Jamming is a first-order transition with quenched disorder in amorphous materials sheared by cyclic quasistatic deformations – Supplementary Information

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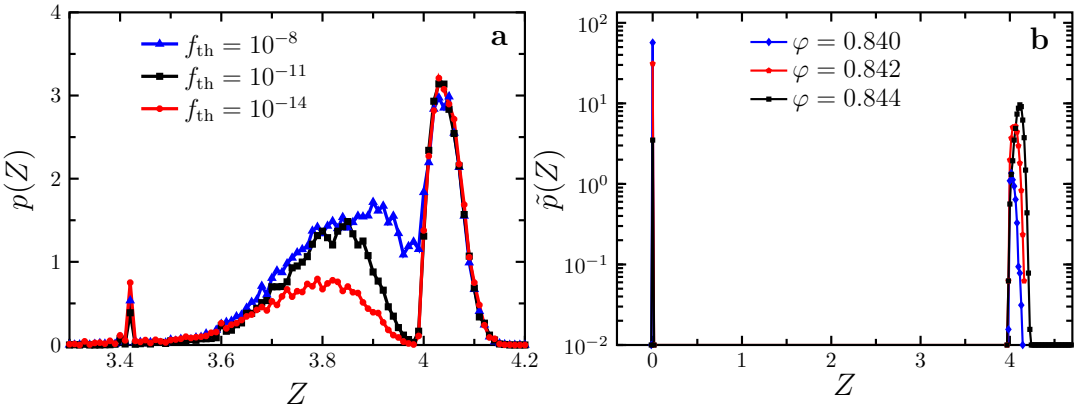

FIG. S1. **Additional data on the distribution $p(Z)$ in two dimensions.** (a) $p(Z)$ for a few different $f_{\rm th}$ ($\varphi = 0.841$ and $N = 1000$). (b) Full probability distribution $\tilde{p}(Z)$ at a few different φ near $\varphi_{\rm J}^{\infty} = 0.8432$ ($N = 1000$ and $f_{\rm th} = 10^{-14}$).

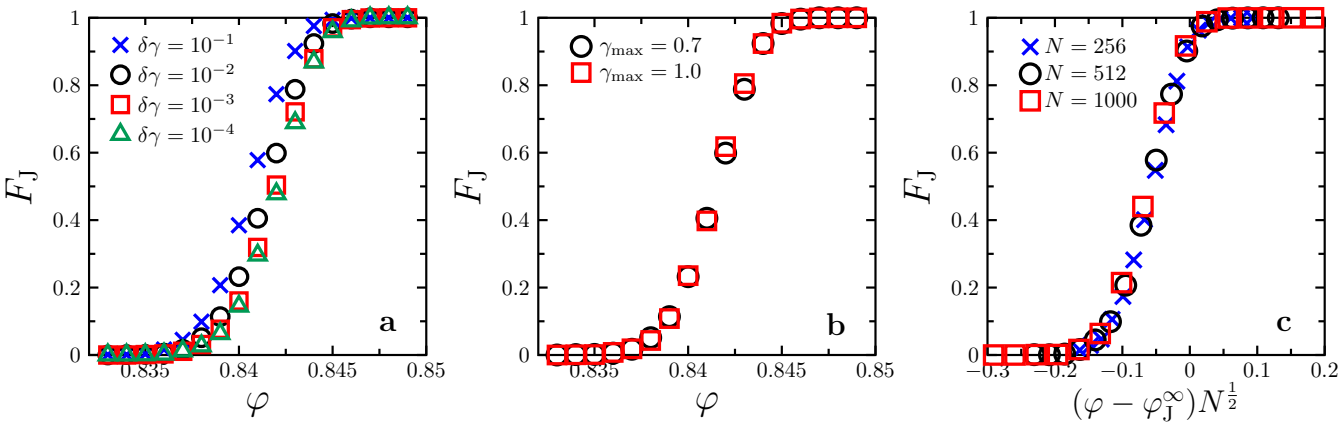

FIG. S2. **Additional data on the jamming fraction $F_{\rm J}$ in two dimensions.** (a) $F_{\rm J}$ as a function of φ, for different strain steps $\delta\gamma$, with fixed $\gamma_{\rm max} = 0.7$ and $N = 512$. (b) $F_{\rm J}$ as a function of φ, for different maximum strains $\gamma_{\rm max}$, with fixed $\delta\gamma = 10^{-2}$ and $N = 512$. (c) Data collapse of $F_{\rm J}(\varphi, N)$ as a function of rescaled variable $(\varphi - \varphi_{\rm J}^{\infty})N^{1/2}$, with $\varphi_{\rm J}^{\infty} = 0.8432$, $\delta\gamma = 10^{-1}$ and $\gamma_{\rm max} = 0.7$.

