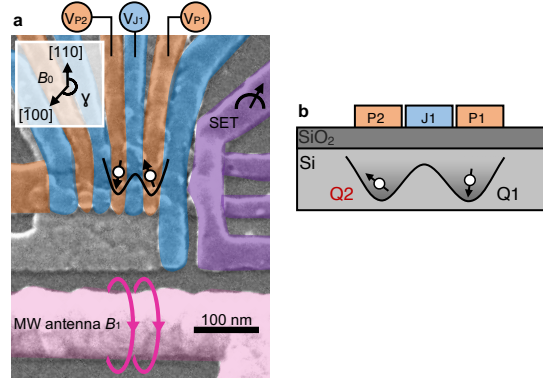
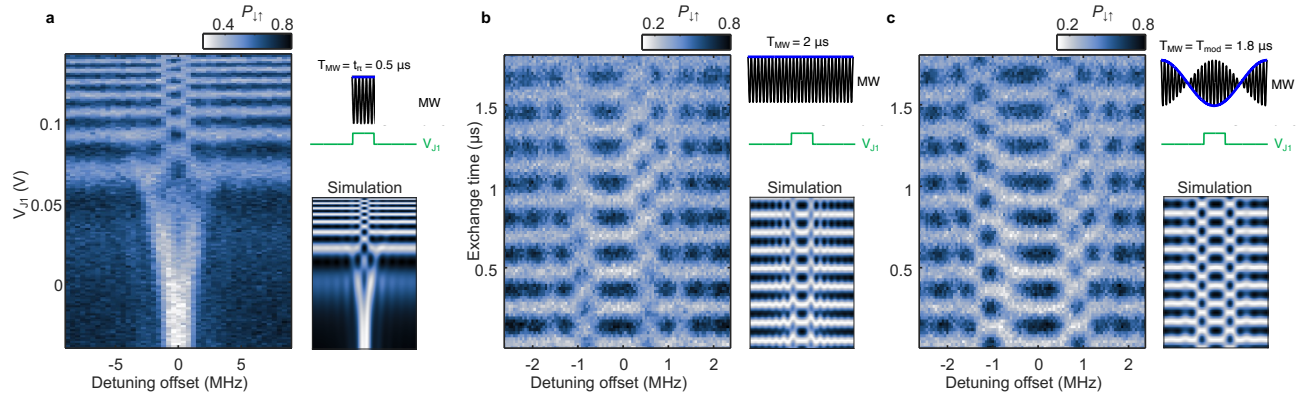


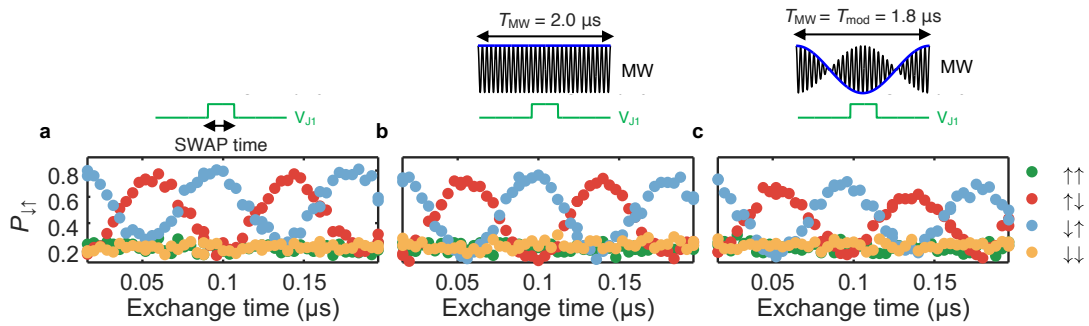
SUPPLEMENTARY INFORMATION



Supplementary Fig. 1. **Device layout.** (a) False-coloured scanning electron microscope (SEM) image of an identical SiMOS quantum dot device. A double dot is confined under gate P1-P2 with electron configuration (3,3). The gate J1 is used to control the exchange coupling between the spins. A global field is generated with an on-chip ESR antenna. A device cross-section view is given in (b).



