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Accretion of Phobos and Deimos in an extended debris disc stirred by transient moons

Pascal Rosenblatt*
Royal Observatory of Belgium,
Av. circulaire 3, B-1180 Uccle, Belgium

Sébastien Charnoz
Institut de Physique du Globe, Université Paris-Diderot, CEA, CNRS, 91191 Gif sur Yvette, France

Kevin M. Dunseath
Institut de Physique de Rennes
UMR6251 CNRS-Université de Rennes 1
Campus de Beaulieu
F-35042 Rennes Cedex, France

Mariko Terao-Dunseath
Institut de Physique de Rennes
UMR6251 CNRS-Université de Rennes 1
Campus de Beaulieu
F-35042 Rennes Cedex, France

Antony Trinh
Royal Observatory of Belgium,
Av. circulaire 3, B-1180 Uccle, Belgium

Ryuki Hyodo
Institut de Physique du Globe, Paris 75005, France. Department of Planetology, Kobe University, Kobe 657-8501, Japan

Hidenori Genda
Earth-Life Science Institute, Tokyo Institute of Technology; 2-12-1 Ookayama, Meguro-ku, Tokyo, Japan

Steven Toupin
Royal Observatory of Belgium,
Av. circulaire 3, B-1180 Uccle, Belgium
Now at: Institut d'Astronomie et d'Astrophysique, Université libre de Bruxelles, CP 226, Boulevard du Triomphe, B-1050 Brussels, Belgium

(*) To whom correspondence should be addressed (rosenb@oma.be)

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SI 1: Distribution of satellite-embryos in the outer disc

We have tested different satellite-embryo distributions in the outer disc by varying its inner edge from 4.2 to 5.6 Mars radii and taking its outer edge at 7 Mars radii. This upper limit is constrained by the absence today of substantial bodies beyond it as they would not have been affected by the MMRs and tidal effects are too weak to displace them over the last 4 Gyr. The number of embryos is between 30 and 112 (Fig. SI1). Embryos at closer distances from Mars are not taken into account since they are expected to be accreted by the inner moon during its migration.

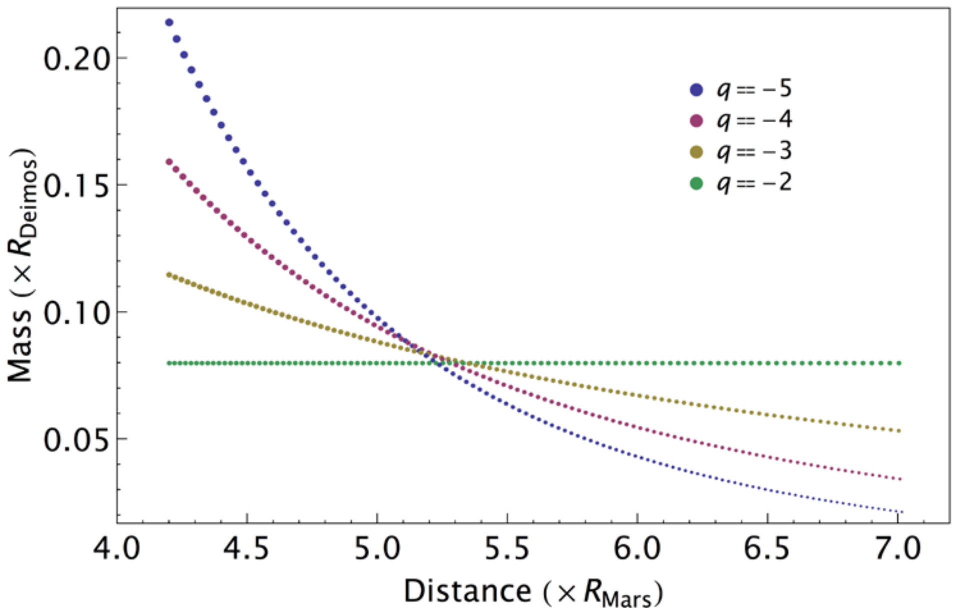

Figure SI1: Distributions of embryo mass used for simulating accretion processes in the circum-Martian outer disc. The size of each solid circle is proportional to each embryo mass.

