1. *Supplementary information:*

A hypergraph model shows the carbon reduction potential of effective space use in housing

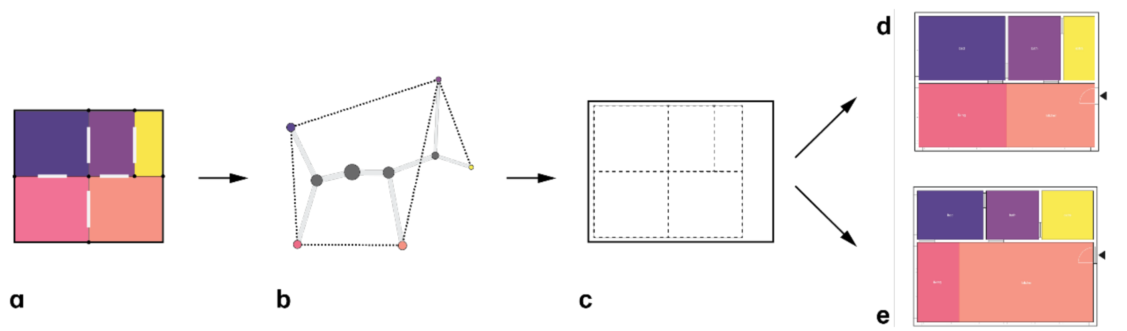
Ramon Elias Weber*[a], Caitlin Mueller[a], Christoph Reinhart[a]

[a] Building Technology Program, Department of Architecture, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139, USA, *reweber@mit.edu

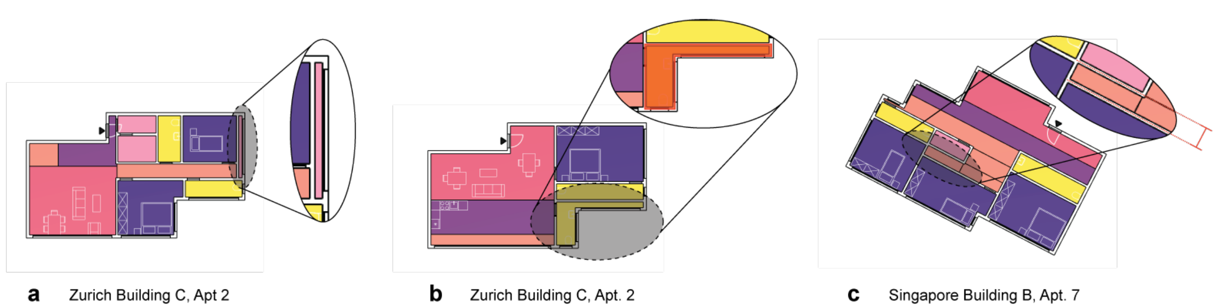
1.1. ***Supplementary note on the hypergraph method***

The following supplemental figures contain additional illustrations to explain the hypergraph method and its limitations. As shown in Supplemental Figure 1, the geometric subdivision can be applied to either retain area ratios or actual areas of the original floor plan, that the hypergraph is based on. For the purposes of this research and the automatic generation of new floor plans the ratio retention method was chosen. In order to make sure that rooms have approximate sizes similar to their original floor plan, only hypergraphs with an original area footprint within a 20% range were used.

Supplementary Figure 2 shows the application of hypergraphs on an apartment boundary that results in an infeasible floor plan layout. Three main failure cases were identified that for successful automated floor plan generation have to be discarded. We outline fast heuristics to discard such infeasible floorplans in "1.2.Supplementary note on heuristics" and "Methods, Apartment validity heuristic".



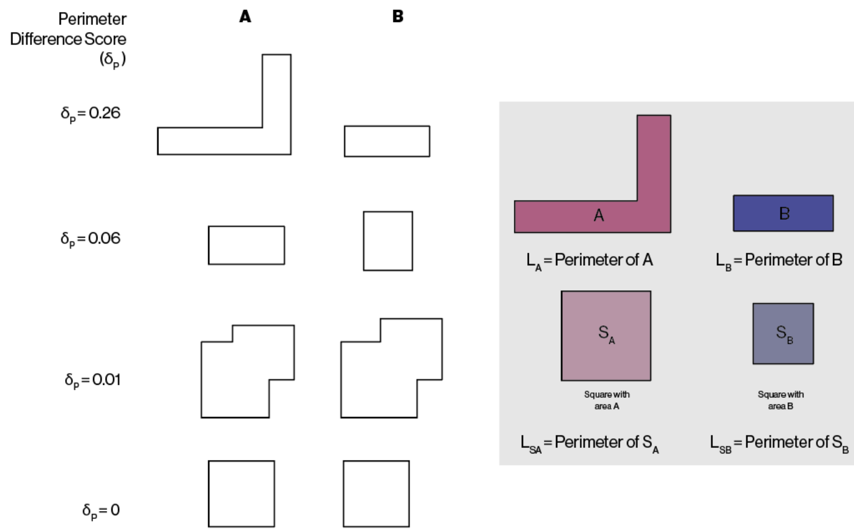
Supplemental Figure 1: A floor plan (a) creates a hypergraph (b) and is applied to a different boundary geometry (c). The new boundary geometry can be subdivided to retain the ratios of the original floor plan (d) or the actual areas of the reference floor plan (e).



