

Supplementary Information - Phase-stabilised self-injection-locked microcomb

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1 Tuning of the offset frequency f_{off}

1.1 Diode laser current

The tuning rate of the diode laser frequency used in our work is approximately 1 GHz mA^{-1} . As illustrated in Figure S1, self-injection locking (SIL) reduces this tuning rate by a factor $\sigma = \partial \xi / \partial \zeta$ due to the feedback from the microresonator. In our case, we estimate $\sigma \approx 40$ when operating in the SIL dissipative Kerr soliton (DKS) regime [1] (the exact value of σ depends on the detuning between diode laser and microresonator), and therefore

$$\frac{\partial f_{\text{off}}}{\partial I_p} \approx 25 \text{ MHz mA}^{-1}. \quad (1)$$

This estimate is in excellent agreement with the measured value of 27 MHz mA^{-1} (c.f. Table 1 in manuscript). Furthermore, we note that SIL reduces the diode laser's linewidth by a factor σ^2 , often known as the SIL stabilisation coefficient. This more than 1000-fold reduction in linewidth significantly reduced the bandwidth requirement for the actuators and is critical in enabling phase-locking via heater control.

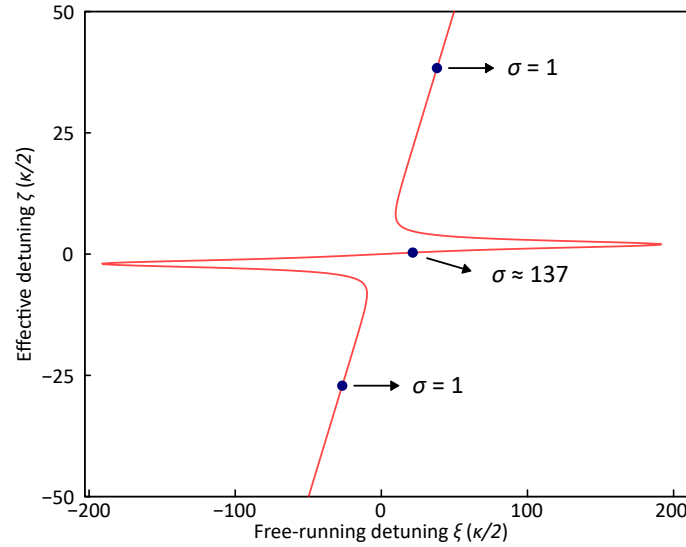


Figure S1 | Conceptual illustration of a SIL tuning curve. The effective detuning $\zeta = 2(\omega_0 - \omega_e)/\kappa$ is shown as a function of the laser-microresonator detuning $\xi = 2(\omega_0 - \omega_{LC})/\kappa$ (ω_0 is the resonance frequency of the microresonator, ω_e the laser emission frequency and ω_{LC} the free-running laser frequency, i.e. the laser's emission frequency in the absence of SIL). Outside of the SIL range, there is a one-to-one dependence between ζ and ξ , whereas, in SIL operation, the laser tuning speed $\partial \zeta / \partial \xi$ is reduced by a factor σ .

1.2 Heater current

Similarly, varying the microheater current modifies the temperature of the microresonator and hence affects the diode laser's emission frequency through the SIL mechanism. In our system, we measure the heater tuning coefficient to be approximately 1 GHz mW^{-1} in our chips (thermally-induced frequency shift per unit of dissipated electrical power), which corresponds to

$$\frac{\partial f_{\text{off}}}{\partial I_{\text{h}}} \approx 210 \text{ MHz mA}^{-1} \quad (2)$$

at our operation point (current bias of 3 mA). This is in good agreement with the measured value of 160 MHz mA^{-1} (c.f. Table 1 in manuscript).

2 Tuning of the repetition rate f_{rep}

To quantify the repetition rate tuning, we need to include the Raman-induced soliton self-frequency shift (SFS) in our considerations. The SFS is given by [2, 3]:

$$\Omega \approx -\frac{64\pi^2}{15} \frac{D_1^2}{2\pi D_2} \delta f_{\text{R}} \tau_{\text{R}}, \quad (3)$$

where D_1 and D_2 , describe the microresonator's mode frequencies $\omega_{\mu} = \omega_0 + D_1\mu + D_2/2\mu^2$ (μ is the relative mode index), $\delta = (\omega_0 - \omega_{\text{p}})/(2\pi)$ the pump to resonance detuning, and f_{R} and τ_{R} the Raman fraction and shock terms respectively, taken to be 20 % and 20 fs in silicon nitride [2]. The SRS translates directly to a change in the repetition rate via the cavity dispersion:

$$\Delta f_{\text{rep}} = \frac{\Omega}{2\pi} \frac{D_2}{D_1} \approx D_1 \delta f_{\text{R}} \tau_{\text{R}}. \quad (4)$$

Hence, the repetition rate is sensitive to the detuning δ via the Raman-induced SFS.

