

Supplementary Information

Small-scale soft-bodied robot with multimodal locomotion

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Supporting videos S1 to S10

S1 - Robot fabrication and characterization

In this section we describe the robot's fabrication process and mechanical properties (section S1A), its surface properties (section S1B), how it can potentially be made biocompatible (section S1C), and also its residual strain energy (section S1D).

A. Fabrication process and mechanical properties of the robot

The base material of the magneto-elastic robot body is an Ecoflex 00-10 polymer matrix (Smooth-On Inc.; density: 1.04 g/cm³) loaded with neodymium-iron-boron (NdFeB) magnetic microparticles (MQP-15-7, Magnequench; average diameter: 5 μm, density: 7.61 g/cm³) according to a mass ratio of 1:1 (mass of NdFeB microparticles to the mass of Ecoflex-10). The NdFeB microparticles' high remanent magnetization enables us to generate effective magnetic actuation for the robots while their high magnetic coercivity ensures that our robot's magnetization profile is maintained during magnetic actuation¹: NdFeB particles requires more than 600 mT to be demagnetized, and our maximum B is limited to 50 mT. The resulting magnetic elastomer has a density (ρ_r) of 1.86 g/cm³.

To fabricate the robots, the pre-polymer is cast onto a flat poly(methyl methacrylate) plate to form a 185 μm-thick film. After thermal polymer curing, the device is demolded by using the laser to cut it into the desired geometry before its removal from the substrate plate. The robot dimensions are shown in Fig. S1b, where L , w and h represent the length, width, and height of the robot, respectively. Unless specified otherwise, $L = 3.7$ mm, $w = 1.5$ mm and $h = 185$ μm are assumed. The robots with these dimensions are defined as the "original" robot in the main text. The harmonic magnetization profile of the robot is implemented by wrapping its soft body around a cylindrical glass rod with a circumference of 3.7 mm and subjecting it to a large, uniform magnetizing field of 1.65 T (Fig. S1a). To create the phase shift, β_R , in the magnetization profile $\mathbf{m}$ (Fig. S1b), the rod is maintained by an angle of β_R during the robot's magnetization process. Unless specified otherwise, β_R is assumed to be 45°. We experimentally evaluated the magnitude of $\mathbf{m}$ to be 62,000 ± 10000 A/m for robots that have a 1:1 mass ratio (Fig. S1c).

The effective Young's modulus (E) of robots, which has a 1:1 mass ratio, is experimentally evaluated to be $8.45 \times 10^4 \pm 2.5 \times 10^3$ Pa through a tabletop test system (MTS Nano Biomix, Agilent Systems) using the accompanying software (Nano Suite V5, Agilent Systems). Figure S1c-d additionally summarizes the characterization of $|\mathbf{m}|$ and E , respectively, for similar magnetic elastomers with different mass ratios. The mean and standard deviation of each data point in these sub-figures are computed from the experimental results of three samples. According to these experimental data, increasing the content of magnetic particles in the polymer matrix increases $|\mathbf{m}|$ more significantly than it increases E . This suggests that robots with higher NdFeB concentration (larger mass ratios) can deform more easily, as they can generate higher magnetic torques, while becoming only slightly stiffer. From the fabrication perspective, it is difficult to further increase the NdFeB mass content beyond the 1:1 mass ratio as the base material Ecoflex-10 no longer cures well. Hence, all the robots used in this work are fabricated based on the 1:1 mass ratio to maximize the amount of NdFeB content while preserving the ease of fabrication.

B. Surface properties of the robot

The native surface of the composite elastomeric material is hydrophobic and microscopically rough. Static advancing ($116^\circ \pm 3^\circ$) and receding ($78^\circ \pm 2^\circ$) water contact angles were measured by the sessile droplet method through an automated goniometer routine (Krüss DSA100 and

accompanying software) that continuously increases and decreases the water droplet volume, respectively. Surface roughness ($R_a = 0.63 \pm 0.02 \mu\text{m}$ and $R_z = 4.37 \pm 0.28 \mu\text{m}$) was measured by laser interferometry (Keyence VK-X200). The as-fabricated robots' surface was not subjected to any treatment before the experiments.

C. Biocompatibility of the robots

While the Ecoflex-10 polymer matrix is itself biocompatible, the presence of NdFeB microparticles within the matrix makes the current composition of our soft robots only partially biocompatible. Full biocompatibility of our soft robots can be recovered by sealing the microparticles safely within the robots². For example, this can be achieved by coating and encasing the robots within an additional, outer thin shell of Ecoflex-10 polymer. This solution would also preserve the surface properties discussed above, as the native Ecoflex-10 is itself hydrophobic and has a microscopically rough surface. Furthermore, because unadulterated Ecoflex-10 has lower E than our composites shown in Fig. S1d, the robot should still be sufficiently compliant to execute all of the proposed locomotion modes and functionalities as long as the additional layer is sufficiently thin. To ensure that we can predict the behavior of such robots, it would then be necessary to characterize the Young's modulus of this new composition.

D. Residual strain energy

We hypothesize that there are two lowest energy configurations because our fabrication process is not perfect and the pre-stress induced from the demolding process will make the robot retain some strain energy. One consequent effect of this phenomenon is that the proposed robots will exhibit two different rest state curvatures when no $\mathbf{B}$ is applied (Fig. S1e). The first rest state curvature, which is relatively smaller, is illustrated in Fig. S1e(I, V) while the second rest state curvature, which is larger, is shown in Fig. S1e(III). We found that the robot can alternate between these two rest state curvatures when it undergoes different shape-change. In particular, the relatively flat curvature can be restored after the robot recovers from the shape specified in Fig. S1e(IV), and the larger rest state curvature is activated after the robot recovers from the shape specified in Fig. S1e(II). Our experiments show that these two different rest state curvatures can be switched repeatedly.

Having two rest state curvatures implies that the robot has two distinct lowest energy configurations. By inducing a shape-change to alter the amount of strain energy within the robot, the robot can switch between these two lowest energy configurations after $\mathbf{B}$ is removed. Theoretically, if there is no residual strain energy, the lowest energy configuration of the robot should be perfectly flat.

As mentioned in the main text and the captions in Fig. 1, the residual strain energy would also induce small errors in the predicted robot shapes. Since this fabrication uncertainty may vary across different robots, it is expected that different robots would have slightly different errors in their predicted shapes. To reduce such uncertainties, we always use the same batch of robots to characterize each locomotion mode (SI sections S4-10).

Despite having residual strain energy, we remark that our robot is still able to achieve multiple modes of locomotion. This is supported by our multi-locomotion demonstration in Fig. 3 where we show that one single robot is able to realize all the proposed modes of locomotion. In the future, we will account for these effects via calibrating $\mathbf{B}$ and we will also improve our fabrication process such that the fabricated robot will have less residual strain energy.

S2 - Magnetic actuation setup

A. Experiment setup

The magnetic actuation setup is composed of three orthogonal pairs of custom-made electromagnets, and it has an inner chamber size of 65 mm $\times$ 65 mm $\times$ 65 mm (Fig. S2). The input currents driving the electromagnets are specified by software control signals through a custom electronic board. We use a calibrated magnetic actuation matrix to map the control current input to the actual magnetic field $\mathbf{B}$ in the working space. With respect to the center of working space, which coincides with the center of the reference system defined in Fig. S2, the 95% homogeneous field region is 16 mm wide along the X - and Y -axis and 50 mm wide along the Z -axis. All demonstrations of individual locomotion modes were conducted in this homogenous field region, except for the multimodal locomotion experiments in Fig. 3a, d, e, for which the maximum displacement of the robot was around 30 mm. In these situations, we would recalibrate our actuation matrix to produce a relatively spatially uniform $\mathbf{B}$ for our robot. All characterizations in sections S4-S11 were conducted in the homogenous field region, except for the characterization of the jumping and jellyfish-like swimming, in which a pair of Helmholtz coil is used for simplicity.

Despite our efforts to provide spatially uniform fields, uncontrolled spatial-gradients in these regions may still exist. For example, we have observed unwanted magnetic forces pulling the walking robot towards the center of the workspace (Fig. 3d and Supporting Video S6). While these forces have induced unwanted disturbances on the walking locomotion, we remark that the robot is able to walk successfully even against the gradient (see SI section S13 for more discussion). We further remark that the current setup was designed for proof-of-concept purposes, showing the possibility of creating highly mobile miniature soft robots that have multiple modes of locomotion. Electromagnetic coil setups with larger uniform workspaces can well be conceived to address these issues and render the whole robotic system more suitable for potential biomedical applications described in SI section S14.

B. Coordinate frames

Here the global frame is defined by the magnetic actuation setup's coordinate frame, and we shall define the axes in this frame with capital letters, *e.g.*, X -, Y - and Z -axes, and XY plane. To distinguish this global frame from the robot's un-deformed local frame shown in Fig. 1, we assign the axes of the local frame with lower case letters, *e.g.* x -, y - and z -axes, and xy plane. As the soft robots can be steered by $\mathbf{B}$, we can orientate the robot's local frame with respect to the global frame accordingly (see SI section S3B). For example, we can make the robot's local xy plane parallel with the global XY plane by applying a $\mathbf{B}$ that only has components in the XY plane.

Except for the main text, the shape change analyses in SI section S3A, and the description for cargo delivery in SI section S14C, which are more intuitive to be described in the robot's local frame, all other analyses and characterizations in the SI are described in the global frame (SI sections S4-S11). In these analyses and characterizations, we have orientated the robot's local xy plane to be parallel with the global XY plane.

S3 - General theory

Here we provide the general theory, which describes how our soft robots respond to external magnetic fields. We begin by providing a quasi-static mathematical analysis of how the actuating fields can generate desired time-varying shapes for a magnetically actuated soft robot (section 3A).

Subsequently, we describe how the actuating fields can be used to rotate the body of the robot (section S3B), and also how to use our theory to determine the operating range of $\mathbf{B}$ (section S3C). SI sections S4-11 then propose more detailed models and discussions for each locomotion mode. A scaling analyses summary for the soft robots is finally presented in SI section S12 and Table S4. We also provide a nomenclature for all the mathematical symbols in Table S1 and a summary of our robot's physical properties in Table S2.

We remark that the analyses and discussions presented here and in the following SI sections aim to facilitate future studies on soft-bodied locomotion. The analyses can also provide design guidelines, which can be used as the first step towards designing and optimizing miniature soft magnetic robots that have multi-locomotion capabilities.

A. Quasi-static analysis

Given the magnetization profile $\mathbf{m}$ prescribed in Fig. S1b, here we use quasi-static analysis to describe how we can use the external magnetic fields, $\mathbf{B} = [B_x \ B_y \ B_z]^T$, to control our robot's shapes. The magnetization profile, $\mathbf{m}$, along the robot's length, s , is described as:

$$\mathbf{m}(s) = \begin{pmatrix} m_x \\ m_y \\ 0 \end{pmatrix} = m \begin{pmatrix} \cos(\omega_s s + \beta_R) \\ \sin(\omega_s s + \beta_R) \\ 0 \end{pmatrix}. \quad (\text{S3.1})$$

The variables m_x and m_y represent the components of $\mathbf{m}$ along the x - and y -axis, respectively, while m and ω_s represent the magnitude and spatial angular frequency of $\mathbf{m}(s)$, respectively, with $\omega_s = \frac{2\pi}{L}$. Using the above $\mathbf{m}(s)$ and without any loss of generality, we use the robot's local reference frame in Fig. S1b to describe $\mathbf{B}$, which is a function of time, t . At a given t , the interaction between $\mathbf{B}$ and $\mathbf{m}(s)$ creates a spatially varying stress in the soft robot to create desirable deflections (Fig. S3). These static deflections on an element can be described by the Euler-Bernoulli equation for beams, *i.e.*, the bending moment $M_b = EI \frac{\partial^2 \theta}{\partial s^2}$, where the rotational deflection along the robot, $\theta(s)$, is expressed explicitly with respect to actuating magnetic torque, τ_m , as³:

$$-\tau_m A = EI \frac{\partial^2 \theta}{\partial s^2}(s), \quad (\text{S3.2})$$

where

$$\tau_m = [0 \ 0 \ 1]\{R\mathbf{m} \times \mathbf{B}\}. \quad (\text{S3.3})$$

The variables $A = hw$, $I = h^3w/12$ and E represent the robot's cross-sectional area, second moment of area and Young's modulus, respectively, while R is the standard z -axis rotational matrix that accounts for the change in direction of $\mathbf{m}$ due to the robot's rotational deflection:

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (\text{S3.4})$$

Physically, the left side of Eq. (S3.2) represents the magnetic actuation that deforms the robot, while its right side is a function of the resultant deflections. Since we do not apply magnetic forces

onto the robot, the shear forces along the robot are null at steady-state and are therefore neglected for the quasi-static analyses (Fig. S3). For the locomotion modes that have small deflections, such as the undulating swimming, Eq. (S3.2) can be simplified by approximating R with a 3×3 identity matrix, $s \approx x$ (along the body frame's x -axis), and $\theta \approx \frac{\partial y}{\partial x}$ where y refer to the vertical deflection. With these simplifications, Eq. (S3.2) then becomes:

$$-[0 \quad 0 \quad 1]\{\mathbf{m} \times \mathbf{B}\}A = EI \frac{\partial^3 y}{\partial x^3}. \quad (\text{S3.5})$$

Using Eq. (S3.5), the robot's shape under small deflection condition can be described via the linear combination of two principal shapes, resembling respectively a sine and a cosine function (Fig. 1b(II,III)). The first principal shape can be enforced when the direction of $\mathbf{B}$ is parallel to $[\cos \beta_R \quad \sin \beta_R \quad 0]^T$, i.e., $\mathbf{B} = B[\cos \beta_R \quad \sin \beta_R \quad 0]^T$. Then, Eq. (S3.5) can be triple integrated with respect to x to become⁴:

$$y(x) = \frac{mAL^3 B}{8\pi^3 EI} \cos\left(\frac{2\pi}{L}x\right) + k_1 x^2 + k_2 x + k_3, \quad (\text{S3.6})$$

where B represents the magnitude of $\mathbf{B}$, and k_1 , k_2 and k_3 are the constants of integration, valid for all B values as long as the robot's deflections are small (i.e., $\theta \approx \frac{\partial y}{\partial x}$), including the case of $B = 0$. When $B = 0$, the LHS of Eq. (S3.6) has to be zero for all x , because the robot does not deform in the absence of $\mathbf{B}$, and this implies that Eq. (S3.6) becomes:

$$k_1 x^2 + k_2 x + k_3 = 0. \quad (\text{S3.7})$$

As x can vary from 0 to L , Eq. (S3.7) can only be satisfied across all x if and only if $k_1 = k_2 = k_3 = 0$. Therefore, Eq. (S3.6) can be rewritten as:

$$y(x) = \frac{mAL^3 B}{8\pi^3 EI} \cos\left(\frac{2\pi}{L}x\right). \quad (\text{S3.8})$$

Similarly, when $\mathbf{B}$ is parallel to $[-\sin(\beta_R) \quad \cos(\beta_R) \quad 0]^T$, i.e., $\mathbf{B} = B[-\sin(\beta_R) \quad \cos(\beta_R) \quad 0]^T$, Eq. (S3.5) can be triple integrated to become:

$$y(x) = \frac{mAL^3 B}{8\pi^3 EI} \sin\left(\frac{2\pi}{L}x\right) + k_4 x^2 + k_5 x + k_6, \quad (\text{S3.9})$$

and using the same reasoning as for k_1 , k_2 and k_3 , we deduce that Eq. (S3.9) can only be valid across all x if and only if $k_4 = k_5 = k_6 = 0$. Therefore, Eq. (S3.9) becomes:

$$y(x) = \frac{mAL^3 B}{8\pi^3 EI} \sin\left(\frac{2\pi}{L}x\right). \quad (\text{S3.10})$$

As mentioned in the main text, if the direction of $\mathbf{B}$ is not aligned along the principal directions, the robot will assume a shape that can be described by the weighted superposition of Eqs. (S3.8) and (S3.10). While Eqs. (S3.8) and (S3.10) can be used to fully describe the small deflection shapes

used in undulating swimming and meniscus climbing, they are insufficient to describe the large deflection shape change necessary for rolling, walking, jumping and jellyfish-like swimming. The large deflection shapes are achieved by applying a large magnitude $\mathbf{B}$ parallel or anti-parallel to $[\cos \beta_R \ \sin \beta_R \ 0]^T$. To predict the robot's shapes under such conditions, we have to solve Eq. (S3.2) numerically with the following free-free boundary conditions:

$$\frac{\partial \theta}{\partial s}(s=0) = \frac{\partial \theta}{\partial s}(s=L) = 0. \quad (\text{S3.11})$$

Physically, these boundary conditions imply that there is zero bending moment applied on the free-ends of the beam. The numerical solutions reveal that the robot creates a large deflection configuration that resembles a 'C'-shape when we apply a large magnitude $\mathbf{B}$ parallel to $-\cos \beta_R \ \sin \beta_R \ 0]^T$ (Fig. 1b(IV)). On the other hand, when we use the same boundary conditions and apply a large magnitude $\mathbf{B}$ parallel to $[\cos \beta_R \ \sin \beta_R \ 0]^T$, the numerical solution of Eq. (S3.2) yields a 'V'-shape (Fig. 1b(V)). In both configurations, the curvature increases as B is increased. Furthermore, if the direction of $\mathbf{B}$ is not aligned along the principal axis shown in Fig. 1b(IV,V), the robot will have a rigid-body rotation until its $\mathbf{M}_{\text{net}}$ is aligned with $\mathbf{B}$. At the end of this rotation, the robot will assume its 'C'- or 'V'-shape (see the next sub-section for more details).

B. Rigid-body rotation

As the net magnetic moment of a robot tends to align with the external magnetic fields, it is possible to use the direction of the fields to control the orientation of the magnetic robot^{1,5}. While our robot does not have a net magnetic moment in the un-deformed state, a net magnetic moment arises once the robot deforms. Mathematically, the net magnetic moment, $\mathbf{M}_{\text{net}}$, is expressed as:

$$\mathbf{M}_{\text{net}} = \int_0^L R \mathbf{m} A \, ds. \quad (\text{S3.12})$$

Once equipped with $\mathbf{M}_{\text{net}}$, we can create rigid-body torques on the robot by controlling $\mathbf{B}$. This magnetic torque, $\boldsymbol{\tau}_{\text{RB}}$, can be represented as:

$$\boldsymbol{\tau}_{\text{RB}} = \mathbf{M}_{\text{net}} \times \mathbf{B}. \quad (\text{S3.13})$$

Physically, Eq. (S3.13) implies that as long as $\mathbf{M}_{\text{net}}$ is not aligned with $\mathbf{B}$, a magnetic torque will be applied onto the robot. If there are no other external torques acting on the robot, the magnetic torque will eventually align $\mathbf{M}_{\text{net}}$ along the direction of the applied $\mathbf{B}$ at steady state conditions. In the following sub-sections, we will discuss how we can rotate the robot about its z - and y -axis.

I. Rotation about z -axis

Here, we will explain why the robot is unable to rotate about its z -axis when it has small deflections (*i.e.*, B is small) but will be able to do so under large deflection conditions (*i.e.*, B is large). We begin this discussion by showing that the robot's $\mathbf{M}_{\text{net}}$ will always be aligned with $\mathbf{B}$ under small deflection conditions, and this in turn will conclude that there will be no rigid-body magnetic torques acting on the robot, making the robot unable to rotate about its z -axis. To establish this mathematical proof, we first show that when the robot is subjected to a $\mathbf{B}$ that is

parallel to the principal directions shown in Fig. 1b(II-III), its resultant deformation will generate an $\mathbf{M}_{\text{net}}$ parallel to the corresponding principal directions. Subsequently, by using the superposition principle, we can generalize and conclude that the robot's $\mathbf{M}_{\text{net}}$ will always be aligned with the applied $\mathbf{B}$ under small deflection conditions.

Based on Eq. (S3.8), the robot will produce a cosine shape when $\mathbf{B}$ is parallel to its first principle direction, i.e., $\mathbf{B} = B[\cos(\beta_R) \ \sin(\beta_R) \ 0]^T$. Since the deformation of this shape is small, its rotational deflection can be approximated as:

$$\theta(x) \approx \frac{dy}{dx} = -\frac{mAL^2B}{4\pi^2EI} \sin\left(\frac{2\pi}{L}x\right), \quad (\text{S3.14})$$

Using the small rotational deflection representation in Eq. (S3.14), we can approximate $\cos\theta$ and $\sin\theta$ to be 1 and θ , respectively. These approximations will in turn simplify the resultant rotational matrix R in Eq. (S3.12) and Eq. (S3.4) to:

$$R(x) \approx \begin{bmatrix} 1 & \frac{mAL^2B}{4\pi^2EI} \sin\left(\frac{2\pi}{L}x\right) & 0 \\ -\frac{mAL^2B}{4\pi^2EI} \sin\left(\frac{2\pi}{L}x\right) & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (\text{S3.15})$$

By substituting Eq. (S3.15) into Eq. (S3.12), the deformed robot's $\mathbf{M}_{\text{net}}$ can be expressed as:

$$\mathbf{M}_{\text{net}} = \int_0^L mA \begin{pmatrix} \cos\left(\frac{2\pi}{L}x + \beta_R\right) + \frac{mAL^2B}{4\pi^2EI} \sin\left(\frac{2\pi}{L}x + \beta_R\right) \sin\left(\frac{2\pi}{L}x\right) \\ -\frac{mAL^2B}{4\pi^2EI} \cos\left(\frac{2\pi}{L}x + \beta_R\right) \sin\left(\frac{2\pi}{L}x\right) + \sin\left(\frac{2\pi}{L}x + \beta_R\right) \\ 0 \end{pmatrix} dx. \quad (\text{S3.16})$$

Based on simple trigonometry identities, $\sin\left(\frac{2\pi}{L}x + \beta_R\right)$ and $\cos\left(\frac{2\pi}{L}x + \beta_R\right)$ can be rewritten as:

$$\sin\left(\frac{2\pi}{L}x + \beta_R\right) = \sin\left(\frac{2\pi}{L}x\right) \cos(\beta_R) + \cos\left(\frac{2\pi}{L}x\right) \sin(\beta_R), \quad (\text{S3.17A})$$

$$\cos\left(\frac{2\pi}{L}x + \beta_R\right) = \cos\left(\frac{2\pi}{L}x\right) \cos(\beta_R) - \sin\left(\frac{2\pi}{L}x\right) \sin(\beta_R). \quad (\text{S3.17B})$$

By substituting Eq. (S3.17A-B) into Eq. (S3.16) and rearranging the equation, the deformed robot's $\mathbf{M}_{\text{net}}$ can be expressed as:

$$\begin{aligned} \mathbf{M}_{\text{net}} = \int_0^L mA \left(\begin{array}{c} \cos\left(\frac{2\pi}{L}x + \beta_R\right) + \frac{mAL^2B}{8\pi^2EI} \sin\left(\frac{4\pi}{L}x\right) \sin(\beta_R) \\ \sin\left(\frac{2\pi}{L}x + \beta_R\right) - \frac{mAL^2B}{8\pi^2EI} \sin\left(\frac{4\pi}{L}x\right) \cos(\beta_R) \end{array} \right) dx \\ + \frac{m^2A^2L^2B}{4\pi^2EI} \left[\int_0^L \sin^2\left(\frac{2\pi}{L}x\right) dx \right] \begin{pmatrix} \cos(\beta_R) \\ \sin(\beta_R) \\ 0 \end{pmatrix}. \end{aligned} \quad (\text{S3.18})$$

As the first component on the right side of Eq. (S3.18) integrates the harmonic functions over one full wavelength, this component is computed to be zero. Therefore, by completing the integration for the second component on the right side of Eq. (S3.18), this equation can be simplified into:

$$\mathbf{M}_{\text{net}} = \frac{m^2A^2L^3B}{8\pi^2EI} \begin{pmatrix} \cos(\beta_R) \\ \sin(\beta_R) \\ 0 \end{pmatrix}. \quad (\text{S3.19})$$

We remark that the direction of $\mathbf{M}_{\text{net}}$ in Eq. (S3.19) is parallel to the first principal direction of $\mathbf{B}$, i.e., $\mathbf{B} = B[\cos(\beta_R) \quad \sin(\beta_R) \quad 0]^T$.

Next, we repeat similar derivations when $\mathbf{B}$ is parallel to the second principal direction, i.e., $\mathbf{B} = B[-\sin(\beta_R) \quad \cos(\beta_R) \quad 0]^T$. Based on Eq. (S3.10), the robot will produce a sine shape with this $\mathbf{B}$. Assuming small deflection conditions, its rotational deflection can be approximated as:

$$\theta(x) \approx \frac{dy}{dx} = \frac{mAL^2B}{4\pi^2EI} \cos\left(\frac{2\pi}{L}x\right), \quad (\text{S3.20})$$

and the corresponding rotational matrix in Eq. (S3.12) becomes:

$$R(x) \approx \begin{bmatrix} 1 & -\frac{mAL^2B}{4\pi^2EI} \cos\left(\frac{2\pi}{L}x\right) & 0 \\ \frac{mAL^2B}{4\pi^2EI} \cos\left(\frac{2\pi}{L}x\right) & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (\text{S3.21})$$

By substituting Eq. (S3.21) into Eq. (S3.12) and use steps similar to those presented in Eq. (S3.16-S3.18), the deformed robot's $\mathbf{M}_{\text{net}}$ can then be expressed as:

$$\mathbf{M}_{\text{net}} = \frac{m^2A^2L^3B}{8\pi^2EI} \begin{pmatrix} -\sin(\beta_R) \\ \cos(\beta_R) \\ 0 \end{pmatrix}, \quad (\text{S3.22})$$

and this shows that the generated $\mathbf{M}_{\text{net}}$ is parallel to the second principal direction of $\mathbf{B}$. Hence, Eqs. (S3.19) and (S3.22) conclude that the deformed robot will not be able to rotate about its z-axis when a $\mathbf{B}$ with small magnitude is applied parallel to either of the principal directions, i.e., there is no rotation when the robot deflections are small. Because superposition principle is applicable for the analysis shown in Eq. (S3.14-S3.22), we conclude that under small deflection

conditions (*i.e.*, B is small), the robot will not be able to rotate about its z -axis. Hence in such situations, the robot will only experience a shape-change in the xy plane as described in the main text and SI section S3A.

Conversely, because the superposition principle will no longer be applicable for a large B , the robot's $\mathbf{M}_{\text{net}}$ in such situations will not necessarily be parallel with the applied $\mathbf{B}$. This eventually will make the robot rotate until its $\mathbf{M}_{\text{net}}$ is aligned with $\mathbf{B}$, and the robot will assume either its 'C'- or 'V'-shape configuration when it stops rotating. The 'C'- or 'V'-shapes are the steady-state configurations because their generated $\mathbf{M}_{\text{net}}$ is always naturally aligned with the applied $\mathbf{B}$ (as shown in Fig. 1b (IV-V)). Hence for large deflection cases, we can adjust the direction of $\mathbf{B}$ in the XY plane to rotate the 'C'- or 'V'-shape robot about its z -axis (Fig. 1c). The ability to control the robot's angular displacement about its z -axis is essential for achieving several modes of locomotion, as it allows the robot to produce the necessary tilting for jumping and walking, as well as enabling the rolling locomotion (see Supporting Video S3 and SI section S5).

Finally, we will also discuss what happens when a mid-range B is applied, *i.e.*, a B that is neither sufficiently small nor big. In this situation, two possible scenarios could occur. When a mid-range B is lower than a specific threshold, the deformed robot will not be able to generate an effective rigid-body rotation. As a result, the obtained shape of the robot can be predicted by solving Eq. (S3.2) numerically. On the other hand, if the mid-range B exceeds this threshold, the robot would rotate until its $\mathbf{M}_{\text{net}}$ is aligned with $\mathbf{B}$ and it will assume either its 'C' or 'V'-shape configuration in steady-state conditions. Theoretically, this threshold represents the minimum required B for the deformed robot to generate a $\mathbf{M}_{\text{net}}$ that is non-parallel to the applied field. However, in practice as our fabrication process is not perfect, it is not easy to quantify this threshold accurately. Hence, avoiding mid-range B and employing either small or large B would be the best to ensure that the robot is able to achieve all modes of locomotion. Indeed, we show that we are able to perform all modes of locomotion by using ranges of B that are sufficiently small or large.

II. Steering strategy (rotation about y -axis)

As the robot's $\mathbf{M}_{\text{net}}$ always resides in its xy plane (Fig. 1b), this implies that when a $\mathbf{B}$ that only has XY plane components is applied on the robot, a magnetic torque can re-orientate the robot until its local xy plane becomes parallel to the global XY plane shown in Fig. S2. Similarly, after making the robot's xy plane to be parallel with the global XY plane, we can use B_z to create a magnetic torque that can rotate the robot about its y -axis and steer it along other desired directions. In contrast to rotating the robot about its z -axis, we can always rotate the robot about its y -axis regardless of the magnitude of $\mathbf{B}$. More information regarding how to use external magnetic fields to steer a magnetic robot with a net magnetic moment along a desired direction can be found in references 1 and 3.

C. Predicting the operating range of B

The theoretical models presented in section 3A-B can help to determine the operating ranges of B such that the robot can generate the required time-varying shapes to enable different modes of locomotion. For locomotion modes enabled by small B , we can use Eqs. (S3.8) and (S3.10) to predict the required $\mathbf{B}$ for generating the necessary time-varying shapes. However, a recursive approach has to be used for determining the required $\mathbf{B}$ for locomotion modes that require a large B . This recursive approach is implemented by first specifying a $\mathbf{B}$, and substitute it into Eq. (S3.2)

to numerically solve for the curvature of the ‘C’- or ‘V’-shape configuration. If the obtained curvature is unsatisfactory, we will change another $\mathbf{B}$, and repeat the same procedures until a suitable $\mathbf{B}$ is obtained.

S4 - Analysis for jumping

To analyze the jumping locomotion, we first specify the required $\mathbf{B}$ signals (section S4A). Subsequently, we provide a theoretical model, which can predict how high the robot can jump when we vary $\mathbf{B}$ (section S4B). Using this model, we then discuss how the jumping height of the soft robot is affected by its dimensions (section S4C).

A. Experimental details

For the robot to jump along a desired direction (Fig. 2h, also in Fig. S4a), we specify the following $\mathbf{B}$ sequence (Fig. S5a):

1. The robot is first curled into a ‘C’-shape configuration by specifying $B = 16.5$ mT and $\alpha = -90^\circ$ (0 ms in Fig. S4a).
2. Subsequently, we use a step function to change the direction of $\mathbf{B}$, rotating its direction 162° clockwise. This step function simultaneously induces a rigid-body rotation (clockwise in this case) as the robot’s $\mathbf{M}_{\text{net}}$ tends to align with the direction of $\mathbf{B}$, as well as the robot’s shape changes shown in 0 – 13.5 ms in Fig. S4a.
3. While $\mathbf{B}$ ($B = 16.5$ mT, $\alpha = 72^\circ$) is maintained, the ends of the robot hit the ground (13.5 ms in Fig. S4a), allowing the robot to jump (13.5 – 16.9 ms in Fig. S4a).

As mentioned, such $\mathbf{B}$ sequence prompts two different mechanisms as the robot performs directional jumping. The first mechanism is the rigid-body rotation of the robot, induced by the rigid-body torque created from the interaction of $\mathbf{M}_{\text{net}}$ and the flipped $\mathbf{B}$ (0 – 13.5 ms in Fig. S4a). The second mechanism is the shape-change, as the robot changes from its initial ‘C’-shape (0 ms in Fig. S4a) to a flatter sheet (10.2 ms Fig. S4a) and then back to its ‘C’-shaped configuration (10.2 - 13.5 ms in Fig. S4a). Both mechanisms help to increase the momentum of the robot until the robot strikes the substrate. Upon impact, the substrate then induces both a vertical and horizontal momentum for the robot to jump.

In the following discussion, we focus on the shape-change mechanism, for which we have devised a ‘straight’ (vertical) jumping strategy (Fig. S4b). Using robots with $\beta_R = -90^\circ$, the required setup for experimental characterization reduces to the single pair of electromagnetic coils along the Y axis (Fig. S2). Robots with $\beta_R = 45^\circ$ can also perform straight jumping but they require two pairs of electromagnetic coils to generate a step $\mathbf{B}$ along the $\alpha = 135^\circ$ direction, and this may induce unnecessary experimental errors. To further ensure a spatially uniform $\mathbf{B}$, we adjusted the distance of the two coils along the Y axis to form a Helmholtz configuration. Using these experimental conditions, the robot jumps when we specify a step change of $\mathbf{B}$ along the Y -axis of the global reference system. Mathematically, $\mathbf{B}$ can then be simply expressed as $\mathbf{B} = [0 \ B \ 0]^T$.

B. Theoretical model

Here we provide a model for the straight jumping strategy. This model is based on an energy conservation approach, which first analyzes the energy states of the soft robot at two salient instants: 1) when it is initially flat on the substrate, and 2) the moment when it leaves the substrate. Based on the principle of energy conservation, the soft robot gains energy from the work done by

the magnetic torque, W , between these two states. This energy is then re-distributed into three components: the change in strain energy, ΔS , and kinetic energy, ΔK , for the robot, and the frictional losses, f_L , during the jump of the robot. Mathematically, this implies that:

$$W - \Delta S - f_L = \Delta K. \quad (\text{S4.1})$$

The frictional losses include energy loss from contact friction with the substrate, air drag, and viscoelastic losses within the soft robot. To simplify our analysis, here we will neglect the effects of frictional losses and assume that f_L is equal to zero. Equation (S4.1) can therefore be simplified into:

$$W - \Delta S = \Delta K. \quad (\text{S4.2})$$

Based on the Euler-Bernoulli equation, ΔS can be expressed as:

$$\Delta S = \frac{1}{2} \int_0^L EI \left(\frac{\partial \theta_f}{\partial s} \right)^2 - EI \left(\frac{\partial \theta_i}{\partial s} \right)^2 ds = \frac{1}{2} \int_0^L EI \left(\frac{\partial \theta_f}{\partial s} \right)^2 ds, \quad (\text{S4.3})$$

where $\theta_f(s)$ represents the final angular deformation of the robot before it leaves the substrate and we assume that the initial shape of the robot is flat, i.e., $\theta_i(s) = 0$.