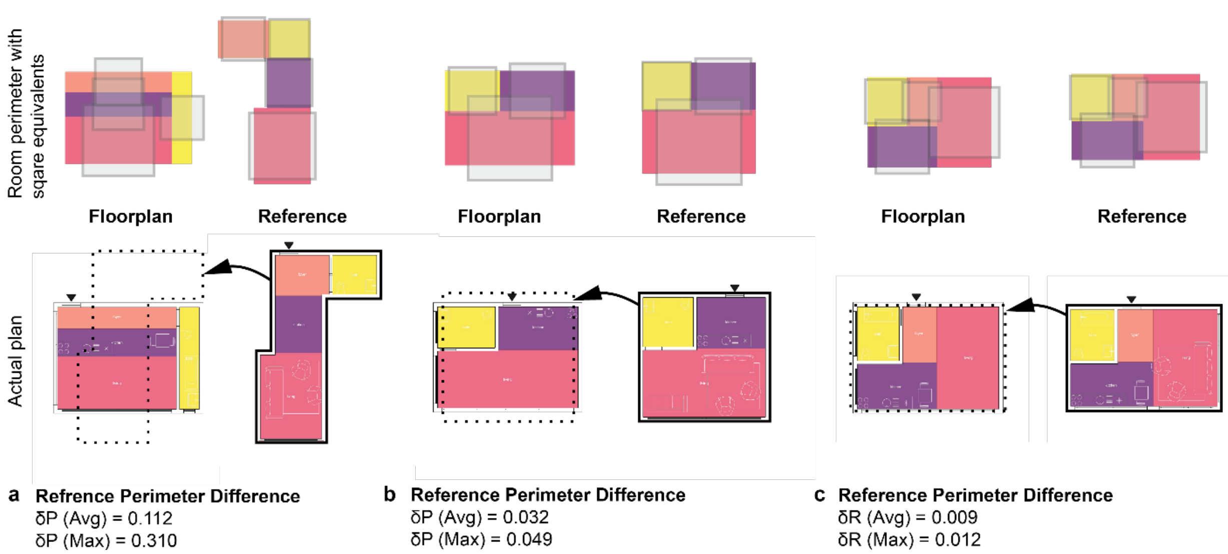
Supplemental Figure 2: Different failure cases where the subdivision algorithm creates a geometrically valid but spatially infeasible floor plan: An infeasible room blocking a bedroom façade access (a), subdivision resulting in infeasible room geometry (b), and subdivision resulting in foyer spaces that are too thin to be passable (c).

1.2. *Supplementary note on heuristics*

For the automatic generation of floor plans via the hypergraph method a series of geometric heuristics were developed that are fast to compute and help discard infeasible geometries. Supplemental Figure 3 illustrates the perimeter difference score δp, described in Equation 1. The perimeter difference score allows a quick assessment of the differences between two geometric shapes based on their boundary length. As shown in Supplemental Figure 4, this can be applied to all rooms in a given floor plan, creating a perimeter score of the source plan (that the hypergraph was derived from) and the resulting floor plan that the hypergraph was applied to. To cull floor plans with infeasible interior layout subdivisions the authors suggest a perimeter difference score of 0.1 and lower.



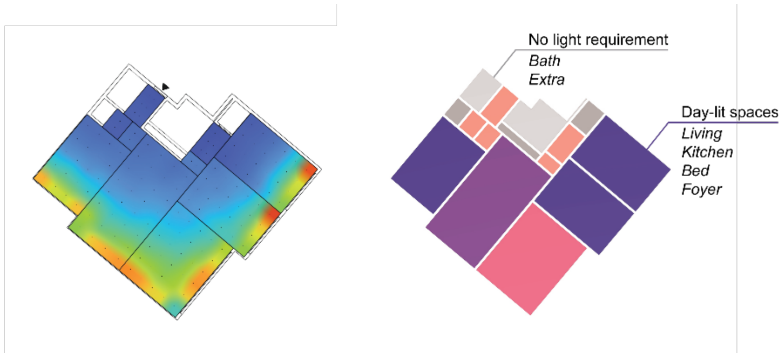
Supplemental Figure 3: Example reference (A) and target (b) boundary room polygons with corresponding perimeter difference scores.



*Supplemental Figure 4: Example of 3 fitted floor plans **a-c**. The outline of rooms of each floor plan is displayed with a square equivalent to the area of the room to calculate the perimeter difference. The actual floor plan of both the reference and the fitted floor plan is displayed. For each plan the corresponding perimeter difference $\boldsymbol{\delta_R}$ is calculated. A low fit with a value of $\boldsymbol{\delta_R}\ (\boldsymbol{average}) = 0.112$ (a), a medium fit with a value of $\boldsymbol{\delta_R}\ (\boldsymbol{average}) = 0.08$ and a very close fit with a value of $\boldsymbol{\delta_R}\ (\boldsymbol{average}) = 0.009$.*