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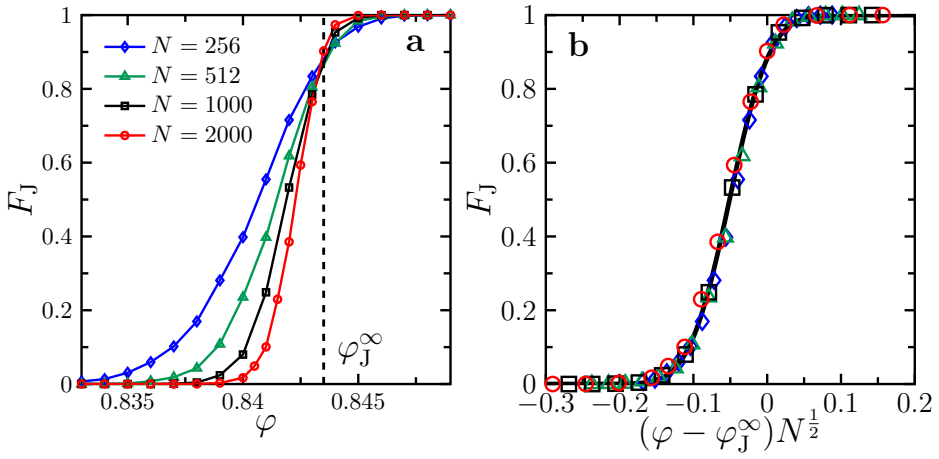

FIG. S3. **Jamming fraction in two dimensions for $\gamma_{\max} = 1$.** (a) Fraction of jammed states $F_{\rm J}$ as a function of φ for a few different N. The intersection of curves gives $\varphi_{\rm J}^{\infty} = 0.8435(1)$. (b) The data points of $F_{\rm J}$ with different N collapse as a function of $(\varphi - \varphi_{\rm J}^{\infty})N^{1/2}$. The solid line represents the fitting according to Eq. (4),with two fitting parameters $u = 0.051(1)$ and $\sigma_{\varphi} = 0.043(1)$.