SI 2: Embryo interactions and accretions in the outer disc

Analysis of impact velocities

Collisions between satellite-embryos in the outer disc are treated as inelastic through the use of a coefficient of restitution. This approach neglects the possibility of shattering one or both the colliding partners and the eventual dispersion of the resulting fragments. If the specific impact energy Q is not too far above the critical energy for catastrophic disruption Q^*, any fragments formed will slowly separate and reassemble relatively quickly. This assumption is inherent in our treatment.
Quantitatively, we have estimated the ratio of the specific impact energy Q to the critical energy for catastrophic disruption Q^* using the Leinhardt and Stewart formalism[1]. For bodies similar in size to Phobos, colliding at about 5 Mars radii, the impact velocity necessary for catastrophic disruption is of the order of 25 to 30 m/s. In our simulations, 87% of the collisions occur with impact velocities below 30 m/s, leaving 13% potentially disruptive (Fig. SI2). Almost 97% of impacts occur with velocities less than 60 m/s, while the highest impact velocity reached was

approximately 400 m/s. Clearly some disruptive collisions should occur, but our statistics suggest that in most cases our assumption that the fragments will reassemble relatively quickly is reasonable. Nevertheless, this question is one that needs to be addressed in further work.

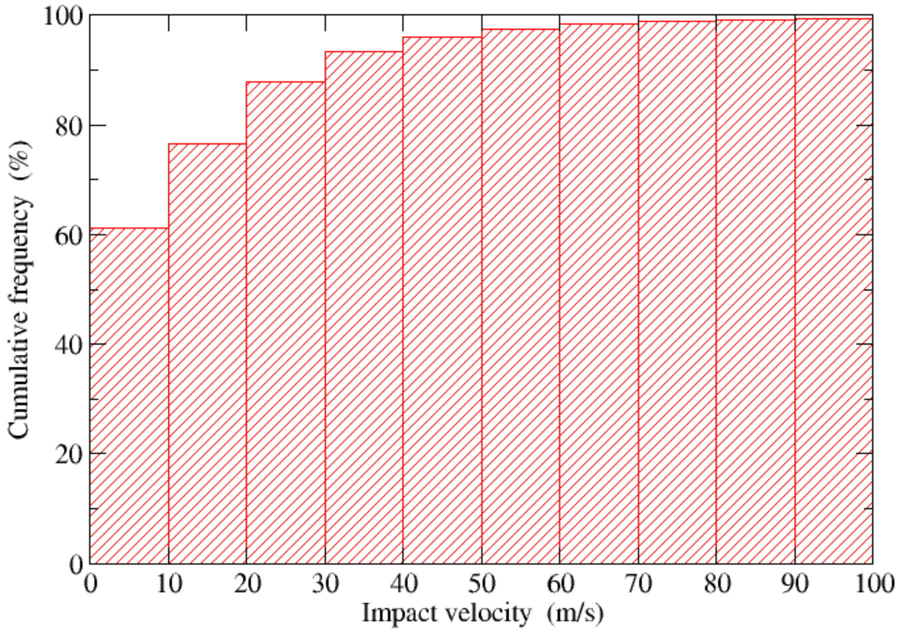

Figure SI2: Cumulative histogram showing the distribution of impact velocities occurring during the numerical simulation of encounters in the outer disc. The critical energy Q^* corresponds to impact velocities of the order of 25 to 30 m/s.

