

Lasing Reporting Summary

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► Experimental design

Please check: are the following details reported in the manuscript?

1. Threshold

Plots of device output power versus pump power over a wide range of values indicating a clear threshold

☒ Yes
☐ No

Shown in Fig 1f of the main manuscript

2. Linewidth narrowing

Plots of spectral power density for the emission at pump powers below, around, and above the lasing threshold, indicating a clear linewidth narrowing at threshold

☒ Yes
☐ No

Shown in Fig 1e and Fig 2d of the main manuscript

Resolution of the spectrometer used to make spectral measurements

☒ Yes
☐ No

See Methods.

3. Coherent emission

Measurements of the coherence and/or polarization of the emission

☐ Yes
☒ No

Coherence and polarization were not measured. Polarization could have been measured if our setup/sample was configured/designed so that light emitted end on from the network fibers could be analyzed.

4. Beam spatial profile

Image and/or measurement of the spatial shape and profile of the emission, showing a well-defined beam above threshold

☐ Yes
☒ No

Far-field top-view image shown in Fig 1b. A well-defined beam cannot be seen as the image is of light scattering out of plane from the network. A well-defined beam could be observed if the light emitted out from the fibers could be imaged end on.

5. Operating conditions

Description of the laser and pumping conditions
Continuous-wave, pulsed, temperature of operation

☒ Yes
☐ No

See Methods.

Threshold values provided as density values (e.g. W cm^{-2} or J cm^{-2}) taking into account the area of the device

☒ Yes
☐ No

See Results Paragraph 3.

6. Alternative explanations

Reasoning as to why alternative explanations have been ruled out as responsible for the emission characteristics
e.g. amplified spontaneous, directional scattering; modification of fluorescence spectrum by the cavity

☒ Yes
☐ No

Yes they have been ruled out.

7. Theoretical analysis

Theoretical analysis that ensures that the experimental values measured are realistic and reasonable
e.g. laser threshold, linewidth, cavity gain-loss, efficiency

☒ Yes
☐ No

Yes, we estimated the threshold by modeling. The estimated values were consistent with our experimental values.

8. Statistics

Number of devices fabricated and tested

☒ Yes
☐ No

More than 16 different networks were tested.

Statistical analysis of the device performance and lifetime (time to failure)

☒ Yes
☐ No

Stability of lasing discussed in Supplementary Note 5.