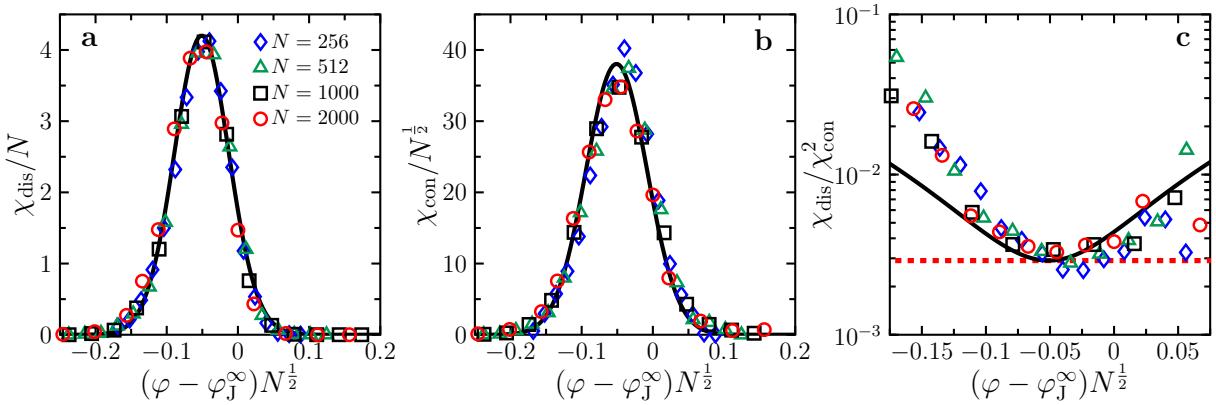

FIG. S4. **Susceptibilities in two dimensions for $\gamma_{\max} = 1$.** (a) The disconnected susceptibility $\chi_{\rm dis}$ rescaled by N, (b) the connected susceptibility $\chi_{\rm con}$ rescaled by $N^{1/2}$, and (c) the ratio between $\chi_{\rm dis}$ and $\chi_{\rm con}^2$ are plotted as functions of $(\varphi - \varphi_{\rm J}^{\infty})N^{1/2}$. The solid black lines in (a-c) are Eqs. (5), (6) and (7) respectively. The dashed red line in (c) is the constant term in Eq. (7), i.e., $\chi_{\rm dis}/\chi_{\rm con}^2 = \pi\sigma_{\varphi}^2/2$. To draw these lines, we use $u = 0.051$ and $\sigma_{\varphi} = 0.043$ obtained from the fitting in Fig. S3b. Symbols in (a-c) have the same meanings as indicated in the legend of (a).

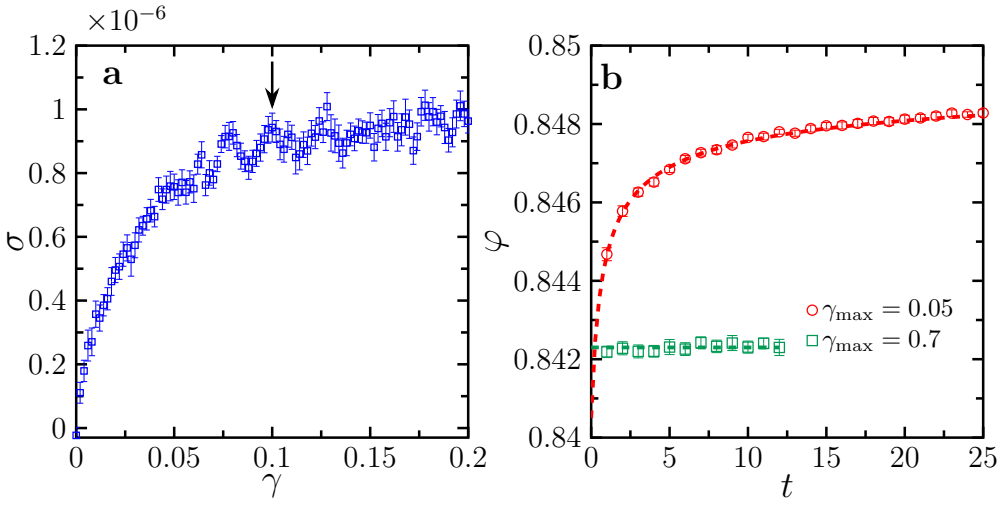

FIG. S5. **Constant-pressure shear simulations in two dimensions.** (a) Stress σ as a function of the strain γ in simple athermal quasistatic shear, under a constant pressure $P = 10^{-5}$. The yield strain is $\gamma_Y \approx 0.1$ (arrow), beyond which the curve becomes nearly a plateau. (b) φ as a function of the number of cycles t in cyclic athermal quasistatic shear (in one cycle γ varies as $0 \to \gamma_{\max} \to -\gamma_{\max} \to 0$), with a fixed $P = 10^{-5}$. The data of $\gamma_{\max} = 0.05$ are fitted to a logarithmic form (dashed line), $\varphi(t) = \varphi_f - \frac{\varphi_f - \varphi_0}{1 + B\log(1+t/\tau)}$, as proposed in Ref. [74]. Here $\varphi_0 = 0.8405$ is the packing fraction of un-strained initial systems; $\varphi_f = 0.8516(5)$, $B = 0.61(15)$ and $\tau = 0.60(18)$ are fitting parameters. The error bar represents the standard error of the mean.