1.3. *Supplementary notes on environmental simulation*

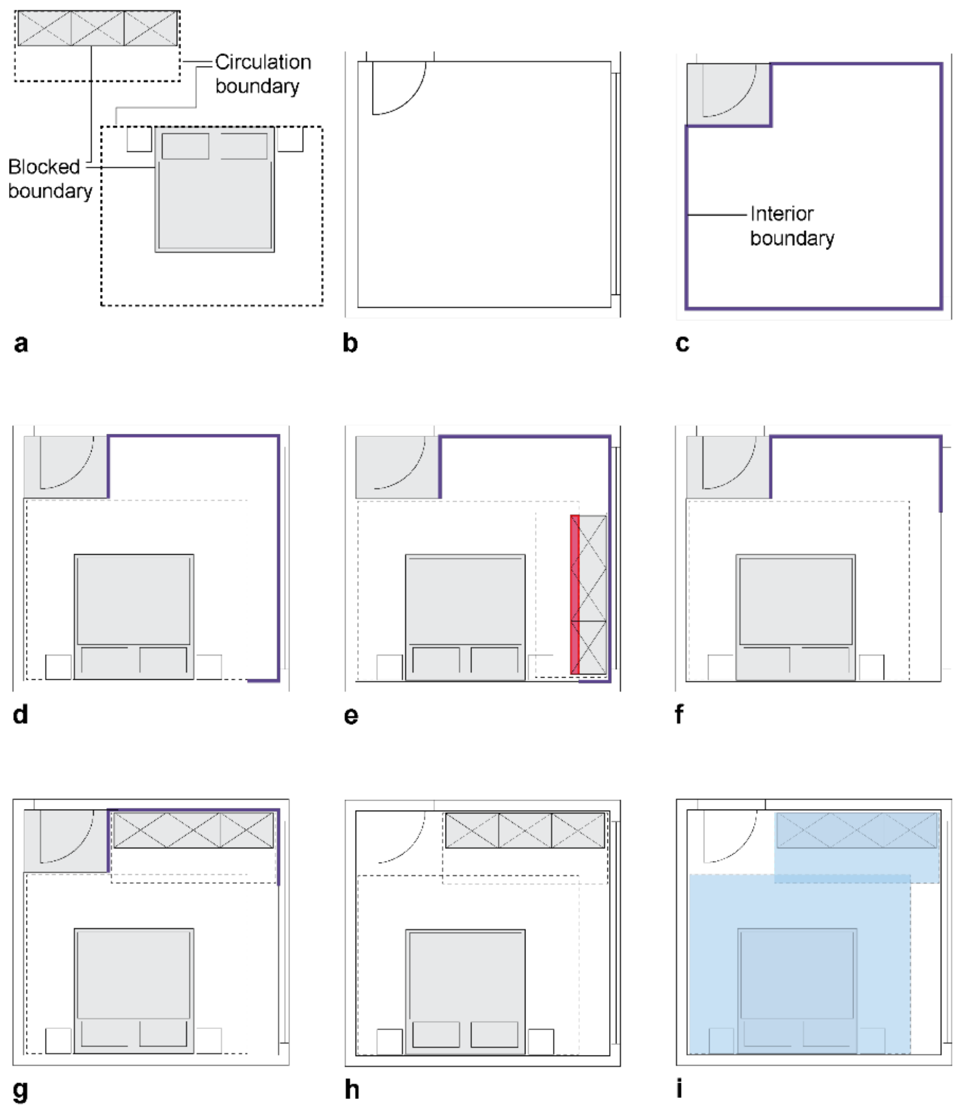
In the scope of this research the daylight simulations are only conducted for rooms that are in need of daylight, based on program. As illustrated in Supplemental Figure 5 this includes living, kitchen, bed and foyer spaces and does not include bath or extra spaces.



Supplemental Figure 5: Example of an sDA calculation of apartment visualizing mean illuminance (left) and their corresponding program (right).

1.4. *Supplementary notes on furniture placement*

Supplemental Figure 6 extends the description of section "5.9. Furniture placement" and shows a sample placement of bedroom furniture in full detail along the interior boundary of a room. Supplemental Figure 7 outlines the minimal furniture requirements of different apartment sizes.



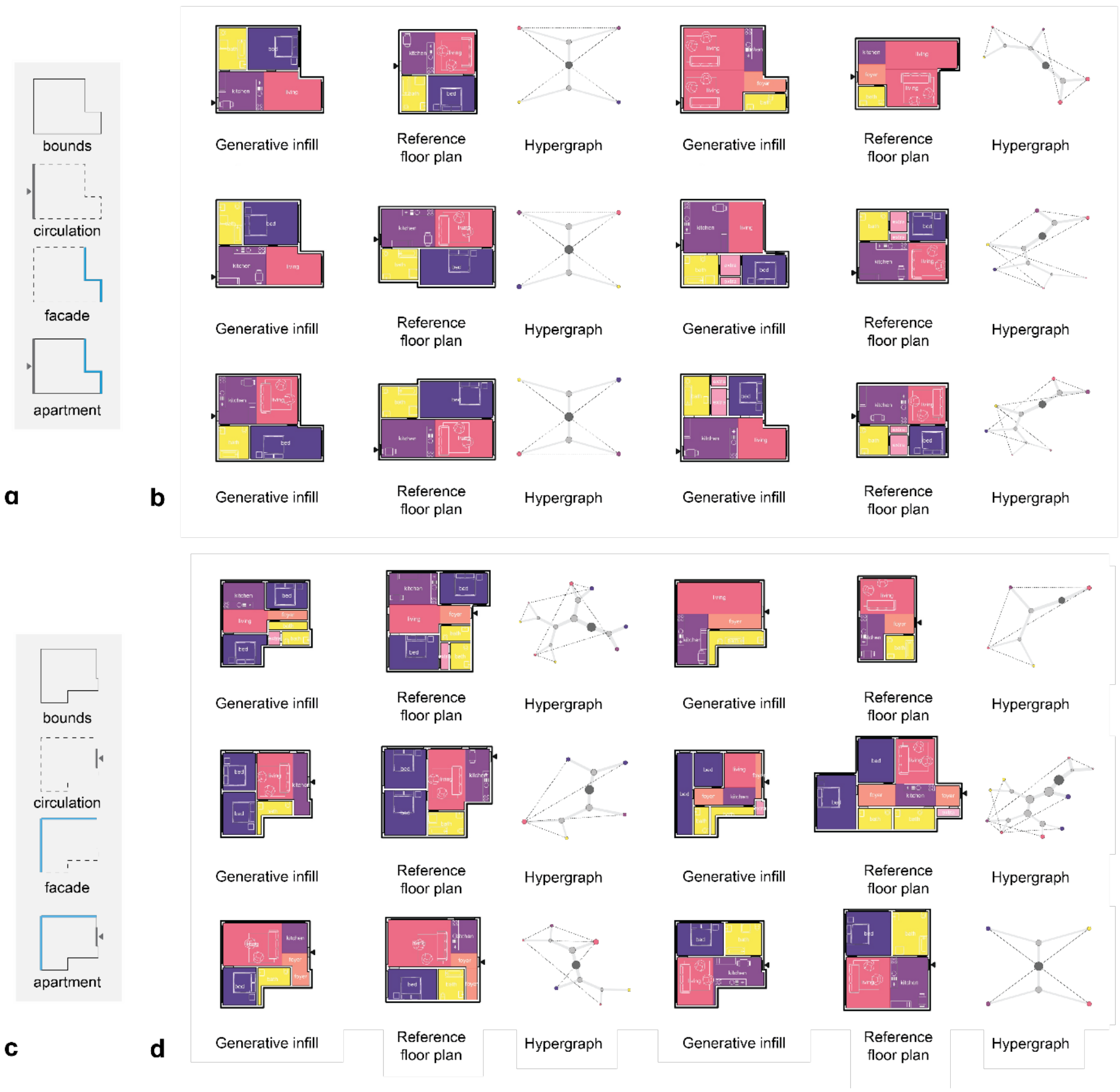
Supplemental Figure 6: Procedure of furniture placement inside a sample room. Furniture objects are defined with a circulation boundary that is allowed to intersect with other circulation boundaries and blocked boundary that is not allowed to intersect with any object (a). A bed and drawer are placed in the bedroom (b). The circulation area of the door is subtracted from the interio r boundary to create the interior placement polygon (c). The first furniture item (the bed) is placed along the first edge of the interior placement polygon (d) after successful placement the circulation bounds are subtracted from the interior boundary. The second furniture item (the drawer) is placed along the next line segment of the interior boundary. The blocked boundary of the drawer intersects with the circulation boundary of the bed (e), which is an infeasible placement. The interior boundary is shortened by the circulation boundary of the drawer (f), and in a second try the drawer is placed on the next line of the interior boundary polygon (g). A successful placement is achieved (h) and the furniture area of the room can be calculated (i).

