

Entangling gates on degenerate spin qubits dressed by a global field



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REVIEWER COMMENTS

Reviewer #1 (Remarks to the Author):

This report presents experiments exploring the use of a global ac magnetic field to control an array of spin qubits in a Si-MOS device. In principle, global control schemes reduce the number of rf-lines needed to control large arrays. The report focuses on a few key experiments that present solutions to some of the key issues. (i) By tilting the dc magnetic field, two qubits can be made close to degenerate in both Larmor and Rabi frequency, allowing one to simultaneously drive both qubits. This overcomes variability due to spin-orbit, and the inhomogeneity of the ac B-field generated by the antenna, although this is unlikely to scale beyond two qubits. (ii) For a pair of degenerate qubits, a SWAP operation is performed by pulsing the exchange-gate. The authors present evidence for an improvement on the damping of the exchange oscillation when a global microwave drive field is applied, that is further enhanced using a SMART-drive (sinusoidal amplitude modulation). (iii) The coherence metrics of two degenerate qubits are reported. (iv) The addressability of single qubits is explored by tuning qubit 2 into and out of resonance with the global drive. This is combined with the SWAP gate to do single qubit control of both qubits.

I like the paper, and think it will influence my own research. I have learnt a lot by reading it, so aside from the mostly presentational comments below, I endorse publication. In particular, I think the use of coherence protection scheme in the case of two-qubit control is a significant new research direction.

1. (p2, 1st col) "The g-factor variability..." How is the B-field oriented with respect to the device? Is the B-field being rotated in or out of plane? In fig. 1c, the B-field definition shows two vectors [100] and [110], which I would expect to be at 45deg. The diagram is also not consistent with final paragraph of p2, "At $\gamma = 224^\circ$..."

2. To understand the scene it is necessary for the reader to read the first two sections, and then the methods. Otherwise the role of gates P1, P2 and J1 are undefined. A short sentence along the lines of “Consider two quantum dots with plunger gates P1, P2, and a barrier gate J1.” Or similar is required.

3. In fig 1, why is J1 scanned? Is this because one desires the Larmor frequencies to be insensitive to J1 for the global swap operation?

4. As I understand, in fig 1e, at $B_0=370\text{mT}$ the Rabi frequency and Larmor frequencies are matched. In fig. 1f, $B_0 = 836\text{ mT}$ for parity readout. Are the Rabi and Larmor frequencies also matched are the conditions same or different? Else, how different are the Rabi frequencies here?

5. (p3, final para of col1) “The adiabatic ST readout yields a periodic oscillation at the Rabi frequency but it is not a simple sinusoidal because only the $|\text{down},\text{up}\rangle$ is unblockaded, while the driving populates all states.” I do not follow. This sounds like the measurement would project along $|\text{down},\text{up}\rangle$, how does this give rise to additional harmonics in the motion? The two spins may be degenerate, but there will still be a residual spin-spin interaction, how does this influence the qubit dynamics? Does a 1d cut through the simulation match the 1d cut through the data in figs 1e,f?

6. In figs. 2b,c, what is the on/off time of the J1- pulse with respect to the microwaves? Is it important? If so, what determines a good on/off time?

7. I think it would be helpful to see an equation expressing the input state used in fig. 2 in terms of θ .

8. I think it would be helpful to see plots of the effective damping time vs θ for the three cases in fig.2. Then the question is does the microwave drive simply produce a damping in the exchange oscillation that is averaged over θ ? For the no microwave drive case, the damping is low at 0, π , whereas it is more uniform in b,c. Also, in fig. 1e, an oscillation between up/down and down/up states is reported but if I understand, in a case that is not

entangled. In fig. 2, an oscillation between up/down and down/up states is also reported. The point I am unclear on is does the combination of microwave drive and J-gate yield the same operation as the J-gate alone? I appreciate that the microwave-drive pulse alone is an identity. However, it is not obvious that the combination of time-overlapped operations, that do not commute at most instants in time would separate to yield an operation equivalent to the J-gate alone. If it does, is there a condition on the timing of the J-pulse? For example, does the J-gate need to be centered on the microwave pulse?

9. (p5, 1st col, para starting “in fig 3d) I think it would be more informative to give ratio of $T_{\text{mod}}/T_{\text{rabi}}$.