Results

A total of 288 numerical simulations over 100 kyr have been performed. Different initial distributions of embryos have been tested (Supplementary Information Section 1). The other orbital parameters of the embryos were generated randomly with initial eccentricities between 0 and 0.001 and inclinations between 0 and 0.001 radians[12]. We have considered one inner moon with final mass of 2.29×10^{19} kg which rises to 4.35 Mars radii in around 8 kyr. In 108 cases (just over 37%), the simulations produce two outer satellites. Of these, 91 (31.6%) yield a favourable outcome where the mass of the innermost satellite is greater than that of the outermost satellite.
The results are summarized in Fig. SI3, which shows the distribution of mass as a function of the semi-major axis a with bars corresponding to $a(1 \pm e)$, i.e. the closest and furthest distances from Mars for each satellite. The size of the bars thus gives an indication of the eccentricities of their orbits.

After an inelastic collision not resulting in accretion, there is a tendency for the two bodies to collide again after only a few orbits. As a result, accretion can occur after two or three collisions involving the same bodies. This also suggests that fragments

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created in a disruptive collision could recombine relatively quickly if they remain in similar orbits.

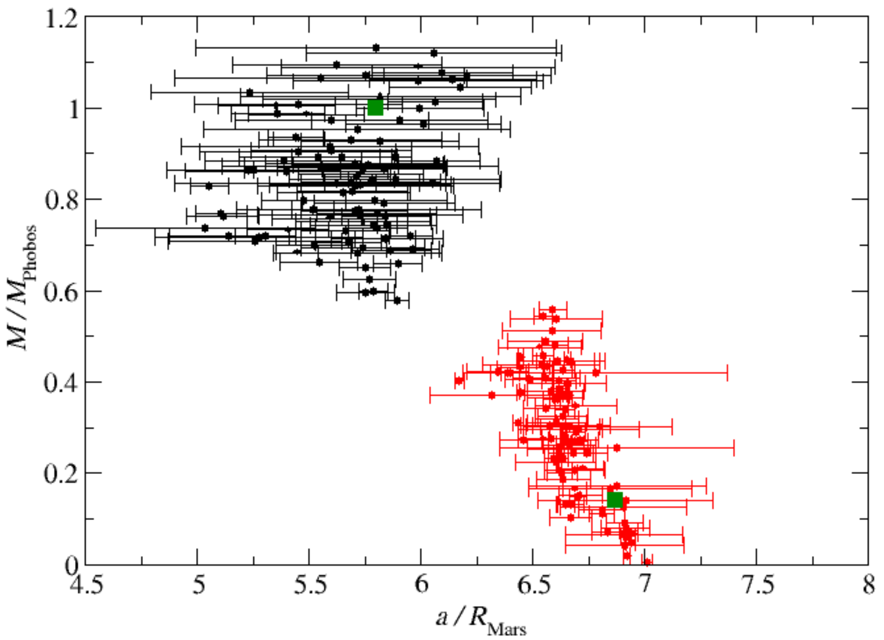

Figure SI3: Mass and distance from Mars of the two satellites obtained in 34.5% of the 288 runs performed by varying the initial conditions of the system, for restitution coefficients ε_n=0.1, ε_t=0.1. The horizontal bar represents the minimum and maximum distances from Mars of the satellite and gives an indication of its orbital eccentricity. Green squares represent the masses of Phobos and Deimos at their expected positions about 4 Gyr ago (Supplementary Information, Section 4).

The results show that it is possible to obtain a more massive satellite at about 5.7 Mars radii with mass similar to that of Phobos and a less massive one close to 6.7 Mars radii with a mass similar to that of Deimos. Eccentricities however tend to be somewhat large although occasionally they can be much smaller. The statistical distributions of eccentricities are shown in Fig. SI4.

We have performed additional simulations with larger initial eccentricities (up to 0.2) and inclinations (up to 10°), reflecting a more excited initial outer disc than assumed in the main text. These give even better percentages of favourable outcomes (between 36% and 50%) compared to the results presented in the main text. The final eccentricities of the outermost satellite are however slightly larger, on average close to 0.05, which makes it more difficult to reach Deimos' current circular orbit by tidal damping. One further caveat is that in a more excited disc, impact velocities tend to be larger, so that disruptive collisions, neglected in our approach, may have a more significant role. It is however arguable (Supplementary Information, Section 3) that the particles in the disc should have orbits with low eccentricity and inclination, as assumed in the main text, so that these problems will not be a concern.