On the other hand, the magnetic work done W can be expressed as a double integral:

$$W = \int_0^L \left[\int_{\theta_i}^{\theta_f} \tau_m A d\theta \right] ds. \quad (\text{S4.4})$$

The inner integral represents the work done by the magnetic torque on an arbitrary infinitesimal volumetric element along the robot (Fig. S6). The upper limit of this integral represents the final angular deformation of the element before the robot leaves the substrate and the lower limit $\theta_i(s) = 0$. Therefore, W can be obtained by the outer integral as we sum up the total work done over all the infinitesimal elements along the robot. By using Eqs. (S3.3-S3.4), the inner integral can be evaluated to be:

$$\int_0^{\theta_f} \tau_m A d\theta = \int_0^{\theta_f} \{ (m_x B_y - m_y B_x) \cos \theta - (m_y B_y + m_x B_x) \sin \theta \} A d\theta. \quad (\text{S4.5})$$

Because the magnetization profile components, m_x and m_y , and the actuating fields, B_x and B_y are independent of θ , Eq. (S4.5) can be evaluated to be:

$$\int_0^{\theta_f} \tau_m A d\theta = \{ (m_x B_y - m_y B_x) \sin \theta_f + (m_y B_y + m_x B_x) (\cos \theta_f - 1) \} A. \quad (\text{S4.6})$$

Finally, by substituting Eq. (S4.6) and our magnetic actuation signals ($\mathbf{B} = [B_x \ B_y \ 0]^T = [0 \ B \ 0]^T$) back into Eq. (S4.4), we can express W as:

$$W = -mBA \int_0^L \cos(\omega_s s + \theta_f) ds. \quad (\text{S4.7})$$

Physically, Eq. (S4.7) implies that the work done is proportional to the magnitude of the magnetization profile and the actuating field, and it is also dependent on the final shape of the robot before it leaves the substrate. By substituting Eqs. (S4.7) and (S4.3) back into Eq. (S4.2), we obtain a mathematical expression for ΔK . Once ΔK is known, we apply the principle of energy conservation again, this time from the moment when the robot leaves the substrate till the moment when it reaches its maximum height, $H_{\max}$. At the maximum height, we assume all the kinetic energy used for jumping is fully converted into gravitational potential energy:

$$M_{\text{Rob}} g H_{\max} = \Delta K, \quad (\text{S4.8})$$

where M_{Rob} and g represent the mass of the robot and the gravitational constant, respectively. We note that Eq. (S4.8) is a simplified model because it does not account for frictional losses and also not all the kinetic energy in Eq. (4.2) is used for jumping, *i.e.*, some of the kinetic energy is converted to excite other modes of vibration of the robot. By substituting Eqs. (S4.3) and (S4.7) into Eq. (S4.8) and by rearranging the equation, $H_{\max}$ can be obtained as:

$$H_{\max} = - \left(\frac{mBA}{M_{\text{Rob}} g} \int_0^L \cos(\omega_s s + \theta_f) \, ds + \frac{EI}{2M_{\text{Rob}} g} \int_0^L \left(\frac{\partial \theta_f}{\partial s} \right)^2 \, ds \right). \quad (\text{S4.9})$$

Using Eq. (S4.9), we can then predict the theoretical $H_{\max}$ of the soft robot across different B . Equation (S4.9) shows that $H_{\max}$ is independent of w but its highly non-linear nature makes it difficult to derive a simple scaling law for h and L . Despite so, we will show in the following subsection that this model is able to predict that the sample robots in Fig. S7 can jump higher if they have a smaller h (Fig. S7a, b) and a larger L (Fig. S7a.c).

C. Discussion

To investigate how the jumping performance varies when the dimensions of the soft robot are changed, we fabricated six additional robots, each having a different L , w or h from the original robot shown in Fig. 1a. For these additional robots, we have measured their maximum jumping height for $13.5 \text{ mT} \leq B \leq 23.5 \text{ mT}$. We used image detection techniques in MATLAB and standard spline fitting methods to extract from the experiments the final shape of the robots before they left the substrate, *i.e.*, the θ_f of the robots. Experimental and model-predicted values of $H_{\max}$ are presented in Fig. S7. Because θ_f may vary in each experimental point, the model predicted points in Fig. S7 also have error bars.

Generally, while the model in Eq. (S4.9) agrees with the experimental trends, the predicted values of $H_{\max}$ in Fig. S7 overestimate the actual values as they assume that there are no frictional losses and all kinetic energy of the robot is used for jumping.

The experimental dependency of jumping height on B parameterized by the dimensions of the robots is shown in Fig. S8. Based on the experimental data, robots can jump higher if they have a longer L or a thinner h and there are no observable effects when w is changed. This trend agrees with the model specified by Eq. (S4.9) (see model predicted curve in Fig. S7).

In summary, our experimental results suggest that the soft robot can jump higher if we increase L or decrease h for the straight jumping strategy. The predictions of the theoretical model agree with the experimental data.

S5 - Analysis for rolling

Here we analyze the rolling locomotion by first specifying the required $\mathbf{B}$ sequence (section S5A) and then providing a theoretical model, which can predict how fast the robot can roll as we vary the rolling frequency f of $\mathbf{B}$ (section S5B). Finally, we discuss how the rolling speed V_{roll} of the robot is affected when we vary the robot's dimensions (section S5C).

A. Experimental details

A schematic drawing for the robot rolling in Fig. 2e is illustrated in Fig. S10. This locomotion is realized by using the following $\mathbf{B}$ sequence in the XY plane (Fig. S9a):

1. An initial $\mathbf{B}$ ($B = 18.5$ mT, $\alpha = 0^\circ$) is imposed to curl the robot into a 'C'-shape as shown in Fig. S9b.
2. The deformed robot possesses an $\mathbf{M}_{\text{net}}$ as specified by Eq. (S3.12), allowing it to be steered by a rotating $\mathbf{B}$ in the XY plane. The rotating frequency is 5 Hz in Fig. 2e.
3. As $\mathbf{B}$ starts to rotate clockwise, a rigid-body magnetic torque, τ_{Roll} , is induced onto the robot as the robot's $\mathbf{M}_{\text{net}}$ tends to align with $\mathbf{B}$. This torque makes the robot roll.
4. Once $\mathbf{B}$ stops rotating, the robot's $\mathbf{M}_{\text{net}}$ fully aligns with $\mathbf{B}$ and τ_{Roll} is reduced to a null vector. Hence, the robot will stop rolling.

Mathematically, the sequence of $\mathbf{B}$ described above can be represented as:

$$\mathbf{B}(t) = B \begin{pmatrix} \cos(2\pi ft) \\ \sin(2\pi ft) \\ 0 \end{pmatrix}, \quad (\text{S5.1})$$

where f represents the frequency of the rotating $\mathbf{B}$ and it is ranged between $0 < f \leq 40$ Hz in all of our experiments. To fairly evaluate V_{roll} for robots with differing dimensions, we had adjusted B for each robot such that all of them could produce the same curvature. This adjustment is described in the next subsection.

B. Theoretical model

To enable the rolling locomotion, the magnitude of the rotating $\mathbf{B}$ has to be large to make the robot curve into a 'C'-shape. For the original robot shown in Fig. 1a, a B larger than 10 mT is selected to guarantee a sufficiently large robot deflection.

We assume the rolling locomotion akin to that of a wheel rolling on a flat substrate. This assumption is supported by the observation that the robot can curl into a 'C'-shape that resembles a circular wheel when we apply the rotating $\mathbf{B}$. We have used MATLAB 2016 (boundary value problem solver) to numerically solve the governing Eq. (S3.2), and we then computed the net magnetic moment, $\mathbf{M}_{\text{net}}$, of the robot in the 'C'-shape configuration with Eq. (S3.12). In the rolling locomotion, $\mathbf{M}_{\text{net}}$ becomes a function of the rotational displacement of the robot, ϕ_R (Fig. S10), and mathematically it can be expressed as:

$$\mathbf{M}_{\text{net}}(\phi_R) = M_{\text{net}} \begin{pmatrix} \cos(\phi_R) \\ \sin(\phi_R) \\ 0 \end{pmatrix}, \quad (\text{S5.2})$$

where the scalar M_{net} represents the magnitude of the net magnetic moment.

Once $\mathbf{B}$ starts rotating, a rigid-body torque, τ_{Roll} , is applied on the robot as its net magnetic moment tries to align with the field. Mathematically, τ_{Roll} can be written as:

$$\tau_{\text{Roll}} = [0 \quad 0 \quad 1](\mathbf{M}_{\text{net}} \times \mathbf{B}) = M_{\text{net}}B \sin(2\pi ft - \phi_R). \quad (\text{S5.3})$$

The magnitude of τ_{Roll} depends on the M_{net} and B , and on the angle between them. As the robot rolls, the ground induces a pushing force, F , which propels the translation of the robot (Fig. S10). The translational equation of motion of the robot is:

$$F - C_T V_{\text{roll}} = M_{\text{Rob}} \dot{V}_{\text{roll}}, \quad (\text{S5.4})$$

where C_T represents the translational damping. Likewise, the equation of motion of the robot about the z -axis can be expressed as:

$$\tau_{\text{Roll}} - C_R \dot{\phi}_R - F r_{\text{eff}} = J \ddot{\phi}_R, \quad (\text{S5.5})$$

where C_R , r_{eff} , and J represent the rotational damping coefficient, radius of wheel and moment of inertia of wheel, respectively. Assuming that the robot does not slip as it rolls on the substrate, the kinematic relationship between the x and ϕ_R can be expressed as:

$$r_{\text{eff}} \dot{\phi}_R = V_{\text{roll}}, \quad r_{\text{eff}} \ddot{\phi}_R = \dot{V}_{\text{roll}}. \quad (\text{S5.6})$$

By substituting this no-slip condition into Eq. (S5.4), multiplying it by r_{eff} , and finally adding it to Eq. (S5.5), the governing dynamic equation for the rolling locomotion can be expressed as:

$$M_{\text{net}}B \sin(2\pi ft - \phi_R) = J_{\text{eff}} \ddot{\phi}_R + C_{\text{eff}} \dot{\phi}_R, \quad (\text{S5.7})$$

where $J_{\text{eff}} = M_{\text{Rob}} r_{\text{eff}}^2 + J$ and $C_{\text{eff}} = C_T r_{\text{eff}}^2 + C_R$. Since the size of the robot is relatively small, its small J_{eff} enables the robot to quickly reach a steady-state velocity, which can be rewritten as:

$$\dot{\phi} = \frac{M_{\text{net}}B}{C_{\text{eff}}} \sin(2\pi ft - \phi_R). \quad (\text{S5.8})$$

A common phenomenon for miniature, magnetically actuated rotating robots is that the angle between the actuating field and the robots' angular displacement becomes constant at steady-state speed^{6,7}. Mathematically, this means that $\sin(2\pi ft - \phi_R)$ becomes a constant value in Eq. (S5.8). Equation (S5.8) also reveals that the maximum angular velocity of the robot occurs when the angle between the field and the robot's angular displacement becomes 90° , i.e., $2\pi ft - \phi_R = \frac{\pi}{2}$ since $\sin(2\pi ft - \phi_R) = 1$. This maximum angular velocity is known as the *step-out* frequency for the soft robot, and it is represented by:

$$\dot{\phi}_{R \text{ step-out}} = \frac{M_{\text{net}}B}{C_{\text{eff}}}. \quad (\text{S5.9})$$

Once the rotating frequency of $\mathbf{B}$ exceeds the step-out frequency, *i.e.*, $2\pi f > \dot{\phi}_{R_{step-out}}$, the frequency of $\mathbf{B}$ is no longer synchronized with the robot's angular velocity and it is difficult to predict $\dot{\phi}_R$. The discussion for such conditions will, however, be beyond the scope of this paper.

We experimentally validated that our robots do not exceed the step-out frequency under the control signals we used. Therefore $2\pi f t - \phi_R = k$, where k is the constant lag angle between $\mathbf{B}$ and ϕ_R , and it does not change with time once the robot reaches its steady-state speed. By differentiating this expression in terms of time, this implies that:

$$\dot{\phi}_R = 2\pi f. \quad (\text{S5.10})$$

Based on Eq. (S5.10) and the no-slip condition in Eq. (S5.6), the governing equation for our rolling locomotion becomes:

$$V_{roll} = 2\pi r_{eff} f. \quad (\text{S5.11})$$

The variable r_{eff} can be determined by first solving Eq. (S3.2) numerically using MATLAB 2016 (boundary value problem solver) to determine the rolling shape, *i.e.*, the 'C'-shape, and then determine the parameters of the circle that best fits it. We remark that the effective circumference of the fitting circle, *i.e.*, $2\pi r_{eff}$, is generally approximately 10% higher than the length of the robot. A simple scaling analysis can also be derived based on Eq. (S5.11) if we approximate $r_{eff} \approx \frac{L}{2\pi}$ and this will suggest that $V_{roll} \propto L$.

As m , B , E and other constants are independent of the dimensions of the robot, we can conclude that a higher B is required to roll stiffer robots³, *i.e.*, robots with smaller L or larger h . Indeed, to enable a fair comparison of rolling locomotion performance for robots of different dimensions, we have adjusted B in such a way that all of the robots could produce the same curvature. For the robots with length L_{new} different from the reference L , the same curvature could be obtained when the magnitude of $\mathbf{B}$, $B_{L,roll}$, became³:

$$B_{L,roll} = \left(\frac{L}{L_{new}}\right)^2 B. \quad (\text{S5.12})$$

Likewise, we could obtain the same curvature for robots that had thickness h_{new} by changing the magnitude of $\mathbf{B}$, $B_{h,roll}$, to:

$$B_{h,roll} = \left(\frac{h_{new}}{h}\right)^2 B. \quad (\text{S5.13})$$

Finally, varying w does not change the curvature if B is unchanged³. Therefore, we used Eq. (S5.12-S5.13) in our experiments to adjust the corresponding B for robots with differing dimensions, allowing all of the robots to create a curvature equivalent to that of the original robot under $B = 18.5$ mT.

C. Discussion

Here we discuss how the rolling speed V_{roll} varies with the frequency f of the rotating $\mathbf{B}$ and the dimensions of the robot. For this purpose, we fabricated six additional robots where each of them has either a different L , w or h compared to the original robot.

The experimental data presented in Fig. S11 show that the V_{roll} has a linear relationship with f at low values, *e.g.*, $f < 10$ Hz in Fig. S11a-b. Generally, our theoretical model agrees with the data, though it underestimates V_{roll} . This discrepancy could be justified by the fact that the actual ‘C’-shape configuration of the robot is not a perfect circle, as assumed in the model. Moreover, the standard deviation of V_{roll} increases when f becomes high, *e.g.*, when $20 \text{ Hz} \leq f \leq 40 \text{ Hz}$ in Fig. S11a-b. This occurs because for such high f the ends of the robot tend to impact the substrate with such a large force that it makes the robot jump. To avoid this less predictable behavior, we recommend constraining V_{roll} within its linear regime.

The experimental data for the dependency of V_{roll} on f parameterized by the dimensions of the robots, presented in Fig. S12, show no significant variations in V_{roll} versus w and h . The V_{roll} , however, increases when the robots have larger L . This trend generally agrees with the predictions of Eq. (S5.11), as r_{eff} becomes bigger when L increases.

In summary, the rolling speeds of the robots have a linear relationship with a low f . Robots with a larger L can roll faster. Rolling is also much more controllable for relatively low f (*e.g.*, $f \leq 4$ Hz for our robots). Finally, according to Eqs. (S5.12) and (S5.13), it is easier to induce the curvature necessary for rolling (*i.e.*, we can induce the same curvature using a lower B) for robots with larger L or smaller h . In this respect, we can use the same B to achieve the same curvature for robots that have different w .

S6 - Analysis for walking

In this section, we first specify a typical sequence of $\mathbf{B}$ required for walking (section S6A) and then provide a model that can predict the walking speed of our robot as we vary f and B_{max} , *i.e.*, the maximum B value in one walking cycle (section S6B). Finally, we discuss how the robot dimensions affect its walking speed in section S6C.

A. Experimental details

Driven by the $\mathbf{B}$ sequence shown in Fig. S13a in the XY plane, the soft robot can walk rightwards as shown in Fig. 2f. The $\mathbf{B}$ sequence can be described as follows:

1. The robot first anchors on its front end and starts to tilt forward. As it rotates, it also starts to deform into a ‘C’-shape (0 ms – 62 ms in Fig. 2f). To realize this motion sequence, the direction of $\mathbf{B}$ is fixed at $\alpha = 150^\circ$ while B is increased linearly until it peaks at $B_{\text{max}} = 10$ mT.
2. The robot maintains the ‘C’-shape and begins to anchor on its back end such that it can tilt backwards (62 ms – 122 ms in Fig. 2f). To realize this motion, the direction of $\mathbf{B}$ changes linearly from $\alpha = 150^\circ$ to $\alpha = 120^\circ$ while maintaining B at 10 mT.
3. Finally, the robot extends its front end, allowing it to make a net stride forward (122 ms – 210 ms in Fig. 2f). To realize this motion, the direction $\mathbf{B}$ is fixed at $\alpha = 120^\circ$ while its magnitude is decreased linearly from 10 mT to 0 mT.

To investigate how the walking locomotion is affected by the dimensions of the soft robot, we fabricated six additional robots that have either a different L , w or h compared to the original robot.

Using these robots, we conducted two types of experiments, where we respectively evaluated the walking speed of the robots, V_{walk} , versus different f (from 2 Hz to 20 Hz) with B_{max} fixed at 10 mT, and V_{walk} versus different B_{max} (from 8 mT to 16 mT) with f fixed at 5 Hz.

B. Theoretical model

Based on our walking strategy, the net stride of the robot in one walking cycle, S_{walk} , is a function of the difference between the robot's curvature in the states shown at times 0 ms and 102 ms in Fig. 2f. More explicitly, S_{walk} can be expressed as:

$$S_{\text{walk}} = S_1 - S_2, \quad (\text{S6.1})$$

where S_1 and S_2 represent the distance between the ends of the robot as illustrated in Fig. S14. Because $B = 0$ in Fig. S14a, S_1 can be approximated as L . On the other hand, S_2 in Fig. S14b can be computed by first solving Eq. (S3.2) numerically with MATLAB 2016 (boundary value problem solver).

We assume that the soft robot can respond to the actuating fields when the frequency of $\mathbf{B}$ is low. In these regime, the walking speed, V_{walk} , can be approximated as the product of S_{walk} and the walking frequency f :

$$V_{\text{walk}} = S_{\text{walk}}f = (L - S_2)f. \quad (\text{S6.2})$$

Validity of Eq. (S6.2) assumes no-slip conditions at the anchored ends. Equations (S6.2) and (S3.2) suggest that V_{walk} does not scale with w , and they also suggest that V_{walk} has a non-linear relationship with L and h , as S_2 can only be solved numerically by Eq. (S3.2). However, Eq. (S3.2) does suggest that robots with larger L and thinner h can deform more to create a smaller S_2 . This in turn suggests that Eq. (S6.2) predicts that robots with larger L and thinner h will walk faster.

C. Discussion

Figure S15 compares the experimental data with corresponding theoretical predictions, based on Eq. (S6.2) and Eq. (S3.2), for V_{walk} versus f . The experimental results suggest that V_{walk} has a linear relationship with f at low frequencies. However, for higher values of f , there appear instances where V_{walk} has a non-monotonic trend, *e.g.* V_{walk} drops to zero after 14 Hz in the $L = 333 \mu\text{m}$ case (Fig. S15b(II)). This happens because, under such conditions, the ends of the robot approach the substrate so quickly (at the end of Step 2 in SI section S6A) that upon contact the robot is bounced backwards. Such bouncing reduces the effective net stride per walking cycle, making the robots slower, and in the case shown in Fig. S15b(II), V_{walk} becomes 0 mm/s.

Figure S16 compares the experimental data with corresponding theoretical predictions, based on Eq. (S6.2) and Eq. (S3.2), for V_{walk} versus B_{max} . The experimental results show that V_{walk} correlates positively with B_{max} . This agrees with our theoretical model, which predicts that a larger B_{max} can induce a larger net stride, *i.e.*, $(L - S_2)$, in each walking cycle.

The experimental data for the dependency of V_{walk} on f and B_{max} parameterized by the dimensions of the robots are respectively shown in Fig. S17 and Fig. S18. The results in Fig. S17 suggest that, in the linear regime where $f \leq 11$ Hz, more compliant robots (*i.e.*, with larger L or smaller h) walk faster when B_{max} is fixed. This happens because the more compliant robots can produce a larger net stride per walking cycle. Additionally, the robots with larger L are expected

to walk faster thanks to a relatively larger body-length. Our experimental data do not show any obvious correlation between V_{walk} with w .

In general, while the theoretical predictions in Eq. (S6.2) and Eq. (S3.2) tend to overestimate V_{walk} , they agree with the trend of the experimental data (Fig. S15 and S16). Indeed, our model predicts that robots with smaller h and larger L walk faster while w does not affect V_{walk} . The predicted speeds are larger than the experimental speeds because the robots are observed to slip at their anchor points on the substrate, and slippage reduces the net stride of the robots in each walking cycle. Furthermore, due to fabrication uncertainties (section S1D), the robots are not perfectly flat even for $B = 0$. Therefore, also by approximating S_1 with L our model tends to overestimate the net stride in each walking cycle.

S7 - Analysis for meniscus climbing

In this section, we analyze the robot's meniscus climbing by specifying a typical required sequence of $\mathbf{B}$ for this locomotion (section S7A) and then proposing a quasi-static, force-based model which can predict the minimum required field magnitude, B_{min} , to effectively let the robot climb up to the top of a positive liquid meniscus (section S7B). Finally, we discuss how the robot dimensions affect meniscus climbing in section S7C.

A. Experimental details

Driven by the $\mathbf{B}$ sequence of Fig. S19 in the XY plane, the water meniscus climbing shown in Fig. 2b that prompts a subsequent landing onto the adjacent platform, is achieved by the following steps:

1. The initial $\mathbf{B}$ ($B = 18.5$ mT, $\alpha = 315^\circ$) is imposed to let the robot assume a 'C'-shape with upward curvature. The robot achieves in 520 ms a stable position along the meniscus.
2. $\mathbf{B}$ is subsequently rotated counter-clockwise toward $\alpha = 157^\circ$ to make the robot climb further up. After 4.6 s of $\mathbf{B}$ is on, the robot reaches contact with the wall of the platform. The subsequent rotation progressively pushes the upper portion of the robot out of water while being supported by the curled lower portion of the robot on the meniscus (4.8 s – 12.1 s).
3. Finally, $\mathbf{B}$ is turned off and the robot rests on the edge of the adjacent platform.

The above description uses a 'C'-shape configuration to climb the meniscus. If there is no need to push the upper portion of the robot out of the water, we remark that the meniscus climbing locomotion can also be realized when the robot has cosine-shaped small deflection (Fig. 1bIII). In the following section, we propose a quasi-static model of the soft robot's water meniscus climbing, which describes the process till the robot touches the wall of the platform. The corresponding experiment we performed is shown in Fig. S20. A $\mathbf{B}$ pointing upwards and of increasing magnitude was imposed to progressively curve the robot, and the smallest $\mathbf{B}$ magnitude B_{min} which made the robot touch the wall at the top of the meniscus was recorded. As the robot's $\mathbf{M}_{\text{net}}$ is always aligned with $\mathbf{B}$ under small deflection conditions (section S3B), the following model does not consider magnetic torque.

B. Theoretical model

When contacting water from air, our soft robots get almost completely immersed in water (Fig. S20) and strongly pinned at the water/air interface, as confirmed by the conformal attachment of the triple contact line to the robots perimeter during meniscus climbing (see *e.g.*, Fig. 3d).

The robots stably reside at the water-air interface thanks to the generalized Archimedes principle⁸, *i.e.*, the combined effect of buoyancy, acting on the surface of the robots in contact with water, and of surface curvature, acting on their contact perimeter. The generalized buoyancy force is quantified by the total weight of water displaced by the robots, which includes the water displaced both by the robots' body (*i.e.*, inside the contact line) and by the menisci along their perimeter (*i.e.*, outside)⁸. Given the conformal contact line pinning along its perimeter, a floating body can move upward along a positive liquid meniscus provided that the quasi-static work the body makes on the system, by extending and/or curving the water interface, compensates the energy required to climb the gravitational potential.

Here we first describe a two-dimensional force-based analytical model for an arc-shaped, soft floating robot (arc-shaped model, in short) to explain the mechanics of meniscus climbing and predict the body curvature that allows the robot, with known geometrical and material parameters, to climb the height of a sloped water meniscus. We then derive a model for cosine-shaped robots (*i.e.*, cosine-shaped model) to predict the minimal value of B , $B_{\min}$, that makes the robot climb the complete height of the meniscus.

I. Arc-shaped model

The two-dimensional geometry of reference is sketched in Fig. S21a. According to experiments, we assume the hydrophobic robot body fully immersed in water with only its top surface in contact with air. The triple contact line is consequently pinned at the perimeter of the top robot surface. Contact line pinning leaves the edge angles θ_1 and θ_2 undetermined, thus free to conform to boundary conditions within the limits imposed by canthotaxis, specifically by the Gibbs' criterion⁹. This has two important consequences. First, edge angles are not imposed by material properties, and so they are largely independent of robot's pose (described by the tilt angle α) and curvature (by the radius of curvature R and the angle ϕ). As a corollary, the physical cause of meniscus climbing is in our case not primarily related to surface tension effects. Second, by extending the canthotaxis sector, the robot's surface hydrophobicity enhances contact line pinning and thus the stability of the robot position on the water surface.

The mechanism underlying a soft robot's meniscus climbing is based on buoyancy. When assuming a curved shape, the robot (of density ρ_r) displaces an additional volume of water (of density ρ_w) from the surface (see Fig. 2b, reproduced in Fig. S19b). In presence of a positive water meniscus, the additional buoyancy enables the robot to move up along the interfacial profile. The five variables in the problem (θ_1 , θ_2 , α , R , and ϕ , see Fig. S21a) can be determined by satisfying conditions for mechanical and hydrostatic equilibrium of the curved robot at a distance d from the adjacent, vertical wall. The wall intersects the water surface profile, $H(x)$, at a fixed contact angle θ_0 . In experiments, both the vertical position $H(0)$ of the contact line at the wall and θ_0 were not affected by the presence of the robot (Fig. S20).

The shape of liquid interfaces in hydrostatic equilibrium is described by the Young-Laplace equation^{9,10}:

$$\gamma \frac{\partial^2 H(X)}{\partial X^2} = \rho g H(X). \quad (\text{S7.1})$$

The unperturbed profile H_u of the meniscus (*i.e.*, in absence of the robot) is determined by the boundary conditions $\frac{\partial H(X)}{\partial X} = H'(X) = -\cot \theta_0$ and $H(+\infty) = 0$, leading to:

$$H_u(X) = L_c \cot \theta_0 e^{-X/L_c}, \quad (\text{S7.2})$$

where we define the capillary length $L_C = \sqrt{\gamma/\rho g} = 2.7$ mm. Hence $H_u(0) = L_C \cot \theta_0$.

The profile of the water surface H_r joining the wall to the closest robot edge is in turn determined by the Young-Laplace equation and the boundary conditions $H'(x) = -\cot \theta_0$ and $H'(d) = \cot \theta_1$, leading to:

$$H_r(X) = L_C \frac{\cot \theta_1 \cosh\left(\frac{X}{L_C}\right) + \cot \theta_0 \cosh\left(\frac{d-X}{L_C}\right)}{\sinh\left(\frac{d}{L_C}\right)}. \quad (\text{S7.3})$$

Assuming $H_u(0) = H_r(0)$ according to experiments, the following condition links d to θ_1 :

$$\cot \theta_1 = -\cot \theta_0 e^{-\frac{d}{L_C}}. \quad (\text{S7.4})$$

Mechanical equilibrium is imposed by normal and tangential force balances, with respect to the direction identified by α , and by torque balance. The three balances are respectively described by:

$$0 = \gamma \left(-\cos\left(\theta_1 - \frac{\pi}{2} - \alpha_m\right) + \cos\left(\theta_2 - \frac{\pi}{2} + \alpha_m\right) \right) + (\Delta\rho g L T - \rho_w g A_{cs}) \sin \alpha_m \quad (\text{S7.5})$$

$$0 = \gamma \left(\sin\left(\theta_1 - \frac{\pi}{2} - \alpha_m\right) + \sin\left(\theta_2 - \frac{\pi}{2} + \alpha_m\right) \right) + (\Delta\rho g L T - \rho_w g A_{cs}) \cos \alpha_m \quad (\text{S7.6})$$

$$0 = \gamma \lambda (\sin \delta_1 - \sin \delta_2) + (R - d_{cs}) \sin \alpha \cdot \rho_w g A_{cs} \quad (\text{S7.7})$$

where $\Delta\rho = \rho_r - \rho_w$, $A_{cs} = \frac{R^2}{2}(\phi - \sin \phi)$, $\lambda = R \frac{\sin(\phi/2)}{\cos \beta}$, $\beta = \text{atan}\left(\frac{1-\cos(\phi/2)}{\sin(\phi/2)}\right)$, $\delta_1 = \beta - \left(\theta_1 - \frac{\pi}{2} - \alpha_m\right)$, $\delta_2 = \beta - \left(\theta_2 - \frac{\pi}{2} + \alpha_m\right)$, and the distance of the centroid of the circular segment from the center of curvature is $d_{cs} = \frac{4R \sin^3(\phi/2)}{3(\phi - \sin \phi)}$. In Eqs. (S7.5-S7.7), the first terms represent the corresponding projection of the surface tension force, the latter term the effect of gravity and buoyancy. Finally, the conservation of robot length provides the last equation as:

$$R\phi = L. \quad (\text{S7.8})$$

The system of nonlinear equations was solved numerically (MATLAB 2016, nonlinear system solver). In Fig. S22, we compare experimental and predicted values of robot's body curvature when first in contact with the wall at the top of the meniscus (*i.e.*, for $d \rightarrow 0$). The actual shape of the deformed robot body was fitted with a circular profile in the same way as in SI section S5b.

II. Cosine-shaped model

To link body curvature to prescribed B , we derived a cosine-shaped version of the previous model to specifically describe a cosine-shaped robot lying at the top of the water meniscus, *i.e.*, for $d \rightarrow 0$. The reference two-dimensional geometry is sketched in Fig. S21b. In this case, we

assume the ends of the robot pinned at the water-air interface, and that the latter has a constant (linear) slope that, from Eq. (S7.2), can be described as $H'(0) = -\cot \theta_0$. From Eq. (7.4) we also get $\theta_1 = \pi - \theta_0$. Under these conditions, horizontal force balance determines $\theta_2 = \theta_0$. Accordingly, the tensile components of the surface tension acting on the ends of the robot are aligned with the water-air interface. Since the vertical projections of the surface tension force are of equal magnitude and opposite in direction, vertical force balance then imposes the equilibrium of weight and buoyancy of the robot. This implies the coincidence of center of gravity and center of buoyancy, as well as balance of torques. The buoyancy of the robot with cosine shape of magnitude D and spatial frequency ω_s can be calculated as:

$$F_B = \rho_w w g \int_0^L [h + D(1 - \cos(\omega_s x))] dx = \rho_w w g (D + h)L, \quad (\text{S7.9})$$

and the balance of robot buoyancy and robot's body weight leads to:

$$D = h \left(\frac{\rho_r}{\rho_w} - 1 \right), \quad (\text{S7.10})$$

where the second factor is always positive for $\rho_r > \rho_w$. Finally, by using Eq. (S3.8) the prescribed $B_{\min}$ to climb the entire meniscus height is computed as:

$$B_{\min} = \frac{8\pi^3 DEI}{mAL^3} = \left(\frac{\rho_r}{\rho_w} - 1 \right) \frac{2\pi^3 E h^3}{3mL^3}. \quad (\text{S7.11})$$

The above equation shows that $B_{\min} \propto L^{-3}, h^3$. Comparison of experimental and model-predicted values of $B_{\min}$ are presented in Fig. S23.

C. Discussion

To discuss how the minimal prescribed B , $B_{\min}$ to climb the entire meniscus height, varies against the robots' dimensions, we include the experimental data of four additional robots that have either a different L , h compared to the original robot (Fig. 1a). As shown in Fig. S22, the arc-shaped model correctly predicts how L and h can impact the meniscus climbing performance: shorter and thicker robots require correspondingly smaller R (i.e., larger curvature) to climb the full meniscus. Similarly, Fig. S23 shows that Eq. (S7.11) of the cosine-shaped model, specific for the robot in contact with the wall at the top of the water meniscus, can provide estimates of upper bounds on the values of $B_{\min}$ in accord with the experimentally recorded values.

With regard to the latter model, we remark that the simplifying assumption of constant linear slope of the water/air interface holds in principle for bodies of length $L \ll L_C$, for which the Taylor series expansion of the exponential meniscus profile (Eq. (S7.2)) can be approximated with its linear term. This condition is admittedly not quite met by our robots. In spite of this discrepancy, the model yields reasonable values for the prescribed $B_{\min}$.

In all cases, w is excluded from consideration, since the 2D nature of the models implicitly assumes infinitely wide robots. In experiments, the finite width of the actual robots introduces an additional curvature of the water surface, in the direction perpendicular to the 2D model plane (Fig. 3d). This transverse curvature is not contemplated in the model. Such inherent discrepancy may partly account for the discrepancy in the results¹¹. An additional approximation was introduced in the arc-shaped model by assuming that the additional volume of displaced water is equal to that of the circular segment subtended by the curved robot. This is admittedly a

simplifying overestimate of the water volume additionally displaced from the surface, which may explain the model's systematic overestimate of R .