10. In fig 3d, the effective Rabi frequency is independent of detuning over a range of 1.27 MHz. In theory, is the value of the detuning tolerance proportional to the Rabi frequency, or is it more subtle than that? In the fig.3c case the bandwidth is about $\Omega_R/4$, is this related to the spread in effective Rabi frequencies? Are the values for single qubit T_2^* and Hahn echo times known? How do they compare?

11. (p5, 2nd col) “....cause (RWA) to break.” The RWA is only likely to breakdown for frequency shifts of similar size to the 10s’ GHz qubit frequency, or am I missing something?

12. In figs 4(d,f) I think it would help to add an additional x-axis of the phase-shift imparted by pulsing the Stark gate, $\text{dnu}_{\{12\}}/dV.V(\text{Stark}) \times (\text{Time duration})$. Also, why do the simulations have no x-axis values? Is there a scaling between voltage applied and delivered to the gate?

13. In methods, is it possible to add a reference to SET stabilization scheme used?

14. Is the difference in ac B-field strength at the two qubits known?

Reviewer #2 (Remarks to the Author):

In this work, the authors report on two-qubit SWAP gate based on degenerate spins that coherently dressed by a global control field. They introduce methods for matching the Larmor frequency and the Rabi frequency, and study the SWAP gate with or without the global control field. Although the two-qubit joint coherence is far from that of the single qubit, it still shows an improvement of SWAP gate, indicating the advantages of global control field. Unfortunately, qubit under gate P1 cannot be directly driven, and there are no results about the control fidelity of SWAP gate, as well as the optimization of its control fidelity by global field. Furthermore, I cannot grasp the advantages of global control field for large-scale quantum computing, and each adjustment of parameters implies a challenge to scalability, such as frequency matching processes. Overall, both highlights and issues coexist in this manuscript. The authors should make reasonable revisions to make it suitable for publication in Nature Communications.

1. In Fig 1c, small variations in the magnetic field angle lead to dramatic changes in δE_z . If there are three or more qubits, does this approach realize Larmor frequency matching of all qubits? “The g-factor variability is expected to be $< 0.1\%$ for a DC magnetic field applied along the [100] direction.” A Larmor frequency of 10 GHz means the δE_z is several MHz, which is questionable for scalability.
2. In section Rabi frequency matching, “we can tolerate the stray electric fields by fine-tuning the magnetic field strength until the Rabi frequencies of both qubits match.” Is there any results that the Rabi frequency is variable with fine-tuning the magnetic field strength? If realizing Rabi frequency matching for two qubits is so difficult, how to ensure scalability?
3. “...which we attribute to degradation of the SET charge readout caused by heating”, how to optimize the heating effect after considering the global control field?
4. T_2^{2Q} and T_2^{2QHahn} are much smaller than that of single spin qubit, is there any explanation of how the two qubits interact with each other?
5. In Fig 2d, when $\theta = 0$ or π , its coherence looks good. Is the enhancement of $\theta = \pi/2$ overemphasized?
6. It’s more compelling to insert content about the control fidelity of SWAP gate.
7. Does the detuning offset in Fig 3c-d represent the change of global microwave frequency?

8. Qubit under gate P1 cannot be directly driven, how to measure the Rabi chevron shown in Fig 1e? Combined with SWAP gate?

9. Is there any error in the writing of $[-100]$ in Extended Fig 1? $[100]$?

Additional question: For global control approach, it seems that the complexity of qubit operation is increased, and it seems challenging to meet scalability requirements across all qubits. So, how to achieve large-scale quantum computing effectively by using the global control approach in the future?

We thank you and the referees for the attention to our manuscript. All the comments from the referees were taken into consideration, and they helped us significantly improve our text to more clearly convey the principles of driven two-qubit gates. Please refer to the attached marked-up PDF file.

We hope these changes elevate the manuscript to the quality level of Nature Communications and provide confidence in the editorial board for publishing our work.