67

68 *Supplemental Figure 7: Minimum furniture blocks by apartment size*

1.5. *Supplementary note on hypergraph samples*

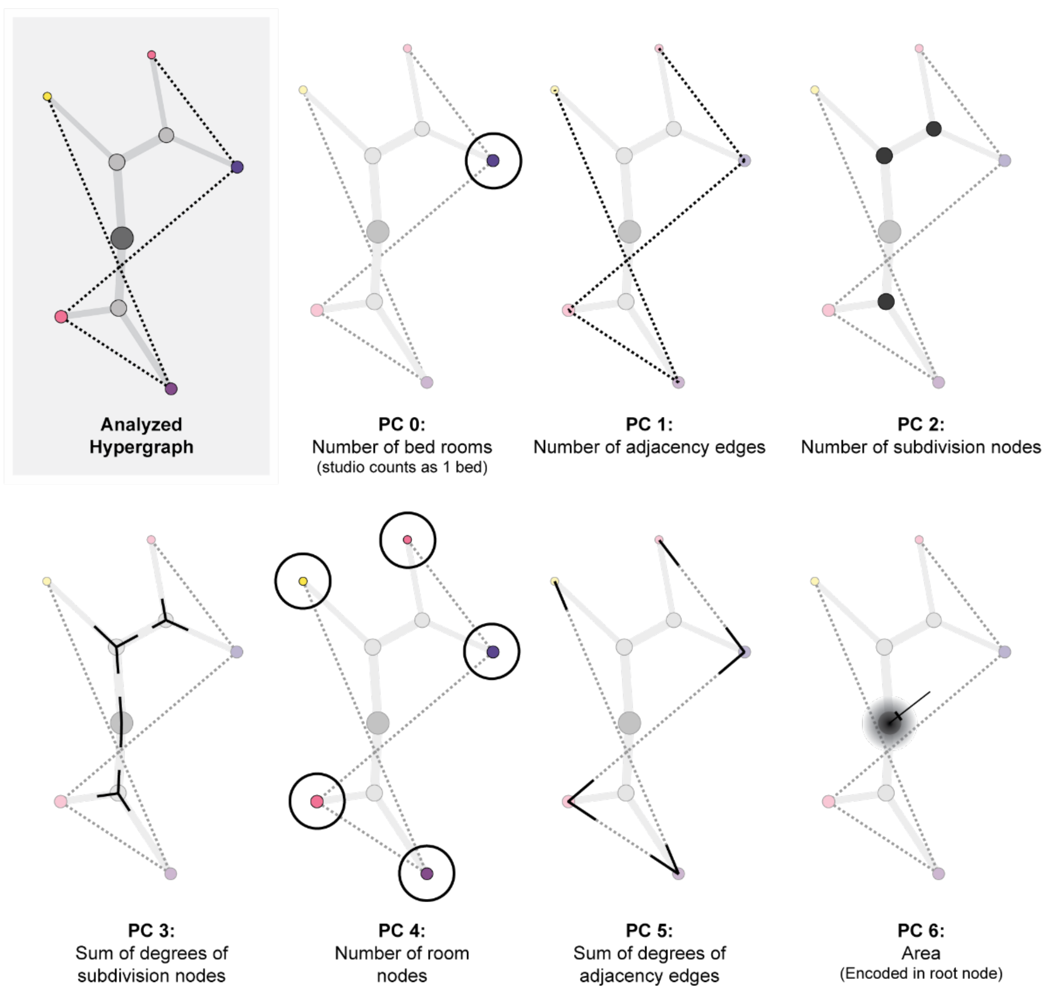
Supplementary Figure 8 shows different examples of artificially generated layout subdivisions using the hypergraph method. The hypergraph subdivision is oriented to match the orientation of the circulation, resulting in different floor plan layouts.



Supplemental Figure 8: Input boundaries (a,c) with applied generative infill (b,d), where from left to right the resulting floor plan, the source floorplan and its corresponding hypergraph are shown.