We have also considered heavier ($3x10^{19}$ kg) and lighter (less than 10^{19} kg) inner moons. The former tends to over-excite the orbits of the embryos so that the final accreted bodies have eccentricities that are too large (up to 0.3). The latter tends to leave more than two bodies since the MMRs are less efficient in favouring accretion amongst the satellite-embryos in the outer disc: the lighter moons spawned by the inner disc migrate more slowly and not sufficiently far (less than 4 Mars radii). The satellite-embryos thus accrete mainly by their mutual interaction, leaving most often 4 or 5 small satellites in the outer disc after 100 kyr. Note that we extended a number of runs up to 500 kyr and observed very few late accretions, which do not significantly modify the statistics.

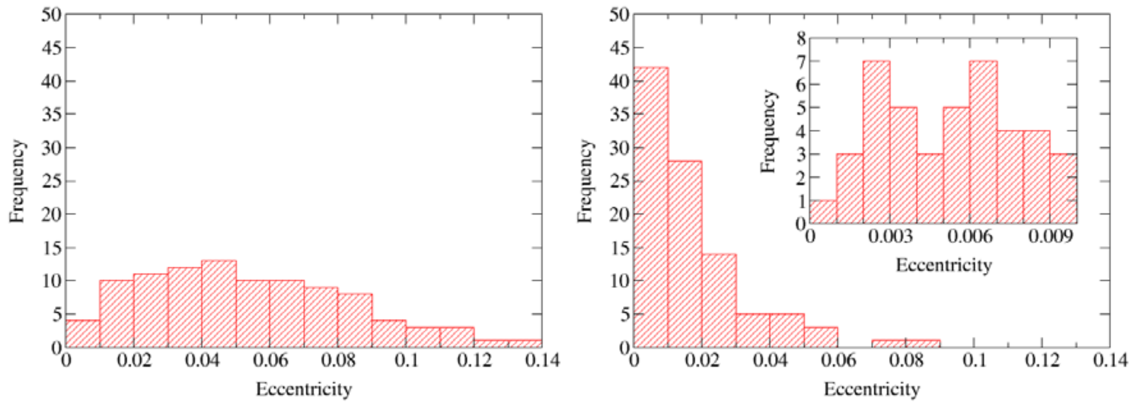

Figure SI4: Histograms showing the distribution of the eccentricities of the two satellites formed in the outer disc, corresponding to 34.5% of the 288 runs with this inner moon. Left and right plots are for the innermost and outermost satellites, respectively.

SI 3: Collision rate and time for disc flattening

In order for a disc to flatten, its constituent particles need to undergo several collisions which damp random velocities while keeping constant their total angular momentum. The collision time is given by $T_c=(\tau\Omega)^{-1}$, where Ω is the total Keplerian frequency and $\tau= N_p \pi r_p^2/S$ the local optical depth, with N_p and r_p the total number of particles and the particle radius respectively and S the surface of the disc. The value of T_c must therefore be much smaller than the time needed to form the largest inner moon, which is about 100 years (see main text). This implies $\tau >> 5\ 10^{-6}$ (at 6 Mars' radii), which is very low and comparable to some faint rings found around Jupiter, for example. Hence even a very faint ring will have enough time to flatten. Assuming that the total outer disc mass is similar to that of Phobos and Deimos combined, and that the average particle density is 2 g/cm^3, the average size of particles in the disc must be much less than a few hundred meters (Fig. SI5). While this appears reasonable in a giant impact, where rocks can be vaporized, the actual size of fragments is not possible to determine in the current SPH simulations due to the lack of resolution. Furthermore, the size distribution of the particles is also neglected. It is therefore arguable that the disc can flatten over a short period of time as the process does not

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require unrealistically small debris. A more definitive answer however would require SPH simulations with several billion particles, which are beyond the computational resources currently available.