All the best,
Ingvild Hansen, Andrew Dzurak and Henry Yang (on behalf of all co-authors)

~~Reviewer #1 (Remarks to the Author):~~ - - - - -

This report presents experiments exploring the use of a global ac magnetic field to control an array of spin qubits in a Si-MOS device. In principle, global control schemes reduce the number of rf-lines needed to control large arrays. The report focuses on a few key experiments that present solutions to some of the key issues. (i) By tilting the dc magnetic field, two qubits can be made close to degenerate in both Larmor and Rabi frequency, allowing one to simultaneously drive both qubits. This overcomes variability due to spin-orbit, and the inhomogeneity of the ac B-field generated by the antenna, although this unlikely to scale beyond two qubits. (ii) For a pair of degenerate qubits, a SWAP operation is performed by pulsing the exchange-gate. The authors present evidence for an improvement on the damping of the exchange oscillation when a global microwave drive field is applied, that is further enhanced using a SMART-drive (sinusoidal amplitude modulation). (iii) The coherence metrics of two degenerate qubits are reported. (iv) The addressability of single qubits is explored by tuning qubit 2 into and out of resonance with the global drive. This is combined with the SWAP gate to do single qubit control of both qubits.

I like the paper, and think it will influence my own research. I have learnt a lot by reading it, so aside from the mostly presentational comments below, I endorse publication. In particular, I think the use of coherence protection scheme in the case of two-qubit control is a significant new research direction.

~~We thank the referee for the positive feedback and interest in our work.~~ - - - - -

~~1. (p2, 1st col) "The g-factor variability ..." How is the B-field oriented with respect to the device? Is the B-field being rotated in or out of plane? In fig. 1c, the B-field definition shows two vectors [1001 and [1101, which I would expect to be at 45deg. The diagram is also not consistent with final paragraph of p2, "At gamma =224deg...."~~

~~The B-field is being rotated in the plane of the qubits. The referee is correct, and the first vector should be [-1001 not [+1001. This has been updated in the figure. Gamma is defined as the positive angle from [1101 to the B-field and at 224 deg the B-field is more or less pointing along [-1001.~~

2. To understand the scene it is necessary for the reader to read the first two sections, and then the methods. Otherwise the role of gates P1, P2 and J1 are undefined. A short sentence along the lines of “Consider two quantum dots with plunger gates P1, P2, and a barrier gate J1.” Or similar is required.

A short sentence has been added in the first section.

3. In fig 1, why is J1 scanned? Is this because one desires the Larmor frequencies to be insensitive to J1 for the global swap operation?

Yes, ideally the Larmor frequencies should not be stark-shifted by J1, only P1 and P2 for single-qubit control. Another reason to scan J1 is to characterise the turn-on of the exchange.

4. As I understand, in fig 1e, at $B_0=370\text{mT}$ the Rabi frequency and Larmor frequencies are matched. In fig. 1f, $B_0 = 836\text{ mT}$ for parity readout. Are the Rabi and Larmor frequencies also matched are the conditions same or different? Else, how different are the Rabi frequencies here?

The Larmor and Rabi frequency for the two qubits are matched in Fig. 1e and Fig. 1f. In Fig. 1e $B_0 = 370\text{ mT}$ with electron configuration (3,3) and a Rabi frequency of $\sim 1.0\text{ MHz}$ for both qubits. In Fig. 1f $B_0 = 836\text{ mT}$ with electron configuration (1,3) and a Rabi frequency of $\sim 0.75\text{ MHz}$ for both qubits. The oscillations in Fig. 1f however oscillate at $\sim 2 \times 0.75 = 1.5\text{ MHz}$ due to the parity readout.

5. (p3, final para of col1) “The adiabatic ST readout yields a periodic oscillation at the Rabi frequency but it is not a simple sinusoidal because only the $|\text{down}, \text{up}\rangle$ is unblockaded, while the driving populates all states.” I do not follow. This sounds like the measurement would project along $|\text{down}, \text{up}\rangle$, how does this give rise to additional harmonics in the motion? The two spins may be degenerate, but there will still be a residual spin-spin interaction, how does this influence the qubit dynamics? Does a 1d cut through the simulation match the 1d cut through the data in figs 1e,f?

