ and in specific cases the projected rotational velocity[21], $v_{\rm eq} \sin i = 22 \pm 1\,{\rm km\,s^{-1}}$as additional constraints. The model was fit by minimizing the χ^2 metric using the Levenberg–Marquardt algorithm[35]. Since the χ^2 surface is at most 5-dimensional (we fix the mass which enters very weakly), and effectively only 3-dimensional ($T_{\rm p}$ is strongly constrained by the V magnitude and the angular diameter is almost 100% correlated with the dimensionless rotation), locating the χ^2 minimum is straightforward.

The near pole-on geometry displayed by Vega - a challenge to detect - has a silver lining: the locations of the zeros in the visibilities, indicated by transitions in the phase angles, are sharply defined in the phase data. Thus, unlike for Altair[34], it is not necessary to add visibility amplitude data to define the spatial scale.

On the other hand, Vega transits essentially at the zenith at the NPOI site. That, combined with the fact that the long baseline is oriented mostly East-West results in only limited additional coverage in the UV plane from Earth rotation even through a reasonable range of hour angles. Figure S1 shows the coverage provided for the observations reported here. The effect is a strong correlation of the the inclination on fractional rotational velocity and, through their strong correlation, angular diameter. Future observations with alternate baselines will be needed to moderate this.

The initial reductions were done with the data selected exactly as described here but with no velocity constraint. The result was a fit with a reduced χ^2_ν of 2.06 (DoF=178). From the residuals it was clear that the majority of the excess χ^2_ν was coming from the red channels where high signal to noise ratios were producing very small error estimates which were not consistent with the jitter observed from one scan to the next. Common practice in such cases is to apply a multiplier to all error estimates to remove this discrepancy. Since phases are exponents in the complex visibility,

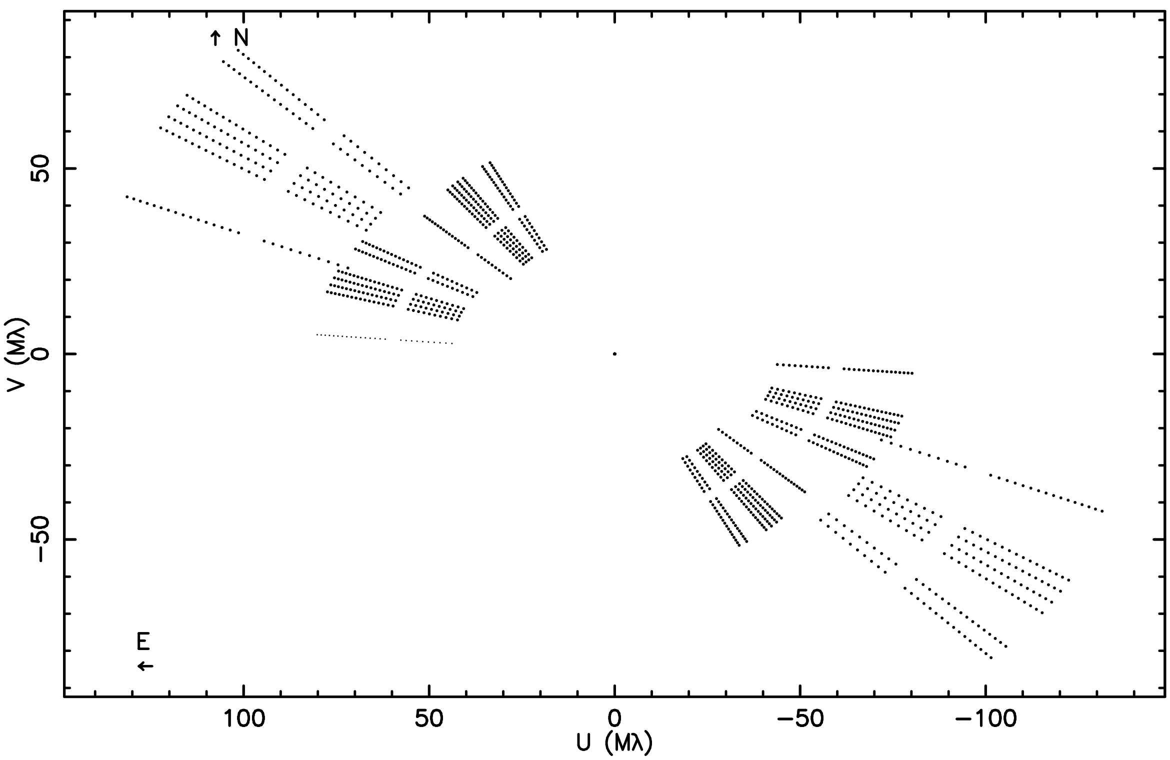

Figure S1 The sky coverage of the observations. The UV coverage provided by the May 25, 2001 observations of Vega. Note the extensive range of spatial scales provided by the nearly octave range in frequencies. On the other hand, rotation synthesis does not fill in the UV plane very completely, even with the large range of hour angles of the observations.

it is appropriate to increase the errors by adding in quadrature a 2° additional error, effectively removing the discrepancy. We had estimated *a priori* that the calibration uncertainties would be of order 1° to $1\overset{\circ}{.}5$, so this is not unreasonable. The parameters from the reduction including the augmented errors are given in column 2 of Table S1.

