nature portfolio

Peer Review File

Creating and controlling global Greenberger-Horne-Zeilinger entanglement on quantum processors

REVIEWER COMMENTS

Reviewer #1 (Remarks to the Author):

The manuscript “Creating and controlling global Greenberger-Horne-Zeilinger entanglement on quantum processors” by Bao et al. presents a series of experiments on generating, preserving, and controlling the GHZ entanglement. The authors create GHZ states up to 60 qubits with fidelities no less than 0.59. As far as I know, this is the largest GHZ state with genuine entanglement generated among various quantum platforms, which sets the current record. This achievement is realized using an efficient generation protocol based on high-fidelity single- and two-qubit quantum gates. What is more impressive is that the authors go a step further to protect and manipulate the fragile GHZ state. They design a novel strategy to stabilize the GHZ states against noises within a many-body scarred discrete time crystal (DTC). This DTC is constructed based on a 36-qubit Ising ring, in which two special Floquet eigenstates are nonthermal, forming the time-crystalline order. They experimentally realize it with digital quantum circuits on a 36-qubit quantum processor, and observe an enhanced lifetime of GHZ state compared with that of a noninteracting dynamical decoupling scheme. They also show the dynamic switching between protecting and destroying the target GHZ state.

This manuscript is written in a clear way. The experimental data and illustrations of figures are of high quality, which nicely demonstrate the main conclusions, and are well supported by detailed supplementary material. In summary, this work tackles the timely scientific question of generating, preserving, and manipulating large-scale entanglement, with novel protocols and state-of-the-art experiments.

I have some comments that the authors should address.

a. Benchmarking of GHZ states. To obtain the fidelity of a GHZ state, one needs to measure two diagonal terms and two off-diagonal terms of the density matrix. The authors estimate the off-diagonal terms with MQC protocol and show their values in Fig. S7 and Fig. S8. However, the data for diagonal terms are absent. Showing them in the supplementary material would make the benchmarking results more convincing.

b. Description of the model. Hamiltonian (2) is a perturbed Ising model under periodical drives and forms a cat scar DTC in the Fock space localized subspace. In addition to the necessary parameters for constructing the Ising ring, many other parameters also appear in this model, such as \phi_1, \phi_2, \lambda_1, and \lambda_2. The authors should explicitly explain why these parameters are introduced in the model.

c. Choice of parameters. In the experimental realization, the authors implement the Hamiltonian (2) with \phi_1=-\pi/2, \phi_2= \pi/2-0.6, \lambda_1= \lambda_2=0.05. The authors should explain more about how they choose the values of these parameters.

d. Free-decay data. The authors claim that the GHZ state phase oscillation of the free-decay case without any protection drops three times more quickly than that of the DTC case. However, the data for the free-decay case is not shown in the main text and supplementary material. I suggest the authors provide these data in the supplementary material.

Reviewer #2 (Remarks to the Author):

This manuscript by Bao et al. presents an experiment where the authors first create large-scale GHZ states with up to 60 qubits and demonstrate their fidelities are above the threshold of 0.5 for multiqubit entanglement. The authors then proceed to evolve the GHZ states under a family of Floquet unitaries which host a few scar states with far-detuned quasienergies. The large overlap between the GHz states and the scar eigenstates allow signatures of the GHz states to be preserved for some 30 cycles of the Floquet drive. The authors believe the techniques demonstrated here are a promising path toward preserving large scale entanglement.

While the technical aspects of the paper mostly appear sound, I cannot recommend the manuscript for publication in Nat Comm, at least not in its present form. My main criticisms of the work are as follows:

1) While large scale GHz state generation is nice to achieve here , there is no surprise to see this capability given the single- and two-qubit fidelities reported by the authors. If this is the first work where such fidelities have been reported, some credit may be given to the impressive hardware advancements. However, similar processors with similar gate fidelities have been used by the same group in recent publications (e.g. Fibonacci anyon work), and a repeated appreciation of the same technology using another text-book demonstration is somewhat lessened in impact.

2) The GHZ protection by evolving with DCT-like Floquet unitary is interesting to see, but I fail to appreciate the impact of this demonstration. From the physics perspective, it is guaranteed that an initial state with large overlap with a scar state would survive for quite some time. What is the insight here?