S8 - Analysis for undulating swimming

In this section, we analyze the undulating swimming by first providing the required sequence of $\mathbf{B}$ (section S8A) and then a model that can approximate our robots' swimming speed (section S8B). Finally, we discuss how the robot's dimensions can affect its swimming speed in section S8C.

A. Experimental details

Similar to the rolling locomotion (SI section S5A), we use a rotating $\mathbf{B}$ in the XY plane to enable undulating swimming (Fig. S24). The key difference between the control signals of these two locomotion modes is that B has to be much smaller here (*e.g.* 1 mT to 5 mT) to prevent the robot from curling up into a 'C'-shape. With this rotating $\mathbf{B}$, the robot is able to create an undulating traveling wave along its body such that it can produce an effective swimming gait similar to a Taylor swimming sheet^{4,12}. For the undulating swimming locomotion, the rotating $\mathbf{B}$ with frequency f is expressed as:

$$\mathbf{B}(t) = \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} = B \begin{pmatrix} \cos(2\pi ft) \\ \sin(2\pi ft) \\ 0 \end{pmatrix}. \quad (\text{S8.1})$$

B. Theoretical model

The speed of the undulating swimming depends heavily on the highest rotational frequency of $\mathbf{B}$ that the robot can respond to. Therefore we need to estimate the robot's mechanical bandwidth, *i.e.*, its two lowest non-zero fundamental natural frequency $\omega_{n,2}$ and $\omega_{n,3}$. This computation can be achieved by a simple vibrational analysis, which assumes small deformations within the robot. Using this assumption, we analyze an arbitrary infinitesimal element of the robot, dx , at time t (see Fig. S25). Based on Newton's force law, the equation of motion of such element along the vertical axis is given as:

$$-Cw\dot{y} + \frac{\partial v}{\partial x} = \rho_r A \ddot{y}. \quad (\text{S8.2})$$

where C and v represent the damping coefficient and internal shear force, respectively, while ρ_r represents the robot's density and y refers to the vertical deflection. Physically, $-Cw\dot{y}$ and $\rho_r A \ddot{y}$ in Eq. (S8.2) represent the damping and inertial forces acting on the element, respectively.

In a similar manner, we can use $\tau = J\alpha$ to obtain the rotational equation of motion for this element about its bending axis:

$$\frac{\partial v}{\partial x} = -\frac{\partial^2 M_b}{\partial x^2} - \frac{\partial \tau_m}{\partial x} A. \quad (\text{S8.3})$$

The variable M_b represents the bending moment acting on the element, and based on the Euler-Bernoulli equation, it can be approximated to be $EI \frac{\partial^2 y}{\partial x^2}$ for small deflections. Therefore, by substituting Eq. (S8.3) into Eq. (S8.2), we can obtain the governing equation for the beam:

$$-Cw\dot{y} - EI \frac{\partial^4 y}{\partial x^4} - \frac{\partial \tau_m}{\partial x} A = \rho_r A \ddot{y}. \quad (\text{S8.4})$$

We utilize the well-established mode-shape analysis to evaluate this partial differential equation¹³. We first perform a free-vibration analysis to obtain the robot's mode shapes, *i.e.*, we temporarily remove the damping forces and magnetic torques from Eq. (S8.4). Subsequently, we use the variable separation method for y such that

$$y(x, t) = F(x)G(t), \quad (\text{S8.5})$$

where $F(x)$ is solely a function of x and $G(t)$ is only a function of time. By substituting Eq. (S8.5) into Eq. (S8.4) (without the damping effects and magnetic actuation) and rearranging the terms, we obtain:

$$-\frac{EI}{\rho_r A} \left(\frac{\partial^4 F}{\partial x^4} \right) \left(\frac{1}{F} \right) = \left(\frac{1}{G} \right) \ddot{G}. \quad (\text{S8.6})$$

Based on classical vibration analysis, both sides of Eq. (S8.6) are independent of x and t , and they can be equated to become a function of the R^{th} natural frequency, $\omega_{n,R}^2$, of the robot¹³:

$$\left(\frac{1}{G} \right) \ddot{G} = -\omega_{n,R}^2, \quad (\text{S8.7})$$

and

$$-\frac{EI}{\rho_r A} \left(\frac{\partial^4 F}{\partial x^4} \right) \left(\frac{1}{F} \right) = -\omega_{n,R}^2, \quad (\text{S8.8})$$

After solving the homogeneous equation in Eq. (S8.8), we can express the R^{th} mode shape of the robot (F_R) with $\omega_{n,R}$:

$$F_R = A_R e^{\frac{\sqrt{\omega_{n,R}}}{\beta} x} + B_R e^{-\frac{\sqrt{\omega_{n,R}}}{\beta} x} + C_R \cos\left(\frac{\sqrt{\omega_{n,R}}}{\beta} x\right) + D_R \sin\left(\frac{\sqrt{\omega_{n,R}}}{\beta} x\right), \quad (\text{S8.9})$$

where $\beta^4 = \frac{EI}{\rho_r A}$ and the variables A_R , B_R , C_R and D_R are constants dependent on the robot's boundary conditions. Because the robot has free ends, we substitute the following free-free boundary conditions into Eq. (S8.9):

$$\begin{aligned} \frac{\partial^2 y}{\partial x^2}(x=0) &= \frac{\partial^2 y}{\partial x^2}(x=L) = 0 \\ \frac{\partial^3 y}{\partial x^3}(x=0) &= \frac{\partial^3 y}{\partial x^3}(x=L) = 0 \end{aligned} \quad (\text{S8.10})$$

Physically, the boundary conditions in Eq. (S8.10) dictate that there are no shear forces and bending moments acting on the free ends of the robots. Based on these boundary conditions, we can obtain the following characteristic equation for the robot:

$$\cos\left(\frac{\sqrt{\omega_{n,R}}}{\beta}L\right)\left(e^{\frac{\sqrt{\omega_{n,R}}}{\beta}L} + e^{-\frac{\sqrt{\omega_{n,R}}}{\beta}L}\right) = 2. \quad (\text{S8.11})$$

Once the characteristic equation has been solved numerically, the robot's three lowest natural frequencies are given as:

$$\frac{\sqrt{\omega_{n,1}}}{\beta}L = 0, \quad \frac{\sqrt{\omega_{n,2}}}{\beta}L = 4.73 \quad \text{and} \quad \frac{\sqrt{\omega_{n,3}}}{\beta}L = 7.85. \quad (\text{S8.12})$$

The first mode, $\omega_{n,1}$, represents the robot's rigid-body motion, while the second and third mode, $\omega_{n,2}$ and $\omega_{n,3}$, are the two lowest non-zero fundamental natural frequencies of the robot. As the first mode does not describe the shape change of the robot with respect to the actuating magnetic field's frequency, we begin our frequency response analysis with $\omega_{n,2}$ and $\omega_{n,3}$. Based on Eq. (S8.12), these two natural frequencies can be expressed as:

$$\omega_{n,2} = 4.73^2 \sqrt{\frac{E}{12\rho_r}} \left(\frac{h}{L^2}\right) \quad \text{and} \quad \omega_{n,3} = 7.85^2 \sqrt{\frac{E}{12\rho_r}} \left(\frac{h}{L^2}\right). \quad (\text{S8.13})$$

The mode shapes corresponding to $\omega_{n,2}$ and $\omega_{n,3}$ affect the small deflection cosine and sine shapes shown in Fig. 1b(II-III), respectively. Therefore, by using Eq. (S8.13), Eq. (S8.9) and Eq. (S8.5), the undulating traveling wave generated along the soft robot's body can be expressed as:

$$\begin{aligned} y(x,t) &= \left(\frac{mAL^3B}{8\pi^3EI}R_1\right)\left[\cos\left(\frac{2\pi}{L}x\right)\cos(2\pi ft)\right] \\ &+ \left(\frac{mAL^3B}{8\pi^3EI}R_2\right)\left[\sin\left(\frac{2\pi}{L}x\right)\sin(2\pi ft)\right], \end{aligned} \quad (\text{S8.14})$$

where the variables R_1 and R_2 represent the asymptotic approximation of the Bode magnitude function of ideal 2nd order systems, which can be expressed as:

$$R_1 = \begin{cases} 1, & f < \frac{\omega_{n,2}}{2\pi} \\ \left(\frac{\omega_{n,2}}{2\pi f}\right)^2, & f \geq \frac{\omega_{n,2}}{2\pi} \end{cases}, \quad R_2 = \begin{cases} 1, & f < \frac{\omega_{n,3}}{2\pi} \\ \left(\frac{\omega_{n,3}}{2\pi f}\right)^2, & f \geq \frac{\omega_{n,3}}{2\pi} \end{cases}. \quad (\text{S8.15})$$

To better analyze Eq. (S8.14), we rearrange its terms to become:

$$y(x, t) = \left(\frac{mAL^3B}{8\pi^3EI} R_1 \right) \left[\cos\left(\frac{2\pi}{L}x\right) \cos(2\pi ft) + \sin\left(\frac{2\pi}{L}x\right) \sin(2\pi ft) \right] + \left(\frac{mAL^3B}{8\pi^3EI} \right) (R_2 - R_1) \left[\sin\left(\frac{2\pi}{L}x\right) \sin(2\pi ft) \right]. \quad (\text{S8.16})$$

Physically, the first and second components on the RHS of Eq. (S8.16) represent the traveling and stationary waves along the robot's body, respectively. As the stationary wave component is a time-symmetrical motion, it does not propel the robot at low Reynolds number regime¹² *i.e.*, $Re \ll 1$. Hence this component can be excluded, and the effective undulating swimming gait of the robot, *i.e.*, the effective traveling wave, can be expressed as:

$$y(x, t) = \left(\frac{mAL^3B}{8\pi^3EI} R_1 \right) \left[\cos\left(\frac{2\pi}{L}x\right) \cos(2\pi ft) + \sin\left(\frac{2\pi}{L}x\right) \sin(2\pi ft) \right]. \quad (\text{S8.17})$$

Based on the Taylor's swimming sheet model, the swimming speed, V_{swim} , of this effective undulation swimming gait can be represented as⁴:

$$V_{\text{swim}} = \frac{2\pi^2}{L} \left(\frac{mAL^3B}{8\pi^3EI} R_1 \right)^2 f, \quad (\text{S8.18})$$

where f represents the traveling wave's frequency (coincident with the rotation frequency of B). The equation implies that: 1) when $f < \frac{\omega_{n,2}}{2\pi}$, $V_{\text{swim}} \propto L^5, h^{-4}$; 2) when $f \geq \frac{\omega_{n,2}}{2\pi}$, $V_{\text{swim}} \propto L^{-3}$. Because this locomotion can be represented as a low pass system, our scaling analysis will be based on the range of f smaller than $\omega_{n,2}$.

Equation (S8.18) is valid for two types of boundary conditions: 1) the robot is swimming in the bulk of a fluid¹², and 2) the robot is swimming on an air-fluid interface. The key assumptions for the Taylor's swimming sheet model in Eq. (S8.18), however, is that it assumes low Reynolds number, *i.e.*, $Re \ll 1$, and also the wavelength is much larger than the traveling wave's amplitude, *i.e.*, $L \gg \frac{mAL^3B}{8\pi^3EI} R_1$.

C. Discussion

To discuss how the undulating swimming speed, V_{swim} , varies against the robots' dimensions, we include the experimental data of six additional robots that have either a different L , w or h compared to the original robot (Fig. 1a). We conducted two types of experiments on these robots, to respectively evaluate their 1) V_{swim} versus f , and 2) V_{swim} versus B . In all of these experiments, our robots were pinned at the air/water interface.

For the first type of experiments, we fixed B at 5 mT and varied f from 40 Hz to 160 Hz. The experimental results are shown in Fig. S26 together with the theoretical predictions based on Eq. (S8.18). The experimental data indicate that, for all robots, V_{swim} peaks at a robot-specific frequency, and these peak frequencies range between 60 Hz to 100 Hz. Our theoretical model predicts the existence of peak swimming speeds for the robots, though it is unable to capture the specific f at which the peak speed occurs. Despite this limitation, the model can approximate V_{swim}

versus f to be around one order of magnitude. The biggest discrepancy between the theoretical predictions and experimental results is illustrated in Fig. S26c(II), for which the experimentally obtained V_{swim} is about 20 times faster than the predicted speed when $f > 100$ Hz.

For the second type of experiments, we varied B from 1 mT to 5 mT while fixing f at 40 Hz. The experimental results are shown in Fig. S27 together with the theoretical predictions based on Eq. (S8.18). Our experimental results show that V_{swim} has a positive correlation with B and this agrees with our theoretical prediction. In general, the model can approximate V_{swim} versus B within the same order of magnitude for all the robots.

The experimental data for the dependency of V_{swim} on f and B parameterized by the dimensions of the robots are shown in Figs. S28 and S29. In general, these experimental data show that robots with larger L can swim faster and that w does not have an obvious correlation with V_{swim} , and these phenomena agree with our theoretical predictions. However, in contrast to our theoretical predictions, our experiments indicate that robots with larger h will swim faster.

The discrepancies between the experimental results and theoretical predictions can be attributed to several factors. First, the Taylor swimming sheet model in Eq. (S8.18) assumes $Re \ll 1$, whereas our characterization experiments displayed in Figs. S28-S29 show that Re is between 4.6 and 190 for the original robot used in the main text, and between 2.54 and 676 for all the robots tested in SI. Since inertial effects are not included in the model, a discrepancy between the model and the data is expected. Indeed, to our best knowledge, precise solution of swimming at high Reynolds number can usually only be solved numerically¹⁴ and it is non-trivial to include the inertia effects in our analytical model. Second, the soft robots violate the model assumption that the swimmer's wavelength is much bigger than its traveling wave's amplitude, as the amplitude and wavelength are in the same order of magnitude in some conditions. Third, uncertainties in our fabrication process cause our robots to have a residual small curvature even in its rest state (SI section 1D and Fig. S1e). Such uncertainty makes the amplitude of the sine shape smaller than predicted (see Fig. 1b(II)). The ensuing non-ideal traveling wave across the robot body introduces an additional discrepancy between the model and the experimental results. Lastly, we did not consider surface tension effects in the model because the dynamic angles formed by water at the edges of the robots, which depend on the amplitude and frequency of the traveling wave as well as the material composition of the robots, are not trivial to be accounted for¹⁵.

In summary, our experimental data suggest that robots with larger h and L can swim faster, while no obvious correlation between V_{swim} and w has been observed. For most cases, our theoretical model provides an order-of-magnitude accurate approximation for V_{swim} . Therefore, the model can be used as a guide for designers to estimate the swimming speed of future soft robots that can produce a similar swimming gait. We will try to develop a more accurate model that can predict V_{swim} at high Re regime and account for the robot's surface effects in the future. We will also further improve our fabrication method such that the robots can produce a better undulating traveling wave along their body.

S9 - Analysis for crawling

To analyze the crawling locomotion, here we specify the $\mathbf{B}$ sequence required to invoke this locomotion (section S9A), and based on our observations from the experiments conducted for seven soft robots with different dimensions (section S9B), we propose a suitable fitting model to account for it (section S9C).

A. Experimental details

The crawling locomotion shown in Fig. 2g was performed in a glass tunnel of rectangular cross-section ($0.645 \text{ mm} \times 2.55 \text{ mm}$) using a rotating $\mathbf{B}$ in the XY plane with frequency f of 20 Hz (counter-clockwise) and B of 30 mT (Fig. S30). The $\mathbf{B}$ sequence here can be described by Eq. (S8.1). While the crawling locomotion is possible for $B > 2 \text{ mT}$ for the original robot, we used $B = 30 \text{ mT}$ for the experiment in Fig. 2g to obtain a noticeable crawling displacement per cycle. Because the crawling robots were constrained by the walls, their deflections were small even when a large B was applied.

To better understand the crawling locomotion, we fabricated six additional robots, having either a different L , w or h compared to the original robot shown in Fig. 1a, and conducted two types of experiments to study respectively 1) crawling speed, V_{crawl} , versus f (from 20 Hz to 140 Hz) while fixing B at 10 mT, and 2) V_{crawl} versus B (from 2 mT to 18 mT) while fixing f at 20 Hz.

B. Discussion

Similar to the Taylor swimming gait presented in section S8, a rotating $\mathbf{B}$ is used for generating a traveling wave along the robots' body to enable the crawling locomotion (Fig. S30). The key difference between crawling and undulating swimming, however, is that the crawling direction is parallel to the propagation direction of the traveling waves while the motion of the swimming gait is anti-parallel to its traveling wave.

The experimental results in Fig. S31 show the variation of V_{crawl} versus f parameterized by the dimensions of the robots. It suggests that V_{crawl} of almost all robots increases linearly with f until $f = 40 \text{ Hz}$. When f reaches 60 Hz, V_{crawl} for the robot with $h = 333 \text{ }\mu\text{m}$ starts to decline. Such phenomenon affects also other robots when f becomes higher. We remark that this happens when the traveling wave can no longer produce constant amplitude: The robot is still able to crawl, but the relationship between V_{crawl} and f is no longer linear.

The experimental data for the dependency of V_{crawl} on B parameterized by the dimensions of the robots are shown in Fig. S32. In general, we observe a positive correlation between V_{crawl} and B , until V_{crawl} starts to saturate for relatively large B . Such saturation of V_{crawl} could be explained by the change of the contact area between the robot and the tunnel as we vary B (Fig. S33). Using for instance the original robot (with dimensions of $3.7 \text{ mm} \times 1.5 \text{ mm} \times 0.185 \text{ mm}$), we observe that V_{crawl} starts to saturate after B exceeds 14 mT, and that this corresponds to a saturation of the contact length between the robot and the inner surface of the tunnel (Fig. S33). This is expected, since the amplitude of the traveling wave across the robot's body length within the tunnel is eventually constrained by the cross-section of the tunnel.

At last, we do not observe an obvious correlation between the robots' crawling speed and their dimensions L , w , or h : For any specific value of f and B , V_{crawl} appears similar for all robots.

C. Fitting model

The observations reported in the previous subsection suggest that V_{crawl} can be approximated with a polynomial function of f and by an exponential function of B . We consequently provide the following fitting model:

$$V_{\text{crawl}} = B^a(k_2 f^2 + k_1 f + k_0), \quad (\text{S9.1})$$

where a , k_0 , k_1 , and k_2 , are the fitting parameters. Dimensions of the robots are not included in the model, since they did not show any obvious co-relationship with the experimental crawling speeds. To estimate the best fitting parameters, we formulated the following optimization problem:

$$\min \sum_{i=1}^n [V_{\text{crawl},i} - B_i^a (k_2 f_i^2 + k_1 f_i + k_0)]^2, \quad (\text{S9.2})$$

where the subscript i represents the corresponding i^{th} data point in the plots of Fig. S31 and Fig. S32. Numerical solutions of Eq. (S9.2) with MATLAB 2016 found the best fitting parameters to be $a = 0.606$, $k_2 = 0.005$, $k_1 = 0$ and $k_0 = -0.009$. The V_{crawl} predicted for each robot using the fitted model is plotted in Figs. S34 and S35. We remark that the applicability of this fitted crawling model is limited to soft robots that have dimensions of $185 \mu\text{m} \leq h \leq 333 \mu\text{m}$, $2.4 \text{ mm} \leq L \leq 5 \text{ mm}$ and $0.75 \text{ mm} \leq w \leq 2 \text{ mm}$, crawling in a glass tunnel that has a rectangular cross-section of $0.645 \text{ mm} \times 2.55 \text{ mm}$ using a rotating $\mathbf{B}$ that has the following range of parameters: $2 \text{ mT} \leq B \leq 18 \text{ mT}$ and $20 \text{ Hz} \leq f \leq 140 \text{ Hz}$.

S10 - Analysis for jellyfish-like swimming

We discuss the jellyfish-like swimming by first specifying a typical required sequence of $\mathbf{B}$ to enable this locomotion (section S10A) and then describing our experimental results for seven robots of different dimensions (section S10B). Deriving a comprehensive analytical model for such complex swimming gait ($Re > 1$) is an open quest¹⁴ and is beyond the scope of this paper. Therefore, here we provide a fitting model that can approximate the observed swimming speed of our robots (section S10C).

A. Experiment details

To perform the jellyfish-like swimming locomotion for the original robot shown in Fig. 2a, we used the sequences of $\mathbf{B}$ given in Fig. S36a in the XY plane, where the maximum B , $B_{\text{max}} = 17 \text{ mT}$ and $f = 25 \text{ Hz}$. Theoretically, $\mathbf{B}$ should oscillate along $\alpha = 135^\circ$ (equivalently to $\alpha = 315^\circ$) to create similar ‘C’-shape and ‘V’-shape configurations that are shown in Fig. 1b. Due to fabrication uncertainty, Fig. S36b however shows that the robot can perform this locomotion when $\mathbf{B}$ is oscillating along the direction $\alpha = 105^\circ$ and $\alpha = 285^\circ$. We have therefore calibrated the applied $\mathbf{B}$ for this specific robot in our experiments.

To characterize the jellyfish-like swimming, six additional robots with different L , w or h compared to the original robot were also fabricated. Like the jumping locomotion described in SI section S4, the characterization here is done with robots that have $\beta_R = -90^\circ$, instead of $\beta_R = 45^\circ$ as this allows us to simplify our setup to become a pair of Helmholtz coil. Using these experimental conditions, the robot could generate a jellyfish-like swimming locomotion when we prescribed a $\mathbf{B}$ sequence that has time-varying magnitude along the Y -axis of the global reference system. Mathematically, $\mathbf{B}$ can then be simply expressed as $\mathbf{B} = [0 \ B \ 0]^T$.

Using these robots, we have conducted two types of experiments to evaluate respectively 1) the jellyfish-like swimming speed of the robots, V_{jf} , versus f (from 20 Hz to 40 Hz) with B_{max} fixed at 20 mT , and 2) V_{jf} versus B_{max} (from 20 mT to 40 mT) with f fixed at 20 Hz .

B. Discussion

The prescribed **B** sequence (Fig. S36a) is meant to make the robot implement a slow recovery stroke where the robot changes from its ‘C’-shape to ‘V’-shape configuration (0 ms – 19.5 ms in Fig. 2a) and a fast power stroke (19.5 ms – 32 ms in Fig. 2a), which makes the robot change back to its ‘C’-shape configuration again. Based on the data in Figs. S38–S39, this swimming gait has Reynolds number that ranges from 74 to 190 for the original robot described in main text, and from 39 to 421 for all the robots tested in the SI. This gait resembles that of an actual jellyfish swimming, *e.g.* as presented in reference 14. According to that study, part of the remarkable efficiency of jellyfish swimming is to be attributed to exploiting the vortex rings produced by the sequence of power and recovery strokes. We evidenced in Fig. S37 the generation of fluid vortices in the wake of our robots’ jellyfish-like swimming locomotion by using 45 μm diameter polystyrene tracer microspheres (Polysciences, Inc).

Dependency of the experimental V_{jf} on f is shown in Fig. S38, where the three sub-figures allow to better identify trends of the robots’ swimming speed as we change their dimensions. As shown, except for the robot with $L = 4.35$ mm (Fig. S38b) and $w = 2$ mm (Fig. S37c), V_{jf} peaks within $30 \text{ Hz} \leq f \leq 40 \text{ Hz}$ and starts to decay afterwards. Indeed for $f > 40 \text{ Hz}$ these robots are observed to become unable to fully complete their recovery and power strokes within each period.

Dependency of the experimental V_{jf} on B_{max} is shown in Fig. S39. In general, we observe that the robots swim faster as we increase B_{max} . However, V_{jf} of more compliant robots starts to decrease when B_{max} exceeds a robot-specific threshold value. For example, the robot with $h = 108 \mu\text{m}$ starts to swim slower after B_{max} exceeds 25 mT (Fig. S39a). This arguably happens because once $B_{\text{max}} > 25 \text{ mT}$ the speed of the robot’s recovery stroke becomes so fast that the robot is observed to produce significant downward motion. Such retarding effect is also observed in other robots, such as the robot with $h = 142 \mu\text{m}$ in Fig. S39a and the robot with $L = 5$ mm in Fig. S39b.

Based on the experimental data shown in Fig. S38 and S39, we observe that more compliant robots (*i.e.*, larger L , smaller h) swim faster for $B_{\text{max}} < 30 \text{ mT}$. This is expected, since these robots can create larger power strokes compared to their stiffer counterparts when subjected to the same B_{max} . Based on Fig. S38c and S39c, w only affects the dependency of V_{jf} on f but not on B_{max} . Based on Fig. S38c, robots with larger w generally have a relatively lower V_{jf} as we increase f .

C. Fitting model

The observations presented above suggest that we can fit polynomial functions for both f and B_{max} . Moreover, as robots with different dimensions have peak speeds for different values of f and B_{max} , the coefficients of the polynomial functions need be a function of h , L and w . In view of these observations, we propose the following fitting model:

$$V_{\text{jf}} = \left[\sum_{i=0}^2 k_i h^{a_i} L^{b_i} B_{\text{max}}^i \right] \left[\sum_{j=0}^3 k_{3+j} h^{a_{3+j}} L^{b_{3+j}} w^{c_j} f^j \right], \quad (\text{S10.1})$$

where k_i , a_i , b_i , and c_i are the fitting parameters. To estimate the best fitting parameters, we formulate the following optimization problem:

$$\min \sum_{k=1}^n \left[V_{\text{jf},k} - \left[\sum_{i=0}^2 k_i h^{a_i} L^{b_i} B_{\text{max},k}^i \right] \left[\sum_{j=0}^3 k_{3+j} h^{a_{3+j}} L^{b_{3+j}} w^{c_j} f_k^j \right] \right]^2, \quad (\text{S10.2})$$

where the subscript k represents the corresponding k^{th} data point in the plots presented in Fig. S38 and Fig. S39. By solving Eq. (S10.2) numerically with MATLAB 2016, we present the obtained fitting parameters in Table S3. The fitting model in Eq. (S10.1) shows that the relationship between V_{jf} and the robot's dimensions is highly nonlinear.

The values of V_{jf} predicted for each robot by the fitted model are shown in Fig. S40 and Fig. S41. We remark that the applicability of this fitted swimming model is for soft robots that have dimensions of $108 \mu\text{m} \leq h \leq 185 \mu\text{m}$, $3.7 \text{ mm} \leq L \leq 5 \text{ mm}$ and $1.5 \text{ mm} \leq w \leq 2.5 \text{ mm}$, using a sequence of $\mathbf{B}$ that has the following range of parameters: $20 \text{ mT} \leq B_{\text{max}} \leq 40 \text{ mT}$ and $20 \text{ Hz} \leq f \leq 40 \text{ Hz}$.

S11 - Transition modes

A. Immersion

Driven by the $\mathbf{B}$ sequence shown in Fig. S42a in the XY plane, the immersion shown in Fig. 2d is realized by the following steps:

1. The initial $\mathbf{B}$ ($B = 20 \text{ mT}$, $\alpha = 90^\circ$) is imposed to curl the robot downward (32 ms in Fig. 2d).
2. The robot rotates counterclockwise to reduce body contact with the water-air interface to a minimum (41 ms in Fig. 2d), leaving a single end of the robot pinned to the water surface (00:29 of the Supporting Video S6).
3. A quick 180° flipping of the external $\mathbf{B}$ disengages the pinned end of the robot from the water surface (66 ms in Fig. 2d, corresponding to 00:30 of the Supporting Video S6). Consequently the robot, denser than water, sinks.

We remark that for our robots, with main dimension $L \approx L_c$, the Bond number $Bo = \rho g L^2 / \gamma = (L/L_c)^2 \approx 1$, therefore the magnitude of surface tension effects cannot be neglected compared to gravitational ones. Immersion in the liquid bulk requires that the robots break the surface tension of water. The minimum force F_{im} necessary to break the contact between a single robot end and the water-air interface (as for the robot configuration in Fig. 3b) is of the order of γp , where $p = 2(w + h)$ is the perimeter of the cross-section of the robot and $\gamma = 72 \text{ mN} \cdot \text{m}^{-1}$ is the surface tension of pure water at room temperature. In the case of the original robot, $F_{\text{im}} \approx 243 \mu\text{N}$. Such force is provided by the rigid-body torque induced by $\mathbf{B}$ in step 3 above.

B. Landing

Driven by the $\mathbf{B}$ sequence shown in Fig. S43 in XY plane, the transition from the top of the water meniscus to the adjacent platform shown in Fig. 2c is realized by the following steps:

1. The initial $\mathbf{B}$ ($B = 11.7 \text{ mT}$, $\alpha = 270^\circ$) is imposed to curl the robot (2.8 s in Fig. 2c, 0 time unit in Fig. S43a). During this motion, water can easily dewet the robot surface (see next subsection).
2. Subsequently the $\mathbf{B}$ starts to rotate clockwise. The $\mathbf{B}$ magnitude decreases to 4.7 mT at the 0.26 time unit in Fig. S43a since most of the robot has peeled away from the water surface and a large B is no longer necessary.

Here we want to highlight that, due to residual strain energy caused by fabrication uncertainty (section S1D), this specific robot in Fig. 2c can perform rigid body rotation even when B ceases to be 4.7 mT. We have therefore calibrated the applied B for this specific robot in our experiments.

C. Effect of robot's surface properties

The native elastomer surface of the robot is hydrophobic and microscopically rough (SI section S1B). The low wettability of the robot surface, and particularly the high receding water contact angle ($\sim 78^\circ$, see SI section S1B), leaves the receding triple contact lines free to move during the rotation of the robot. Hence, the robot can easily peel away from the water surface to stand on the adjacent solid substrate (Fig. 2c).

Surface hydrophobicity also enhances the pinning of the robot at the water-air interface, because it extends the angular range that can be spanned by the edge angles without displacing the position of the contact line on the robot body. In combination with its microscale roughness, surface hydrophobicity promotes the capture and adhesion of microscopic air bubbles on the robot surface upon immersion in water from air. The presence of a *plastron*, *i.e.*, a sparse layer of microbubbles, is revealed by the shiny appearance of the robot underwater (Supporting Video S1), caused by the light refracted at the water-air interface of the bubbles. The *plastron* marginally increases the water volume displaced by the submerged robot, and thus its buoyancy.

Moreover, surface hydrophobicity energetically promotes the emersion of the robot from the water bulk to the water surface, since the replacement of the elastomer-water interface with a full elastomer-air interface during emersion decreases the interfacial energy of the system. On the other hand, for the same reason, hydrophobicity also makes it harder for the robot to disengage from the water surface during immersion.

S12 - Scaling analyses summary

A. Scaling analysis for robot's physical parameters

Here, we provide a simple scaling analysis for the robot's physical parameters. First, the magnitude of the magnetization profile for the infinitesimal elements shown in Fig. S3 scales proportionally with h and w , while the robot's mass and the magnitude of net magnetic moment, M_{net} , scales proportionally with h , L and w . Second, the second moment of inertia of the robot, I , has the following scale laws: $I \propto h^3$ and $I \propto w$. The parameter I will be important for describing the bending stiffness of the robot.

B. Scaling analyses for locomotion modalities

Based on the modeling and experimental results discussed in SI sections S4-10, this sub-section and Table S4 will provide an overview scaling analyses for each locomotion modality. We also suggest the operational range of f for each locomotion modality. As long as f is within this range, all of our tested robots are able to execute their desired locomotion successfully. Table S4 also includes the best performance index observed from each locomotion modality.

We remind that our analyses and discussions are intended to facilitate future studies on soft-bodied locomotion. In the future, we will also expand these analyses such that they can be used for optimizing the performance of miniature robots with multi-locomotive capabilities.

1. **Jumping.** Performance index: maximum jumping height, H_{max} .

Theoretically, $H_{\max}$ is independent of w , and its relationships with L and h are highly non-linear. The theory, however, still suggests that $H_{\max}$ has a positive correlation with L and a negative correlation with h .

Experimentally, we observe that robots with larger L and thinner h can jump higher. The data do not show observable trends for w . Hence, these experimental results agree with the trend predicted by the theory.

2. **Rolling.** Performance index: rolling speed, V_{roll} .

Theoretically, V_{roll} is independent of w and h when the rolling robots have similar curvatures. By using simple approximations, we can deduce that $V_{\text{roll}} \propto L$.

Experimentally, good control over rolling is achieved for all the robots when f is between 0 Hz and 4 Hz (Fig. S12). Our data also suggest that when the robots have similar curvatures, robots with larger L will roll faster. There are no observable effects for w and h . These experimental results agree with the trend predicted by the theory.

3. **Walking.** Performance index: walking speed, V_{walk} .

Theoretically, V_{walk} is independent of w and its relationship with L and h is non-linear. However, the theory does imply that V_{walk} has a positive correlation with L and a negative correlation with h .

Experimentally, good control over walking is achieved for $2 \text{ Hz} \leq f \leq 11 \text{ Hz}$ (Figs. S17). Robots with larger L and thinner h are observed to walk faster when subjected to the same $B_{\max}$ (Fig. S18). The data do not show observable trends for w . These experimental results agree with the trend predicted by the theory.

4. **Meniscus climbing:** Performance index: minimum required B , i.e., $B_{\min}$ to reach the top of the meniscus.

Theoretically, $B_{\min} \propto L^{-3}, h^3$. The current model does not account for the effects of w .

Experimentally, all the tested robots can climb to the top of the water meniscus shown in Fig. S20 for $B_{\min} > 6 \text{ mT}$ (Fig. S23). Robots with a larger L and thinner h require a smaller $B_{\min}$ to ascend to the top. These experimental results agree with the trend predicted by the theory.

5. **Undulating swimming:** Performance index: swimming speed, V_{swim} .