In Supplementary Fig. 1b this is explained in more detail. There the spin state of two qubits is displayed on a single Bloch sphere, assuming exchange off. When looking at the weight of the blockaded/unblockaded states, only the south pole of the Bloch sphere is 100% unblockaded giving rise to the non-sinusoidal oscillations. A simulation of the 1d cut through the data in Fig. 1 e,f is also given in Supplementary Fig. 1 confirming that the data corresponds to the expected oscillation pattern.

6. In figs. 2b,c, what is the on/off time of the J1- pulse with respected to the microwaves? Is it important? If so, what determines a good on/off time?

The ON time of the square J1 pulse is given by the exchange time on the x-axis. Longer exchange time gives more exchange oscillations. We have updated the label in the cartoon from “SWAP time” to “Exchange time” to make this more clear.

7. I think it would be helpful to see an equation expressing the input state used in fig. 2 in terms of theta.

We have added an equation expressing the input state in the figure caption.

8. I think it would be helpful to see plots of the effective damping time vs theta for the three cases in fig.2. Then the question is does the microwave drive simply produce a damping in the exchange oscillation that is averaged over theta? For the no microwave drive case, the damping is low at 0, pi, whereas it is more uniform in b,c.

Also, in fig. 1e, an oscillation between up/down and down/up states is reported but if I understand, in a case that is not entangled. In fig. 2, an oscillation between up/down and down/up states is also reported.

The point I am unclear on is does the combination of microwave drive and J-gate yield the same operation as the J-gate alone? I appreciate that the microwave-drive pulse alone is an identity. However, it is not obvious that the combination of time-overlapped operations, that do not commute at most instants in time would separate to yield an operation equivalent to the J-gate alone. If it does, is there a condition on the timing of the J-pulse? For example, does the J-gate need to be centered on the microwave pulse?

At theta=0 and theta=pi SWAP oscillations happens between protected eigenstates of the system which are not exposed to detuning noise. We therefore would expect low damping at these two values of theta. The loss of visibility in b,c and e,f we attribute to degradation of the SET readout due to heating from the microwave.

In Fig. 1e both spins are driven by a single microwave pulse from up/down to down/up with exchange OFF. In Fig. 2 exchange is ON and the microwave always performs an identity by itself, hence the up/down oscillations are caused by SWAPs and not the microwave.

The J-gate is not required to be centered on the microwave pulse. The only condition is that the duration of the microwave pulse must correspond to an identity operation. The global field is the system clock and everything must be timed accordingly (multiples of T_{mod}). When qubits are driven at the same Larmor and Rabi frequency they are effectively under the same rotating frame such that the J operator commutes with the global field whenever the global field performs an identity operation $[IX+XI, \text{SWAP}] = 0$.

9. (p5, 1st col, para starting "in fig 3d) I think it would be more informative to give ratio of $T_{\text{mod}}/T_{\text{rabi}}$.

We have added the ratio in the text.

0. In fig 3d, the effective Rabi frequency is independent of detuning over a range of 1.27 MHz. In theory, is the value of the detuning tolerance proportional to the Rabi frequency, or is it more subtle than that?

In the fig.3c case the bandwidth is about $\Omega_R/4$, is this related to the spread in effective Rabi frequencies?

Are the values for single qubit T_2^* and Hahn echo times known? How do they compare?

The width of the detuning range is proportional to the Rabi frequency and hence faster Rabi is better also for the modulated case.

In Fig 3c the smaller robustness to detuning is due to less noise decoupling from a constant power modulation compared to a sinusoidal modulation. The Rabi frequencies are not perfectly matched and will therefore cause drift between the two qubits over time for the square modulation. For the sinusoidal modulation, on the other hand, no drift will accumulate because rotation differences will be cancelled due to the positive followed by negative amplitude modulation.

The single qubit T_2^* and Hahn echo are not known at the exact same configuration due to the difficulty of measuring these metric with the matched condition. The same device under different configurations has, however, measured longer T_2^* and Hahn echo times.

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1. (p5, 2nd col) “....cause (RWA) to break.” The RWA is only likely to breakdown for frequency shifts of similar size to the 10s’ GHz qubit frequency, or am I missing something?

For the regular RWA it is the frequency detuning compared to the B_0 splitting that matters. In that case the quantisation axis is along the B_0 axis. For dressed qubits we are concerned about the double-rotating frame where the ratio between Rabi frequency and the frequency detuning is what matters. In our case the quantisation axis is along the Rabi/microwave field axis.