The bottom row of Table S1 shows the projected rotational velocity for this model, $v_{\rm eq} \sin i \sim 15\,{\rm km\,s^{-1}}$which, as we note in the main text, is a bit below the observed values. Particularly, Gulliver, Hill and Adelman[20–22] have obtained Roche model fits for Vega using only spectral and line profile information. In the process they obtain the most precise measure of the projected velocity currently available in the literature. The early fits[20,21] were obtained using an unrealistically high estimate of the breakup velocity, and in more recent reports these authors[22] have adopted a much lower equatorial velocity than we report here, in each case leading to a reduced gravity darkening. Significant gravity darkening leads to narrower line profiles than pure geometrical projection would predict. Therefore $\sim\ 22\,{\rm km\,s^{-1}}$ is a lower limit to the actual value of $v_{\rm eq} \sin i$ if our results are correct, and in turn our initial model, column 2 above, is probably off by even more than suggested for this parameter.

On the other hand, these are the only modern determinations of the projected velocity that make any attempt to include the effects of gravity darkening. Combined with the extraordinary quality of their data, it is unlikely these projected velocity determinations are off by a large amount. Because of this and because we would like to reduce the effects of the strong correlations in these model fits we have decided to include the projected rotation in our reductions as an "observable", but with softened precision, $v_{\rm eq} \sin i = 22 \pm 1.0\,{\rm km\,s^{-1}}$. With this additional constraint we have reconverged the model, yielding the parameters and errors given in columns 3 and 4 of Table S1. This is our preferred model. This model provides an only slightly poorer fit to the data ($\Delta\chi^2 \sim 3.7$, i.e., $\chi^2 = 191.9$ versus 195.6, but with an additional degree of freedom) than that without the velocity constraint.

Owing to the limited rotational synthesis described above there are strong correlations among the model parameters (which have been accounted for when propagating errors of derived quantities, reported in the main text). The correlation matrix from the final solution is provided in Table S2.

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Table S1 | Roche Model Fits for Vega

Parameter	ϕ_{cl}	$+v_{\mathrm{eq}}\sin i$	Error
$\omega = \Omega/\Omega_{\mathrm{B}}$	0.946	0.926	$\pm$0.021
θ_{p} (mas)	2.720	2.767	0.037
T_{p} (K)	10006	9988	61
i (deg)	3.02	4.54	0.33
PA (deg)	29.8	8.6	2.7
χ^2_ν	1.078	1.093	–
$v_{\mathrm{eq}}\sin i$ (km s^{-1})	15.1	21.7	–

The Roche model parameters fit during the χ^2 minimization are the angular velocity as a fraction of breakup, the polar angular diameter, the polar effective temperature, the inclination of the rotational axis ($i = 0$ for pole-on) and the position angle of the pole (measured North through East). The quality of the fits is indicated followed by the predicted projected equatorial velocity. The first column gives the values for an unconstrained fit to the closure phases and V magnitude (DoF=178). The second column is the fit, with the additional constraint of $v_{\mathrm{eq}}\sin i = 22 \pm 1$ km s^{-1} (DoF=179). Errors given are for the second fit and are the estimated standard deviations[35].

Table S2 | Correlation Matrix for the Final Fit

	ω	θ_{p}	T_{p}	i	PA
ω	1.0000	-0.9992	0.7761	-0.7953	-0.2449
θ_{p}	-0.9992	1.0000	-0.7970	0.7932	0.2489
T_{p}	0.7761	-0.7970	1.0000	-0.6015	-0.1728
i	-0.7953	0.7932	-0.6015	1.0000	-0.2504
PA	-0.2449	0.2489	-0.1728	-0.2504	1.0000

The correlation matrix for the Roche model fit given in columns 3 and 4 of Table S1. The angular diameter, θ_{p}, is almost completely dependent on the fractional angular velocity, ω. Beyond that there are strong correlations between i and (ω and θ_{p}) because of the limited range of projected baselines provided by the rotational synthesis. A more complete set of baselines would presumably improve this substantially.