Theoretically, we have: 1) when $f < \frac{\omega_{n,2}}{2\pi}$, $V_{\text{swim}} \propto L^5, h^{-4}$; 2) when $f > \frac{\omega_{n,2}}{2\pi}$, $V_{\text{swim}} \propto L^{-3}$. As the robotic system is a low-pass system, we will consider only the first condition (i.e., when $f < \omega_{n,2}$) for the scaling analysis. V_{swim} is always independent of w .

Experimentally, all the tested robots can swim successfully when f is between 40 Hz and 160 Hz (Fig. S28). Robots with larger L and thicker h can swim faster (Fig. S29). The experimental data do not show observable trends for w . There exists a discrepancy between the experimental and theoretical scaling for h .

6. **Crawling:** Performance index: crawling speed V_{crawl} .

Experimentally, good control over crawling is achieved for $20 \text{ Hz} \leq f \leq 40 \text{ Hz}$ (Figs. S31). The data suggests that V_{crawl} has no observable dependency on L , w , and h .

Based on the experiment data, the fitting model suggests that V_{crawl} is independent of w , L and h .

7. **Jellyfish-like swimming:** Performance index: swimming speed V_{jf} .

Experimentally, all the tested robots can swim successfully for $30 \text{ Hz} \leq f \leq 40 \text{ Hz}$ (Figs. S38). It is observed that V_{jf} may peak at certain B_{max} (Fig. S39), which depends on specific robot design. After exceeding this B_{max} , the robots may produce very fast recovery strokes that reduce V_{jf} . Before reaching this B_{max} , we observe that robots with larger L and thinner h swim faster. The dimension w only affects the dependency of V_{jf} on f , but not on B : robots with larger w generally have a relatively lower V_{jf} as we increase f .

Based on the experimental data, the fitting model suggests that the V_{jf} has a highly non-linear relationship with L , h and w . The values for the fitting parameters are provided in Table S3.

In the dimension range we have investigated, our theoretical and fitting models predict that a larger L and a smaller h are always preferred for multimodal locomotion as it can help the robot to move faster and jump higher. The models also suggest that w will only affect the jellyfish-like swimming locomotion and minimizing w can help to increase this swimming speed.

C. Upper and lower bounds for the L , w , and h

Based on our models discussed above, the robot may not be able to fulfill all modes of locomotion if its L is too small, or when its h and w are too large. In addition, the upper bound of L , w , and h is typically constrained by the size requirements of specific applications as well as the maximum allowable workspace of the electromagnetic coil setup that generates the spatially uniform $\mathbf{B}$. The lower bound of h and w is set by our current fabrication limits where we could only demold robots with $h > 40 \text{ }\mu\text{m}$ and $w > 0.3 \text{ mm}$ repeatedly. The lower bound of L is 1 mm because it is very challenging to manually wrap sub-millimeter beams onto the similar circular jigs as that shown in Fig. S1a, during the magnetization process.

S13 - Multimodal locomotion

Here, we describe the procedures to implement the robot's multimodal locomotion shown in Fig. 3. In Fig. 3a-d, the environment is composed of laser-milled, poly(methyl methacrylate) platforms. The platform is treated with oxygen-plasma and has a water contact angle $< 10^\circ$. The platform is encased in a Plexiglas box ($52 \text{ mm} \times 32 \text{ mm} \times 25 \text{ mm}$). The liquid in this environment is de-ionized water.

In Fig. 3a, the robot first uses the rolling locomotion to fall onto the water surface. After dipping into water, the robot drifts slightly away from the platform along the meniscus and starts swimming after it has stabilized. A clockwise $\mathbf{B}$ ($B = 3 \text{ mT}$, 20 Hz) is applied to let the robot swim rightwards. In Fig. 3b, the robot sequentially bends downwards on the water surface and rotates

counterclockwise to sink into the water, and subsequently it uses the jellyfish-like locomotion to swim back to the water surface in Fig. 3c.

Figure 3d shows the robot using four locomotion modes to navigate through liquid and solid surfaces. After climbing onto the solid substrate, the robot uses the directional jumping strategy to overcome the otherwise unsurmountable obstacle. By exploiting the robot's net magnetic moment, we rotate the robot to its required initial pose for implementing the directional jumping. Once the robot has jumped across this obstacle, it starts to walk. In the Supporting Video S6, it can be seen that the robot incurs slight slipping before and after jumping across the obstacle. This slipping is induced by unwanted magnetic gradient-based pulling forces generated by the spatial gradients of $\mathbf{B}$, which are present because the robot has exceeded the 95% homogeneous region that can provide spatially-uniform $\mathbf{B}$ for the robot (see SI section S2). We also remark that this slipping generally happens when the robot has only one end in contact with the substrate. Since the friction is the smallest at this moment, even a small magnetic gradient-based pulling force can translate the robot. However, while there exists such an external disturbance for our robot, it is worth noting that the robot can still walk successfully even against the gradient after jumping over the obstacle. Moreover, since the deformed robot has a net magnetic moment, we are able to steer the robot along a desired direction even after it was disrupted by the magnetic gradient-based pulling forces (see SI section S3B).

In Fig. 3e, the robot first walks towards a glass tunnel (inner diameter: 1.62 mm), which eventually blocks its path. As the tunnel is too small for the robot to walk in it, the robot switches to its crawling locomotion by changing the magnetic field input to a rotating $\mathbf{B}$. After the robot has crawled through the tunnel, it resumes its walking locomotion until it leaves the working space.

S14 - Toward biomedical applications

A. Locomotion in a surgical phantom

Here we demonstrate that our robot can use a combination of meniscus climbing, landing, rolling and jumping to fully explore a surgical human stomach phantom (Fig. 4a and Supporting Video S7).

In the Supporting Video S7, the robot moves back to the starting point very quickly at around 00:34 because it is pulled by unwanted magnetic gradient-based pulling forces generated by the spatial gradients of $\mathbf{B}$, which are present because the robot is outside the region that can provide spatially-uniform $\mathbf{B}$ (see SI section S2). While there exists such an external disturbance for our robot, it is worth noting that the robot can still roll successfully under such conditions.

B. Ultrasound-guided locomotion

Here we demonstrate that our robot has the potential to be integrated with biomedical imaging systems towards realizing potential *in-situ* applications like minimally invasive surgery¹⁶. In particular, we demonstrate ultrasound-guided locomotion (Supporting Video S8, Fig. 4b, Fig. S44). For the experiments, we have used three different biological phantoms to mimic biomedical landscapes, *i.e.*, a water reservoir, tunnels and gaps. We inserted the ultrasonic scanning head of the system (Fujifilm Vevo3100 Imaging System) from the top of our magnetic setup and put it in contact with the phantoms through a thin layer of ultrasonic gel. The ultrasound system was found able to track the position of our robots while they were performing jellyfish-like swimming, crawling, and rolling within the narrow spaces concealed by chicken muscle tissue. This allowed

us to image and track the robots as they navigated in the phantoms. We were able to produce clear images of the robots while having no disruptions to our actuation system.

We also report that while the technicians who worked with us on the ultrasonic setup can easily detect the motions of the robot from the ultrasound images, it is possible to develop imaging algorithms to automate this process in the future.

C. Cargo delivery

The design of the modified robot, presented in Fig. 4d, for selective, magnetically-triggered drug release, presented is shown in Fig. S45. An extra strap is added to the original robot body. For clarity, we only mark out the m_y component of the strap's magnetization profile, as it is the dominant and functional component in releasing the drug. During locomotion, the cargo is mechanically bound on the robot by inserting the strap head into a hole on the robot's body. When a large $\mathbf{B}$ along the y -axis, B_y (>13.4 mT), is applied, the magnetic torque bends and unlocks the strap to release the cargo. Due to the restriction of the hole, a smaller B_y (<12.7 mT in our case) is not strong enough to open the strap and hence this allows us to preserve our locomotion modes. More details pertaining to cargo delivery are illustrated in Supporting Video S10. In the future, we will also optimize the design such that the robot can also carry liquid or powder based loads.

S15 - Spatial gradient of $\mathbf{B}$

By creating a spatially non-uniform $\mathbf{B}$, we can exert magnetic forces on miniature devices that have a net magnetic moment $\mathbf{M}_{\text{net}}$. The mathematical description for these applied forces can be expressed as:

$$\mathbf{F} = (\mathbf{M}_{\text{net}} \cdot \nabla) \mathbf{B}. \quad (\text{S15.1})$$

The method that uses the spatial gradients of $\mathbf{B}$ to translate the miniature robots is typically known as magnetic gradient-based pulling or gradient pulling^{1,5}. As our robots can possess an effective $\mathbf{M}_{\text{net}}$ when they deform, we can also use the spatial gradients of $\mathbf{B}$ to exert magnetic gradient-based pulling on the robots such that their locomotion capabilities can be enhanced. For instance, we can use magnetic gradient-based pulling to modulate the robot's jump height for the jumping locomotion (see Supporting Video S5).

Theoretically, magnetic gradient-based pulling may also levitate a magnetic robot and make it traverse across different terrains and obstacles. However, this approach is not practical as the dynamics of this actuation mode is inherently unstable, especially if the small-scale robot has to be levitated against gravity¹⁷. Furthermore, Abbot *et al.* has shown that time-asymmetrical locomotion like the undulating swimming gait is more energy efficient than simple magnetic gradient pulling actuation methods, *i.e.*, they produce higher propulsion forces¹⁸. It is also easier to actuate miniature magnetic robots using $\mathbf{B}$ than its spatial gradients when their distance from the external electromagnets increases¹⁸. As many biomedical applications require the external electromagnets to be placed outside of the human body, the considerable distance between the electromagnets and the proposed robots would make magnetic gradient-based pulling methods less appealing.

Supporting figures

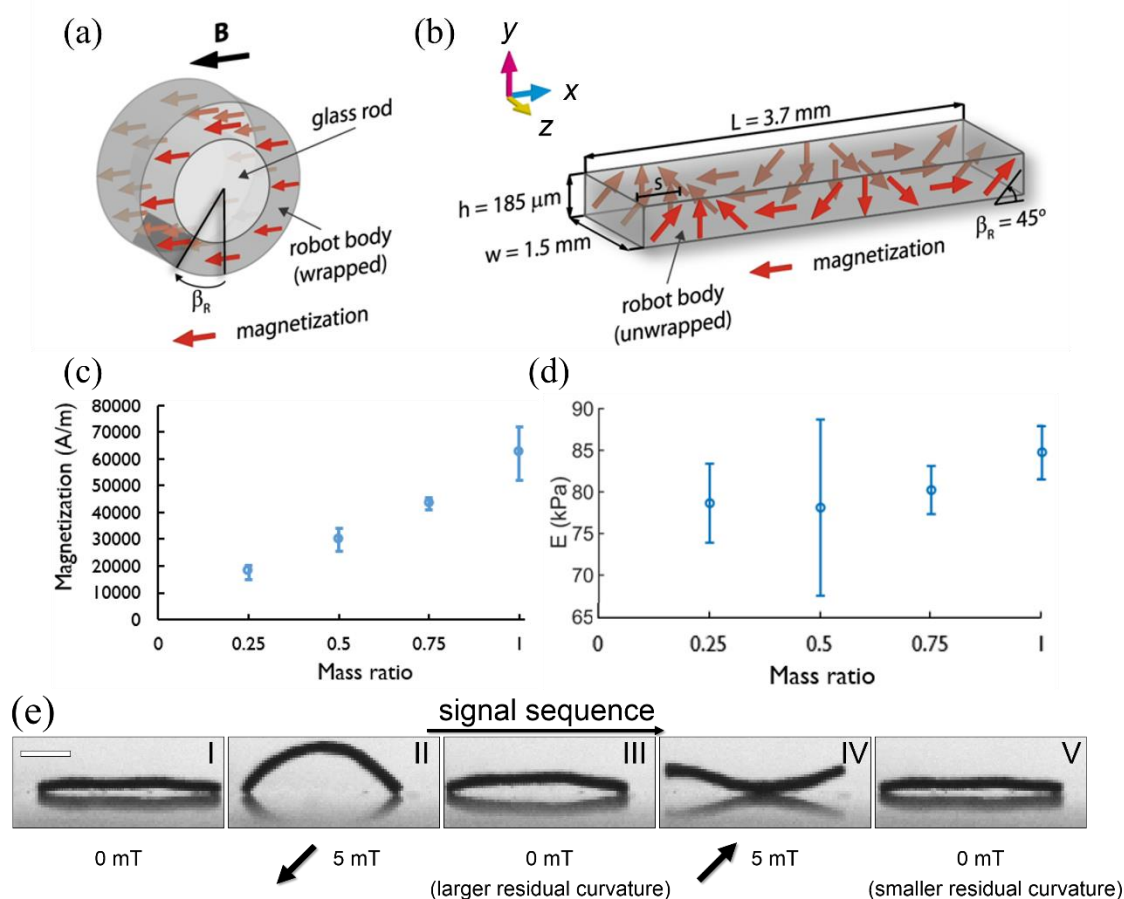


Fig. S1. Magnetization process of the soft magneto-elastic robot, the experimental results for the magnetization magnitude and Young's modulus of similar magnetic elastomers with differing mass ratios, and the experiment for showing different rest state residual curvatures. (a) Sketch of the elastomeric beam, loaded with NdFeB microparticles, wrapped around a cylindrical glass rod (perimeter: 3.7 mm) and magnetized by a uniform 1.65 T magnetic field B produced by a vibrating sample magnetometer (VSM). The relative orientation β_R determines the phase shift in the resulting magnetization profile. (b) The programmed magnetization profile in the unwrapped robot body in its local xyz frame ($\beta_R = 45^\circ$). (c) Experimental values of the magnetization magnitude for soft magnetic materials with different mass ratios. (d) Experimental values of E for soft magnetic materials with different mass ratios. The mass ratio is defined as the mass ratio of NdFeB particles to the mass of Ecoflex-10, e.g., 0.25:1, 0.5:1, and we normalize these ratios such that it is always one part on the Ecoflex-10 side. Each data point in (c) and (d) was evaluated with three samples ($n = 3$). (e) The robot has two rest state curvatures (shown in III, V), which can be alternated via shape change. The five shapes of the robot are achieved sequentially by subjecting the robot to the B field sequence specified below the snapshots.

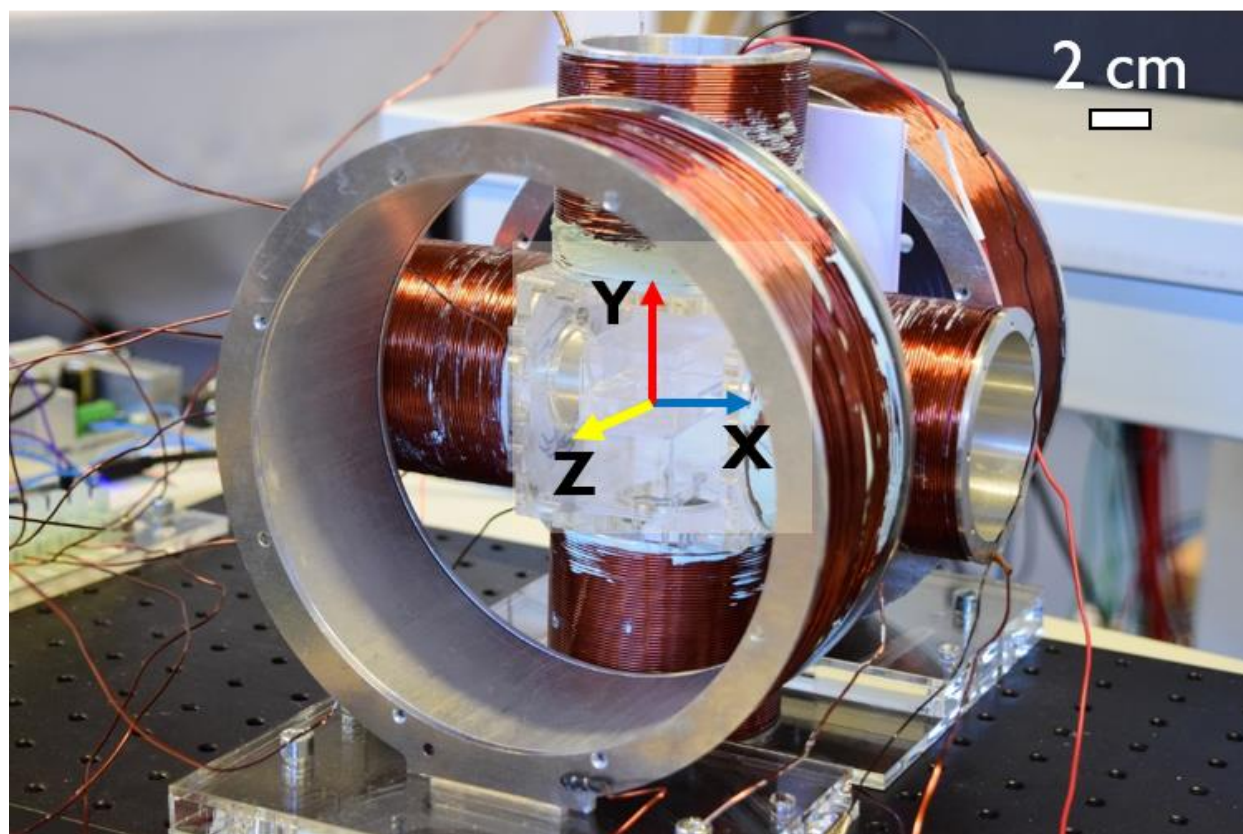


Fig. S2. Photo of the custom-made, six-coils electromagnetic setup used to actuate the magneto-elastic robots. The origin of the global coordinate system is located at the center of the setup.

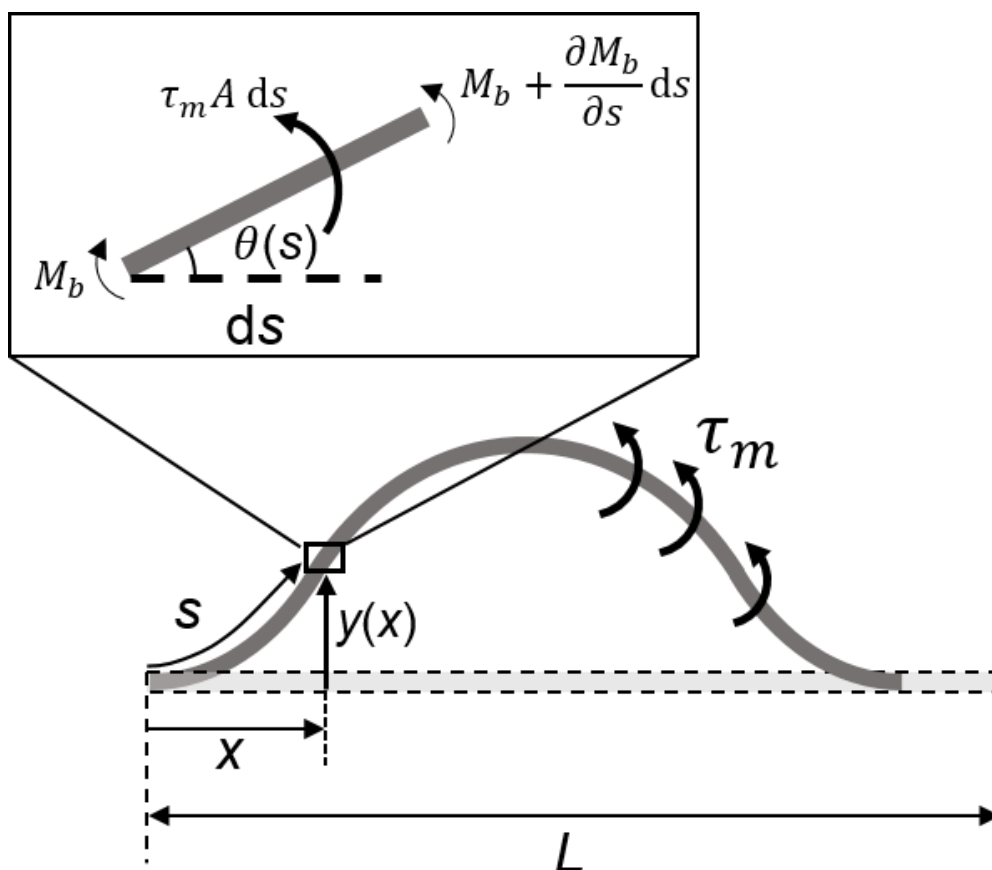


Fig. S3. Quasi-static analysis of the soft robot. Sketch of the geometry is not to scale for representation purposes. The bending moment acting on an infinitesimal element of the robot under steady-state deformation is shown in the inset.

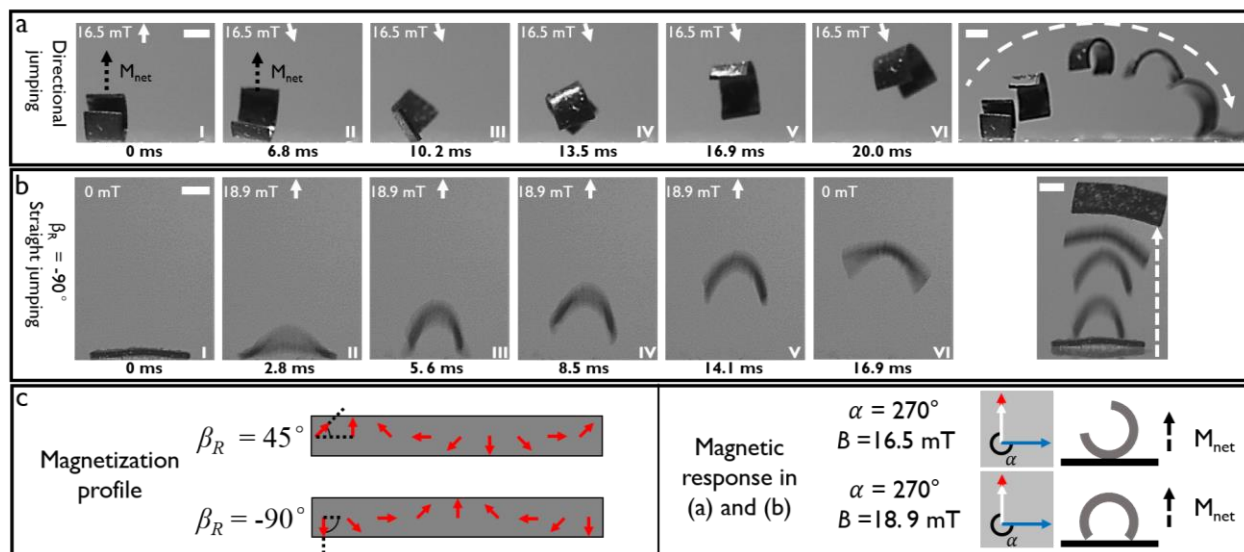


Fig. S4. Soft robots implementing jumping locomotion. (a) Directional jumping by a soft robot with $\beta_R = 45^\circ$ (also shown in Fig. 2h). (b) Straight jumping by a soft robot with $\beta_R = -90^\circ$. The magnetization profiles of the robots shown in (a,b) and their corresponding responses to the applied field are sketched in (c). Scale bars: 1 mm.

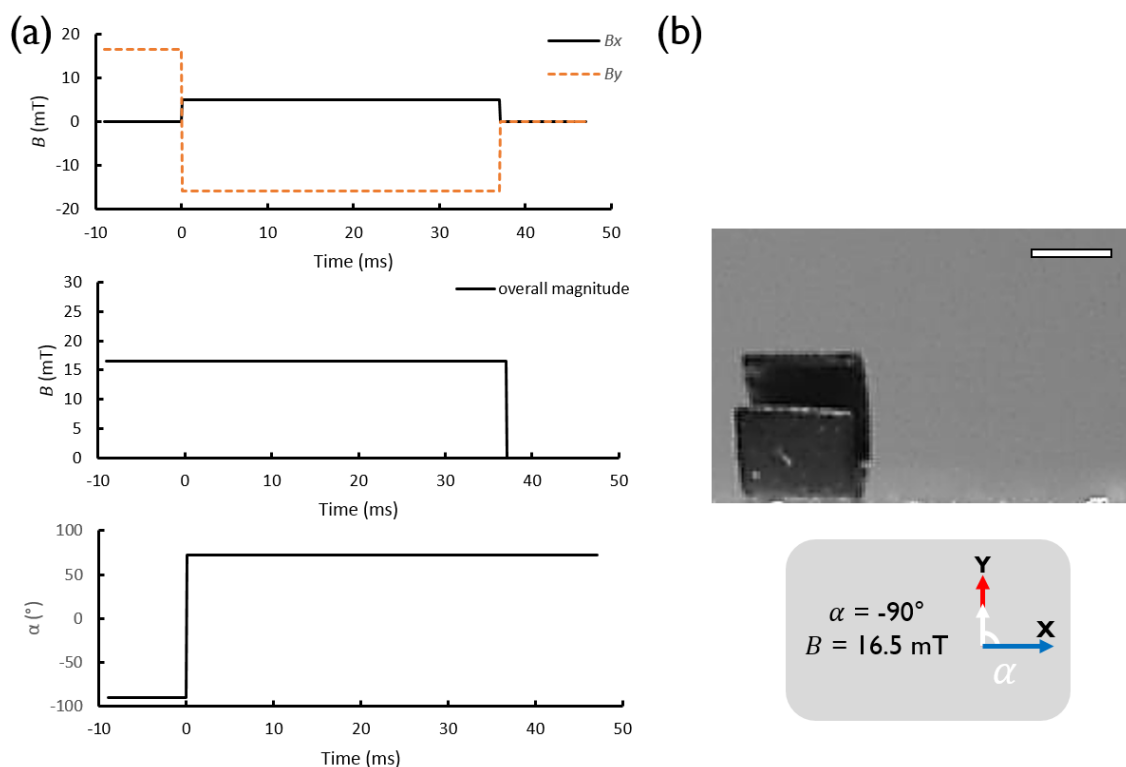


Fig. S5. The required B for the directional jumping locomotion. (a) Time sequence of B . B_x and B_y represent the magnitude of B projected along the X - and Y -axis of the global coordinate system. (b) shows the initial shape of the soft robot before it jumps ($\beta_R = 45^\circ$ and reproduced from Fig. 2h). Scale bar: 1 mm.

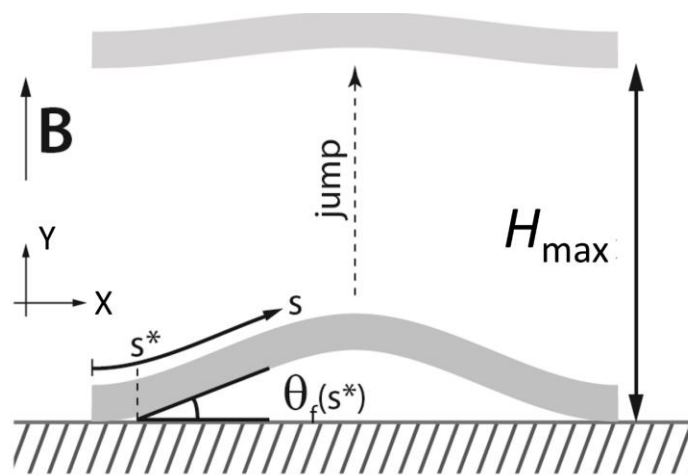


Fig. S6. Sketch for the analysis of the straight jumping locomotion (not to scale).

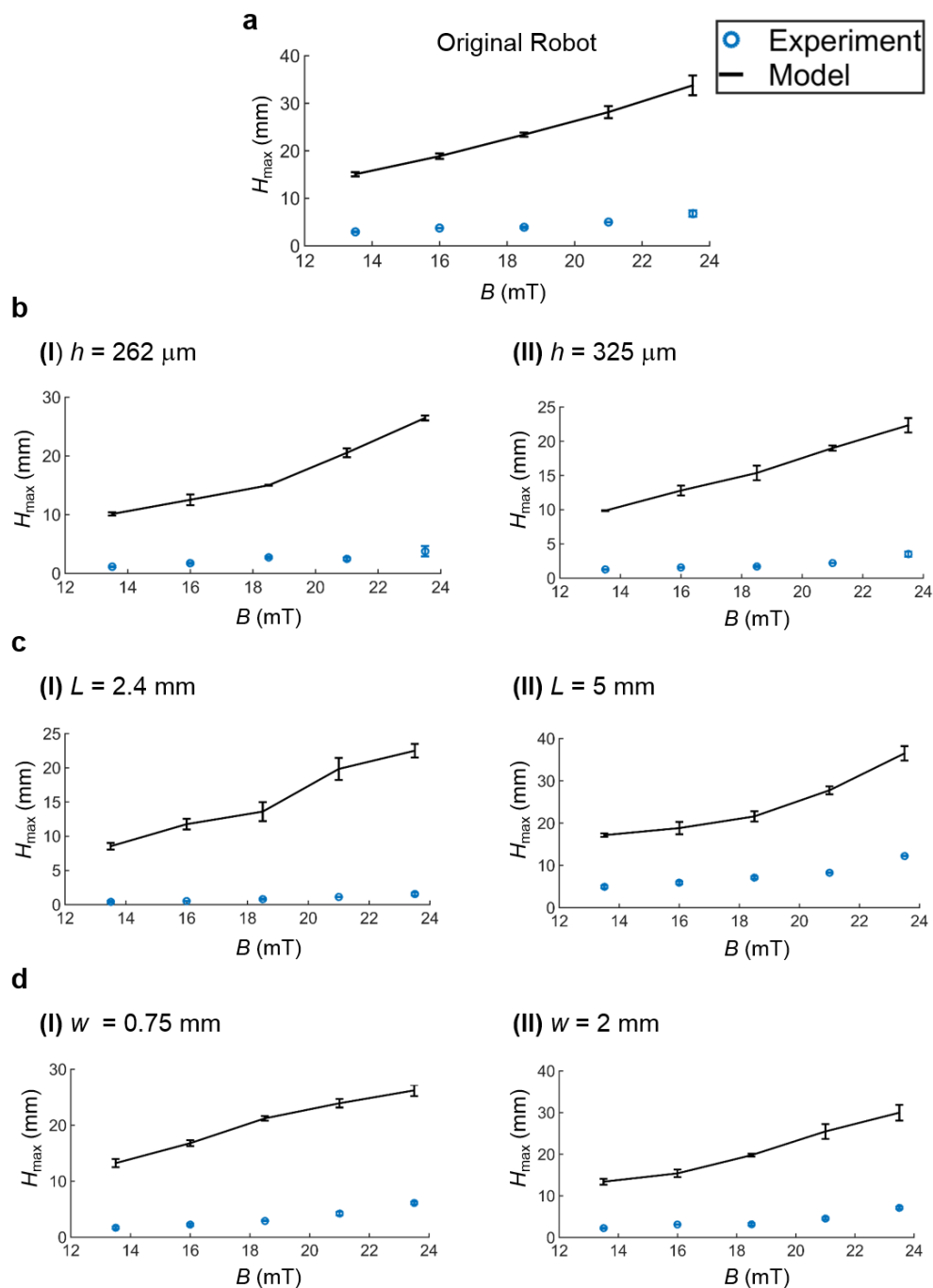


Fig. S7. Experimental and model-predicted results for H_{max} in the straight jumping locomotion. (a) Results for the original robot (dimensions defined in Fig. 1a). (b) Results for robots that have a different h from the original robot. (c) Results for robots that have a different L from the original robot. (d) Results for robots that have a different w from the original robot. Each experimental data point presents the mean and standard deviation of three trials. Error bars in model-predicted values reflect the dispersion in image analysis ($n = 3$).

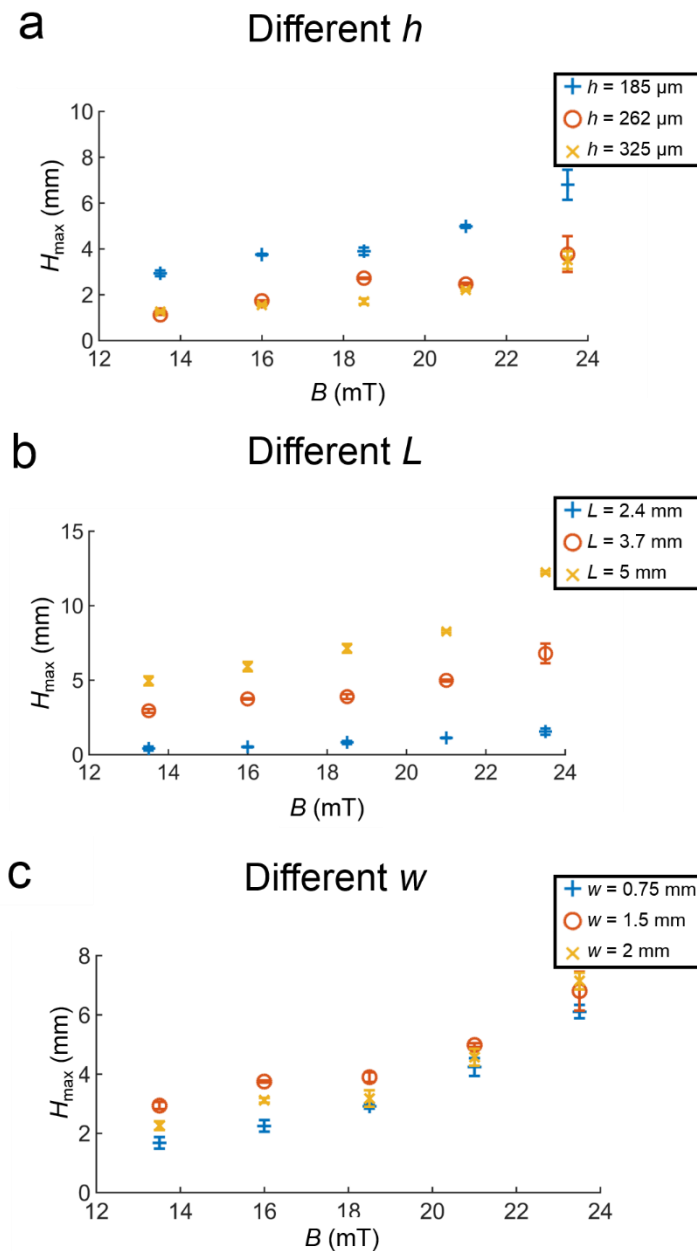


Fig. S8. The compiled experimental data, displaying how the robot's dimensions will affect $H_{\max}$ for the straight jumping locomotion ($H_{\max}$ versus B). (a) Compiled experimental results for robots that have different h . (b) Compiled experimental results for robots that have different L . (c) Compiled experimental results for robots that have different w . Each experimental data point presents the mean and standard deviation out of three trials.