2.1 Diode laser current

As discussed in Section 1.1, the detuning δ can be tuned through the diode laser current at a rate of 25 MHz mA^{-1} . Using eq. 4, we find

$$\frac{\partial f_{\text{rep}}}{\partial I_{\text{p}}} \approx 190 \text{ kHz mA}^{-1}, \quad (5)$$

which is in good agreement with the measured value of 160 kHz mA^{-1} (c.f. Table 1 in manuscript).

2.2 Heater current

The detuning δ can also be adjusted via the heaters, although the tuning rate is reduced through the SIL mechanism by σ to approximately 5 MHz mA^{-1} . Again using eq. 4, we find

$$\frac{\partial f_{\text{rep}}}{\partial I_{\text{h}}} \approx 38 \text{ kHz mA}^{-1}. \quad (6)$$

On the other hand, thermal actuation directly affects the ring's FSR (via the thermorefractive effect and to a lesser extent, through thermal expansion) at a rate

$$\frac{1}{2\pi} \frac{\partial D_1}{\partial I_{\text{h}}} \approx 325 \text{ kHz mA}^{-1}. \quad (7)$$

These two different contributions add up to about $\partial f_{\text{rep}}/\partial I_{\text{h}} = 360 \text{ kHz mA}^{-1}$, which is again in good agreement with the measured value of 400 kHz mA^{-1} (c.f. Table 1 in the main manuscript).

3 Actuator linearity

The dependence of the microcomb repetition rate f_{rep} and offset frequency f_{off} were recorded as a function of the diode laser current I_{p} and heater current I_{h} (see Figure S2). All tuning curves are monotonic and, to good approximation, linear, which ensures stable locking conditions.

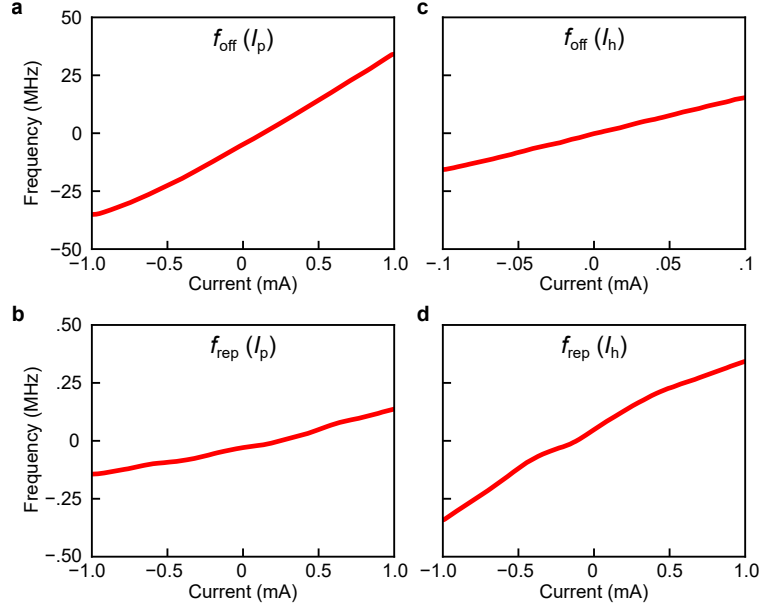


Figure S2 | Actuator tuning curves. Values are shown around the system's operating point. **a**, Offset frequency f_{off} as a function of the diode laser current I_p . **b**, Repetition rate frequency f_{rep} as a function of the diode laser current I_p . **c**, Offset frequency f_{off} as a function of the microheater current I_h . **d**, Repetition rate frequency f_{rep} as a function of the microheater current I_h .

4 Microheater frequency response

We measure the open-loop microheater frequency response by using a slow side-of-fringe lock to stabilise a continuous wave laser on a resonance of the microresonator. A sinusoidal tone is then applied to the microheater, which modulates the resonance frequency and, thereby, the transmission of the continuous wave laser. We can extract the microheater's frequency response by recording the amplitude and phase relation between the input and output signal (see Figure S3).