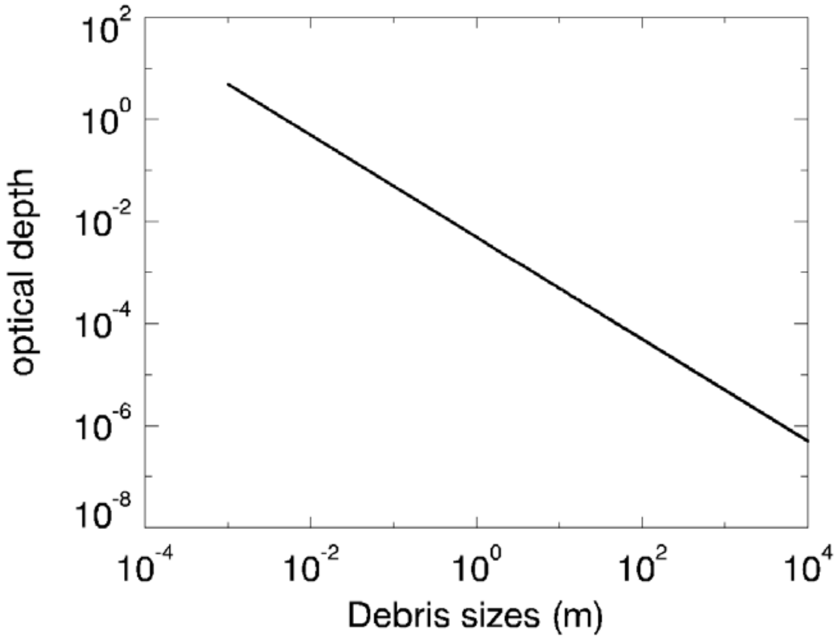

Figure SI5: Optical depth of the outer disc as a function of the average particle size.

SI 4: Tidal orbital evolution of Phobos and Deimos-like satellites

The tidal dissipation rate is given by the ratio k_2/Q, where k_2 is the body's potential Love number and Q its tidal quality factor. The current values for Mars are $k_2=0.164 \pm 0.009$[2] and $Q=82.8 \pm 0.2$[3]. We explored different values of these numbers for the small satellite, finding a range of values from 10^{-5} to about 10^{-4}, which allow the current orbit of Deimos to be reached by tidal damping, i.e. semi-major axis between 6.853 and 6.872 Mars' radii, eccentricity between 0.00037 and 0.045 (Fig. SI6), consistent with the range obtained in our accretion model (Fig. SI3 and SI4). Much less dissipation ($k_2/Q \leq 6\text{x}10^{-7}$) is needed over 4 Gyr to damp the orbit of the Phobos-like satellite to the current orbit of Phobos, since the eccentricity of Phobos (0.0151)[3] is much larger than that of Deimos (0.00024)[3]. The tidal dissipation ratio in our Phobos-like satellite is however three orders of magnitude smaller than in our Deimos-like satellite.