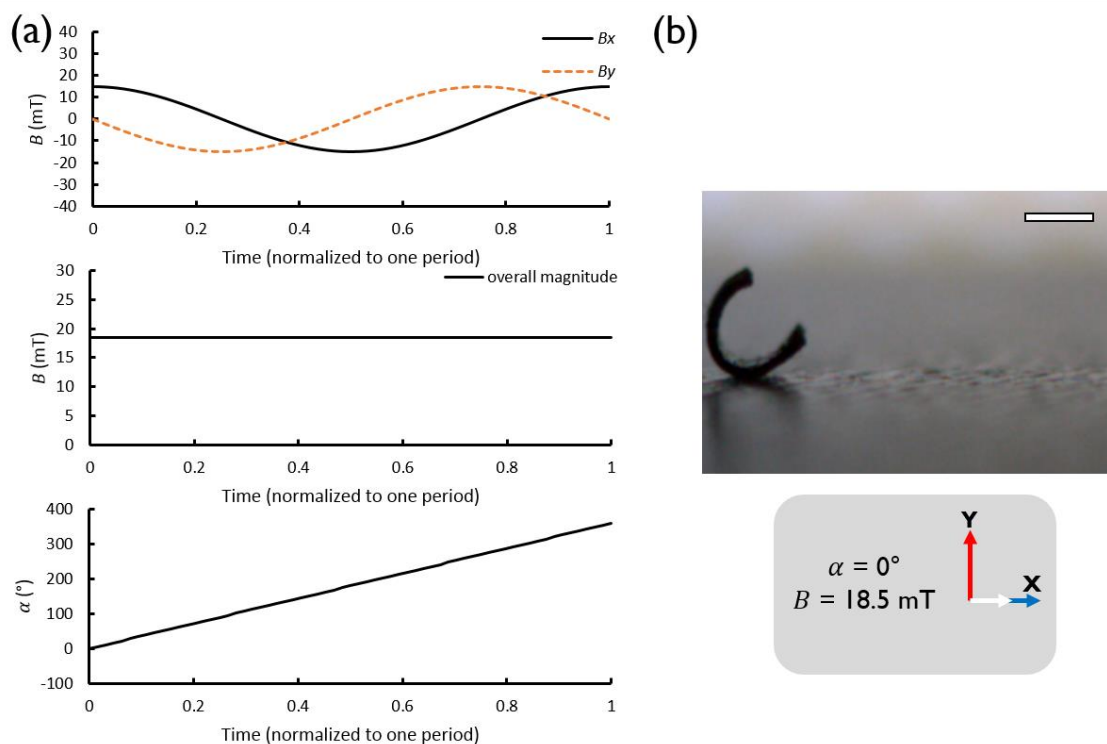


Fig. S9. The required B for the rolling locomotion. (a) Time sequence of the rotating B . B_x and B_y represent the magnitude of B projected along the X- and Y-axis of the global coordinate system. (b) The video snapshot shows the initial 'C-shape' configuration of the robot before it rolls (reproduced from Fig. 2e). Scale bar: 1 mm.

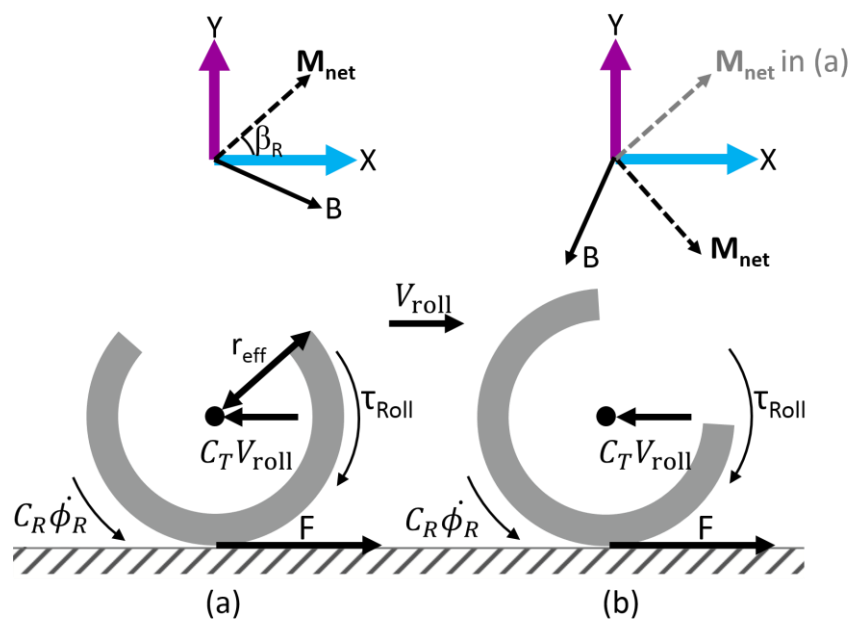


Fig. S10. Sketch for the analysis of the rolling locomotion (not to scale). The illustration shows two instances of the robot as it enables its rolling locomotion. The center of mass is marked out by the black dot. This sketch also shows the applied forces and torques on the robot as it rolls.

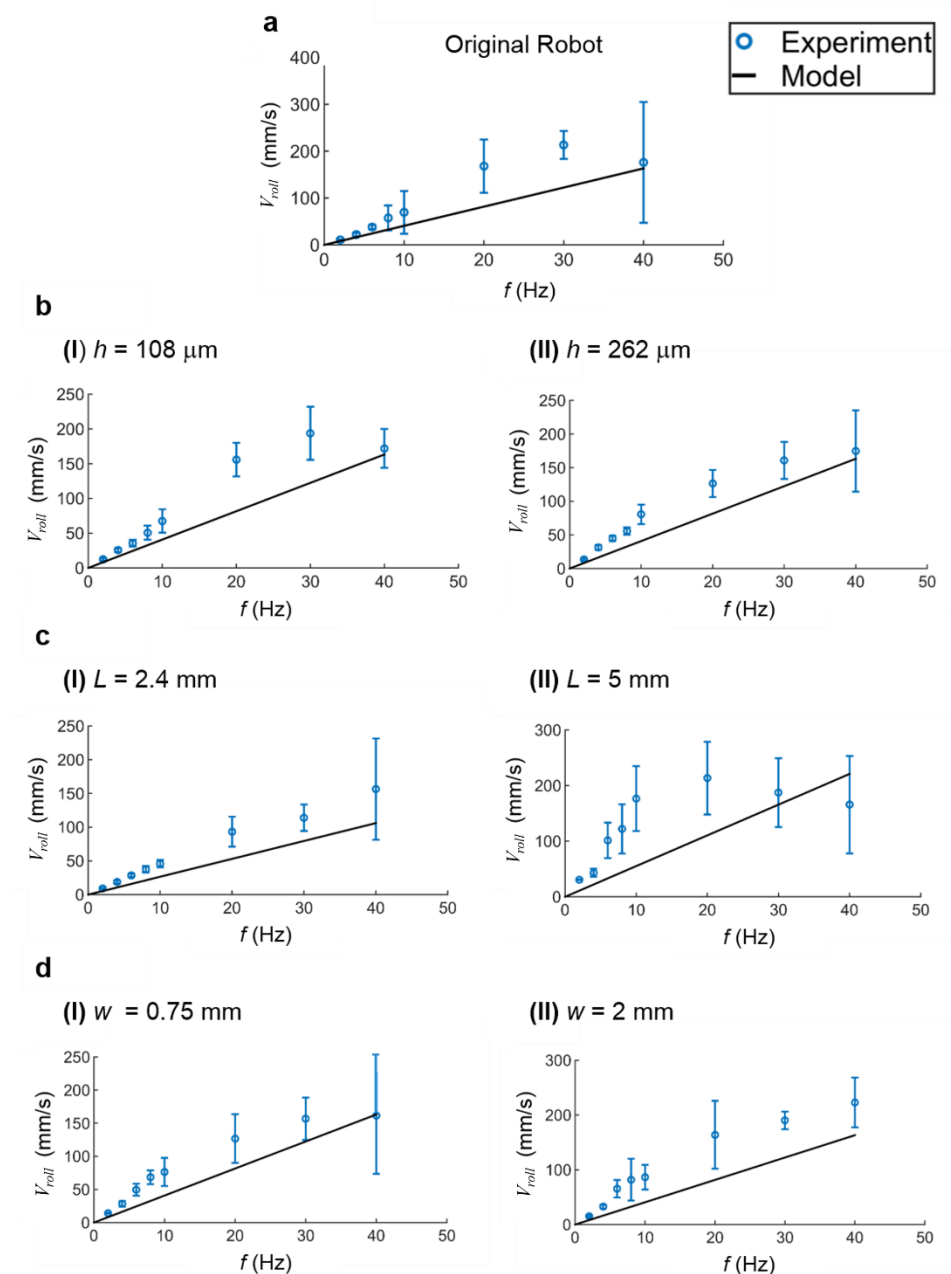


Fig. S11. Experimental and model-predicted results for the rolling locomotion (V_{roll} versus f).

The applied B for all the robots is adjusted in such a way that all these robots can produce a curvature similar to that of the original robot when it is subjected to $B = 18.5$ mT. (a) Results for the original robot (defined in Fig. 1a), where the applied $B = 18.5$ mT. (b) Results for robots that have a different h from the original robot. (I) $B = 6.3$ mT (II) $B = 37$ mT. (c) Results for robots that have a different L from the original robot. (I) $B = 44$ mT (II) $B = 10$ mT. (d) Results for robots that have a different w from the original robot. $B = 18.5$ mT for both (I) and (II). Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

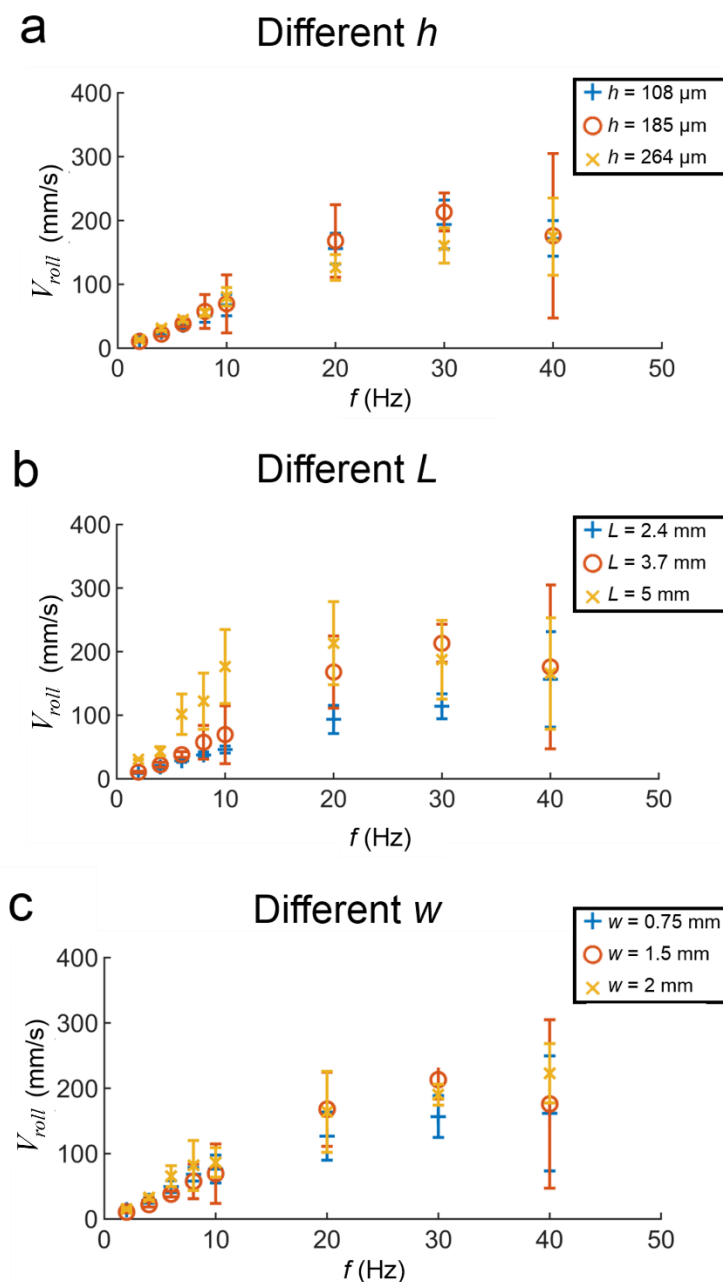


Fig. S12. The compiled experimental data, displaying how the robot's dimensions will affect V_{roll} for the rolling locomotion (V_{roll} versus f). B is adjusted to make all the soft robots have similar curvature as that of the original one when it is subjected to $B = 18.5 \text{ mT}$. (a) Compiled experimental results for robots that have different h . $B = 18.5 \text{ mT}$ ($h = 185 \mu\text{m}$), $B = 6.3 \text{ mT}$ ($h = 108 \mu\text{m}$), $B = 37 \text{ mT}$ ($h = 264 \mu\text{m}$). (b) Compiled experimental results for robots that have different L . $B = 18.5 \text{ mT}$ ($L = 3.7 \text{ mm}$), $B = 44 \text{ mT}$ ($L = 2.4 \text{ mm}$), $B = 10 \text{ mT}$ ($L = 5 \text{ mm}$). (c) Compiled experimental results for robots that have different w . $B = 18.5 \text{ mT}$ for all the three cases. Each experimental data point presents the mean and standard deviation out of three trials.

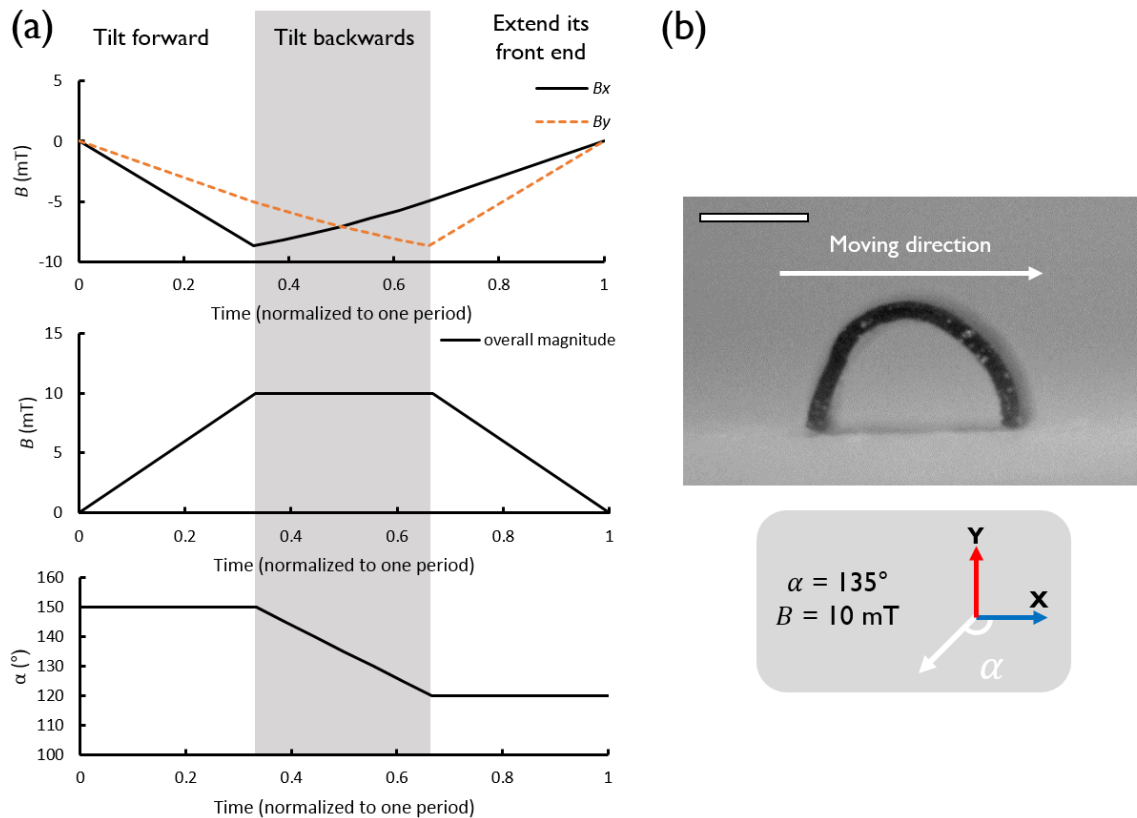


Fig. S13. The required B for the walking locomotion. (a) Time sequence of B . This sequence can be sub-divided into three phases, *i.e.*, 1) tilt forward, 2) tilt backwards, and 3) extend its front end. B_x and B_y represent the magnitude of B projected along the X - and Y -axis of the global coordinate system. (b) The video snapshot shows the 'C'-shape configuration while it is walking (reproduced from Fig. 2f). Scale bar: 1 mm.

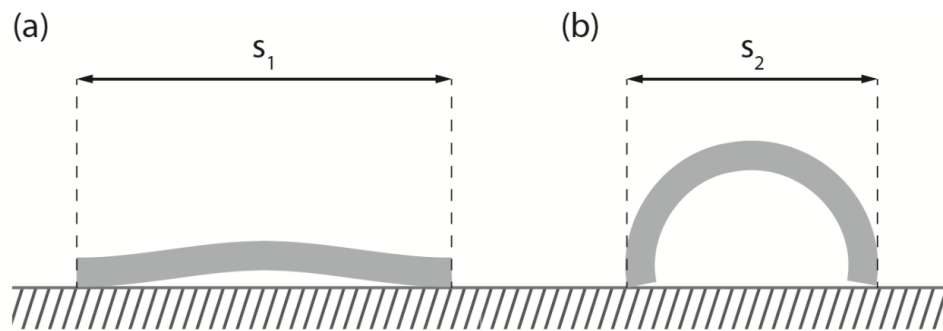


Fig. S14. (a) Stretched and (b) curled state of the robot during a walking cycle (not to scale).

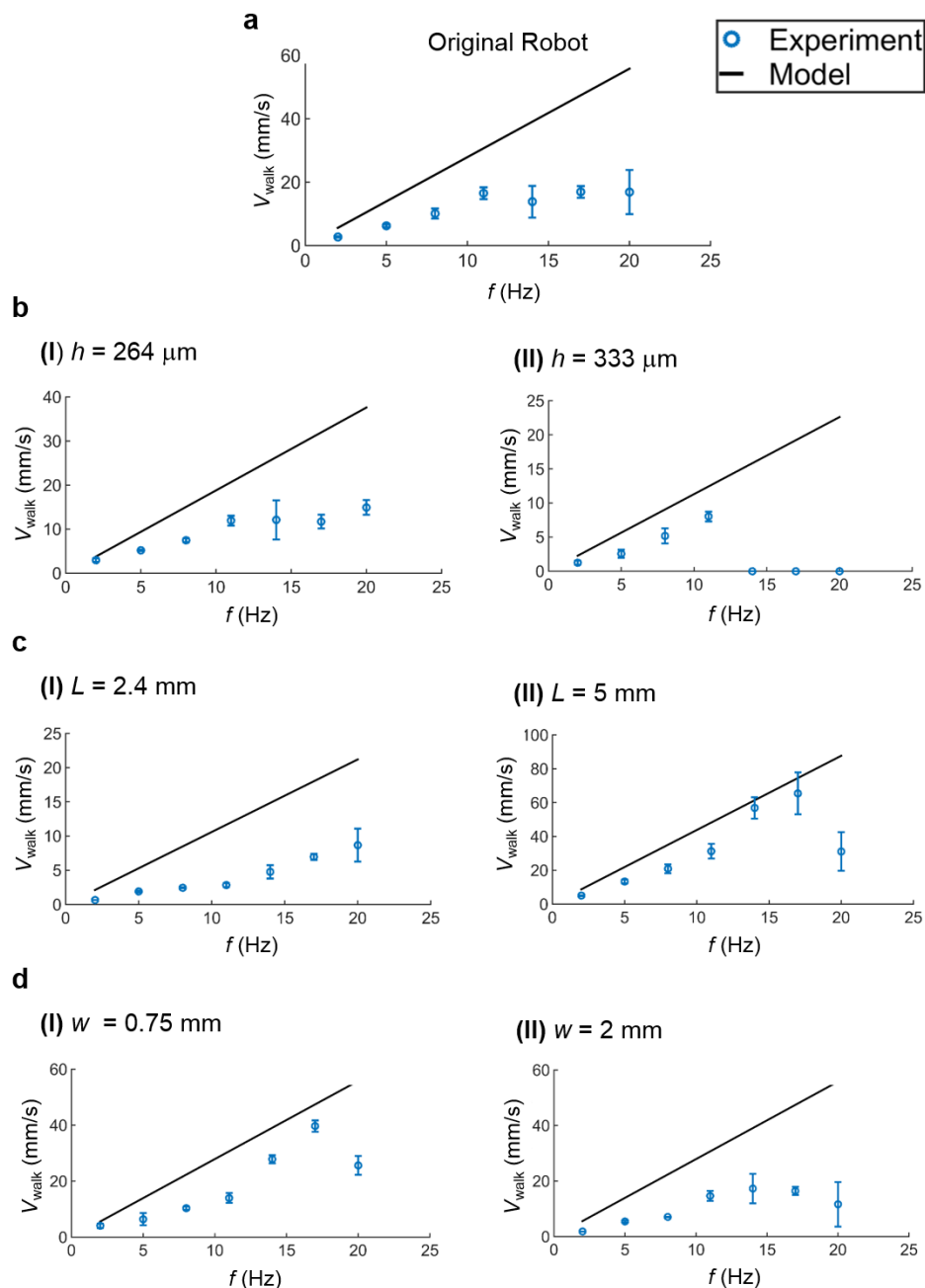


Fig. S15. Experimental and model-predicted results for the walking locomotion (V_{walk} versus f while B_{max} is fixed at 10 mT). (a) Results for the original robot (defined in Fig. 1a). (b) Results for robots that have a different h from the original robot. (c) Results for robots that have a different L from the original robot. (d) Results for robots that have a different w from the original robot. Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

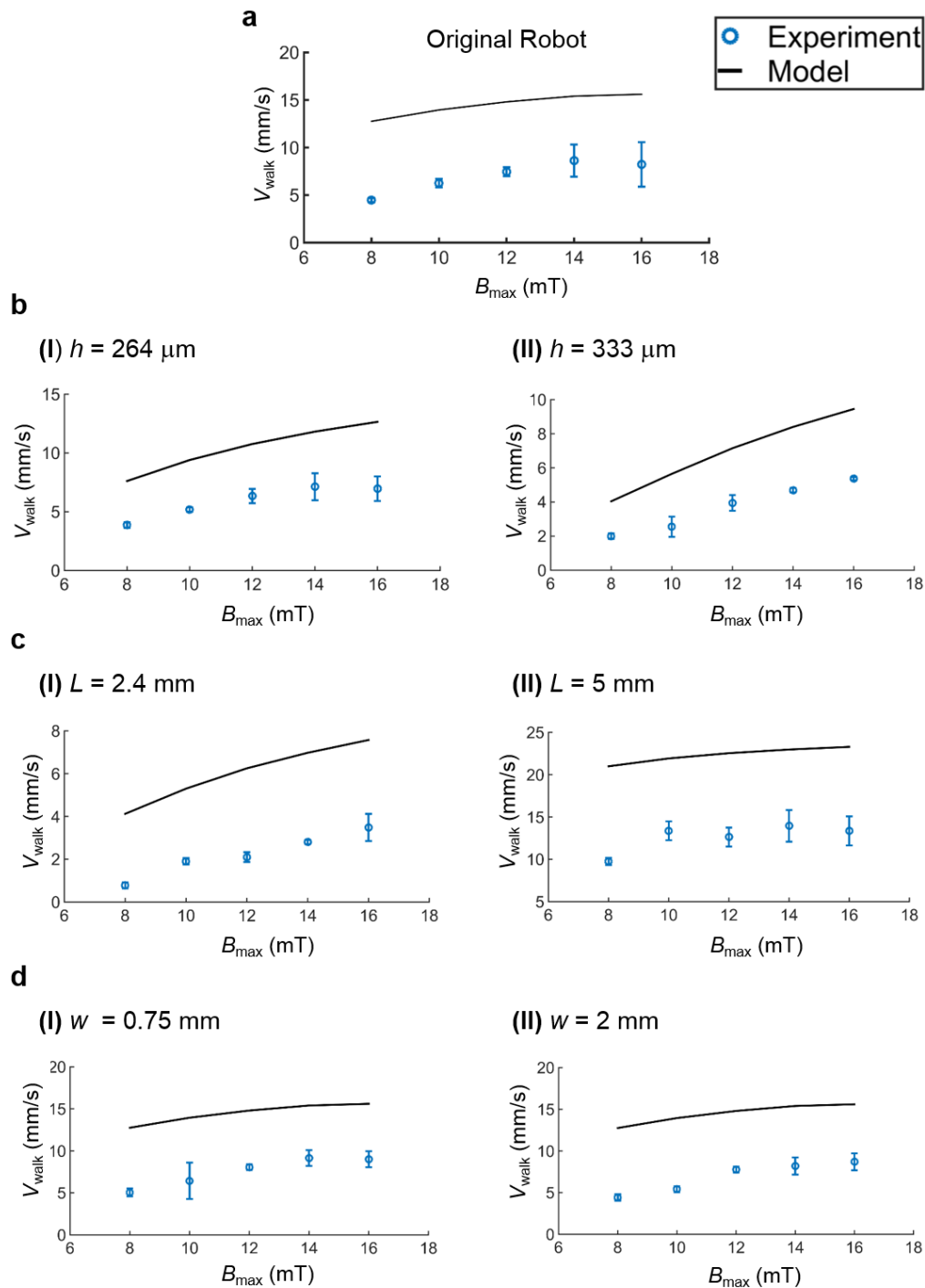


Fig. S16. Experimental and model-predicted results for the walking locomotion (V_{walk} versus B_{max} while f is fixed at 5 Hz). (a) Results for the original robot (defined in Fig. 1a). (b) Results for robots that have a different h from the original robot. (c) Results for robots that have a different L from the original robot. (d) Results for robots that have a different w from the original robot. Experimental data points present mean and standard deviation out of three trials ($n = 3$).

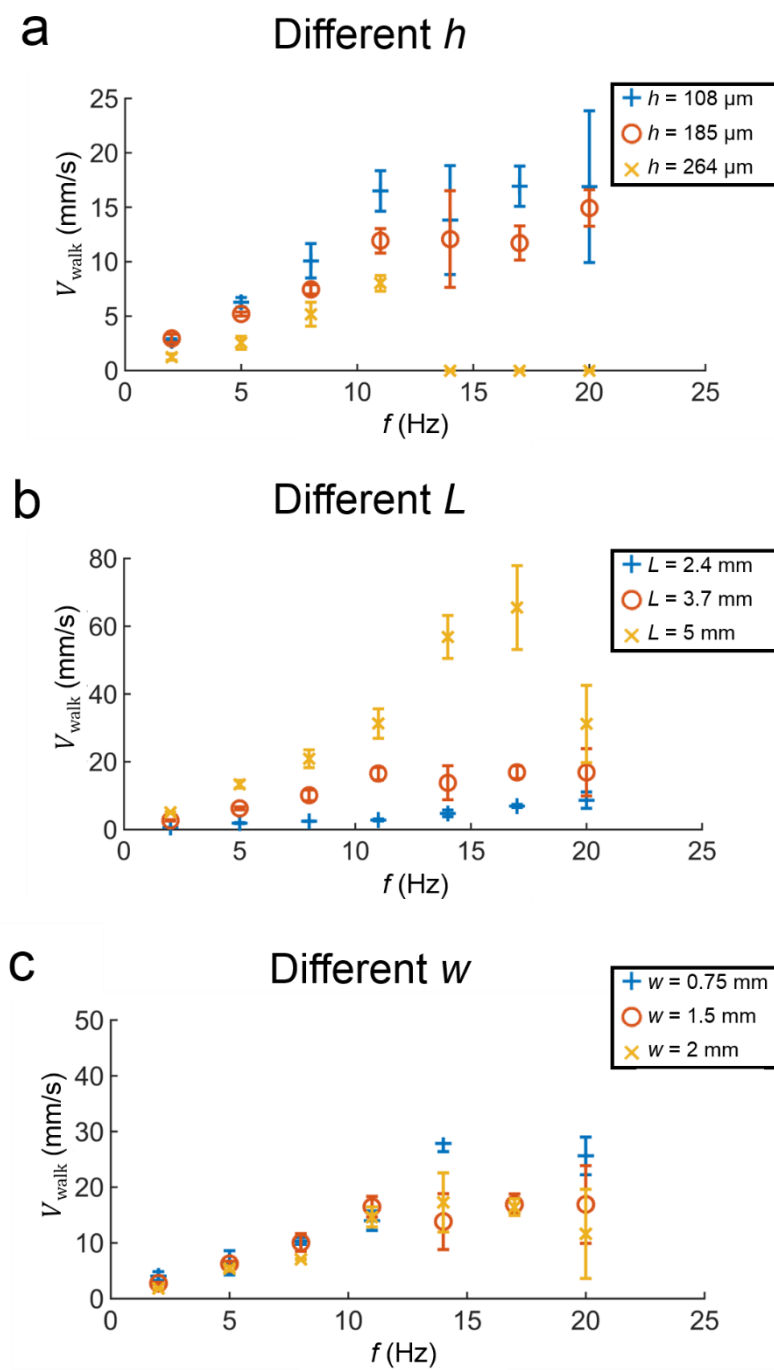


Fig. S17. The compiled experimental data, displaying how the robot's dimensions will affect V_{walk} for the walking locomotion (V_{walk} versus f while B_{max} is fixed at 10 mT). (a) Compiled experimental results for robots that have different h . (b) Compiled experimental results for robots that have different L . (c) Compiled experimental results for robots that have different w . Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

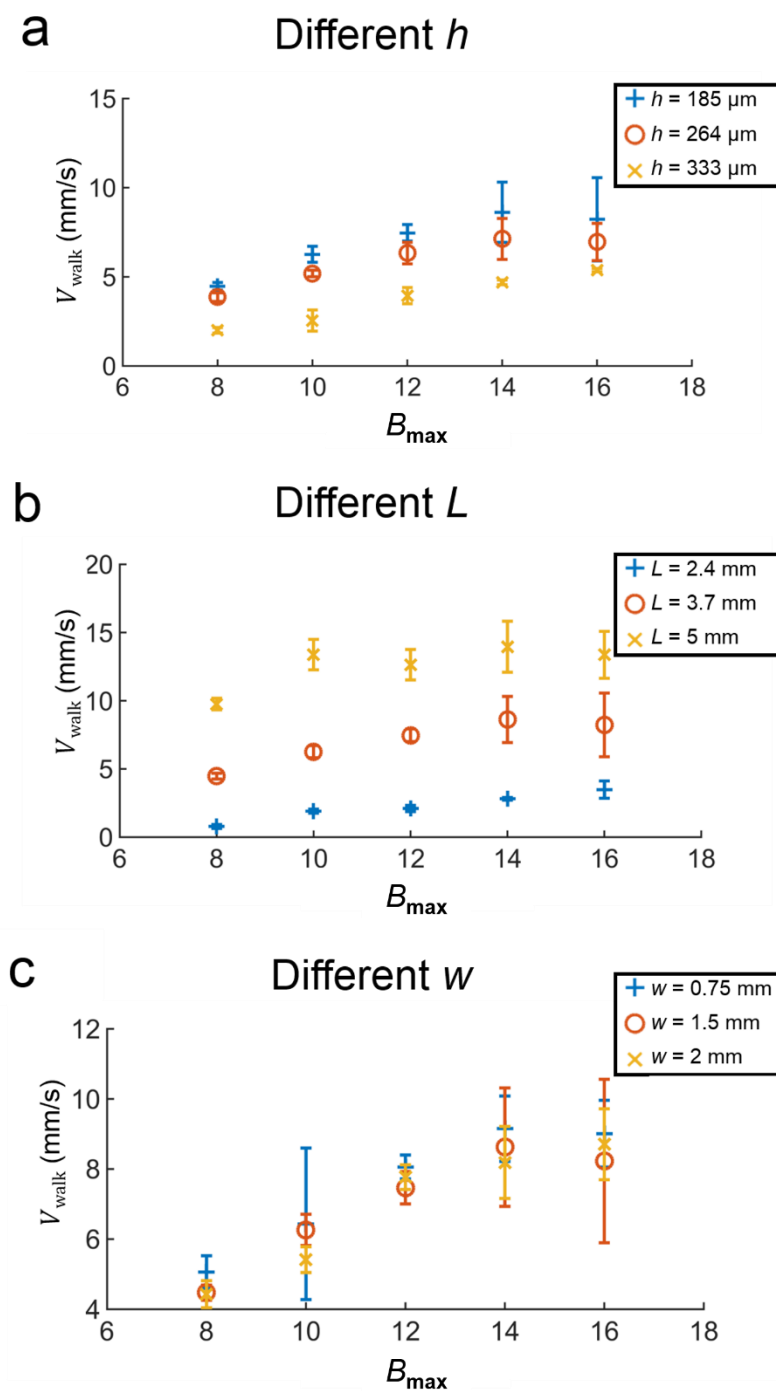


Fig. S18. The compiled experimental data, displaying how the robot's dimensions will affect V_{walk} for the walking locomotion (V_{walk} versus B_{max} while f is fixed at 5 Hz). (a) Compiled experimental results for robots that have different h . (b) Compiled experimental results for robots that have different L . (c) Compiled experimental results for robots that have different w . Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

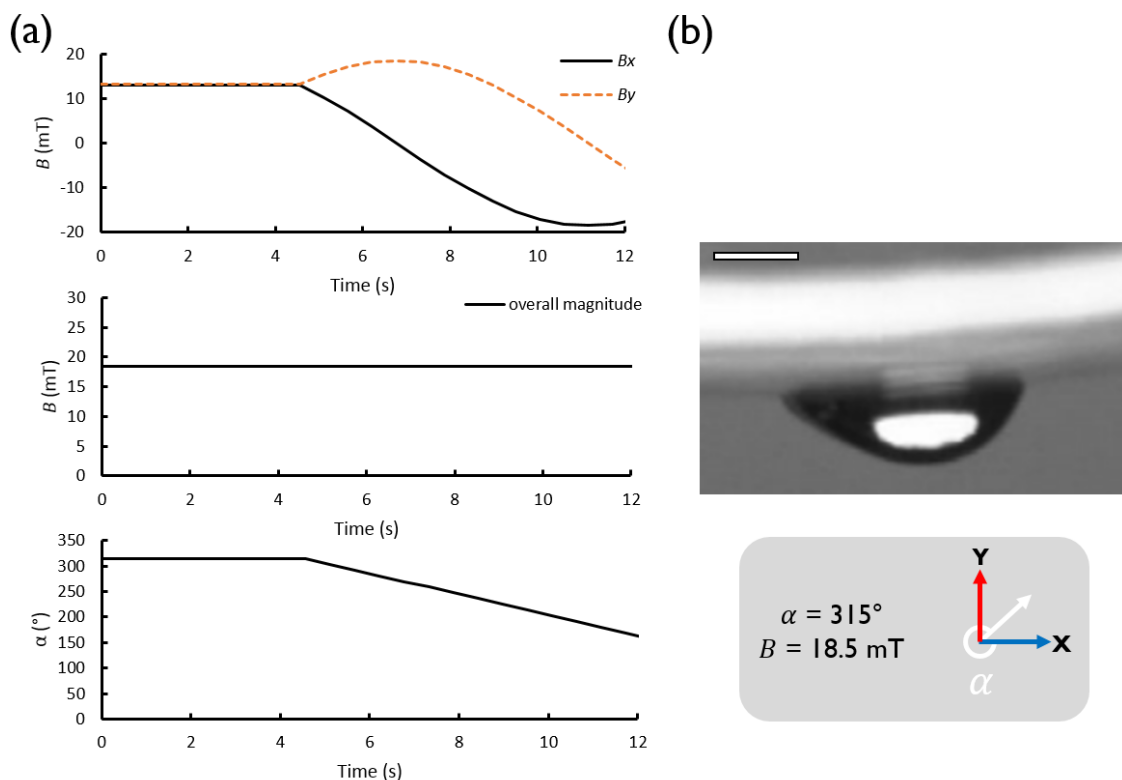


Fig. S19. The required B for the meniscus climbing locomotion. (a) Time sequence of B . B_x and B_y represent the magnitude of the B projected along the X- and Y-axis of the global coordinate system. (b) The video snapshot shows that the robot in a 'C'-shape configuration (for B with $\alpha = 315^\circ$) displaces water from the water-air interface, as evidenced by the air engulfed within its body, while remaining pinned to the water surface. Scale bar: 1 mm.