1.6. *Supplementary note on hypergraph PCA analysis*

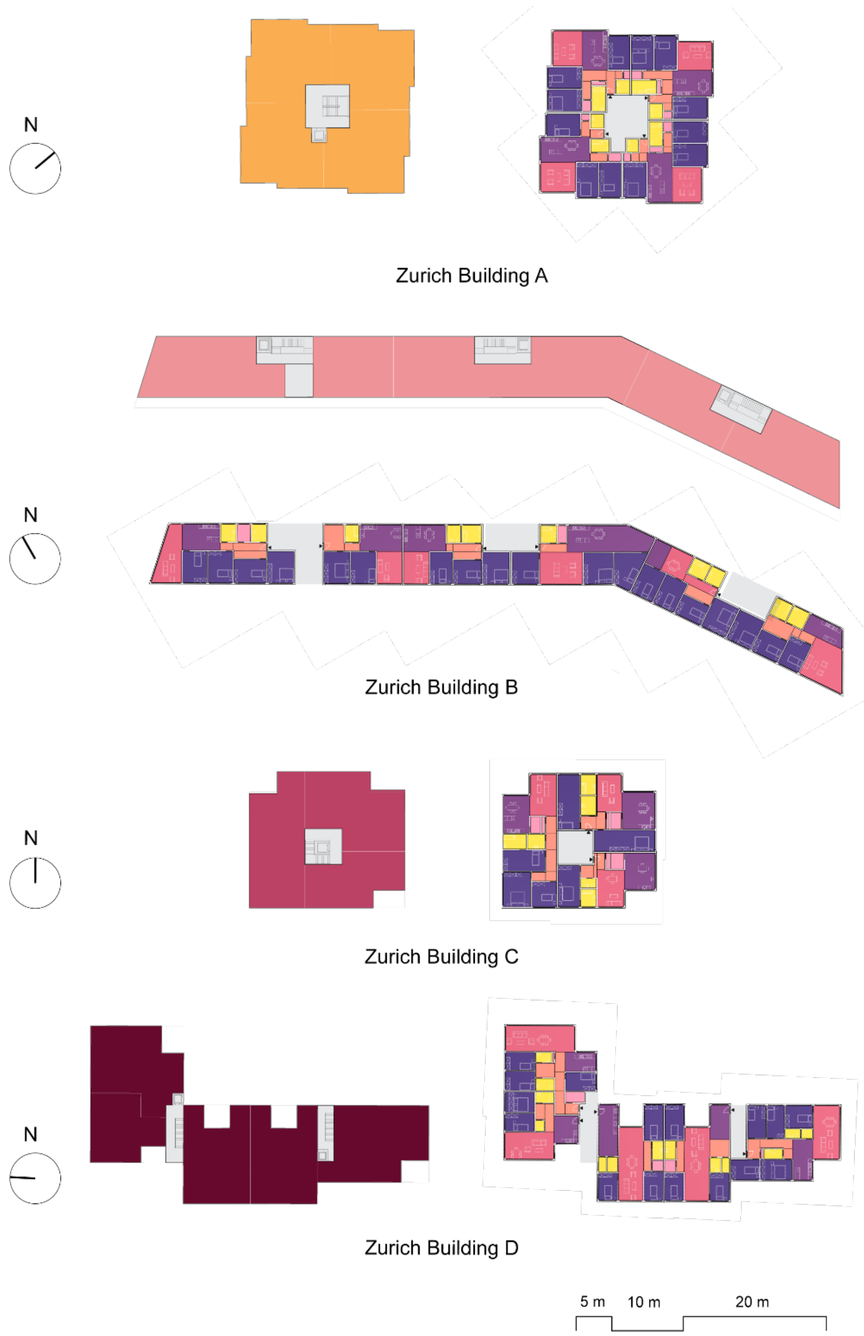
To compare the hypergraphs, a principal component analysis (PCA) is conducted. As hypergraphs have different numbers of nodes and edges they can't be directly used for comparison. Instead, key attributes of the graphs of different lengths are converted into a one-dimensional vector. Supplemental Figure 9 outlines different attributes of the hypergraph that can be extracted and that capture the differences between different hypergraphs.



