nature physics

SUPPLEMENTARY INFORMATION

DOI: 10.1038/NPHYS1329

Coherent manipulation of Bose-Einstein condensates with state-dependent microwave potentials on an atom chip: Supplementary Information

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(Dated: May 18, 2009)

Simulation of microwave dressed-state potentials

Here we describe our simulation of the trapping potential [1], which extends the theory of [2] to multi-level atoms. Consider a ^{87}Rb atom in the $5S_{1/2}$ ground state at a fixed position $\mathbf{r}$. The atom interacts with the local static magnetic field $\mathbf{B}$ and the microwave magnetic field $\mathbf{B}_{\rm mw}(t) = \frac{1}{2}[\mathbf{B}_{\rm mw}e^{-i\omega t}+\mathbf{B}^*_{\rm mw}e^{i\omega t}]$. The atomic hyperfine states are described by the Hamiltonian

$$H = (\hbar\omega_{\rm hfs}/2)\mathbf{I}\cdot\mathbf{J} + \mu_B\left(g_J\mathbf{J} + g_I\mathbf{I}\right)\cdot\left(\mathbf{B} + \mathbf{B}_{\rm mw}(t)\right).$$

The first term describes the hyperfine coupling between electron spin $\mathbf{J}$ and nuclear spin $\mathbf{I}$ ($J = 1/2$, $I = 3/2$, and $\omega_{\rm hfs}/2\pi = 6.834\,682\,611$ GHz). The second term is the coupling to the static and microwave magnetic fields, with $g_J = 2.002\,331$ ($g_I = -0.000\,995$) the electron (nuclear) Landé g-factor. If high accuracy of the simulation is required, e.g. for comparison with spectroscopic measurements, we numerically determine the eigenstates and -energies of the full Hamiltonian H. However, in most cases, a number of approximations can be made: (1) We neglect the coupling of $\mathbf{I}$ to the external fields because $g_I \ll g_J$. (2) We treat the coupling of $\mathbf{J}$ to $\mathbf{B}$ perturbatively because $\mu_B B \ll \hbar\omega_{\rm hfs}$. (3) We transfer to a frame rotating at frequency ω and make the rotating-wave approximation, valid because $\mu_B B_{\rm mw}, \hbar|\Delta_0| \ll \hbar\omega$, where $\Delta_0 = \omega - \omega_{\rm hfs}$. With these approximations, we express H in the basis $|F, m_F\rangle$ of the eigenstates of the hyperfine spin $\mathbf{F} = \mathbf{J} + \mathbf{I}$, with the quantization axis chosen along the local direction of $\mathbf{B}$:

$$\begin{aligned} H \approx &\sum_{m_2}\left(-\tfrac{1}{2}\hbar\Delta_0 + \hbar\omega_L m_2\right)|2,m_2\rangle\langle 2,m_2| \\ &+\sum_{m_1}\left(\tfrac{1}{2}\hbar\Delta_0 - \hbar\omega_L m_1\right)|1,m_1\rangle\langle 1,m_1| \\ &+\sum_{m_1,m_2}\left[\tfrac{1}{2}\hbar\Omega^{2,m_2}_{1,m_1}|2,m_2\rangle\langle 1,m_1| + \text{c.c.}\right]. \end{aligned} \tag{1}$$

We have approximated $g_J \approx 2$ and introduced the Larmor frequency $\omega_L = \mu_B B/2\hbar$. The microwave couples the transition $|1,m_1\rangle \leftrightarrow |2,m_2\rangle$ with Rabi frequency

$$\Omega^{2,m_2}_{1,m_1} = (2\mu_B/\hbar)\langle 2,m_2|\mathbf{B}_{\rm mw}\cdot\mathbf{J}|1,m_1\rangle$$

and detuning

$$\Delta^{2,m_2}_{1,m_1} = \Delta_0 - (m_2+m_1)\omega_L.$$

We numerically diagonalize H in equation (1) to determine the local dressed states $|\bar{n}\rangle$ and dressed state energies $E(n)$ of the atom at position $\mathbf{r}$, where $n = 0\ldots7$ is a suitable labelling of the states. For position-dependent fields $\mathbf{B}(\mathbf{r})$ and $\mathbf{B}_{\rm mw}(\mathbf{r},t)$, an energy landscape $E(n,\mathbf{r})$ is calculated in this way. In our experiment, the microwave dressing field is always turned on adiabatically with respect to the internal-state dynamics. The bare qubit states $|0\rangle$ and $|1\rangle$ smoothly transform into the dressed states $|\bar{0}\rangle$ and $|\bar{1}\rangle$ with potentials $V_{|\bar{0}\rangle}(\mathbf{r}) = E(0,\mathbf{r}) - \frac{1}{2}\hbar\Delta_0$ and $V_{|\bar{1}\rangle}(\mathbf{r}) = E(1,\mathbf{r}) + \frac{1}{2}\hbar\Delta_0$, respectively. If the motion of the atomic wave function in these potentials is sufficiently slow, as in our experiments, the internal state follows the spatially varying fields adiabatically, and motion-induced transitions between different states $|\bar{n}\rangle$ are suppressed.