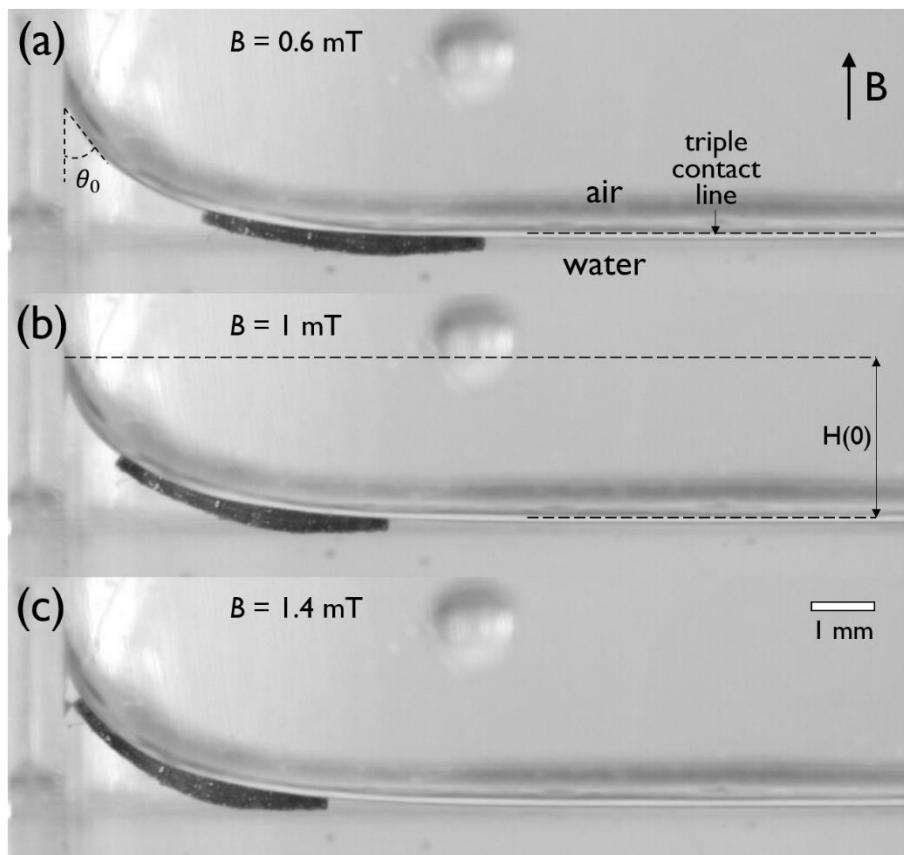


Fig. S20. Meniscus climbing by a magnetic soft robot that has an L of 5 mm. The sequential snapshots show side-views of selected, stable positions assumed by the robot along a positive water meniscus for increasing values of body curvature induced by the external $\mathbf{B}$. The robot is fully immersed in water, its shape conforms to the local interface profile, and the perimeter of its top surface is pinned at the water-air interface.

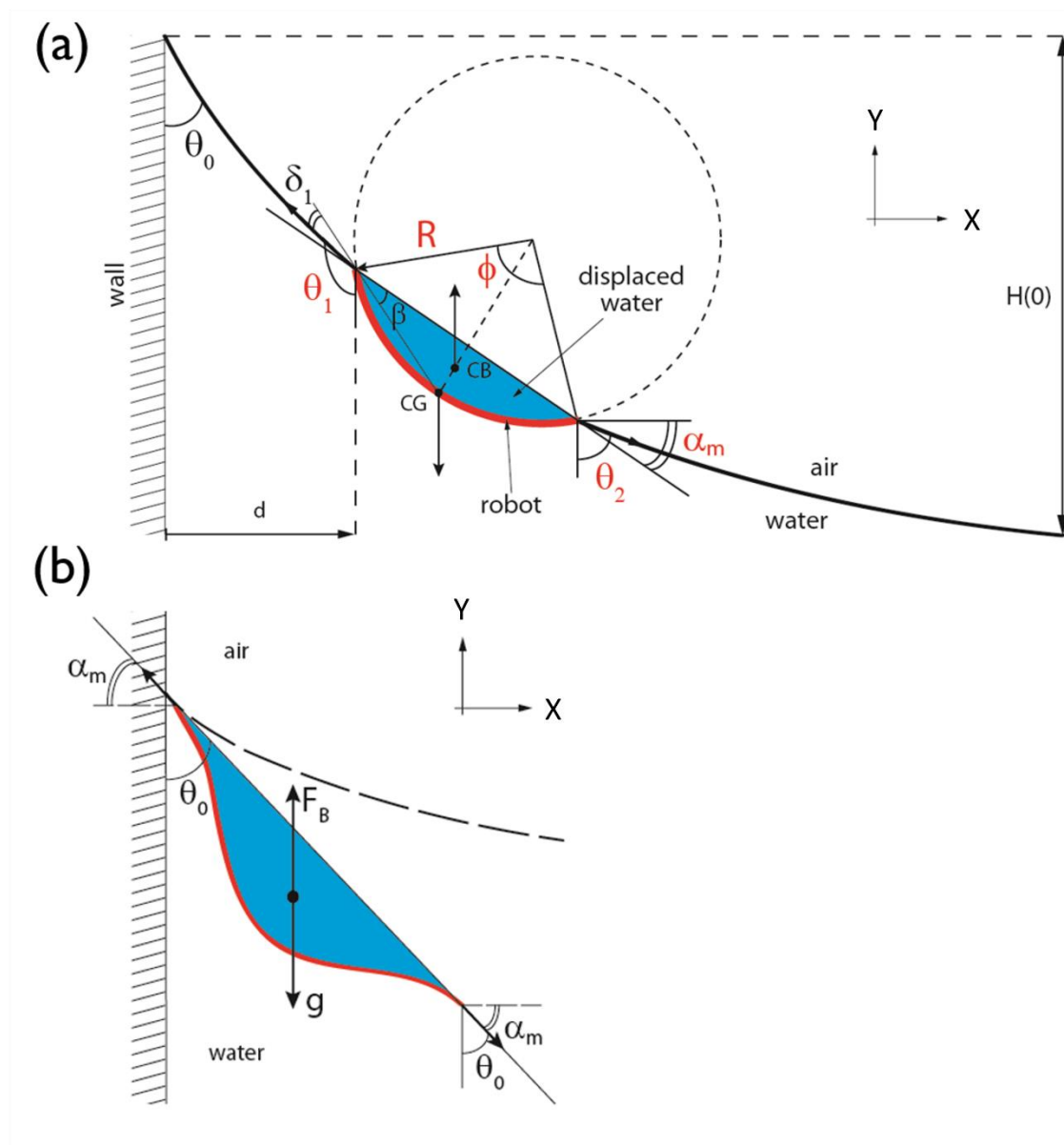


Fig. S21. Sketches of the geometries for the meniscus-climbing models. a) Geometry for the arc-shaped meniscus-climbing model. CG and CB represent respectively the center of gravity of the immersed robot and the center of buoyancy of the displaced water volume. b) Geometry for the cosine-shaped meniscus-climbing model. The model specifically describes the soft robot in close proximity of the adjacent wall at the top of the positive water meniscus, and assumes coincidence of gravity and buoyancy centers. The sketches are not to scale for illustration purposes.

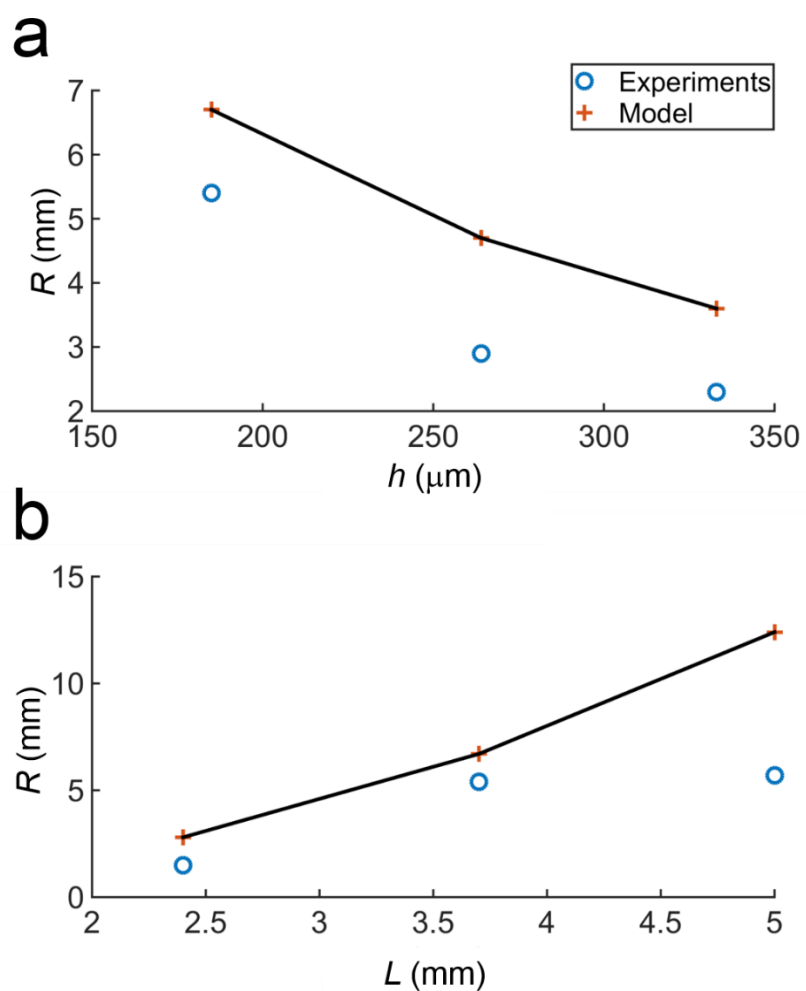


Fig. S22. Results for the meniscus-climbing locomotion (robot curvature versus robot dimensions). Comparison of experimental and model-predicted values of curvature radius for soft robots climbing the entire height of the water meniscus (*i.e.*, for $d \rightarrow 0$, **Fig. S21a**).

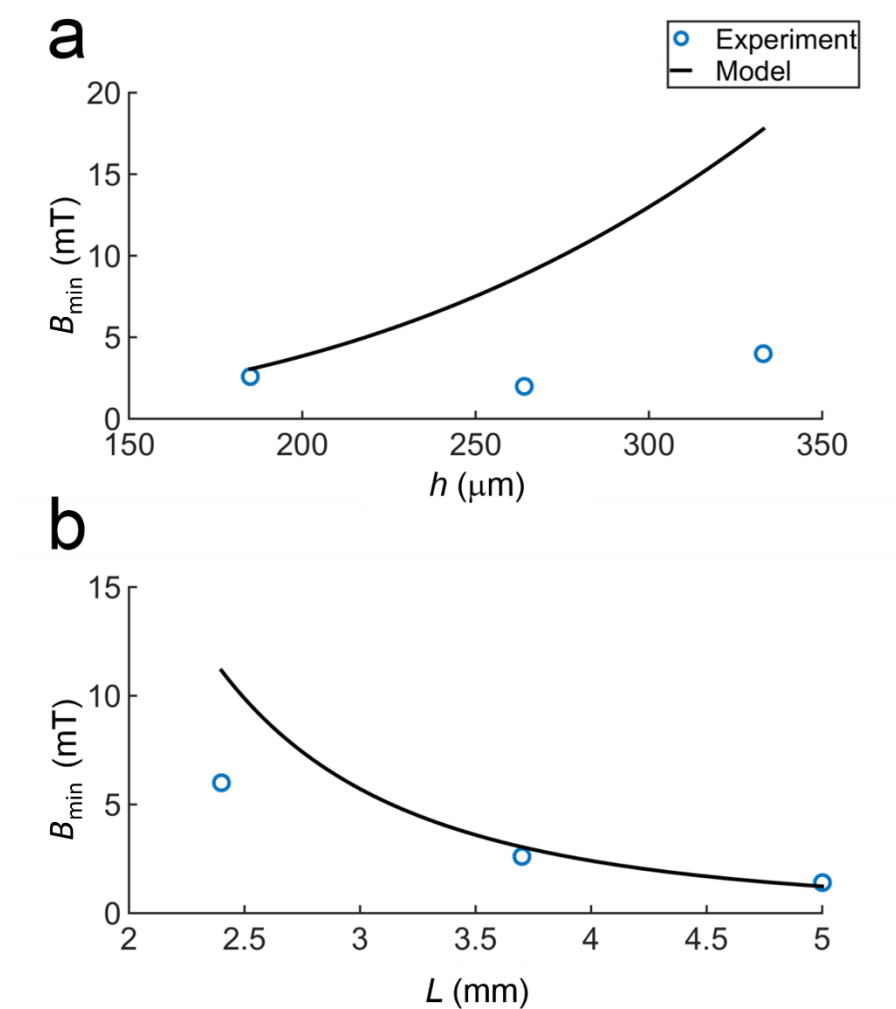


Fig. S23. Results for the meniscus-climbing locomotion ($B_{\min}$ as a function of robot dimensions). Comparison of experimental and model-predicted values of $B_{\min}$ required for soft robots to climb the entire height of the water meniscus (*i.e.*, for $d \rightarrow 0$, **Fig. S21a**).

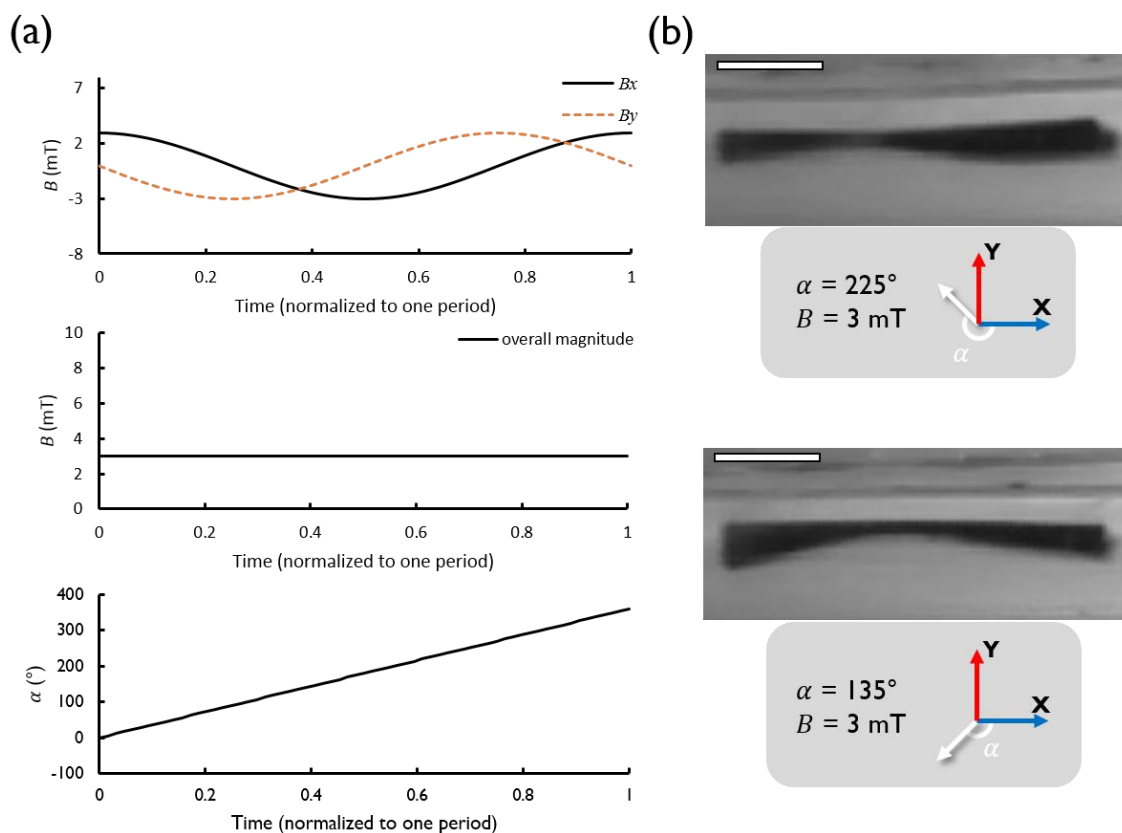


Fig. S24. The required B for the undulating swimming locomotion. (a) Time sequence of B ($B = 3 \text{ mT}$). B_x and B_y represent the magnitude of B projected along the X- and Y-axis of the global coordinate system. (b) The video snapshot shows the corresponding small-deformation shapes of the original robot (Fig. 1a) during undulating swimming locomotion. Scale bar: 1 mm.

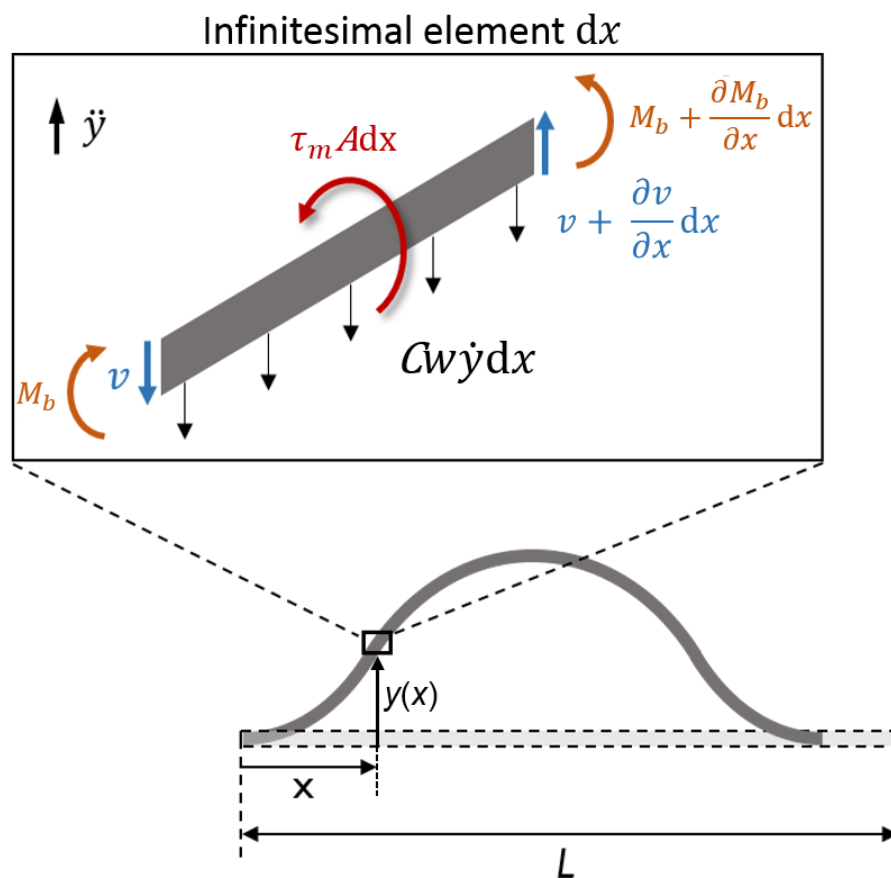


Fig. S25. Vibrational analysis of a soft robot. Sketch of the geometry highlighting forces and torques acting on an infinitesimal element of the robot (not to scale).

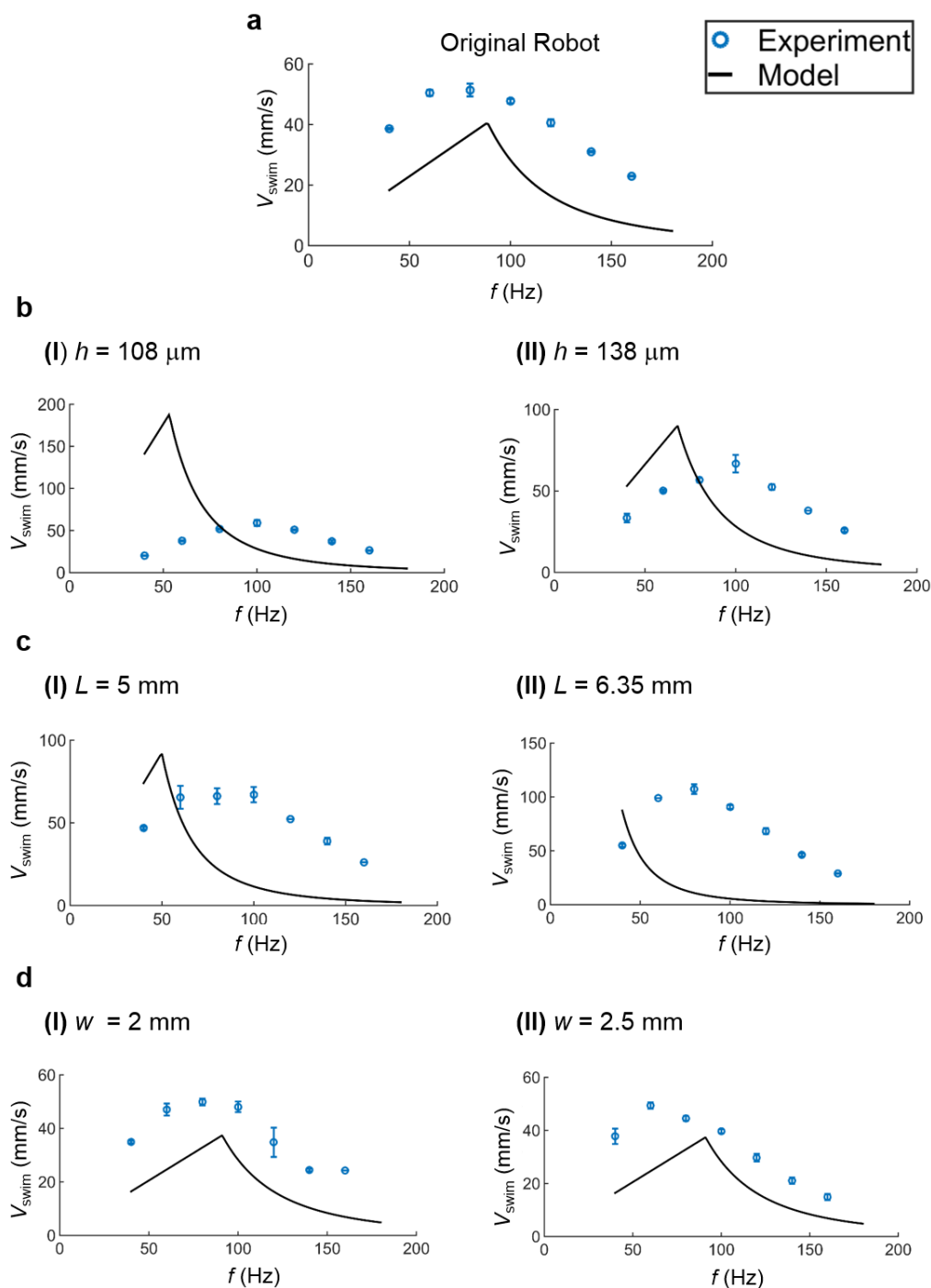


Fig. S26. Experimental and model-predicted results for the undulating swimming locomotion (V_{swim} versus f while B is fixed at 5 mT). (a) Results for the original robot (defined in Fig. 1a). (b) Results for robots that have a different h from the original robot. (c) Results for robots that have a different L from the original robot. (d) Results for robots that have a different w from the original robot. Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

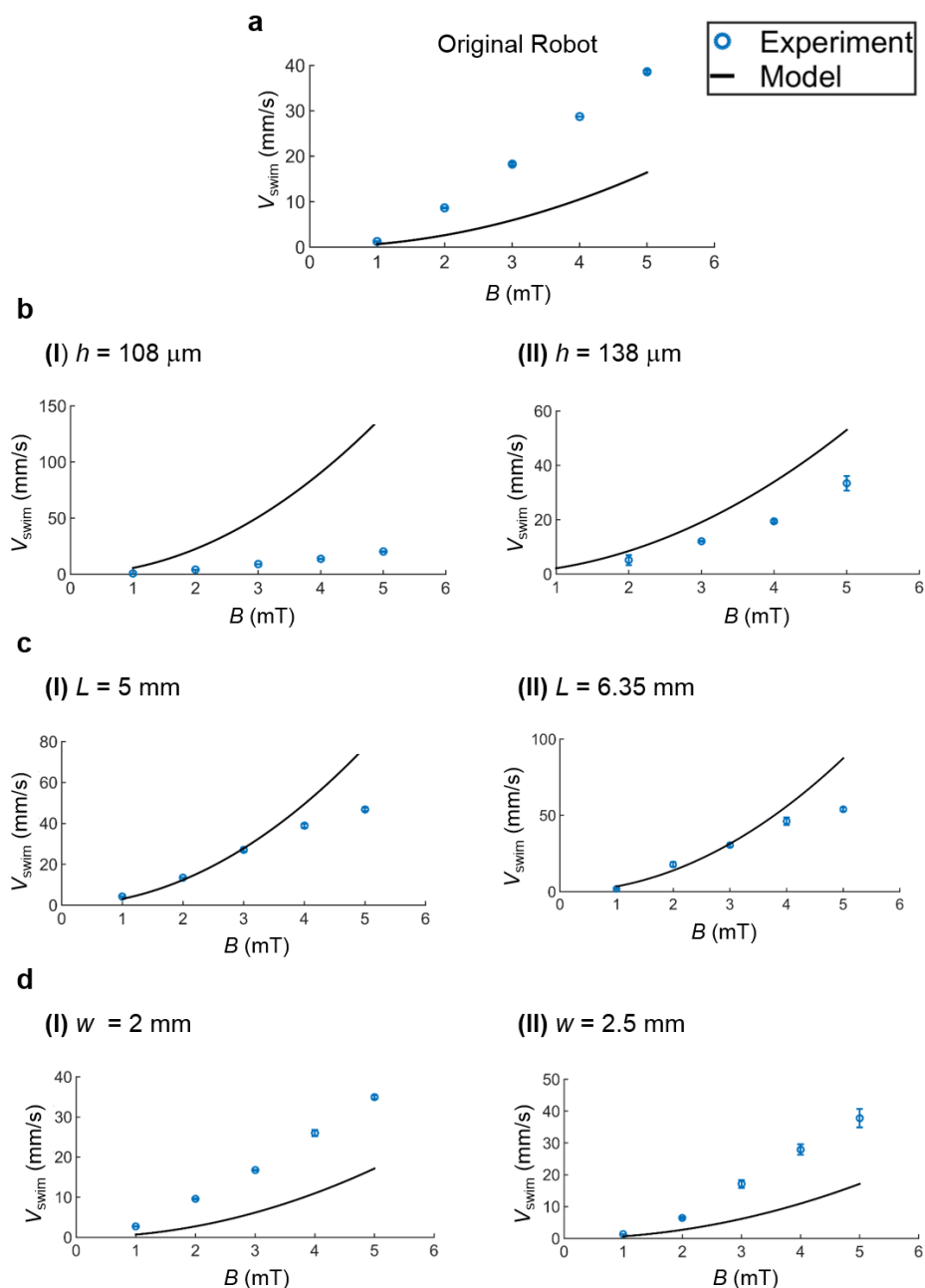


Fig. S27. Experimental and model-predicted results for the undulating swimming locomotion (V_{swim} as a function of B while f is fixed at 40 Hz). (a) Results for the original robot (defined in Fig. 1a). (b) Results for robots that have a different h from the original robot. (c) Results for robots that have a different L from the original robot. (d) Results for robots that have a different w from the original robot. Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

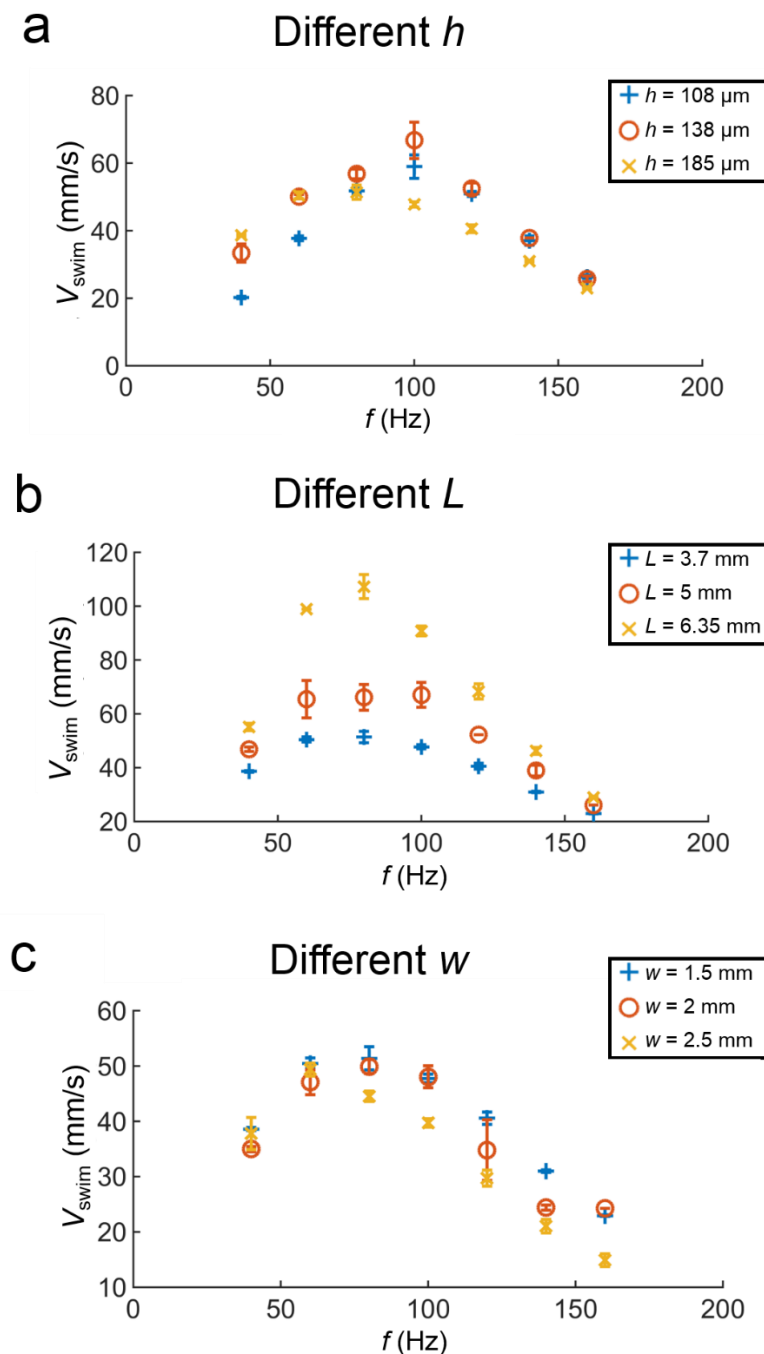


Fig. S28. The compiled experimental data, displaying how the robot's dimensions will affect V_{swim} for the undulating swimming locomotion (V_{swim} as a function of f while B is fixed at 5 mT). (a) Compiled experimental results for robots that have different h . (b) Results for robots that have different L . (c) Compiled experimental results for robots that have different w . Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