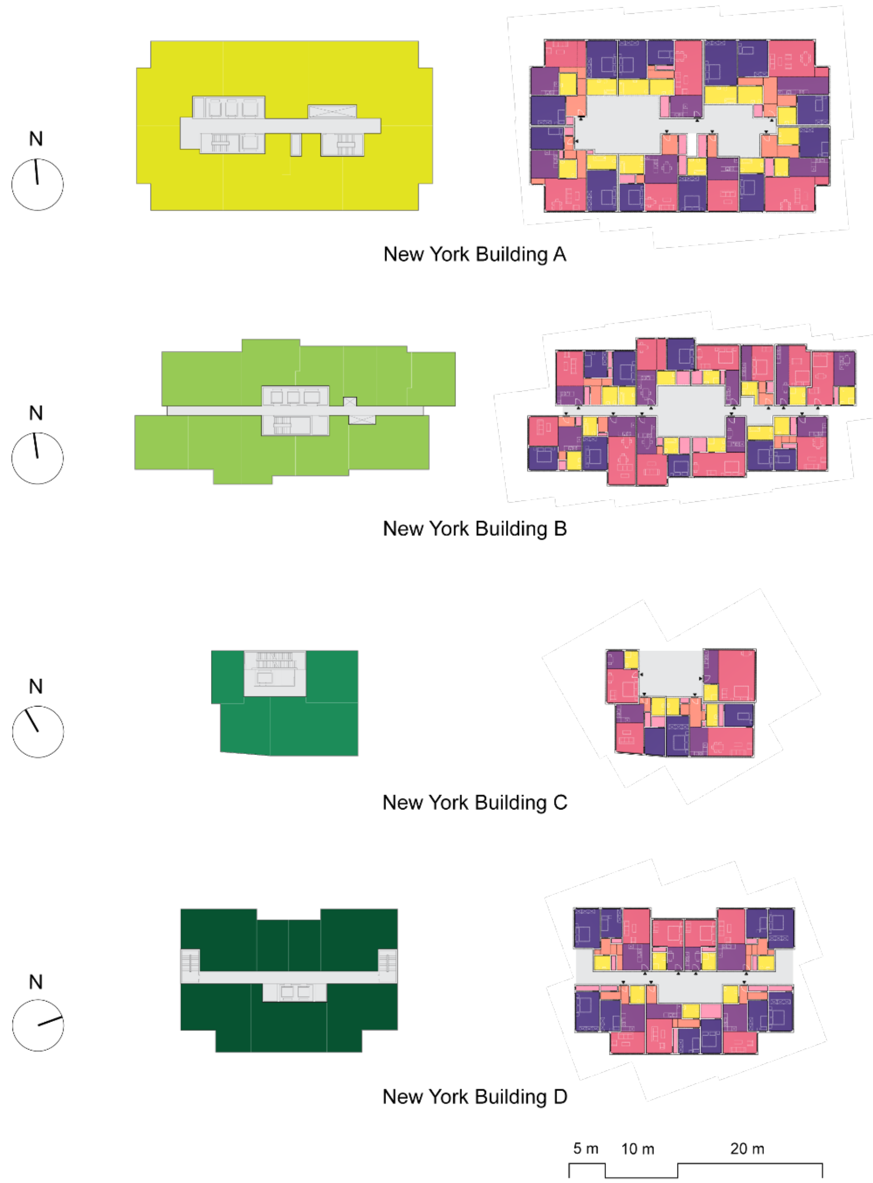
Supplemental Figure 9: Visualization of graph properties that were used for inputs into PCA.

1.7. *Supplementary note on artificially generated floor plans*

For testing of the hypergraph method to generate artificially generated floor plans for real world buildings a dataset of 12 buildings from Zurich, Singapore and New York was created. The buildings have been anonymized and are shown in Supplementary Figures 10-12.