If the microwave is far detuned from all transitions, $|\Delta^{2,m_2}_{1,m_1}|^2 \gg |\Omega^{2,m_2}_{1,m_1}|^2$, the microwave coupling can be treated perturbatively. Each dressed state $|\bar{n}\rangle$ can then be identified with one of the bare states $|F,m_F\rangle$, and each transition connecting to $|F,m_F\rangle$ contributes a potential $\pm\hbar|\Omega^{2,m_2}_{1,m_1}|^2/4\Delta^{2,m_2}_{1,m_1}$, where the $+$ ($-$) sign is for $F = 1$ ($F = 2$) [3]. This is the regime relevant to our experiment. In the main text of our paper, $\Omega_R \equiv \Omega^{2,-1}_{1,-1}$ and $\Delta \equiv \Delta^{2,-1}_{1,-1}$. Figure 1 shows the spatial dependence of $|\Omega_R(\mathbf{r})|$, calculated for an ideal CPW mode.

In addition to $V_{|\bar{n}\rangle}(\mathbf{r})$, our simulation includes several contributions to the potential which are identical for all states of the ground state hyperfine manifold: the quasi-electrostatic potential $V_e(\mathbf{r}) = -(\alpha_0/4)|\mathbf{E}(\mathbf{r})|^2$ due to the electric field of the microwave [4], where $\mathbf{E}(\mathbf{r})$ is the electric field amplitude and α_0 is the ground state polarizability, the potential due to gravity, and the Casimir-Polder surface potential relevant at atom-surface distances below a few micrometers [5].

Characterization of on-chip microwave propagation

We generate the microwave for the on-chip waveguide using an Agilent 8257D microwave generator whose output is amplified and amplitude stabilized using a feedback loop with a directional coupler and an Agilent 8471E detector directly in front of the chip. We measure a relative long-term drift in $P_{\rm mw}$ of $< 5\cdot10^{-4}$ peak-to-

 DOI: 10.1038/NPHYS1329

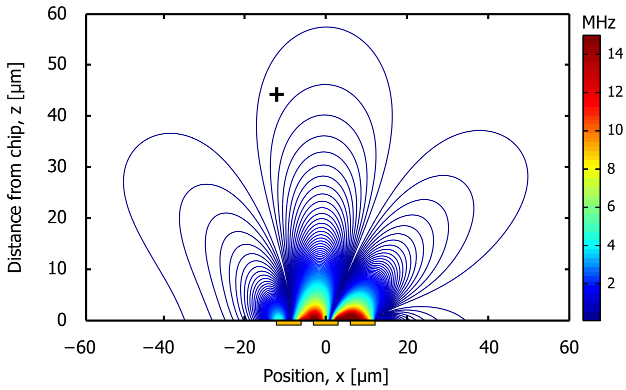

FIG. 1: **Plot of the equipotential lines of $|\mathbf{\Omega_R}|$ (corresponding to Figure 2 of the main text).** Shown is $|\Omega_R(x, y_m, z)|/2\pi$, the outermost line as well as the line spacing are 70 kHz. $\Omega_R(\mathbf{r})$ is calculated from the microwave field $\mathbf{B}_{\rm mw}(\mathbf{r})$ of an ideal CPW mode with $\mathrm{I}_{\rm mw} = 76$ mA and the static magnetic trapping field $\mathbf{B}(\mathbf{r})$. The asymmetry in $|\Omega_R(\mathbf{r})|$ with respect to $x = 0$ is due to the spatial dependence of $\mathbf{B}(\mathbf{r})$. The position of the CPW wires is indicated below the abscissa, the trap position is marked by a black cross.

peak. The relative drift of the power launched into the chip is the same as that of the transmitted power. This indicates that there is no significant long-term drift of the CPW's transmission properties. The chip temperature is stabilized by water cooling of the carrier chip.

The characteristic impedance of the CPW, obtained from quasistatic simulations including conductor and dielectric losses [1], smoothly changes from 50 Ω at the ports to 70 Ω at the chip center. The measured power transmission through the chip is -6 dB. Because of symmetry of the CPW, we expect that approximately -3 dB of the injected power reach the chip center. Simulations suggest that the observed loss is dominated by conductor loss in the chip center where the lateral extent of the CPW is only a few micrometers. The absence of resonances in the transmission spectrum in the investigated frequency range ($\leq$ 8.5 GHz) suggests that the bond wires and other discontinuities do not lead to strong standing waves on the chip. In any case, the atoms would experience only very small microwave potential gradients due to standing waves, because the size of the atomic wave function is much smaller than the microwave wavelength.

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