Supplementary Fig. 2. **Larmor frequency robustness of driven SWAP oscillations.** (a) ESR spectrum as a function of V_{J1} where the on-resonance microwave performs a π -rotation. (b) Square modulation and (c) sinusoidal modulation SWAP oscillation at a fixed V_{J1} level for different microwave frequencies and increasing time, acquired in an interleaved manner. The total microwave duration for the square modulation is $2 \mu\text{s}$ and for the sinusoidal modulation $1.8 \mu\text{s}$. Simulations are shown for each sub-panel.



Supplementary Fig. 3. **SWAP oscillation for different initial states.** SWAP oscillations with initialisations $\{|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\uparrow\uparrow\rangle\}$ for (a) no microwave, (b) continuous wave microwave and (c) sinusoidal modulated microwave.

Supplementary Note 1: Hamiltonian. The Hamiltonian used for all simulations is shown below, where $\hat{\sigma}_{x,y,z}$ and $\hat{\mathbb{I}}$ are the Pauli operators,

$$\hat{H}_{2Q}(t) = \Delta\nu_1(t)(\hat{\sigma}_z \otimes \hat{\mathbb{I}}) + \Delta\nu_2(t)(\hat{\mathbb{I}} \otimes \hat{\sigma}_z) + \Omega_{R1}(t)(\hat{\sigma}_x \otimes \hat{\mathbb{I}}) + \Omega_{R2}(t)(\hat{\mathbb{I}} \otimes \hat{\sigma}_x) + \sum_{i=x,y,z} J(t)\hat{\sigma}_i \otimes \hat{\sigma}_i \quad (1)$$

derived from Refs^{5,7}. The detuning of the Larmor frequency of qubit 1,2 from the external magnetic field B_0 is $\Delta\nu_{1,2}(t)$, and the Rabi frequencies are $\Omega_{R1,2}(t)$ for qubits 1,2. The exchange amplitude is $J(t)$.

The driving field determines the time dependency of $\Omega_{R1,2}(t)$. If the field is constant then $\Omega_{R1,2}(t) = \Omega_{R1,2}$. If the driving field has a sinusoidal modulation, then $\Omega_{R1,2}(t) = \Omega_{R1,2}\sqrt{2}\cos(2\pi f_{\text{mod}}t)$ ^{6,7}. Ideally, $\Delta\nu_{1,2}(t) = 0$, except when performing single qubit operations.

From the Hamiltonian, it can be seen that when $\Delta\nu_{1,2}(t) \approx 0$ and all other parameters are greater than 0, the eigenstates of the system are $|S(1,1)\rangle$, $|T_0(1,1)\rangle$, $|\downarrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$. This means that oscillations between $|\downarrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$ are natural, giving rise to Heisenberg SWAP oscillations, as discussed in Refs. 5, 25, 31–34. In this work, we use the $|S(1,1)\rangle$ notation to refer to the singlet states in the (3,3) charge configurations, as the first two electrons in each dot form a closed-spin shell and can be disregarded⁴⁴. Similarly, $|S(2,0)\rangle$ refers to singlet states in the (4,2) charge configurations.