Supplemental Figure 10: Zurich Building A-D

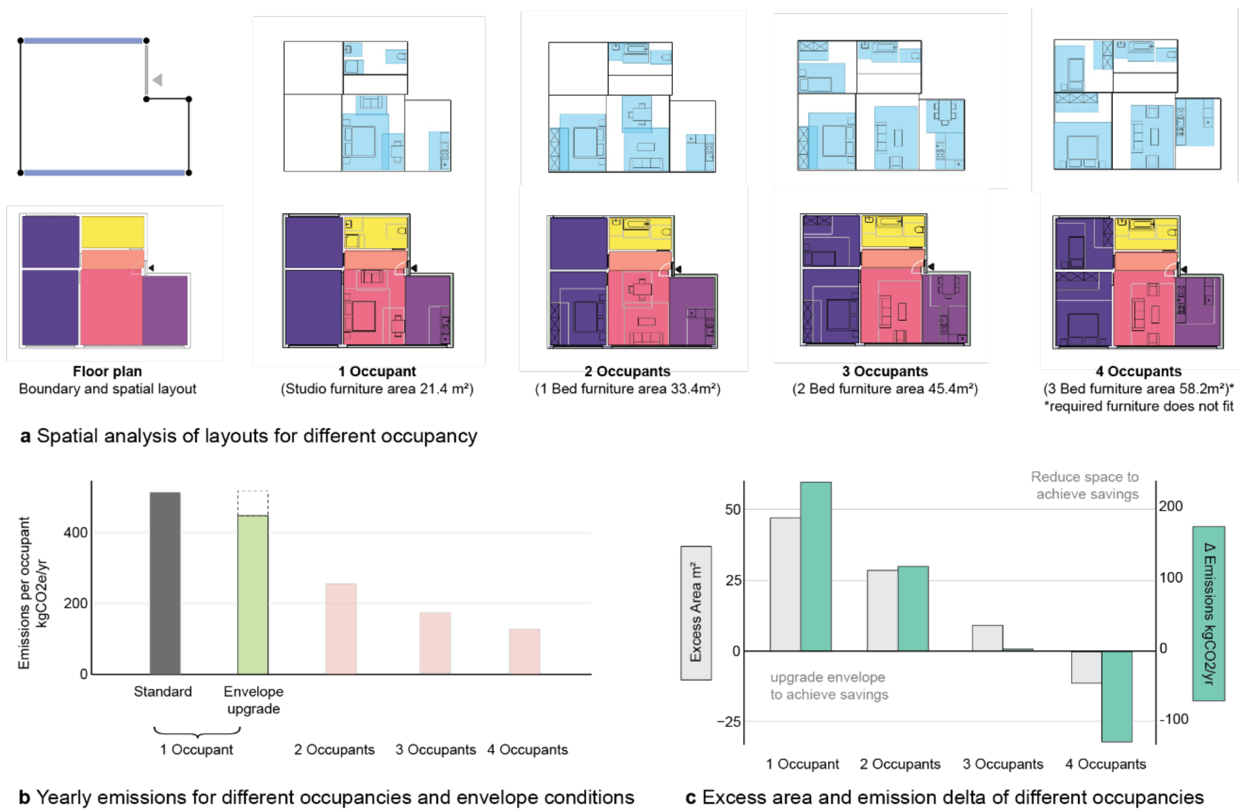


Supplemental Figure 11: New York Building A-D

Supplemental Figure 12: Singapore Building A-D

1.8. *Supplementary note on occupancy*

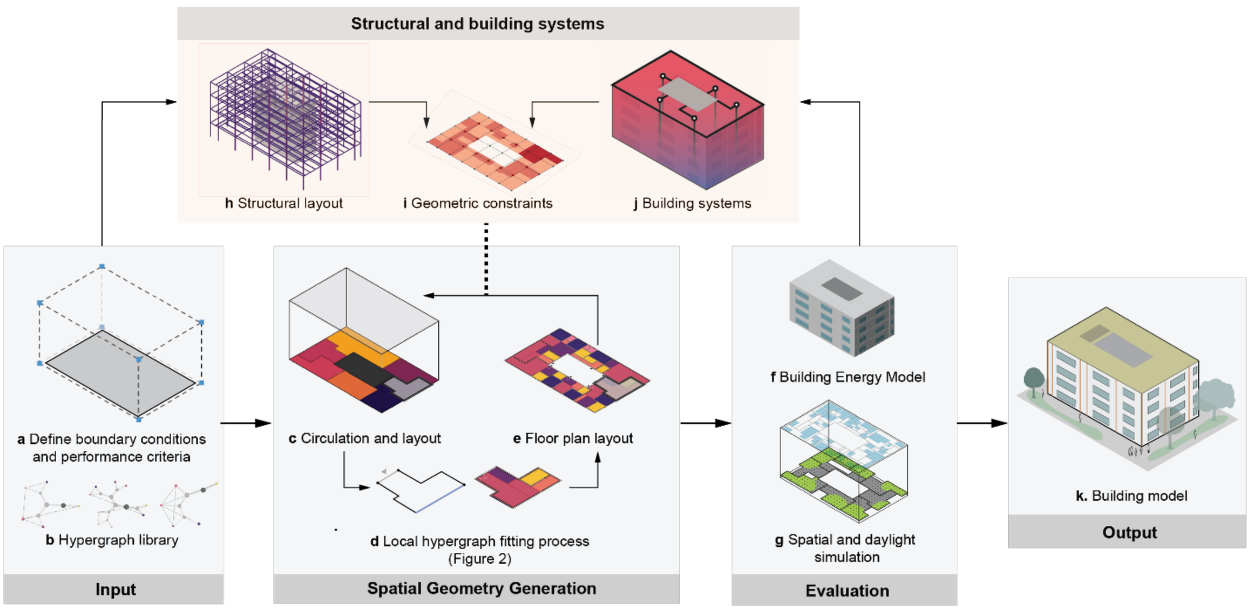
The furniture placement procedure can be used to evaluate and compare apartment layouts from an occupancy perspective. The appropriate reference apartment can be chosen automatically via number of rooms of an input floor plan layout or chosen manually via expected occupancy. As shown in Supplemental Figure 13a different furniture blocks are chosen to populate an apartment for different occupancies. This reveals different yearly emissions per occupant (Supplemental Figure 13b) and different excess delta emissions per apartment, depending on the number of occupants (Supplemental Figure 13c).



Supplemental Figure 13: Impact on emissions of normalizing different occupancies of an apartment unit. The same apartment unit is filled with furniture blocks for different occupancies (a). Emissions are shown normalized by the number of occupants (b). Different occupancies result in different excess emissions from excess floor area (c).

1.9. *Supplementary note on integration of hypergraph into design automation workflow*

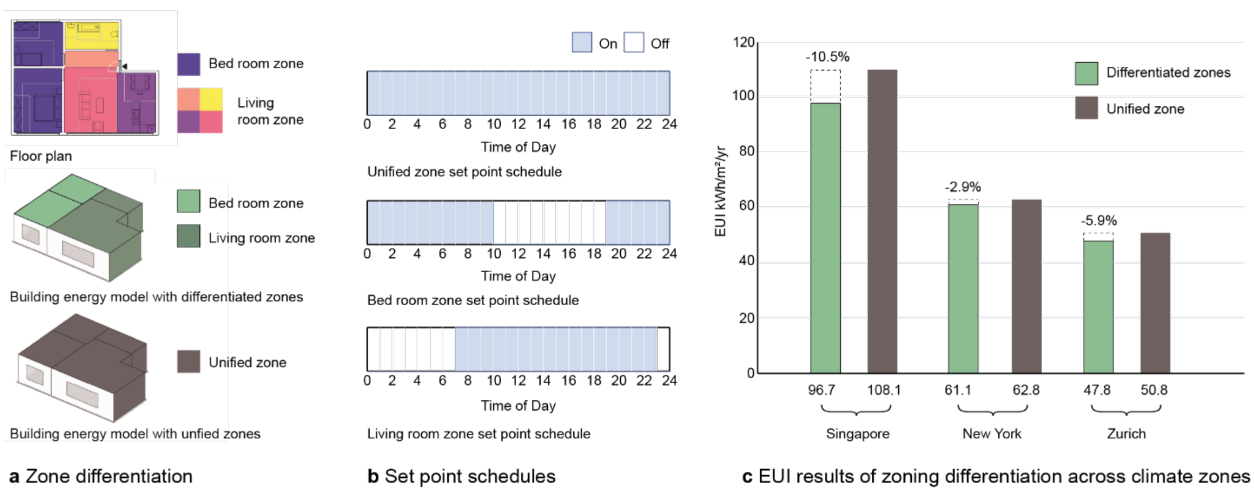
The hypergraph framework can be used stand-alone to evaluate different apartment floor plans of a single unit. Furthermore, it could be integrated into larger design and analysis workflows to evaluate or artificially create whole building designs. A sample workflow and its integration with the hypergraph framework is shown in Supplemental Figure 14. Geometric constraints from structure and building systems can inform the hypergraph generated interior layouts. The detailed building energy model can be used to dimension and inform the MEP system or benchmark low-carbon alternatives to cooling, such as natural ventilation. The Building Energy Model (BEM) created through the hypergraph method can be used to both dimension the requirements of MEP systems and size heating and cooling equipment.