-
2. In figs 4(d,f) I think it would help to add an additional x-axis of the phase-shift imparted by pulsing the Stark gate, $\frac{d\phi}{dV} \cdot V(\text{Stark}) \times (\text{Time duration})$. Also, why do the simulations have no x-axis values? Is there a scaling between voltage applied and delivered to the gate?

We have added an additional x-axis to Fig. 4d,f. We thank the referee for this suggestion. The simulations should have the same axis as the data using the same lever-arm (scaling between applied voltage and effect on gate).

-
3. In methods, is it possible to add a reference to SET stabilization scheme used?

The SET feedback protocol has now been referenced in the methods.

-
4. Is the difference in ac B-field strength at the two qubits known?

The ac B-field strength is reflected in the Rabi frequencies which are on purpose matched as best as possible (here different by <5% of Rabi frequency). The Rabi frequencies are seen to change with $|B_0|$, gamma (caused by spin-orbit coupling) and electron configuration.

Reviewer #2 (Remarks to the Author):

In this work, the authors report on two-qubit SWAP gate based on degenerate spins that coherently dressed by a global control field. They introduce methods for matching the Larmor frequency and the Rabi frequency, and study the SWAP gate with or without the global control field. Although the two-qubit joint coherence is far from that of the single qubit, it still shows an improvement of SWAP gate, indicating the advantages of global control field. Unfortunately, qubit under gate P1 cannot be directly driven, and there are no results about the control fidelity of SWAP gate, as well as the optimization of its control fidelity by global field. Furthermore, I cannot grasp the advantages of global control field for large-scale quantum computing, and each adjustment of parameters implies a challenge to scalability, such as frequency matching processes. Overall, both highlights and issues coexist in this manuscript. The authors should make reasonable revisions to make it suitable for publication in Nature Communications.

[We thank the referee for the feedback.](#)

1. In Fig 1c, small variations in the magnetic field angle lead to dramatic changes in ΔE_z . If there are three or more qubits, does this approach realize Larmor frequency matching of all qubits? "The g-factor variability is expected to be $< 0.1\%$ for a DC magnetic field applied along the [100] direction." A Larmor frequency of 10 GHz means the ΔE_z is several MHz, which is questionable for scalability.

[We agree with the referee that the trick of changing the magnetic field angle is only suitable for the two-qubit case. This is emphasised in the text. For more than two qubits, smaller \$B_0\$ field magnitude and larger Rabi frequency will help. This has been emphasized in the conclusion.](#)

2. In section Rabi frequency matching, "we can tolerate the stray electric fields by fine-tuning the magnetic field strength until the Rabi frequencies of both qubits match." Is there any results that the Rabi frequency is variable with fine-tuning the magnetic field strength? If realizing Rabi frequency matching for two qubits is so difficult, how to ensure scalability?

[Due to the transmission characteristics of the ESR antenna, the power delivered to the qubits, and therefore the Rabi frequency, vary with magnetic field strength accordingly. Due to inhomogeneity of the ac magnetic field and spurious electric components of the microwave field, the change in Rabi frequency due to magnetic field strength is not necessarily equal for the two qubits. This allows for the fine-tuning. To ensure scalability one needs to engineer microwave fields with minimal electric components. This has been emphasised in the conclusion.](#)

3. "...which we attribute to degradation of the SET charge readout caused by heating", how to optimize the heating effect after considering the global control field?

[Current scaled-up architectures require transmission of millions of microwave control](#)

signals to the quantum processor. This constitutes a substantial heat load. For a global control field a single microwave control signal is required which should have a positive impact on the heat budget.

Other methods to reduce the effect of heating include pre-pulsing and more active/frequent SET feedback protocols.

4. T_{2A2Q} and $T_{2A2QHahn}$ are much smaller than that of single spin qubit, is there any explanation of how the two qubits interact with each other?

Single-qubit T_{2^*} and T_{2AHahn} times vary from device to device and this device did not exhibit great coherence times compared to other devices. Also, during the T_{2A2Q} and $T_{2A2QHahn}$ both qubits are exposed to noise during the dephasing. In a regular Ramsey and Hahn echo measurement one of the qubits (ancilla qubit) is initialised in a protected eigenstate while the other is measured. We expect this to roughly half the coherence times.