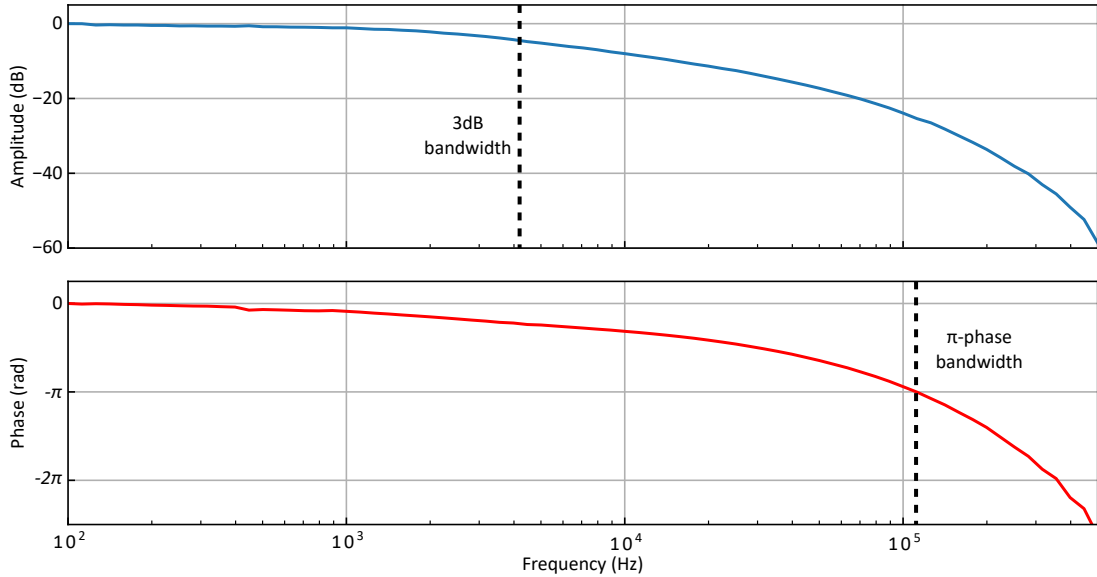


Figure S3 | Open-loop microheater frequency response. Amplitude (top) and phase (bottom) frequency response of the microheater. While the 3 dB bandwidth is approximately 5 kHz a phase delay of π is reached at ~ 100 kHz.

As can be seen, the 3 dB of the microheater is approximately 5 kHz. Despite this, a phase delay of π is reached at ~ 100 kHz. This allows us, in a closed-loop configuration, to effectively extend the heaters's bandwidth to more than 100 kHz. As the heater bandwidth dictates the overall system bandwidth, this step is critical in achieving both offset and repetition rate locks.

5 Comparison of fully phase-locked microcombs

A comparison of fully phase-locked microcombs to date (i.e. ν_p and f_{rep} are both phase-coherently stabilised to an external frequency reference) is provided in Table 1, where we list resonator platform, form factor, the employed actuators as well as important characteristics such as pump laser power, electrical power consumption, volume (excluding driving electronics) and approximate unit cost.

Ref.	Platform	Form factor w/ actuators	1 st actuator	2 nd actuator	Pump ^a (mW)	Elec. ^b (W)	Volume ^c (cm ³)	Cost ^d (USD)
[4]	SiO ₂ WGM	Table-top	ECDL detuning	EDFA power	200	>10	>1000	>10000
[5]	MgF ₂ WGM	Table-top	FL detuning	Aux. FL detuning	240	>10	>1000	>10000
[6]	SiO ₂ WGM	Table-top	ECDL detuning	EDFA power	100	>10	>1000	>10000
[7]	Si ₃ N ₄ PIC	Table-top	ECDL detuning	AOM power	2000	>10	>1000	>10000
[8]	Si ₃ N ₄ PIC	Table-top	SSB modulator	ECDL/EDFA power	200	>10	>1000	>10000
This work	Si ₃ N ₄ PIC	Chip-scale	SIL DFB current	Microheater	25	<0.5	<1	≈100

WGM, whispering-gallery-mode resonator; PIC, photonic integrated circuit microring resonator; ECDL, external cavity diode laser; EDFA, erbium-doped fibre amplifier; FL, fibre laser; AOM, acousto-optic modulator; SSB single sideband; SIL, self-injection locking; DFB, distributed feedback laser diode. ^aCoupled optical pump power. ^bEstimated electrical power consumption of the optical frequency comb source. ^cEstimated volume excluding driving electronics. ^dEstimated unit cost, when fabricated at scale.

Table 1 | Comparison of fully phase-locked microresonator frequency combs.

References

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