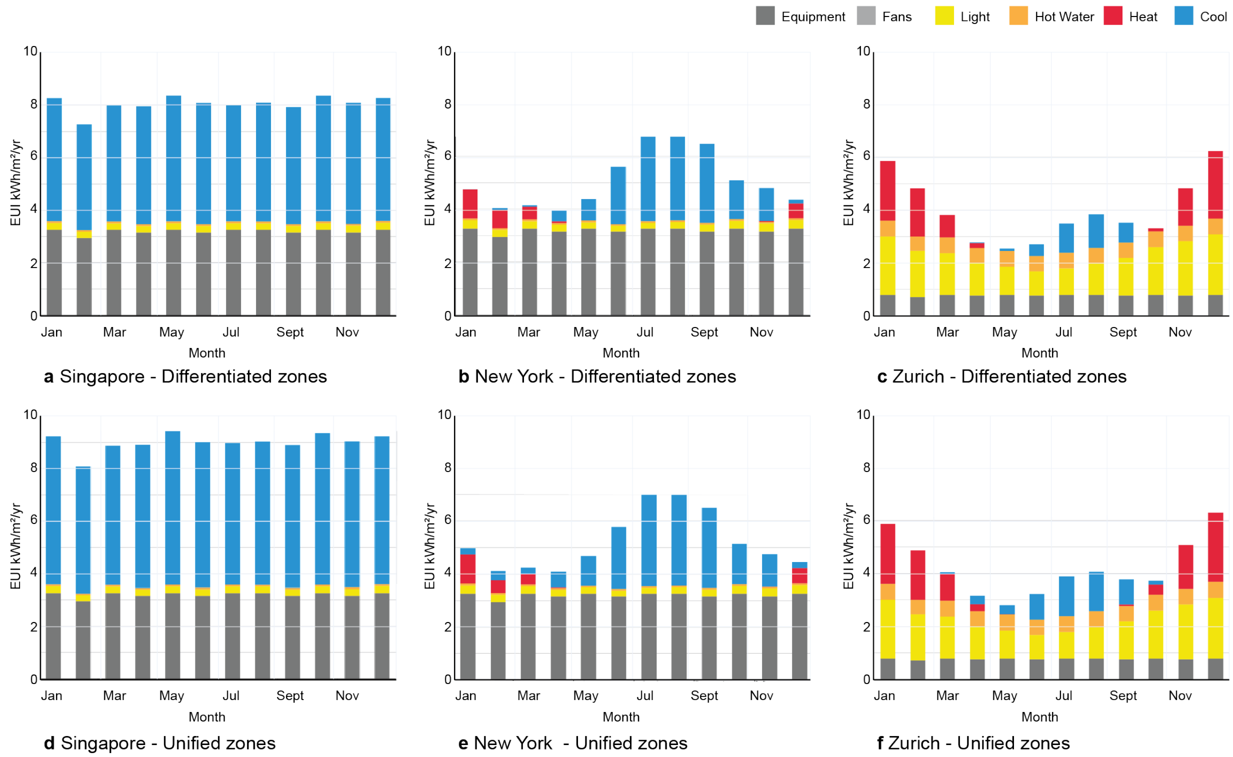
Supplemental Figure 14: Integration of the hypergraph into larger design workflow for a residential building. Boundary conditions of the design problem, including building massing are defined (a) and a hypergraph library selected (b). The circulation and unit mix are defined (c) and for each unit a hypergraph is fitted (d) (as shown in Figure 2) to create a floor plan layout (e). From the floor plan a volumetric Building Energy Model (BEM) can be created (f). The layout is evaluated spatially and for daylight (g). If the performance criteria are satisfied a final building model (k) can be output. The floor plan layout can be further evaluated and from a structural and MEP perspective, where a structural layout (h) can impose geometric constraints (i) on the floor plan. The BEM can be used to estimate energy use and dimension the building's building systems (including heating, cooling and ventilation) (j), MEP systems can further add geometric constraints (i) to the floor plan and influence the fitting process.

1.10. *Supplementary note on differentiating zones in building energy model by occupancy*

In the building energy model (BEM) of the apartments, we do not differentiate usage patterns and occupancy of different rooms. Understanding interactions between occupant behavior is poorly understood and typically not applied to BEM in early stages of design. In order to compare the impact of a more detailed BEM with differentiated zoning by occupancy, we simulate a sample apartment across different cities. As shown in Supplemental Figure 15, the two-bedroom apartment was simulated with unified zones, where set points for cooling and heating were set at all times of the day. In a second simulation round the zones were split into bed room and living room zones, where the set points are turned on and off, depending on the use of the space. This differentiation of zones creates lower heating and cooling demands (Supplemental Figure 15c) due to the fact that in the unoccupied times, temperatures are allowed to drop and rise above the set point temperatures inside the zones. The modest difference of unified and differentiated zoning throughout the year can be seen in further detail in Supplemental Figure 16. In addition to the different climate zones and prevalent weather in Zurich, Singapore, and New York, the different load profiles can be attributed to different equipment usage and light power densities, as defined in the energy simulation templates of ASHRAE 90.1 and SIA 2024, as utilized for the energy simulation.



Supplemental Figure 15: Impact of differentiated zones in building energy model. Visualization of zones corresponding to bed and living areas (a). Set point schedule in energy model (b) for heating and cooling set to 19-26 Degrees Celsius. A bar chart of the results of the yearly energy simulation (c) comparing differentiated and unified zoning in the energy model.



Supplemental Figure 16: Detailed results of the yearly energy simulation for differentiated zoning by use type (a-c) and unified zones across use type (d-f).