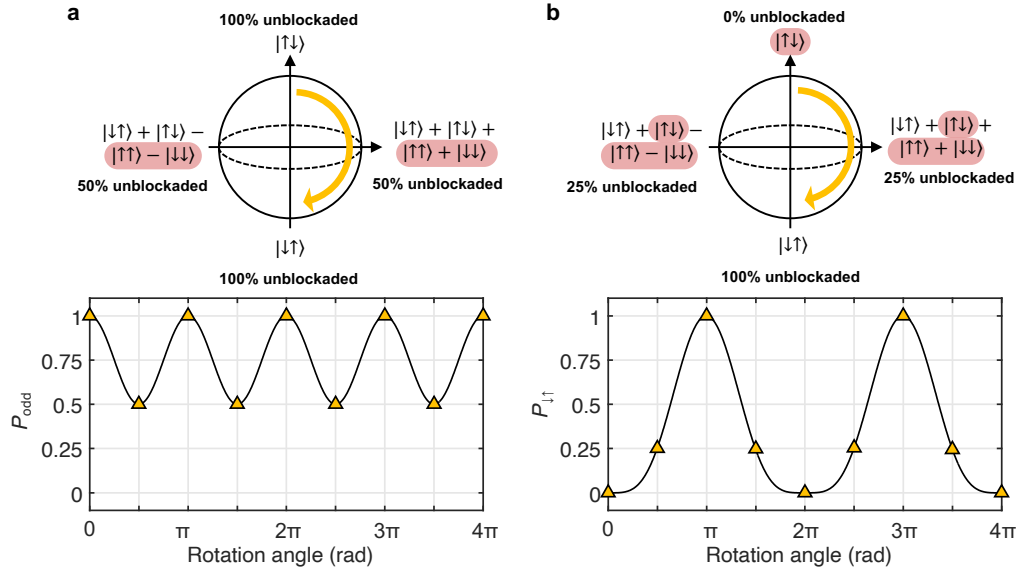
Supplementary Note 2: Optimal sinusoidal modulation. The sinusoidal microwave modulation is configured according to Ref. 7 (referred to as the SMART protocol) where the Rabi frequency and the modulation period follows

$$\Omega_R T_{\text{mod}} = j_i / \sqrt{2}. \quad (2)$$

Here, j_i is the i -th Bessel root of zeroth order. Using the first root $j_1 = 2.404826$ we find the optimal modulation period to be $T_{\text{mod}} = 1.8\mu\text{s}$ when $\Omega_R \sim 1\text{ MHz}$. By driving the spin qubits with this specific waveform, second order noise cancellation can be achieved.

Supplementary Note 3: Readout. In this work, the readout method is based on Pauli spin blockade²⁹. This is achieved by tuning the energy levels of the two neighbouring quantum dots, each containing a spin 1/2 qubit, such that only the $|S(1,1)\rangle$ state tunnels into the $|S(2,0)\rangle$ state, while no other spin state can – following the Pauli exclusion principle. In our case, due to the coupling of $|\downarrow\uparrow\rangle$ with $|S(1,1)\rangle$, only the $|\downarrow\uparrow\rangle$ state transitions into $|S(2,0)\rangle$, which we call adiabatic ST readout. Alternatively, if there is strong coupling between $|T_0\rangle$ and $|S(1,1)\rangle$, set by the Zeeman energy difference, the readout results in the parity readout regime where only $|T_0\rangle$ and $|S(1,1)\rangle$ transition into $|S(2,0)\rangle$. The charge movement from (1,1) to (2,0) distinguishes between a state that is blockaded and a state that is unblockaded. In this work, we were able to achieve the adiabatic ST and parity regime by changing the Zeeman energy difference at the read point – the larger the Zeeman energy difference, the more likely $|T_0\rangle$ will decay into $|S(2,0)\rangle$, resulting in parity readout. To make sense of the readout mechanisms, we discuss the Rabi oscillations observed when both qubits have the same Larmor frequency at the control point. In Supplementary Fig. 4, an initial state $|\uparrow\downarrow\rangle$ is left to evolve on resonance with a driving field, rotating the spins from $|\uparrow\downarrow\rangle$ to $|\downarrow\uparrow\rangle$ as shown by the Bloch spheres. Below the Bloch spheres are the projections of the states into (a) the parity readout basis and (b) the ST basis. In the parity basis, the probability of measuring a charge movement P_{odd} oscillates between 1 (either $|\downarrow\uparrow\rangle$ or $|\uparrow\downarrow\rangle$) and 0.5 (an equal mixture of $|\downarrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\downarrow\rangle$, $|\uparrow\uparrow\rangle$). In the ST basis, the probability of measuring a charge movement $P_{\downarrow\uparrow}$ starts at 0 since $|\uparrow\downarrow\rangle$ is blockaded, then evolves to an equal mixture of all four states, resulting in $P_{\downarrow\uparrow} = 0.25$ since only a quarter of the states are blockaded. When the evolution results in $|\downarrow\uparrow\rangle$, the system is fully unblockaded, and $P_{\downarrow\uparrow} = 1$. Therefore, adiabatic ST readout exhibits oscillations that decay to 0.75, while with parity the oscillations decay to 0.5.