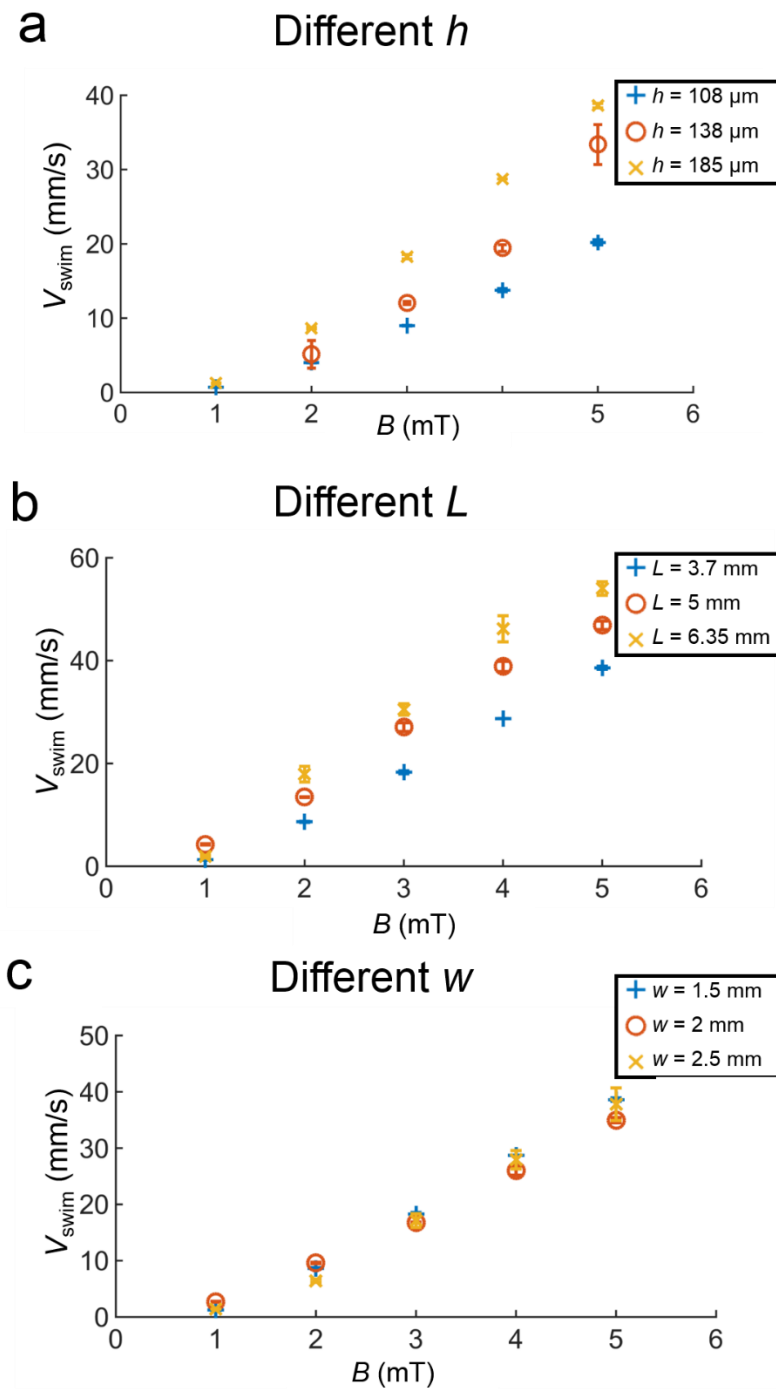


Fig. S29. The compiled experimental data, displaying how the robot's dimensions will affect V_{swim} for the undulating swimming locomotion (V_{swim} as a function of B while f is fixed at 40 Hz). (a) Compiled experimental results for robots that have different h . (b) Compiled experimental results for robots that have different L . (c) Compiled experimental results for robots that have different w . Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

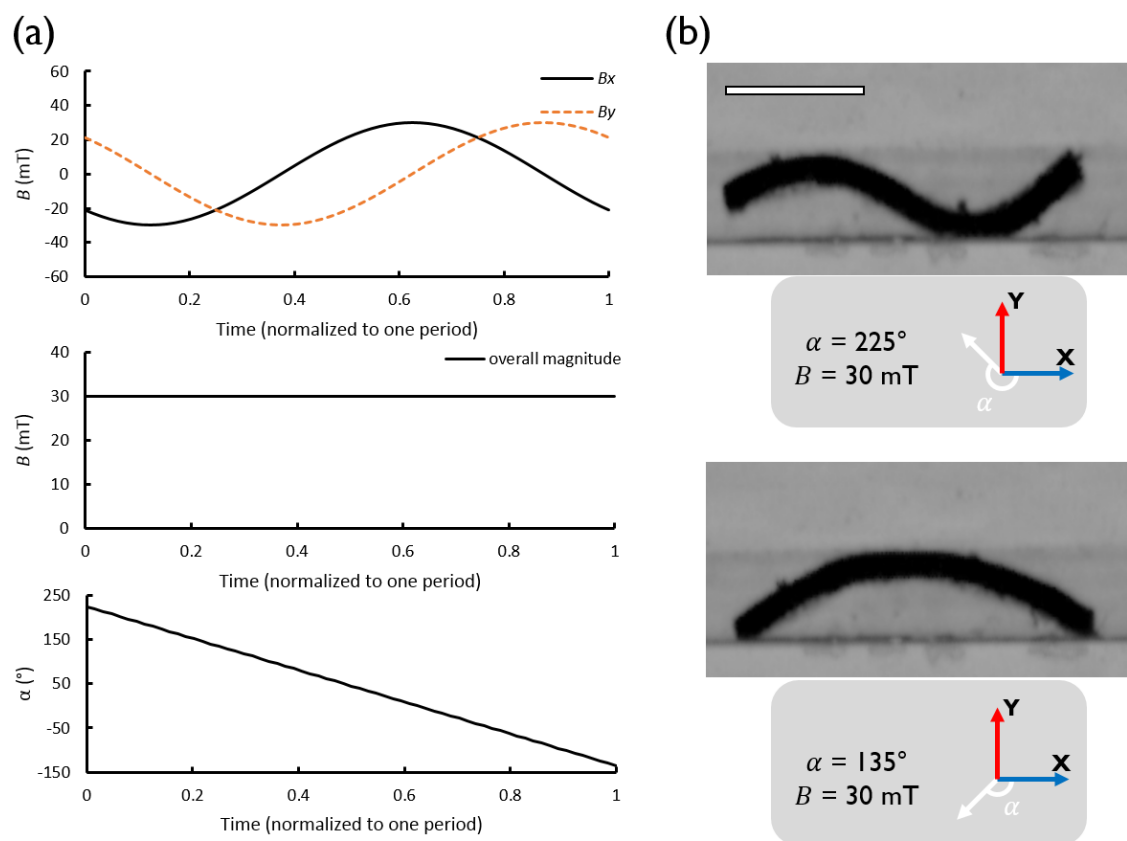


Fig. S30. The required B for the crawling locomotion. (a) Time sequence of B . B_x and B_y represent the magnitude of B projected along the X- and Y-axis of the global coordinate system. (b) shows the instants of the robot when it creates the deformed sine (top) and the cosine (bottom) shapes during a crawling cycle. Scale bar: 1 mm.

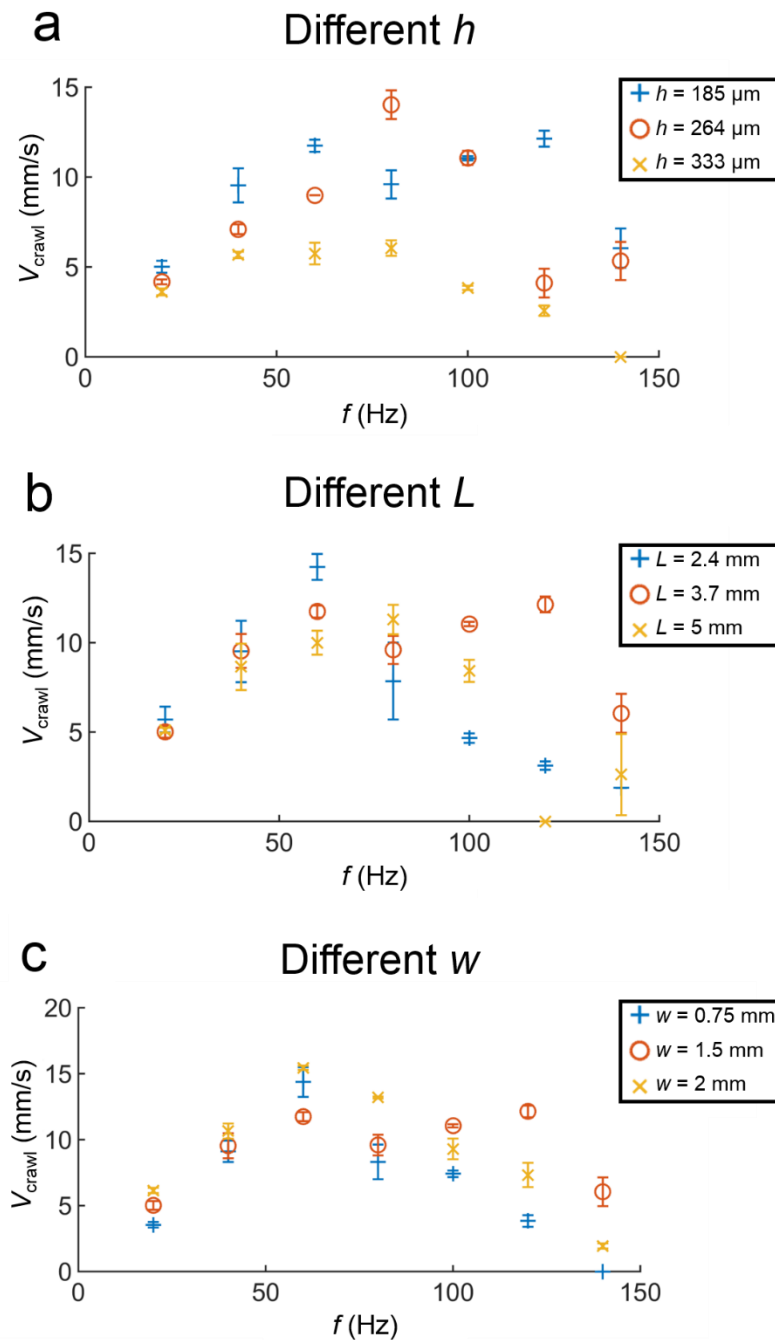


Fig. S31. The compiled experimental data, displaying how the robot's dimensions will affect V_{crawl} for the crawling locomotion (V_{crawl} as a function of f while B is fixed at 10 mT). (a) Compiled experimental results for robots that have different h . (b) Compiled experimental results for robots that have different L . (c) Compiled experimental results for robots that have different w . Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

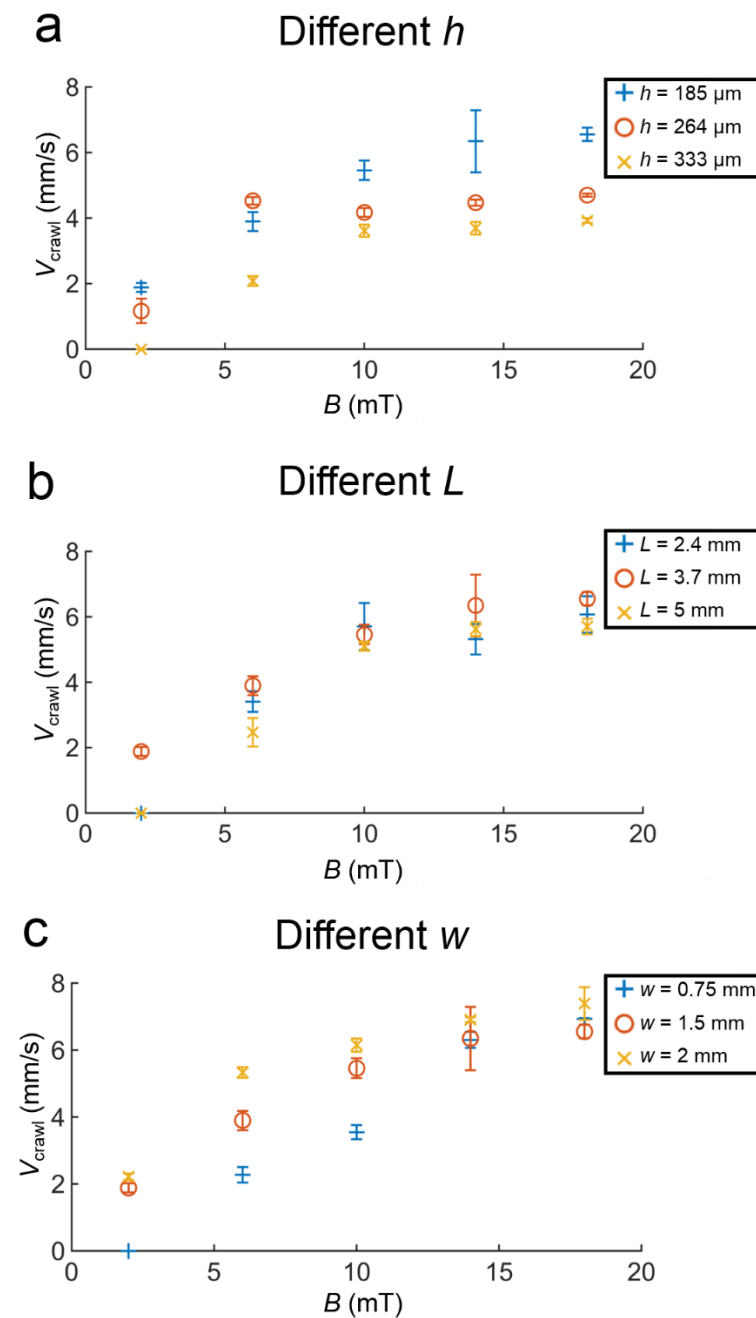


Fig. S32. The compiled experimental data, displaying how the robot's dimensions will affect V_{crawl} for the crawling locomotion (V_{crawl} as a function of B while f is fixed at 20Hz). (a) Compiled experimental results for robots that have different h . (b) Compiled experimental results for robots that have different L . (c) Compiled experimental results for robots that have different w . Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

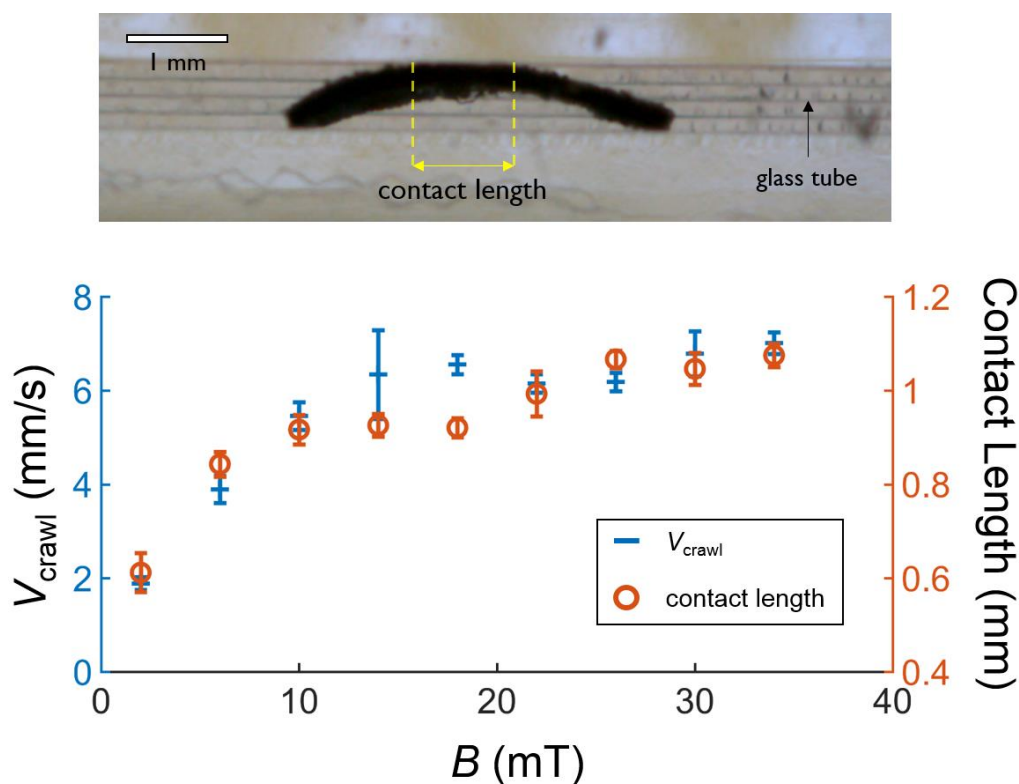


Fig. S33. Crawling speed and contact length between the original robot and the inner tunnel surface as a function of B (f is fixed at 20 Hz). The contact length is defined in the top subfigure, and is assumed uniform across the width of the robot, *i.e.*, contact area = contact length $\times$ w . The flat lining of the contact length correlates with the saturation of the crawling speed. Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

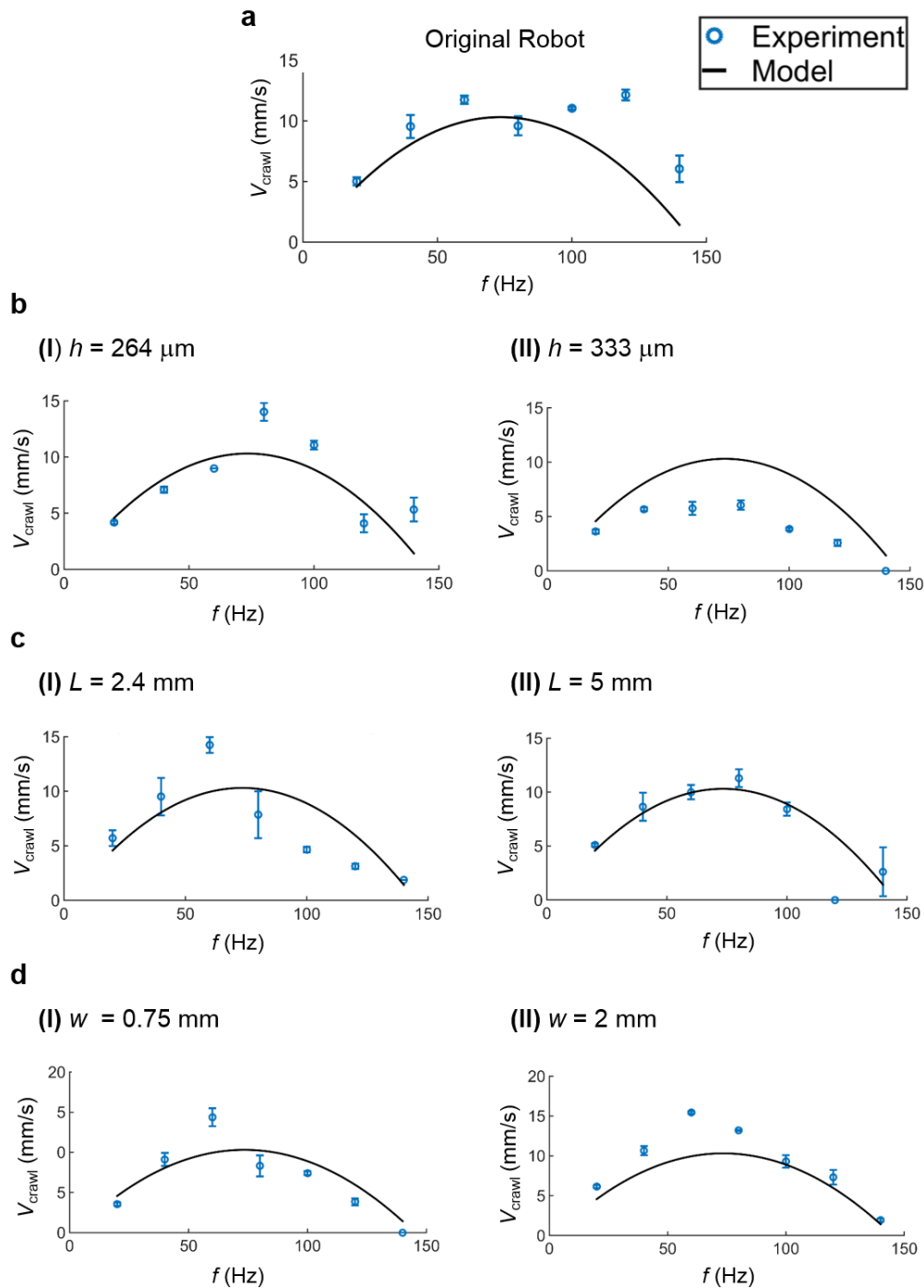


Fig. S34. Experimental and model-predicted results for the crawling locomotion (V_{crawl} as a function of f while B is fixed at 10 mT). (a) Results for the original robot (defined in Fig. 1a). (b) Results for robots that have a different h from the original robot. (c) Results for robots that have a different L from the original robot. (d) Results for robots that have a different w from the original robot. Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

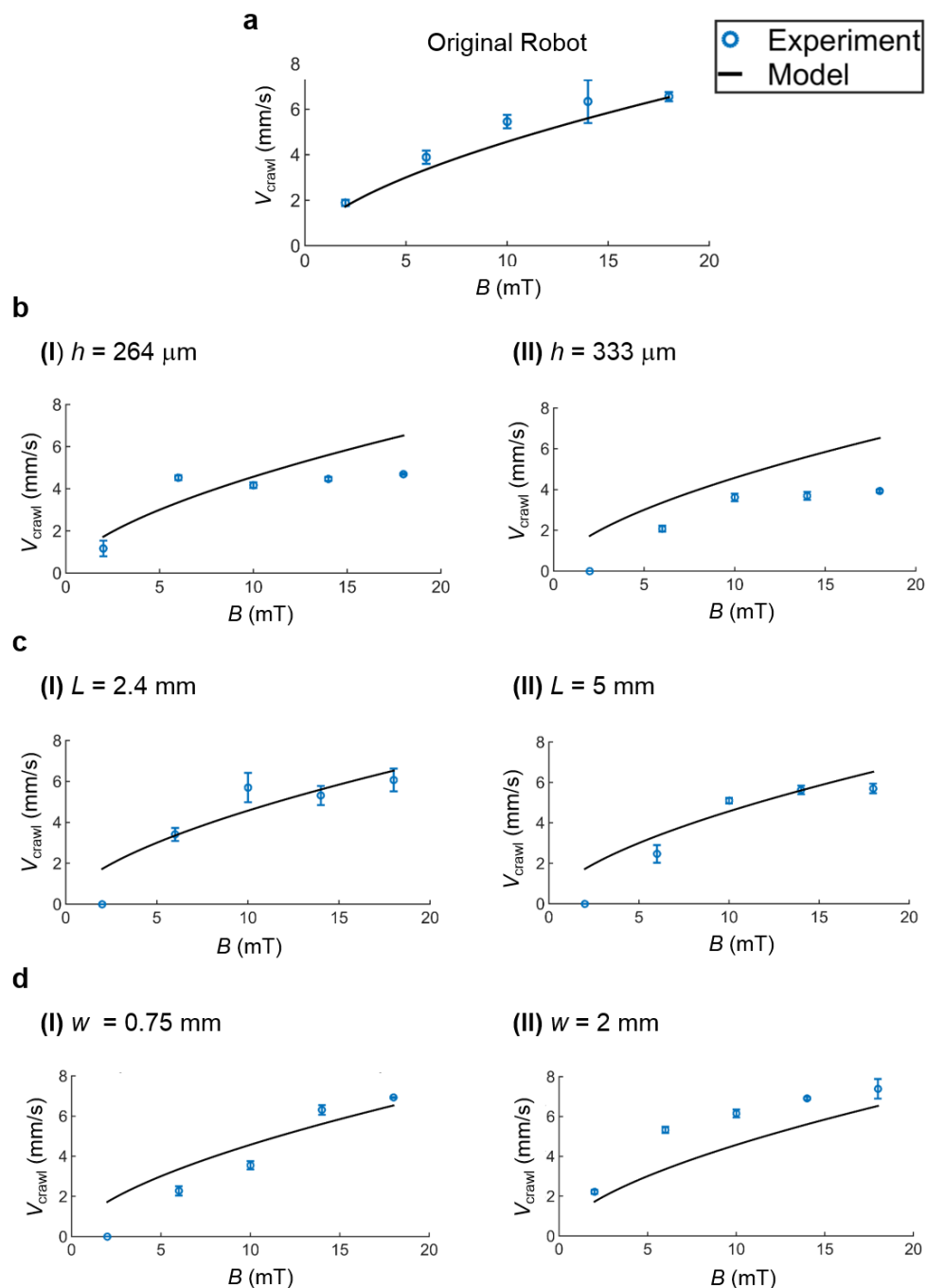


Fig. S35. Experimental and model-predicted results for the crawling locomotion (V_{crawl} as a function of B while f is fixed at 20 Hz). (a) Results for the original robot (defined in Fig. 1a). (b) Results for robots that have a different h from the original robot. (c) Results for robots that have a different L from the original robot. (d) Results for robots that have a different w from the original robot. Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

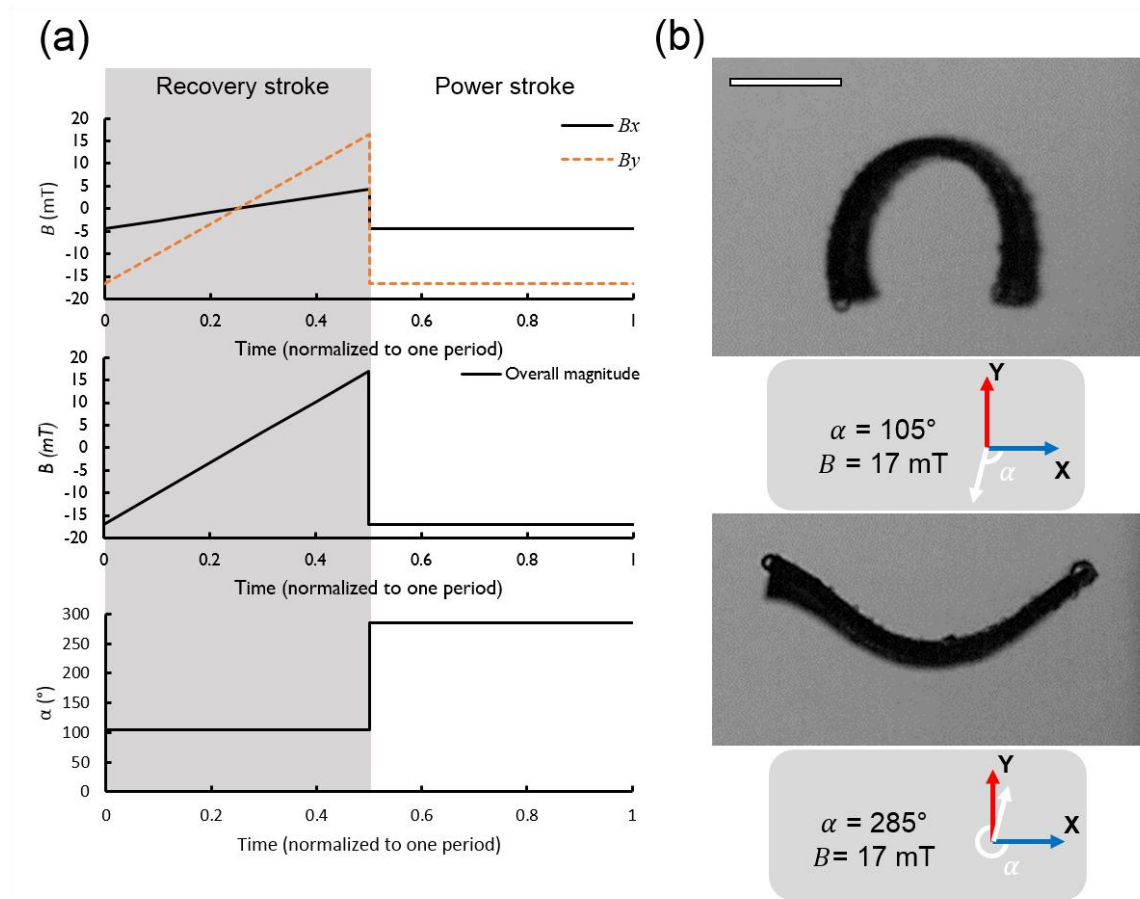


Fig. S36. The required B for the jellyfish-like swimming locomotion. (a) Time sequence of B . B_x and B_y represent the magnitude of B projected along the X - and Y -axis of the global coordinate system. (b) The video snapshots show the 'C'- and 'V'-shape configurations assumed by the robot before the recovery stroke (top) and before the power stroke (bottom), respectively. Scale bar: 1 mm.

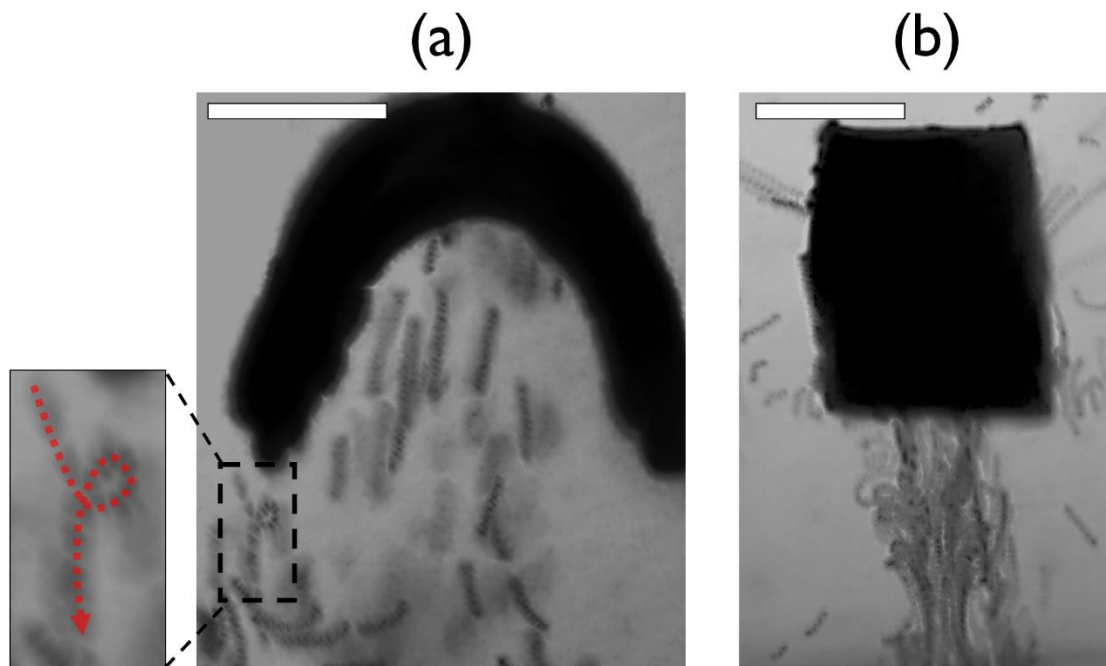


Fig. S37. Fluid vortices produced by the power stroke during the jellyfish-like swimming locomotion. The video snapshots in (a) and (b) respectively show the front and lateral views of two experiment trails recorded separately. The flow was traced using polystyrene particles (diameter: 45 μm). The original robot (dimensions defined in Fig. 1a) was used for this experiment. 40 frames recorded at 2000 fps (corresponding to a time period of 20 ms) were overlapped to produce the images. See also Supporting Video S1. Scale bars: 1 mm.