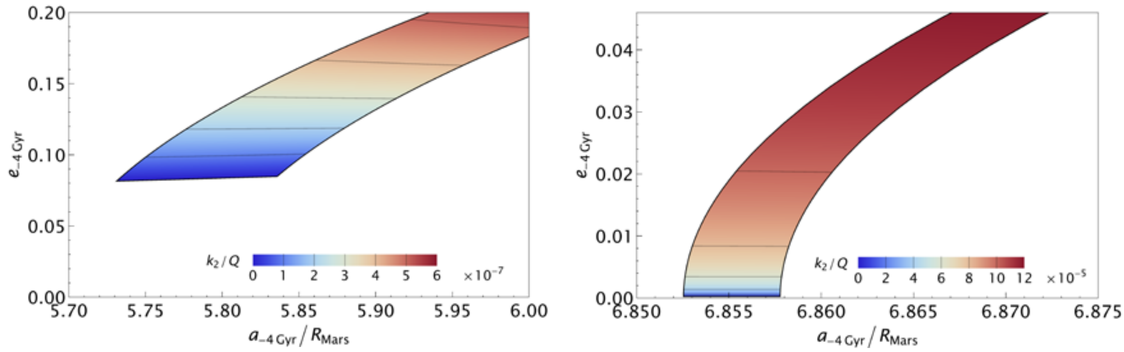

Figure SI6: Range of initial semi-major axis and eccentricity of the orbit of an outer satellite with same size and mass as Phobos (left) and Deimos (right) that can reach the values of the present orbit of the Martian satellites after 4 Gyr of tidal orbital evolution. We have used the current k_2/Q ratio for Mars within its error bar.

The eccentricity of Phobos' orbit can however be increased by resonances with the orbital and rotational motion of Mars over the last 1 Gyr[15]. In this scenario, Phobos' orbital eccentricity would have been close to zero at 3.8 Mars radii 1 Gyr ago, which corresponds to about 3 Gyr after the formation of the small satellites in the outer disc. We have thus recomputed the tidal evolution of a Phobos-like satellite in order to reach this circularized orbit in 3 Gyr, obtaining initial distances between 5.56 and 6 Mars radii and eccentricities between 0 and 0.1, which is consistent with the orbits of the satellites formed in the outer disc (Fig. SI3 and SI4). Nevertheless, the k_2/Q value is larger than 10^{-5}, which is not consistent with the k_2/Q value of the resonance model of Phobos' orbital evolution ($k_2/Q \leq 10^{-6}$)[15].

The k_2 value for Phobos has been estimated at about 10^{-4} assuming a value for the rigidity of a carbonaceous chondritic analog material ($k_2 \sim \rho g R/\mu$, where ρ, g and R are the density, the gravity and the radius of the body, respectively, and μ the rigidity of its material)[4]. The k_2/Q range used in this study implies a Q value larger than 1,000 for the satellite with the size and mass of Phobos. The k_2/Q range for the case of a Deimos-like satellite reduces Q to 2 (or less), which is too small for rocky bodies whose Q values are expected to be larger than 100[15].

Small bodies formed by accretion of smaller debris are however expected to have a significant amount of porosity in their interior (up to 30% of the volume)[16]. The rigidity of such a highly porous small body can be reduced by a factor 10 with respect to a monolithic rocky body[5], hence increasing the Love number k_2 by the same factor. Therefore, the range of values for Q, corresponding to the k_2/Q ratios of this study, could be about 20 for a Deimos-like satellite and larger than 10,000 for a Phobos-like satellite. These higher values of Q for a Deimos-like satellite are closer to those expected for rocky materials, although still low. However, small bodies may also have a lower tidal quality factor than the material they are made of[5], again increasing the k_2/Q factor toward the range of values found in our study. In addition, the value of Q for Mars might have been lower during earlier stages of its evolution, thus requiring less dissipation in the outer satellites (and so larger Q values) to produce the same tidal orbital changes. The behaviour of the tidal quality factor Q over Mars' history and for a porous satellite has still to be explored, especially at the scale of the tidal periods (a few hours). In addition, the material of Phobos and Deimos has not yet been clearly identified and hence its rigidity is not very well constrained.

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SUPPLEMENTARY INFORMATION REFERENCES

1. Leinhardt Z.M. & Stewart S.T. Full numerical simulations of catastrophic small body collisions. *Icarus* **199**, pp. 542-559 (2009).
2. Konopliv, A.S. et al. Mars high resolution gravity fields from MRO, Mars seasonal gravity, and other dynamical parameters. *Icarus* **211**, 401-428 (2011).
3. Jacobson, R.A. The orbits and masses of the Martian satellites and the libration of Phobos. *Astron. J.* **139,** 668-679 (2010).
4. Lambeck, K. On the orbital evolution of the Martian satellites. *J. Geophys. Res* **84,** 5651-5658 (1979).
5. Goldreich, P. & Sari, R. Tidal evolution of rubble piles. *Astrophys. J.* **691,** 54-60 (2009).