The ability to distinguish between $|\downarrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$, i.e. adiabatic ST readout, is useful when observing SWAP operations. To measure SWAP oscillations with parity readout, $|\downarrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$ are no longer distinguishable, so an additional CNOT gate is needed before readout.



Supplementary Fig. 4. **Rabi probability amplitudes for different readout mechanisms.** (a) Parity and (b) adiabatic ST readout shown on two superimposed Bloch spheres with the corresponding Rabi probabilities. Here, 1 corresponds to unblocked and triangle markers show the probabilities of unblocked at $\pi/2$ intervals.

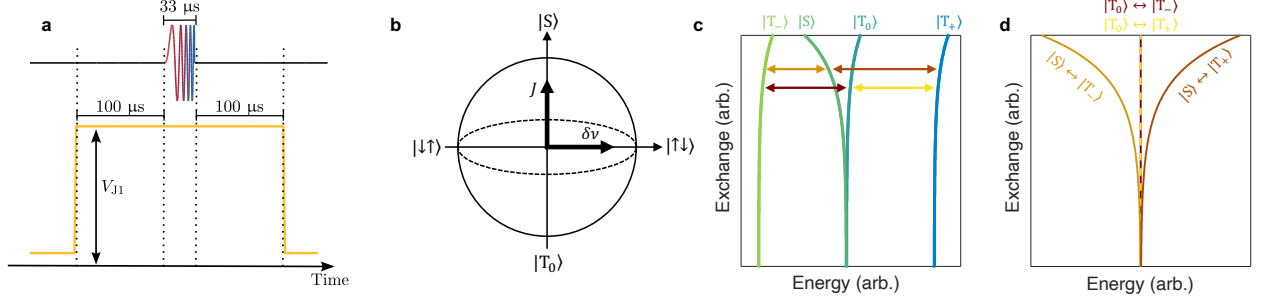
Supplementary Note 4: Adiabatic inversion exchange maps.

Fig. 1(d) shows a frequency scan of the qubit energies as a function of the gate voltage V_{J1} , which controls the exchange interaction. To achieve spin inversion, a chirped microwave pulse (i.e., an pulse whose frequency increases linearly in time) is applied to adiabatically drive spin transitions. The frequency chirp corresponds to a 650 kHz frequency shift over 33 μs .

The system is initialized in a $|\downarrow\uparrow\rangle$ state. However, after V_{J1} is ramped to the control point, there is a wait time of 100 μs before the adiabatic microwave chirped pulse is applied during which the qubits are left idle and undergo exchange oscillations [see Supplementary Fig. 5(a)]. In the case where V_{J1} is sufficiently high and exchange is large compared to the Larmor frequency difference ($J > \delta E_Z$), the initialized $|\downarrow\uparrow\rangle$ state undergoes SWAP oscillations. Consequently, at high V_{J1} the spin state before the adiabatic microwave chirped pulse becomes a 50/50 mixture of $|\downarrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$ or, equivalently, of $|S(1,1)\rangle$ and $|T_0\rangle$.