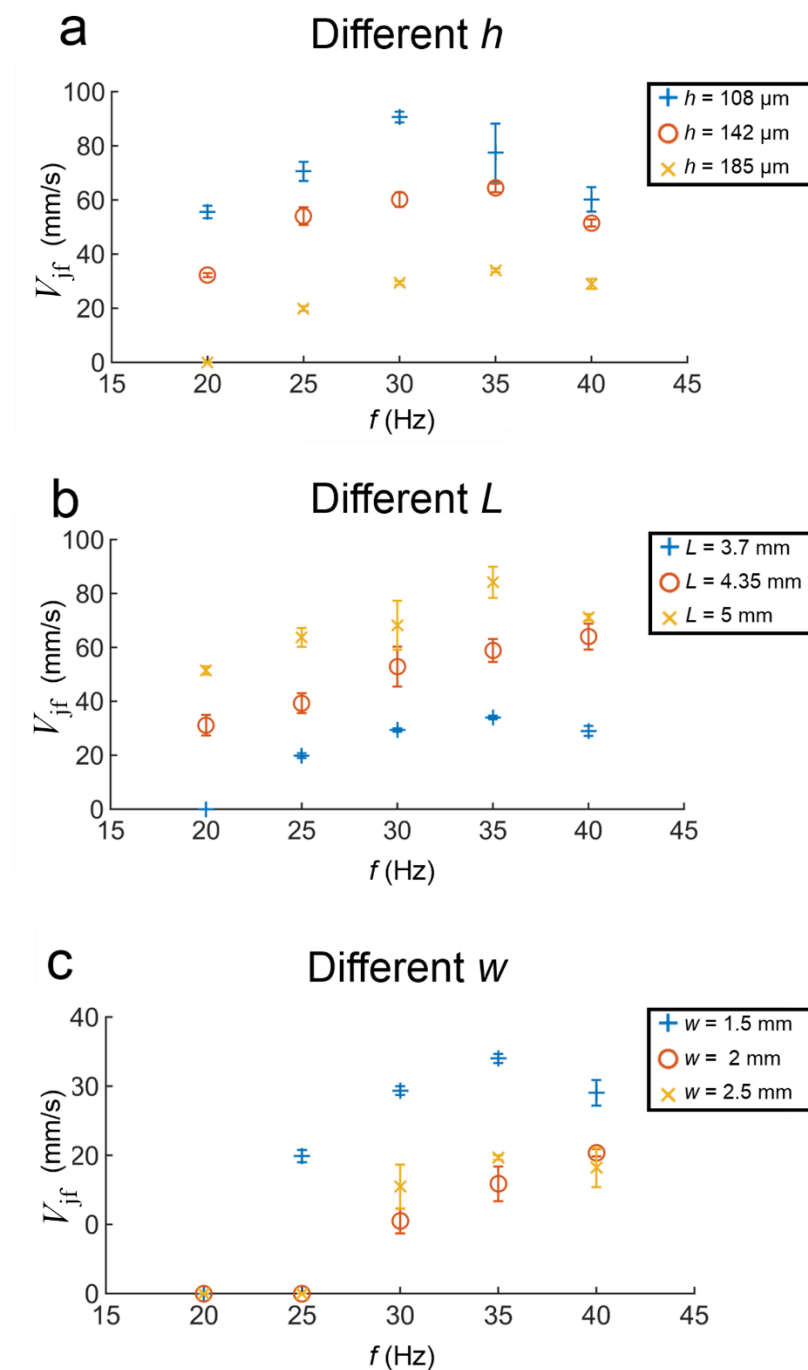


Fig. S38. The compiled experimental data, displaying how the robot's dimensions will affect V_{jf} for the jellyfish-like swimming locomotion (V_{jf} as a function of f while B_{max} is fixed at 20 mT). (a) Compiled experimental results for robots that have a different h from the original robot. (b) Compiled experimental results for robots that have a different L from the original robot. (c) Compiled experimental results for robots that have a different w from the original robot. Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

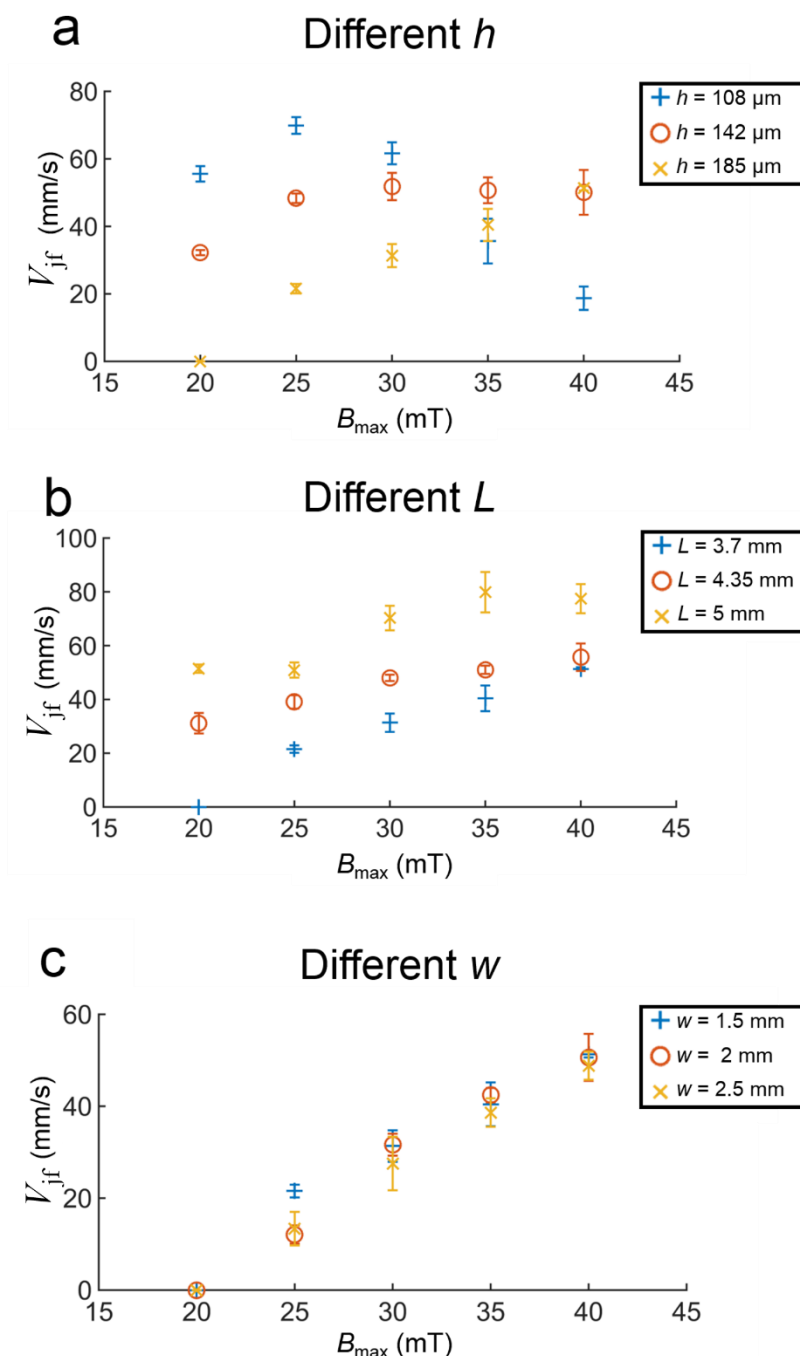


Fig. S39. The compiled experimental data, displaying how the robot's dimensions will affect V_{jf} for the jellyfish-like swimming locomotion (V_{jf} as a function of B_{max} while f is fixed at 20Hz). (a) Compiled experimental results for robots that have a different h from the original robot. (b) Compiled experimental results for robots that have a different L from the original robot. (c) Compiled experimental results for robots that have a different w from the original robot. Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

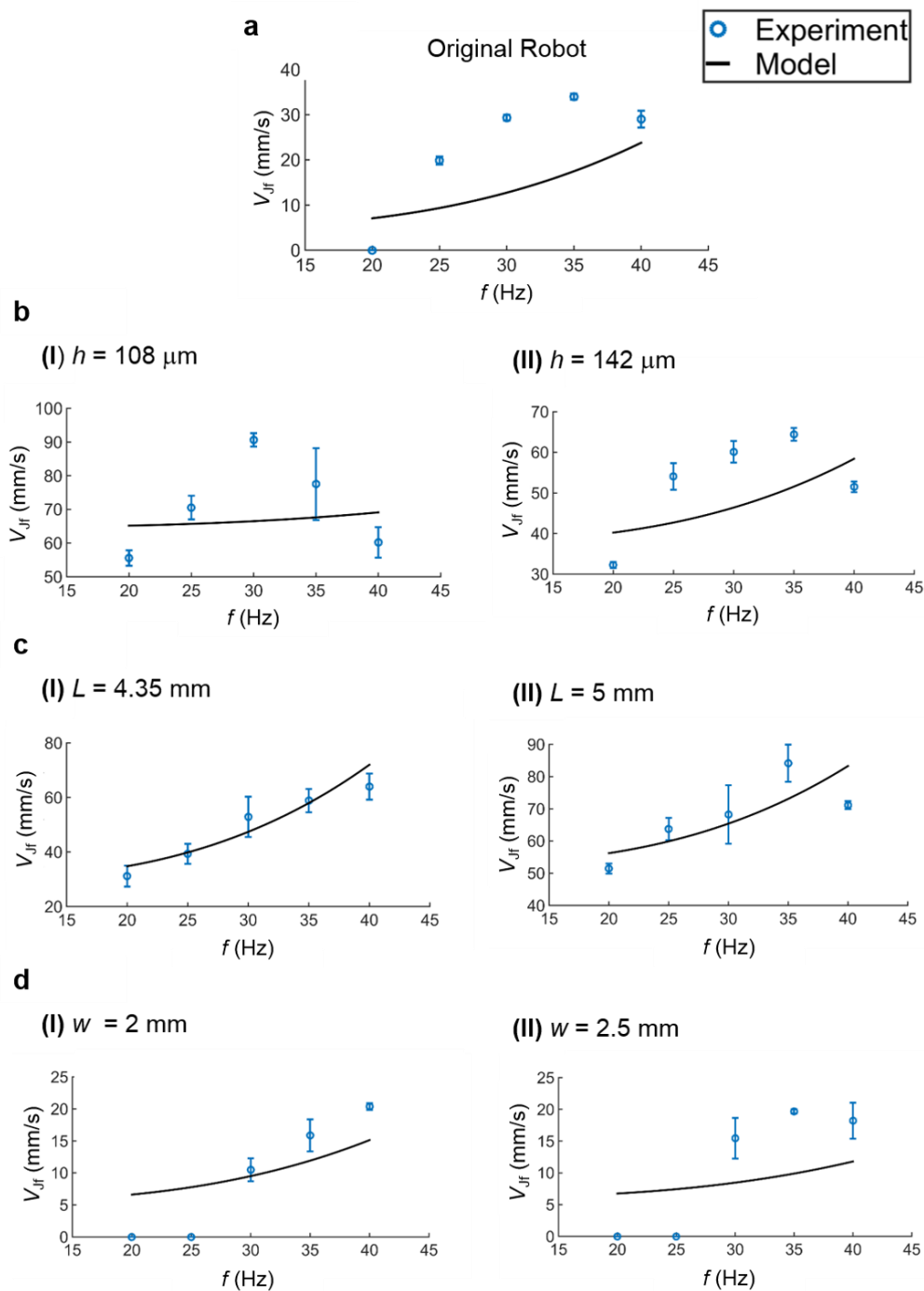


Fig. S40. Experimental and model-predicted results for the jellyfish-like swimming locomotion (V_{jf} as a function of f while B_{max} is fixed at 20 mT). (a) Results for the original robot (dimensions defined in Fig. 1a). (b) Results for robots that have a different h from the original robot. (c) Results for robots that have a different L from the original robot. (d) Results for robots that have a different w from the original robot. Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

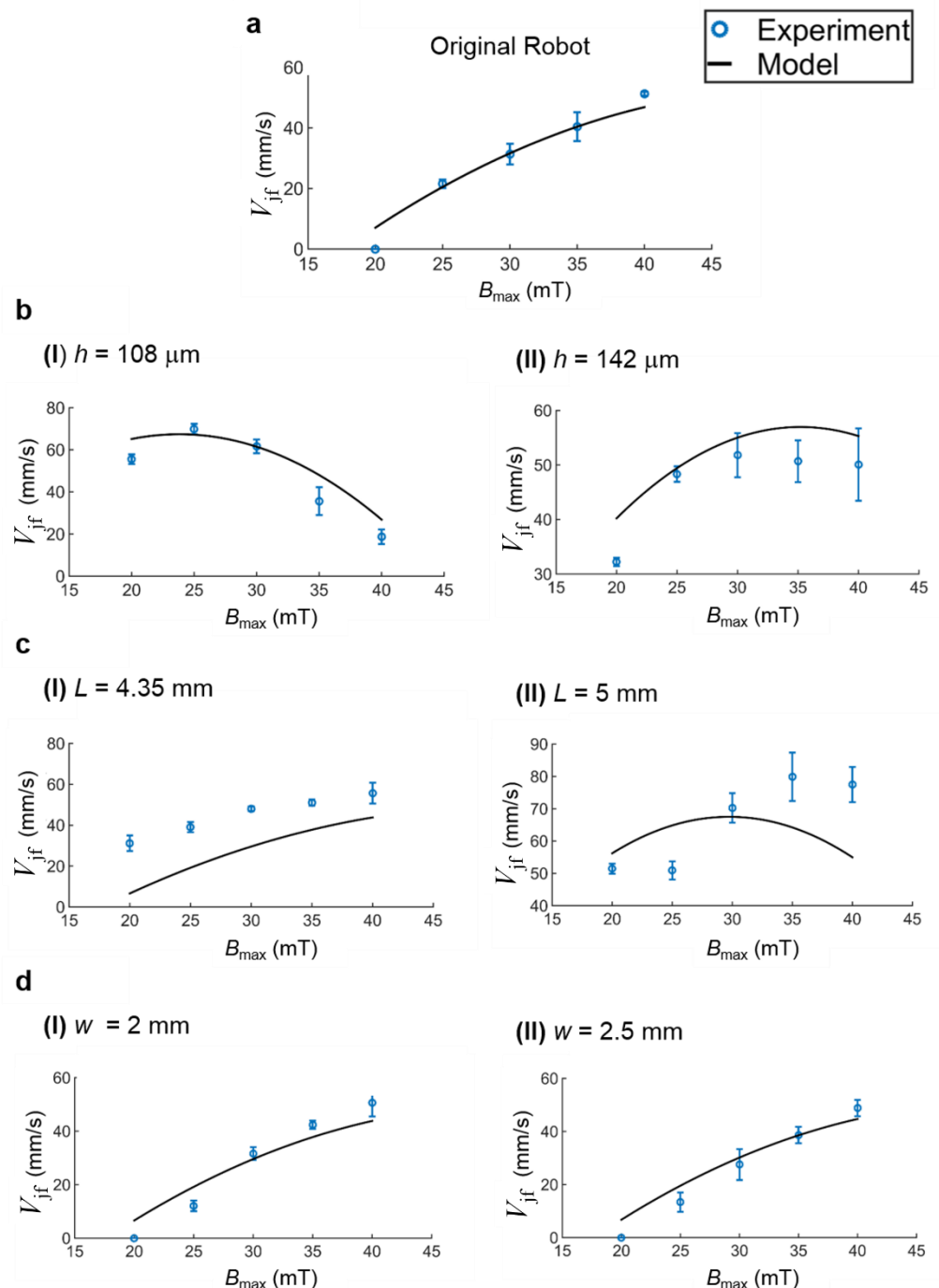


Fig. S41. Experimental and model-predicted results for the jellyfish-like swimming locomotion (V_{jf} as a function of $B_{\max}$ while f is fixed at 20 Hz). (a) Results for the original robot (dimensions defined in Fig. 1a). (b) Results for robots that have a different h from the original robot. (c) Results for robots that have a different L from the original robot. (d) Results for robots that have a different w from the original robot. Each experimental data point presents the mean and standard deviation out of three trials ($n = 3$).

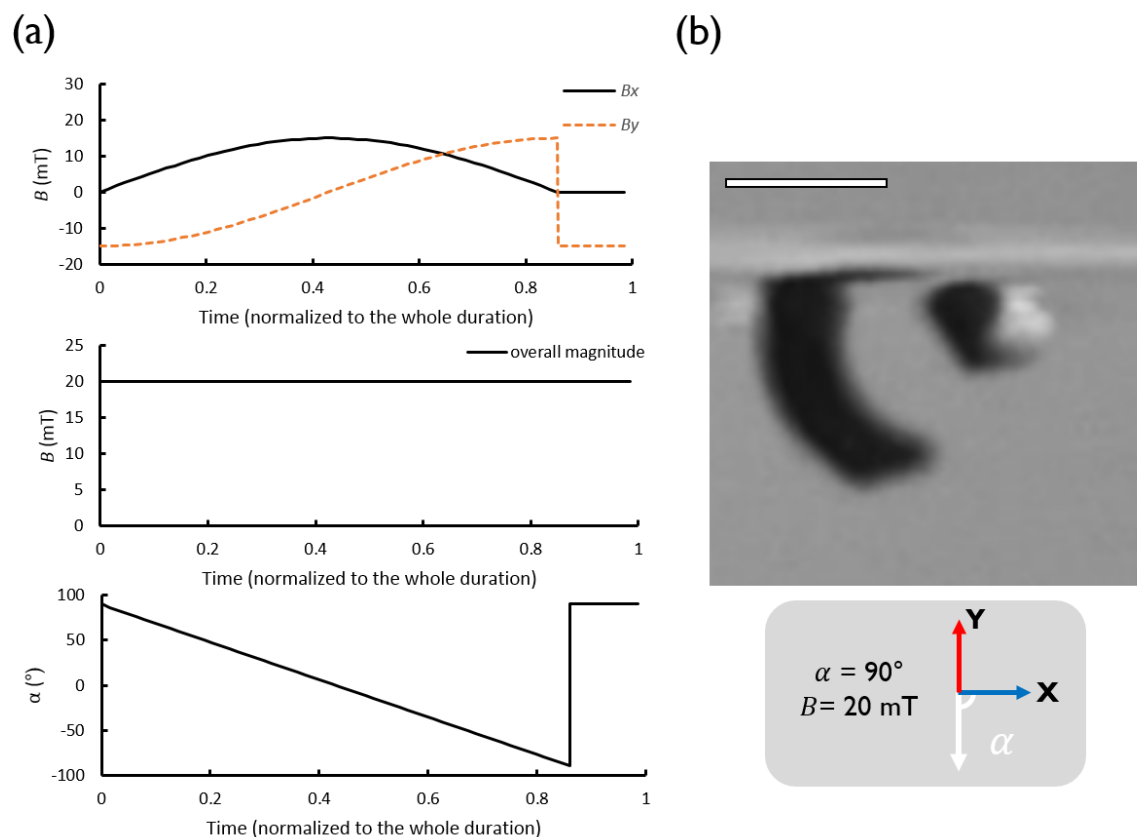


Fig. S42. The required B for the immersion locomotion. (a) Time sequence of B . B_x and B_y represent the magnitude of B projected along the X- and Y-axis of the global coordinate system. (b) The video snapshot shows the initial configuration of the robot for the immersion (reproduced from Fig. 2d). Scale bar: 1 mm.

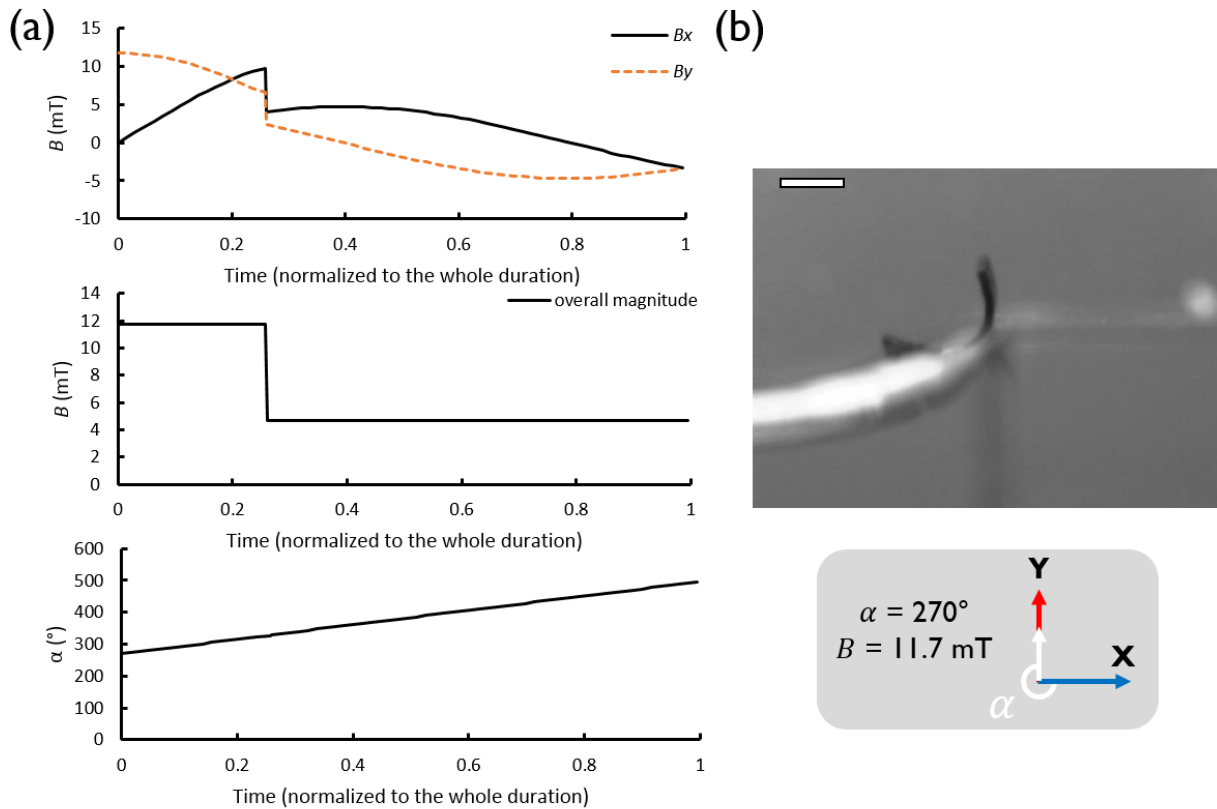


Fig. S43. The required B for the landing locomotion. (a) Time sequence of B . B_x and B_y represent the magnitude of B projected along the X- and Y-axis of the global coordinate system. (b) The video snapshot shows the initial configuration of the robot for the landing (reproduced from Fig. 2c).

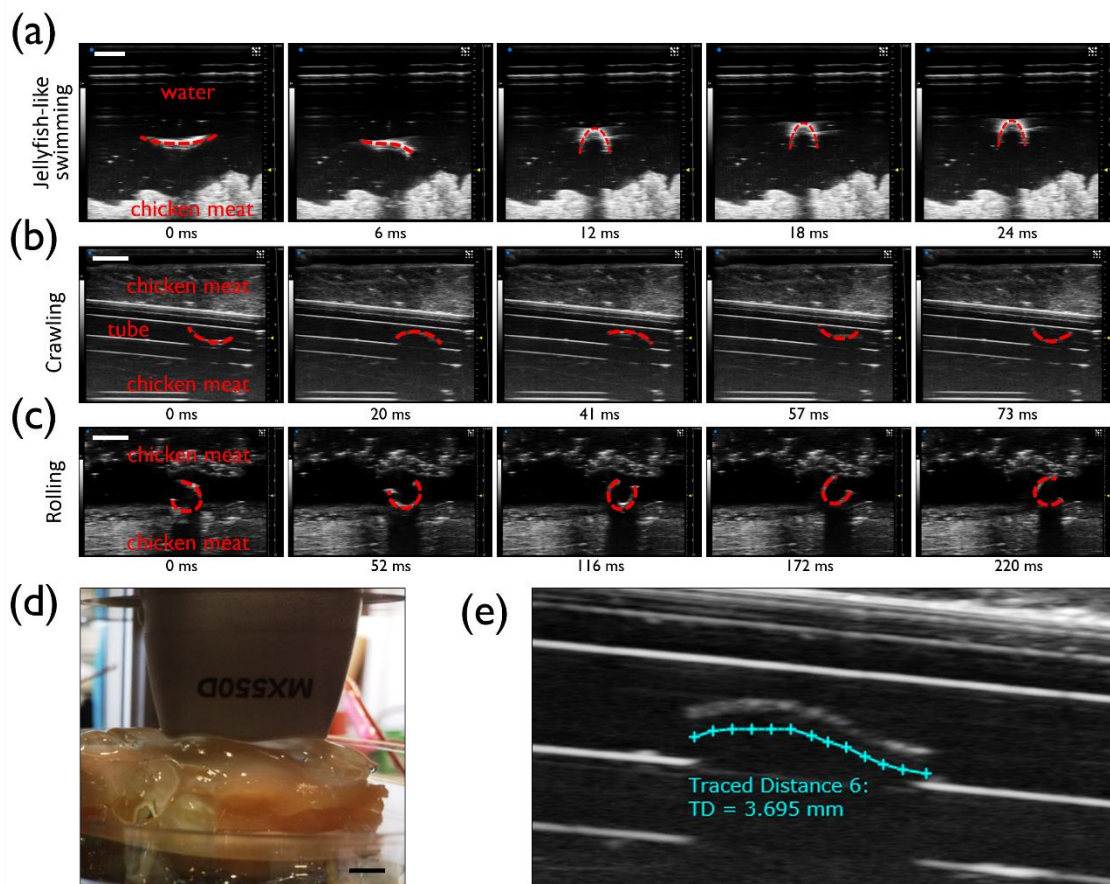


Fig. S44. Ultrasound-guided locomotion. Here, the dimensions of the robots are similar to the original robot (Fig. 1a), *i.e.*, $L = 3.7$ mm, $w = 1.5$ mm and $h = 0.185$ mm. The robot is marked out by red dotted line in a-c. (a) The robot with $\beta_R = -90^\circ$ uses the jellyfish-like swimming locomotion with the following parameters of $\mathbf{B}$ sequence: $f = 30$ Hz and $B_{\max} = 30$ mT. This experiment is conducted in a phantom where the bottom and top parts of the swimming reservoir were made by chicken muscle tissue. (b) The robot with $\beta_R = 45^\circ$ crawls by using the following parameters of rotating $\mathbf{B}$ sequence: $f = 10$ Hz and $B = 10$ mT. This experiment was performed in a silicone tube (inner diameter of 2 mm) buried inside the chicken muscle tissue (see (e) for an enlarged image). (c) The robot with $\beta_R = 45^\circ$ rolls with the following parameters of rotating $\mathbf{B}$ sequence: $f = 2$ Hz and $B = 15$ mT. These snapshots are from the same experiment trial as that in Fig. 4b. The rolling experiment is conducted in the gap created between two layers of chicken muscle tissue. (d) The ultrasound scanning head and the chicken muscle tissue phantom when they are placed in the electromagnetic setup shown in Fig. S2. (e) An enlarged image of the robot when it is within the buried tube. Scale bars are: 1 mm (a-c) and 1 cm (d).

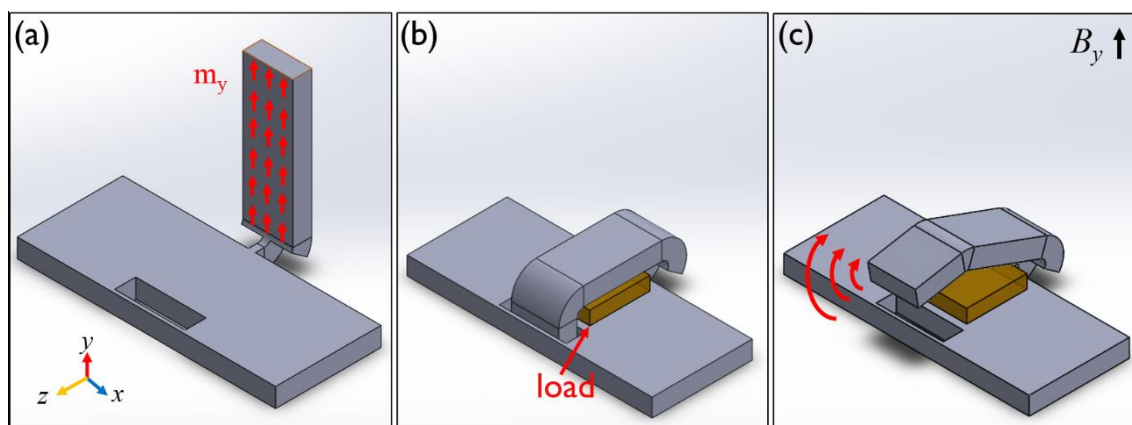


Fig. S45. Design of the modified robot for magnetically triggered cargo release (not to scale).

(a) The strap added to the robot. The y component of the strap's $\mathbf{m}$ is marked out. The magnetization of the robot's main body is the same as that the one shown in Fig. S1b. (b) The strap is locked into the robot body, securing the cargo at its bottom. (c) The strap can be opened by the B_y component of the external magnetic field.

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Supporting tables

General theory	
L	Length of the robot
w	Width of the robot
h	Thickness of the robot
t	Time
$\mathbf{B}$	Magnetic field
$\mathbf{B}_{xy}$	A vector representing the xy plane components of $\mathbf{B}$
B_i	Projection of $\mathbf{B}$ along the i -axis
B	Magnitude of $\mathbf{B}$
$B_{\max}$	Maximum magnetic field magnitude in the prescribed $\mathbf{B}$ sequence
α	Field angle of $\mathbf{B}$ defined in Fig. 1
β_R	Phase shift of the robot on the starting magnetization
$\mathbf{m}$	Magnetization profile
m_i	Projection of $\mathbf{m}$ along the i -axis
m	Magnitude of $\mathbf{m}(s)$
$\mathbf{M}_{\text{net}}$	Net magnetic moment
M_{net}	Magnitude of the net magnetic moment
M_b	Bending moment acting on the element
ω_s	Spatial angular frequency of $\mathbf{m}(s)$
A	Robot's cross-sectional area
I	Second moment of area
E	Young's modulus
y	Vertical deflection defined in Fig. S3
s	Coordinate along beam's length

τ_m	Actuating magnetic torque on infinitesimal element shown in Fig. S3
τ_{RB}	Magnetic torque that induces rigid-body motion
f	Frequency of the rotating $\mathbf{B}$
k_i	Constants of integration
Analysis for jumping	
W	Magnetic work done
ΔS	Change in strain energy
f_L	Frictional losses
ΔK	Change in kinetic energy
θ_f	Final angular deformation of the element before the robot leaves the substrate
θ_i	Initial angular deformation of the element of the robot
$H_{\max}$	Maximum jumping height
M_{Rob}	Mass of the robot
g	Gravitational constant
Analysis for rolling	
V_{roll}	Rolling speed
ϕ_R	Rotational displacement of the robot
$\dot{\phi}_{R\text{step-out}}$	Step-out frequency
τ_{Roll}	Rigid-body torque for rolling
F	Pushing force from the ground
C_T	Translational damping
C_R	Rotational damping
C_{eff}	Effective rotational damping
r_{eff}	Effective radius for rolling

J	Moment of inertia
J_{eff}	Effective moment of inertia
$B_{L,\text{roll}}$	B required to maintain a curvature similar to the original robot when L varies
$B_{h,\text{roll}}$	B required to maintain a curvature similar to the original robot when h varies
Analysis for walking	
V_{walk}	Walking speed
S_i	The distance between the ends of the robot as illustrated in Fig. S14
Analysis for meniscus climbing	
$H_i(x)$	Profile of water-air interfaces
ρ_r	Density of the robot
ρ_w	Density of water
θ_0	Water contact angle on the wall
θ_1, θ_2	Edge angles at the robot's ends
R, ϕ	Radius and sector angle of the circle fitting the deformed robot shape
α_m	Angular orientation of the robot
A_{cs}	Area of the circular segment
d_{cs}	Distance of the centroid of the circular segment from the center of curvature
F_B	Buoyancy force of the robot
D	Magnitude of the cosine shape
B_{min}	Minimum B required to make the robot to climb to the meniscus top
Analysis for undulating swimming	
V_{swim}	Speed of undulating swimming
C	Damping coefficient along the y-axis
v	Vertical shear force
$\omega_{n,R}$	The natural frequency of the R^{th} mode shape of the robot

R_1 and R_2	Asymptotic approximation of the Bode magnitude function of an ideal 2 nd order system for the 1 st and 2 nd mode-shapes
F_R	R^{th} mode shape of the robot
Analysis for crawling	
V_{crawl}	Speed of crawling locomotion
a, k_i	Fitting parameters
Analysis for jellyfish-like swimming	
V_{jf}	Speed of jellyfish-like swimming
k_i, a_i, b_i	Fitting parameters

Table S1. Nomenclature.

Physical properties (Original soft robot)	Value
Average diameter of NdFeB microparticles	5 μm
Average density of NdFeB microparticles	7.61 g/cm^3
Ecoflex-10 polymer matrix density	1.04 g/cm^3
Effective density of the robots	1.86 g/cm^3
Magnetization magnitude	$62,000 \pm 10000 \text{ A/m}$
Effective Young's modulus of the robots	$8.45 \times 10^4 \pm 2.5 \times 10^3 \text{ Pa}$
Contact angle	$116^\circ \pm 3^\circ$ (Static advancing) $78^\circ \pm 2^\circ$ (Static receding)
Capillary length L_c	2.7 mm
Surface roughness	$0.63 \pm 0.02 \mu\text{m}$ (R_a) $4.37 \pm 0.28 \mu\text{m}$ (R_z)

Table S2. Physical properties of the robot.

Fitting Parameters	Optimal Fitted Values
a_0	7.1968
a_1	6.744
a_2	8.6248
a_3	4.9297
a_4	11.7241
a_5	1.5778
a_6	-2.6213
b_0	-8.914
b_1	4.6897
b_2	28.960
b_3	5.7409
b_4	-11.0260
b_5	-10.1775
b_6	-2.4962
c_0	-2.3454
c_1	25.2537
c_2	8.6974
c_3	0.4930
k_0	-7.4319

k_1	6.3250
k_2	-242.4006
k_3	39.9464
k_4	39.9464
k_5	10.5661
k_6	16.3457

Table S3. Summary of the fitting parameters for the jellyfish-like swimming locomotion.

	Performance indicator	Best observable performance index	Suggested f (Hz)	L	h	w
Jumping	$H_{\max}$	12.2 mm	N/A	↑	↓	—
Rolling	V_{roll}	213 mm/s	(0, 4]	↑	—	—
Walking	V_{walk}	65.5 mm/s	[2, 11]	↑	↓	—
Meniscus climbing	$B_{\min}$	1.4 mT	N/A	↑	↓	N/A
Undulating swimming	V_{swim}	107 mm/s	[40, 160]	↑	↑ (Disagree with theory)	—
Crawling	V_{crawl}	15.4 mm/s	[20, 40]	—	—	—
Jellyfish-like swimming	V_{jf}	90.6 mm/s	[30, 40]	↑	↓	↓
I. ‘↑’, ‘↓’, ‘—’ are based on experimental results. ‘↑’ means the corresponding dimension should be increased to improve the specified locomotion. ‘↓’ means the corresponding dimension should be decreased to improve the specified locomotion. ‘—’ means that the corresponding dimension has no obvious correlation with the specified locomotion. II. The dimension h for the undulating swimming locomotion is marked grey as it disagrees with the theoretical prediction. III. We take the mean value for the data point representing the best observable performance index.						

Table S4. Scaling analyses summary. We provide the performance indicator, best observable performance index, operating range of f , and robot dimension effects for each locomotion mode. The suggested operating range of f is based on our tested robots.