

## Supplementary Information

### I. STEP LENGTH DISTRIBUTION

In a homogeneous opaque system both the scatterers density  $N$  and the scattering cross section  $\sigma$  are position independent and therefore the distribution of the step length between two successive scattering events is given by the Lambert-Beer distribution  $P(x) = N\sigma e^{-N\sigma x}$ . Its first moment  $\ell = \frac{1}{N\sigma}$  is known as the scattering mean free path and gives the characteristic scale length of the transport process. Since all moments of this distribution are finite the central limit theorem states that, in the limit of a large number of steps, the total step distribution converge toward a Gaussian and henceforth the transport is diffusive.

In order to have a Lévy flight it is necessary to have a step distribution with diverging moments. We can obtain this modulating the density of scatterers in the sample. As a simplified model we can think of a step distribution that is still given by an exponential decaying function but with an *empty* region where scattering is impossible (see Fig.1 panel a). We can therefore write the new step distribution as

$$P^*(x) = \frac{1}{\ell} \left( H(a-x)e^{-\frac{x}{\ell}} + H(x-(a+b))e^{-\frac{x-b}{\ell}} \right) \quad (1)$$

where  $H(x)$  is the Heaviside step function,  $a$  is the distance after which the hole is met (i.e. the average distance between spheres) and  $b$  the width of the hole.

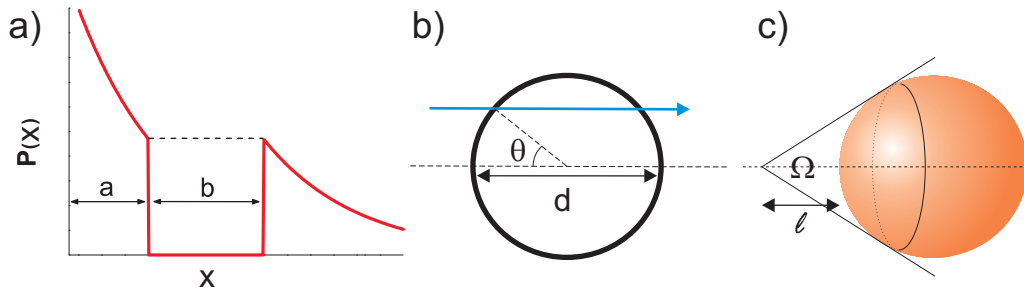


FIG. 1: a) Step length distribution in the presence of a void of size  $b$ . Inside the void no scattering event is possible. b) A ray of light entering a spherical void of diameter  $d$  will see an empty region of size  $b \leq d$  making  $P(b) \neq P(d)$ . c) The total solid angle  $\Omega$  subtended by a sphere determines the probability that the light will encounter that particular spherical void.

If  $b$  is constant this just leads to a renormalization of  $\ell$  and the central limit theorem still holds. But if  $b$  is distributed as  $P(b)$  we must reinterpret eq.1 as  $P(x|b)$ , i.e. the conditional probability of  $x$  with fixed  $b$ . The joint probability  $P(x, b)$  is then given by  $P(x|b)P(b)$  and therefore the moments  $\mu_n$  of the step distribution are given by

$$\mu_n = \int_0^\infty x^n P(x) dx = \int_0^\infty x^n \left( \int_0^\infty P(x, b) db \right) dx = \int_0^\infty x^n \left( \int_0^\infty P(x|b) P(b) db \right) dx. \quad (2)$$

If we write  $P(b) = 1/b^{\alpha+1}$  with  $0 < \alpha < 2$ , we see that all moments  $\mu_n$  with  $n \geq \alpha$  diverge (in the case  $\alpha \geq 2$  all moments of the distribution are finite and the sum converge toward a Gaussian) unless we are in the limit of  $a \gg \ell$ . In this latter case we have  $\mu_1 = \ell$  for any  $\alpha > 0$  and the presence of a void distribution has no effect. We can therefore obtain a Lévy flight choosing the right value for  $\alpha$ .

Experimentally we cannot directly select the distribution  $P(b)$  but we have access only to the diameter distribution of the voids  $P(d)$ , which are 3D spherical objects. Since any ray of light impinging not perfectly perpendicular on the sphere surface will make a step smaller than  $d$  (see Fig.1 panel b) this effect needs to be taken into account. To obtain a relation between  $P(b)$  and  $P(d)$  we can write  $b = d \cos \theta$  with  $P(\theta) = 1$  and  $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$  so that we can apply the following transformation

$$\begin{cases} b = d \cos \theta \\ \theta = \theta \end{cases} \Rightarrow \begin{cases} d = \frac{b}{\cos \theta} \\ \theta = \theta \end{cases} \Rightarrow \|J\| = \sec \theta \quad (3)$$

where  $\|J\|$  is the determinant of the Jacobian matrix of the transformation. We can than write

$$P(b, \theta) = \|J\| P(d) P(\theta) = \sec \theta P(d) \rightarrow P(b) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec \theta P(d) d\theta. \tag{4}$$

If we choose  $P(d) \propto \frac{1}{1+d^{\alpha+1}}$  we obtain

$$P(b) \propto \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sec \theta \frac{1}{1 + \left(\frac{b}{\cos \theta}\right)^{\alpha+1}} d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sec(\theta)}{\left(1 + (b \sec(\theta))^{\alpha+1}\right)} d\theta. \tag{5}$$

In the limit of large  $b$  this integral can be solved and we get  $P(b) \propto 1/b^{\alpha+1}$ , i.e. if  $P(d)$  asymptotically decays as  $1/d^{\alpha+1}$ , also  $P(b)$  asymptotically decays as  $1/b^{\alpha+1}$ .