When the adiabatic microwave pulse is applied, we believe three types of transitions can be driven: a first branch that decreases exponentially in frequency corresponds to $|S(1,1)\rangle$ states transitioning into $|T_-\rangle$ [light orange arrow in Supplementary Fig. 5(c)]; a second branch that increases exponentially in frequency corresponds to $|S(1,1)\rangle$ states transitioning into $|T_+\rangle$ [dark orange arrow in Supplementary Fig. 5(c)]; finally, a third branch that remains constant in frequency in the middle of the map corresponds to $|T_0\rangle$ states transitioning into $|T_-\rangle$ and $|T_+\rangle$ (see Supplementary Fig. 5(c-d) for the energy diagram corresponding to our model of this system). Due to the nature of the parity readout, only $|T_-\rangle$ and $|T_+\rangle$ will be blocked, and any mixture of $|S\rangle$ and $|T_0\rangle$ will be unblocked.

The branches corresponding to $|S(1,1)\rangle$ transitions are present. However, instead of the expected blocked peak associated with $|T_0\rangle$ transitions, we observe a central unblocked peak in between two blocked peaks. For Fig. 1(d) it is possible to explain this peak at high exchange in a similar fashion as in the case of low exchange interaction, where both spins are being resonantly driven via a multi-photon process and $|\downarrow\uparrow\rangle$ ($|\uparrow\downarrow\rangle$) turns into $|\uparrow\downarrow\rangle$ ($|\downarrow\uparrow\rangle$), maintaining the even parity. When the microwave frequency is slightly off-resonant, partial adiabatic inversion is achieved, leading to a proportion of the spins being in an even state and resulting in the side fringes. Although it can be phenomenologically explained for Fig. 1(d), the absence of a central branch cannot be explained for Fig. 1(c) at $\gamma = 224^\circ$, therefore it does not match our theoretical model and evidences our incomplete understanding of the dynamics of degenerate spins in a condition of high exchange.



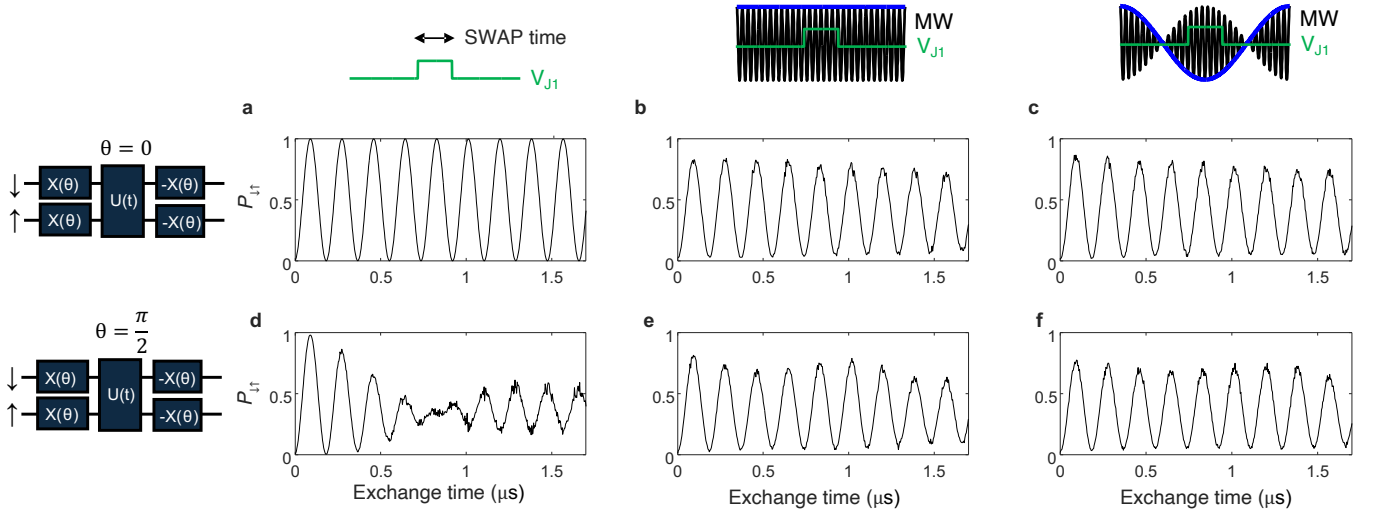
Supplementary Fig. 5. **Adiabatic inversion exchange maps.** (a) Pulse diagram for the control sequence in the adiabatic scans. The spins are left to freely evolve for $100\mu\text{s}$ before and after the chirped pulse. (b) A ST Bloch sphere indicating the quantisation axis as a function of the ratio of exchange and difference in Larmor frequencies. (c) The energy diagram of a two-qubit system with degenerate Larmor frequencies. The arrows show the different energy splittings in the system. (d) The energy splittings in the two-qubit system as a function of exchange, corresponding to the arrows in (c).