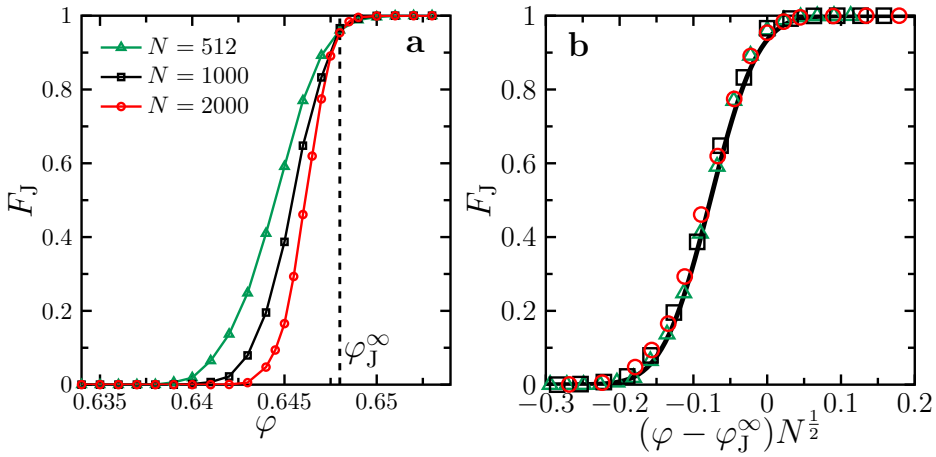

FIG. S6. **Jamming fraction in three dimensions.** (a) Fraction of jammed states F_J as a function of φ for a few different N. The intersection of curves gives $\varphi_J^\infty = 0.6479(2)$. (b) The data points of F_J with different N collapse as a function of $(\varphi - \varphi_J^\infty)N^{1/2}$. The solid line represents the fitting according to Eq. (4), with two fitting parameters $u = 0.077(2)$ and $\sigma_\varphi = 0.051(1)$.

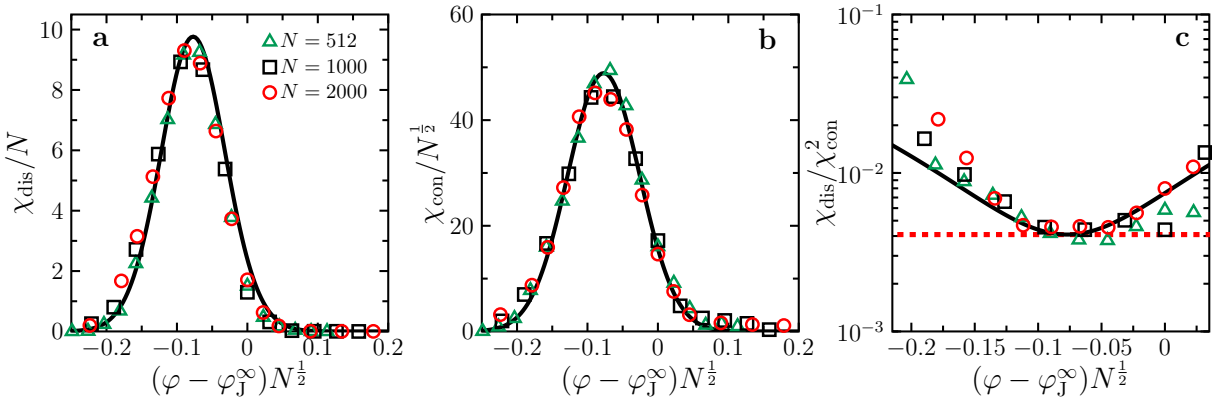

FIG. S7. **Susceptibilities in three dimensions.** (a) The disconnected susceptibility $\chi_{\rm dis}$ rescaled by N, (b) the connected susceptibility $\chi_{\rm con}$ rescaled by $N^{1/2}$, and (c) the ratio between $\chi_{\rm dis}$ and $\chi^2_{\rm con}$ are plotted as functions of $(\varphi - \varphi_{\rm J}^{\infty})N^{1/2}$. The solid black lines in (a-c) are Eqs. (5), (6) and (7) respectively. The dashed red line in (c) is the constant term in Eq. (7), i.e., $\chi_{\rm dis}/\chi^2_{\rm con} = \pi\sigma^2_{\varphi}/2$. To draw these lines, we use $u = 0.077$ and $\sigma_{\varphi} = 0.051$ obtained from the fitting in Fig. S6b. Symbols in (a-c) have the same meanings as indicated in the legend of (a).