Moving to 3D we need to take into account also the probability that a step starting at an average distance  $\ell$  from a given spherical void will actually enter on that void and will not go in another direction. To do that we can calculate the solid angle  $\Omega$  subtended by the sphere (see Fig.1 panel c) as a function of  $d$ . Some simple geometry considerations show that  $\Omega = \left(\frac{d}{2}\right)^2 / (\ell^2 + d\ell)$  so that asymptotically  $\Omega$  scales as  $d$ . Therefore in order to obtain a Lévy flight with parameter  $\alpha$ , we need a diameter distribution of voids that asymptotically decays as  $d^{-\beta}$  with  $\beta = \alpha + 2$ .

II. SAMPLE FABRICATION

The sample were made by suspending Rulite Titanium Dioxide in liquid Sodium Silicate ( $SiO_2 : NaOH$ ). In the suspension, soda lime glass microspheres of different diameter were introduced. This allows to create the required step length distribution of a Levy Flight. As seen above, we have chosen the distribution of diameter to be  $P_s(d) \propto 1/d^3$ , giving a constant volume per sphere category. The following data shows the quantity of spheres for a typical sample. To these spheres were added 10mg of  $TiO_2$ , and 57ml of  $SiO_2 : NaOH$ . The sample was then kept half an hour pressend between two glass plates. In this way, the sample thickness was given by the diameter of the biggest sphere family.

| Diameter ( $\mu m$ ) | Quantity (mg) | Diameter ( $\mu m$ ) | Quantity (mg) |
|----------------------|---------------|----------------------|---------------|
| 550                  | $15 \pm 0.75$ | 100                  | $15 \pm 0.75$ |
| 480                  | $15 \pm 0.75$ | 70                   | $15 \pm 0.75$ |
| 400                  | $15 \pm 0.75$ | 50                   | $15 \pm 0.75$ |
| 330                  | $15 \pm 0.75$ | 40                   | $15 \pm 0.75$ |
| 280                  | $15 \pm 0.75$ | 30                   | $15 \pm 0.75$ |
| 230                  | $15 \pm 0.75$ | 20                   | $15 \pm 0.75$ |
| 200                  | $15 \pm 0.75$ | 15                   | $15 \pm 0.75$ |
| 170                  | $15 \pm 0.75$ | 10                   | $15 \pm 0.75$ |
| 140                  | $15 \pm 0.75$ | 8                    | $15 \pm 0.75$ |
| 120                  | $15 \pm 0.75$ | 5                    | $15 \pm 0.75$ |

### III. TRANSMISSION PROFILE

In order to verify that the transmission profiles we have obtained were not due only to the presence of big spheres, but instead by their precise diameter distribution, we realised a sample containing only one family of spheres, of diameter  $280\text{ }\mu\text{m}$ , and recorded the transmission profile. This sample presented an enhanced transmission through the spheres, causing important fluctuations of intensity between different realisations of the disorder, as for the Lévy samples. However, the intensity profile, averaged over the disorder realisations presents a Gaussian lineshape, showing that transport is diffusive in that case.

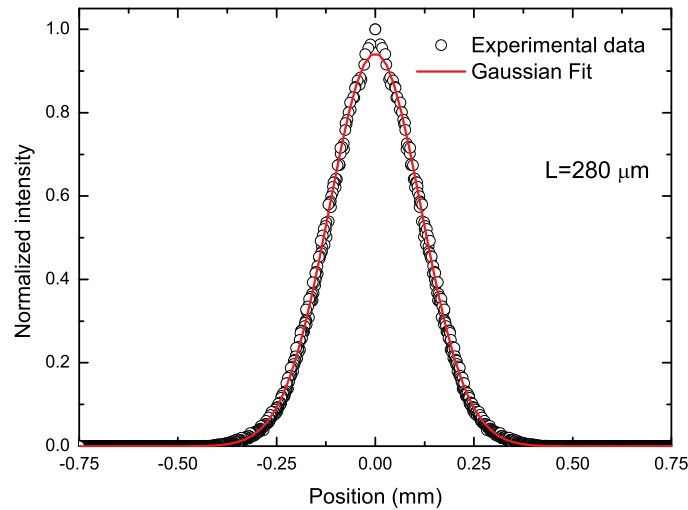


FIG. 2: Averaged transmission profile, through a slab containing only one family of spheres (diameter:  $280\mu\text{m}$ ). The presence of the spheres, in average, does not modify the lineshape of the profile, keeping it equivalent to a diffusive sample. One needs an adequately chosen distribution of diameters to observe the superdiffusive profile, characteristic of a Lévy Flight.