Supplementary Note 5: Noise robustness simulations. During the exchange pulse there is a slight Stark shift from gate J1 that affects the two-qubit system – consistent with Supplementary Fig. 2. The Stark shift will affect any qubit state that is perpendicular to this shift, meaning that any state that is not $|\uparrow\rangle$ or $|\downarrow\rangle$ will be perturbed by the Stark shift. This is seen in Fig. 2(d). For the case of no microwave drive and $\theta = 0$, the qubits are not affected by the Stark shift when exchange is on – confirmed by the coherent SWAP oscillation in Fig. 2(d). However, when $\theta = \pi/2$ the state before the exchange is switched on is $|y\bar{y}\rangle$ which is affected by Stark shift. Fig. 2(g) shows a line cut of $\theta = \pi/2$ from (d). The Stark shift has the effect of bringing the qubit system out of sync with the rotating frame frequency, causing beating – consistent with the revival of the SWAP oscillations after $1\mu\text{s}$. Tuning the device to remove the Stark shift is needed to remove this beating.

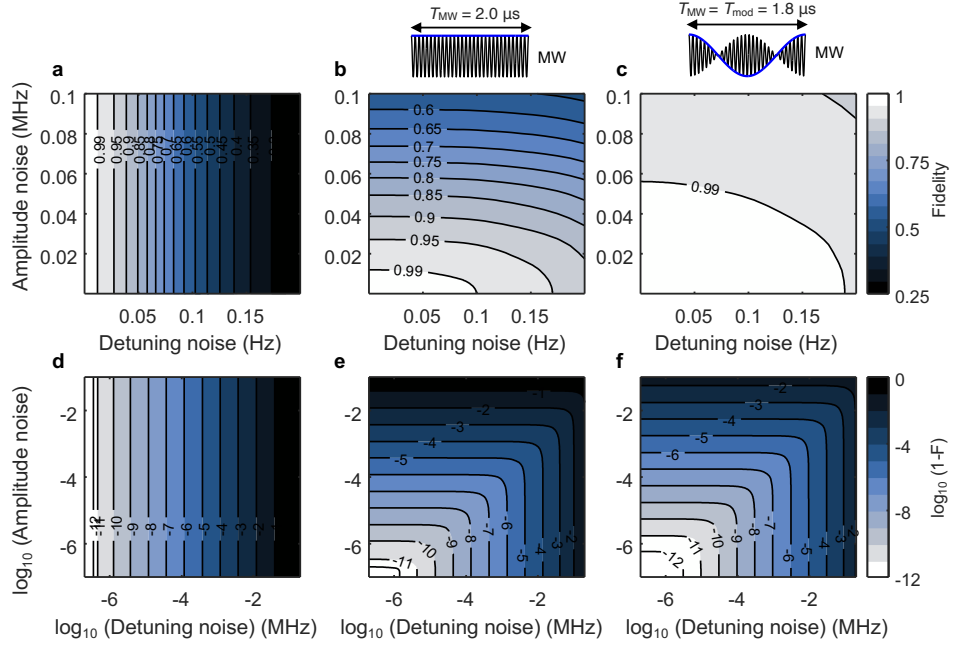
For the case of the driven system with a continuous microwave, when $\theta = \pi/2$ in Fig. 2(e) there is no beating seen. This demonstrates robustness against shifts in the Larmor frequencies. There is a slight reduction in the SWAP oscillation visibility for longer SWAP times when comparing $\theta = \pi/2$ to $\theta = 0$, demonstrating some sensitivity to offsets in the Larmor frequencies. Further robustness is seen when the system is driven by the sinusoidal modulated microwave. Comparing the SWAP oscillations in Fig. 2(f) $\theta = \pi/2$ to $\theta = 0$, the visibility remains constant.

