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Contents

Model description

The mathematical description of I_K and I_{Na} are achieved through a Hodgkin-Huxley style derivation of forward and backward rate equations[1]:

$$I_K = g_K n^4 (V - E_K)$$

$$I_{Na} = g_{Na} m^3 h (V - E_{Na})$$

where g_K and g_{Na} are the conductance of fast K^+ and Na^+ channel; E_K and E_{Na} are the equilibrium potential for K^+ and Na^+. The gate variables m, h, and n are dimensionless activation and inactivation variables, which describe the activation and inactivation processes of the sodium and potassium channels, each of which is governed by the following differential equations:

$$dn / dt = \alpha_n(V) (1 - n) - \beta_n(V) n$$

$$dm / dt = \alpha_m(V) (1 - m) - \beta_m(V) m$$

$$dh / dt = \alpha_h(V) (1 - h) - \beta_h(V) h$$

where the forward and the backward rate α and β describe the transition between the closed and open state of gate. The function of α and β are given by:

$$\alpha_n(V) = 0.01 (V + 26) / (1 - \exp[(-V - 26) / 10])$$

$$\beta_n(V) = 0.36 \exp[(-V - 36) / 80]$$

$$\alpha_m(V) = 0.1 (V + 11) / (1 - \exp[(-V - 11) / 10])$$

$$\beta_m(V) = 4 \exp[(-V - 36) / 18]$$

$$\alpha_h(V) = 0.07 \exp[(-V - 36) / 20]$$

$$\beta_h(V) = 1 / (\exp[(-V - 6) / 10] + 1)$$

The kinetics of activation and inactivation of I_T are modified from a previous model[2]:

$$I_T = g_T {m_T}^2 h_T (V - E_{Ca})$$

$$dm / dt = (m_\infty(V) - m) / \tau_m(V)$$

$$dh / dt = (h_\infty(V) - h) / \tau_h(V)$$

where g_T is the conductance of the T-type Ca^{2+} current, E_{Ca} is the equilibrium potential for Ca^{2+}. The steady-state activation or inactivation variables are described by:

$$m_\infty(V) = 1 / (1 + \exp[-(V + 52) / 7.4])$$

$$h_\infty(V) = 1 / (1 + \exp[(V + 80) / 5])$$

The time constant τ of activation or inactivation are described by:

$$\tau_m(V) = 1 + 0.33 / (\exp[(V + 27) / 10] + \exp[(V + 102) / -15])$$

$$\tau_h(V) = 28.3 + 0.33 / (\exp[(V + 48) / 4] + \exp[(V + 407) / -50])$$

The K^+ and Na^+ leak current are described by:

$$I_{Kleak} = g_{Kleak} (V - E_K)$$

$$I_{Naleak} = g_{Naleak} (V - E_{Na})$$

g_{Kleak} is set to 0.101 and g_{Naleak} is set to 0.024, giving a resting membrane potential of -56 mV and an input resistance of 800 MΩ.

A pulse of transmitter is released when a presynaptic spike occur. The inter-release intervals are fitted by exponential distribution with time constant 250 ms[3].

Synaptic currents are modeled by a kinetic scheme of transmitter binding to postsynaptic receptors[4].

$$dr / dt = \alpha T (1 - r) - \beta r$$

where T is the concentration of transmitter, α and β are the forward and the backward rate of the transmitter binding.

The AMPA current is given by:

$$I_{AMPA} = g_{AMPA}\, O\, (V - E_{AMPA})$$

where g_{AMPA} is the maximal conductance, [O] is the fraction of receptors in the open state and $E_{AMPA} = 0$ mV is the reversal potential. The rate constant were $\alpha = 1.1\ mM^{-1}\cdot ms^{-1}$ and $\beta = 0.19\ ms^{-1}$.

The NMDA current is described by:

$$I_{NMDA} = g_{NMDA}\, B(V)\, O\, (V - E_{NMDA})$$

where g_{NMDA} is the maximal conductance, [O] is the fraction of receptors in the open state and $E_{NMDA} = 0$ mV is the reversal potential. The rate constant were $\alpha = 0.072\ mM^{-1}\cdot ms^{-1}$ and $\beta = 0.0066\ ms^{-1}$. The factor B(V) describes an extra voltage dependence due to the magnesium block, which could be fitted as:

$$B = 1 / (1 + 0.35\, Mg\, \exp(-0.062\, V))$$

where Mg is the external magnesium concentration.

The GABA current is described by:

$$I_{GABA} = g_{GABA}\, O\, (V - E_{GABA})$$

where g_{GABA} is the maximal conductance, [O] is the fraction of receptors in the open state and $E_{GABA} = -80$ mV is the reversal potential. The rate constant were $\alpha = 5\ mM^{-1}\cdot ms^{-1}$ and $\beta = 0.19\ ms^{-1}$.

Simulations were constructed in Matlab 9.0 using Euler method with a time step (dt) of 0.01 ms. The parameters used in the model are presented as aforementioned. The burst probability was evaluated across ten independent trials with different synaptic input.

Reference

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Model parameters

	Value	Unit	Parameter
R	8.31	$J \cdot mol^{-1} \cdot K^{-1}$	Gas constant
T	300	K	Absolute temperature
F	96485	$C \cdot mol^{-1}$	Faraday constant
dt	0.01	ms	Step time
c	1	$\mu F \cdot cm^{2}$	Membrane capacitance
S	1000	μm^{2}	Surface Area
g_K	36	$mS \cdot cm^{-2}$	Delayed rectifier K^{+} channel conductance
g_{Na}	120	$mS \cdot cm^{-2}$	Transient Na^{+} channel conductance
g_{Kleak}	0.106	$mS \cdot cm^{-2}$	K^{+} leak channel conductance
g_{Naleak}	0.019	$mS \cdot cm^{-2}$	Na^{+} leak channel conductance
g_T	2.8	$mS \cdot cm^{-2}$	T-type Ca^{2+} channel conductance
g_{NMDA}	0.61	$mS \cdot cm^{-2}$	NMDA Receptor Conductance
g_{AMPA}	0.008	$mS \cdot cm^{-2}$	AMPA Receptor Conductance
g_{GABA}	0.1	$mS \cdot cm^{-2}$	GABA Receptor Conductance
Mg	0.8	mM	Extracellular Mg^{2+} concentration
E_k	-90	mV	K^{+} Equilibrium Potential
E_{Na}	50	mV	Na^{+} Equilibrium Potential
E_{Ca}	120	mV	Ca^{+} Equilibrium Potential
α_{AMPA}	1.1	$mM^{-1} \cdot ms^{-1}$	forward rate of the glutamate binding to the AMPA receptor
α_{NMDA}	0.072	$mM^{-1} \cdot ms^{-1}$	forward rate of the glutamate binding to the NMDA receptor
α_{GABA}	5	$mM^{-1} \cdot ms^{-1}$	forward rate of the GABA binding to the GABA receptor
β_{AMPA}	0.19	ms^{-1}	backward rate of the glutamate binding to the AMPA receptor
β_{NMDA}	0.0066	ms^{-1}	backward rate of the glutamate binding to the NMDA receptor
β_{GABA}	0.19	ms^{-1}	backward rate of the GAGA binding to the GABA receptor
E_{AMPA}	0	mV	reversal potential of AMPA receptor
E_{NMDA}	0	mV	reversal potential of NMDA receptor
E_{GABA}	-80	mV	reversal potential of GABA receptor