5. In Fig 2d, when $\theta = 0$ or π , its coherence looks good. Is the enhancement of $\theta = \pi/2$ overemphasized?

Qubits can be in arbitrary states during algorithms and error-correcting codes. Therefore, we need noise protection for all possible states, not just eigenstates. When $\theta = 0$ or π the system is initialised in a protected eigenstate which is not exposed to dephasing. In fact, at $\theta=0$ the continuous swapping serves as an echoing sequence doing an effective Hahn echo in the J basis, echoing out dephasing noise. This has been emphasised in main text. At $\pi/2$, on the other hand, the qubits are initialised in the equatorial plane and exposed to dephasing. For the undriven case in (d) this results in significant damping. By driving both qubits during the exchange oscillations in (e-f) the effect is less dramatic due to noise decoupling.

6. It's more compelling to insert content about the control fidelity of SWAP gate.

Comprehensive fidelity studies were not performed here as we do not have good single qubit gate fidelities to assess the full SWAP operator fidelity. This work serves as a proof-of-principle and in the future it will definitely be important to characterise these gates with more comprehensive fidelity studies.

7. Does the detuning offset in Fig 3c-d represent the change of global microwave frequency?

Yes. This has been emphasised in the figure caption.

8. Qubit under gate P1 cannot be directly driven, how to measure the Rabi chevron shown in Fig 1e? Combined with SWAP gate?

The qubit under gate P1 cannot be driven **electrically** with P1. However, we managed to combine P2 with a SWAP gate to achieve single-qubit control of the qubit under gate P1. In Fig. 1e a single microwave field is driving the two qubits simultaneously **magnetically**.

9. Is there any error in the writing of [-100] in Extended Fig 1? [100]?

No, but this is inconsistent with Fig. 1. The error is in Fig. 1. We thank the referee for catching this. We have updated Fig. 1.

Additional question: For global control approach, it seems that the complexity of qubit operation is increased, and it seems challenging to meet scalability requirements across all qubits. So, how to achieve large-scale quantum computing effectively by using the global control approach in the future?

The complexity of qubit operation is slightly increased for the strategy described in this manuscript. However, the same complexity is what allows us to significantly relax the architecture requirements. Only a single microwave signal is required on the quantum computing chip. This will have a positive impact on the heat budget. Exchange and single-qubit control is fully electrical and can be done in parallel instead of sending a train of microwave pulses at different frequencies. By engineering better microwave fields and by driving the qubits faster we believe we can overcome challenges like qubit uniformity and Larmor/Rabi matching conditions.

REVIEWERS' COMMENTS

Reviewer #1 (Remarks to the Author):

The authors have listened to the feedback, and I am satisfied with the response.

I endorse publication. The report proposes and demonstrates a scheme to perform entangling operation in a way that is both compatible with a global control field, and with coherence protection scheme. This is an important milestone in the use of global control fields for control of qubit arrays, and in my opinion meets standard of Nature Comms.

Reviewer #2 (Remarks to the Author):

The authors have made several revisions to address the comments. I do think this has increased the rigor of the paper. However, I believe some of my concerns have not been addressed very well, especially the discussion of multi-qubit scalability.

1. For response 3:

The heating effect is not only caused by the microwaves added to the antenna. The microwaves added to P1 and P2, which are used to induce the Stark shift with optimized waveforms, also produce heating effects (for example, the single-qubit gate microwaves applied to the plunger. In EDSR control, the microwaves add to the plungers has observed heating effects.). Compared to the original ESR control method, this part of the global control field brings additional heating and is not comprehensively superior to original ESR control in terms of heating.

2. For response 5:

If Figure 2 is intended to illustrate that this operation method can provide better noise robustness over a wider range, it would be more convincing to additionally provide the

average T_2^* under different θ in both three figures rather than just a comparison at a single angle.

3. For response 6:

If fidelity characterization cannot be performed using methods like GST and only oscillations are shown, does this overlook potential issues of subspace phase accumulation during the operation?