To confirm that this observation matches our theoretical interpretation, we simulated the SWAP oscillations for $\theta = \pi/2$ and $\theta = 0$, shown in Supplementary Fig. 6. The simulation follows the Hamiltonian in Eq. 1, and includes quasi-static Gaussian noise on the amplitude of the Rabi frequency and the Larmor frequency. On top of this, a Stark shift is implemented when the exchange is switched on. The simulation in Supplementary Fig. 6(d) shows a revival in the SWAP oscillation for the undriven system when $\theta = \pi/2$, and no effect when $\theta = 0$. This is consistent with the experimental data, other than the overall visibility of the SWAP oscillations being affected by more general white noise, or $1/f$ noise. Both the continuously driven cases were also simulated with the same offsets in Supplementary Fig. 6(b) and (c) for $\theta = 0$ and (e) and (f) for $\theta = \pi/2$. Both simulations confirm that neither of the driven cases are strongly affected by the Stark shift. Other noise sources would need to be introduced in the model to further improve the match of the simulation to the experiment.

To demonstrate the improvement in noise robustness for on-resonance driven SWAPs we also simulate the operator fidelity with detuning and microwave noise according to Ref. 7 in Supplementary Fig. 7. Again, we assume quasi-static Gaussian noise. The operator fidelity for the case with no microwave driving, Supplementary Fig. 7(a) and (d), shows that the SWAP operation is highly sensitive to detuning noise compared to the driven cases in (b,c) and (e,f). This agrees with our interpretation of Fig. 2(d-f).



Supplementary Fig. 6. **Simulation of SWAP oscillations with noise.** Simulations of SWAP oscillations for two different initial states of the qubits for (a) no microwave drive, (b) continuous wave microwave of duration $T_{\text{MW}} = 2.0 \mu\text{s}$, and (c) sinusoidal modulated microwave of duration $T_{\text{MW}} = T_{\text{mod}} = 1.8 \mu\text{s}$ with $\theta = 0$. Initialisation with $\theta = \pi/2$ are shown in (d-f).



Supplementary Fig. 7. **Simulation of $\sqrt{\text{SWAP}}$ with noise.** Operator fidelity of a $\sqrt{\text{SWAP}}$ gate during (a) no microwave driving, (b) continuous wave microwave and (c) sinusoidal modulated microwave with detuning noise and microwave amplitude noise in linear scale. The same is given in (d-f) in logarithmic scale.

Supplementary Note 6: Curve fitting of a periodic oscillation. The coherence time of the SWAP oscillations in Fig. 2(g-i) is fitted according to

$$F(A, b, f, g, \phi, T, t) = A(b + (\cos(ft + \phi) \times \cos(gt) \times e^{-t/T})). \quad (3)$$

The Ramsey and Hahn echo times in Fig. 3(a-b) are fitted according to

$$F(A, a, b, c, d, e, f, g, T, t) = A(a - b(c + d \cos(ft) + e \cos(2ft) + g \cos(3ft)) \times e^{-t/T}). \quad (4)$$

The data in Fig. 3(c-d) is fitted similarly.