4. For response 8:

If qubit1 can only be driven by magnetic field and cannot be electrically driven by P1, does it mean that the position of the qubit is not very good and not strictly under P1?

5. About the question of scalability :

Regarding the newly added section in your conclusion, for scalable global field manipulation, reducing the magnetic field strength can be used instead of rotating the magnetic field angle. Why wasn't this method used in the experiment? Additionally, is there more experimental evidence proving the existence of this tunability? If so, maybe it is better to add references.

Reviewer #1 (Remarks to the Author):

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I endorse publication. The report proposes and demonstrates a scheme to perform entangling operation in a way that is both compatible with a global control field, and with coherence protection scheme. This is an important milestone in the use of global control fields for control of qubit arrays, and in my opinion meets standard of Nature Comms.

We thank the referee for the feedback and for endorsing publication.

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The voltage signals sent to P1 and P2 for electric control in this work are in the MHz range, as opposed to GHz range for magnetic control with an antenna or EDSR control. This has been emphasized in the text. Moreover, for a scaled-up architecture multiple antennas would be required for conventional ESR control. For a global control scheme, a single dielectric resonator or cavity can be used instead.

"The aim of a global control strategy using dressed qubits is to have arrays of quasi-degenerate spins that can be driven on-resonance by a global microwave field (GHz range) and addressed individually by local electrodes (MHz range), as illustrated in fig1(b). As opposed to conventional electron-spin-resonance (ESR) control, as well as electric-dipole spin resonance (EDSR) control, individual addressing allows for signals in the MHz range, instead of in the GHz range."

2. For response 5:

If Figure 2 is intended to illustrate that this operation method can provide better noise robustness over a wider range, it would be more convincing to additionally provide the average T_2^* under different θ in both three figures rather than just a comparison at a single angle.

The comparison is done at $\theta = \pi/2$ because this is the worst-case scenario for dephasing.

3. For response 6:

If fidelity characterization cannot be performed using methods like GST and only oscillations are shown, does this overlook potential issues of subspace phase accumulation during the operation?

Fidelity characterization with GST is in principle possible. However, in this specific case we were limited by the single-qubit fidelities.

4. For response 8:

If qubit1 can only be driven by magnetic field and cannot be electrically driven by P1, does it mean that the position of the qubit is not very good and not strictly under P1?

Qubit 1 could not be electrically driven by P1. This can be due to small Stark shift from P1. In the future, this can potentially be tuned with the magnetic field angle, once the Rabi frequency is increased which allows for larger Larmor frequency spread. A sentence about this has been added to the text. We did get around this problem here by using two SWAP gates and driving with P2. This indicated that qubit 1 sits under P1 and qubit 2 under P2.

*"We find that the effect of gate P2 on Q2 is 4 times stronger than on Q1. This strong differential Stark shift control is sufficient to achieve good addressability of Q2. We note that the inverse was not true and we did not perform operations using P1. **In the future the g-factor can potentially be tuned by the magnetic field angle, once larger microwave driving amplitudes can be applied and the magnetic field angle is not required as a knob to reduce Larmor frequency spread. This would allow for direct control of Q1 with P1.**"*

5. About the question of scalability :

Regarding the newly added section in your conclusion, for scalable global field manipulation, reducing the magnetic field strength can be used instead of rotating the magnetic field angle. Why wasn't this method used in the experiment? Additionally, is there more experimental evidence proving the existence of this tunability? If so, maybe it is better to add references.

By reducing the magnetic field strength, the energy splitting between spin up and spin down reduces. For an ensemble of spins, this means that at lower magnetic field strengths, the spread of Larmor frequencies will be smaller. When reducing this energy splitting, however, eventually temperature effects will come into play.

The magnetic field strength chosen in this work is based on many factors. For example, readout visibility, exchange characteristics etc. In the future, it will be important to characterize this tunability with magnetic field strength. Having a stronger ESR drive (Rabi frequency) using a dielectric resonator or cavity can help here since it allows for larger Larmor frequency spread. This has been emphasized in the conclusion.

*"To overcome variability in Larmor and Rabi frequencies, smaller static magnetic field magnitudes and larger microwave driving amplitudes can be applied. Improved microwave engineering will also be important to produce a **stronger and more uniform** global magnetic field."*