3) From an application perspective, the authors seem to be making the case that the protocol offers a way to protect the GHz states against noise. My question here is what if you just do careful spin echo on the state, for example using an XY4 type of sequence which is designed to be also robust against small perturbations in the pulses? Would you get much better protection that way, since it's much simpler and doesn't involve any two-qubit gates at all? Without seeing a clear comparison with state-of-the-art dynamical decoupling sequences, I do not think this convoluted scheme for state preservation has much value beyond an interesting demonstration.

4) Lastly, since the Hamiltonian has to be in 1D, the protocol seems to be even further limited in use since generically it is not possible to deform any 2D geometry into a 1D closed ring form as the authors have done.

Reviewer #3 (Remarks to the Author):

Bao et al. experimentally prepare many-qubit (60 or 36) Greenberger-Horne-Zeilinger (GHZ) states in an array of superconducing transmon qubits. They verify that their prepared states have high fidelity with a perfect GHZ state. Further, the authors apply driving to their prepared state such that the GHZ state is a time-crystalline steady state embedded in the spectrum of the evolution operator. They claim this procedure can be used to stabilize the GHZ state, and demonstrate that gates can be performed on the state without interrupting the stabilization procedure.

This work seems to have the most relevance to the continuing efforts to simulate interesting quantum dynamical systems in synthetic devices. The preparation of the GHZ steady state of a discrete time crystal is a noteworthy achievement, and should be published. The authors convincingly demonstrate that the time-crystalline realization of the GHZ state survives longer than a GHZ state which evolves under non-interacting Rabi drives when subjected to a particular perturbation.

Meanwhile, the claimed relevance to future applications of quantum technologies is insufficiently substantiated. GHZ states are indeed very useful for metrology, and I am somewhat familiar with models of measurement-based quantum computation which use GHZ states as sources of entanglement to replace strong interactions. However, both of these applications seem unrelated to a GHZ state of transmons. Strong interactions are already available in this case, and I am not familiar with any proposal for metrology using highly entangled states of superconducting qubits. The references accompanying the claim that GHZ states "play vital roles… in future quantum technologies such as fault-tolerant quantum computation" are too broad to understand what the authors are alluding to.

Addressing the following questions and comments would improve the paper.

1. Clarify or remove the claims of application to quantum computation, as outlined above.

2. When the authors compare the DTC evolution to Rabi driving, what is meant by the brief comparison to "free decay"? Is this the decay of the qubits with no circuit applied, due only to uncontrolled interactions among the qubits or with the environment, or is this U_1 of equation (2) without the pi pulse?

2a. "Protection" of the GHZ state in this context somewhat implies that the lifetime of the GHZ state with driving is longer than with no driving. Clarifying whether or not this is actually the case is important.

3. The x axis in Figure 3 is the number of drive cycles. This is a fair comparison between Rabi and DTC only when the primary source of error is the number of gates, and not the native relaxation and dephasing of the transmons, without gates applied. Can the authors verify that this is the case? Relatedly, how long is t/T=30 in seconds? Is it the same for Rabi and DTC?

List of changes

(All changes are marked in red in the main text and the Supplementary Information.)

1. We have added a new supplementary section (Supplementary Section 5D3) to compare quantum coherence including the case of DTC, Rabi oscillators, and free-decay. New figures (Fig. S22, Fig. S23) are included.

2. We have added a new supplementary section (Supplementary Section 5E) to extend cat scar DTCs into higher dimensions and open boundary cases. New figures (Fig. S24, Fig. S25) are included.

3. We have added a new supplementary section (Supplementary Section 5F) to discuss GHZ states' role in benchmarking spectral-paired cat eigenstates. New figures (Fig. S26, Fig. S27) are included.

4. We have added a new figure (Fig. S6) to illustrate the diagonal elements of density matrices in Supplementary Section 3B.

5. Various minor typos have been fixed, and other text modifications prompted by the Reviewers. These are left colorized for easier identification.

Point-by-Point Response

Response to Reviewer # 1

We thank the Reviewer for highlighting that our work *"sets the current record"* in generating the *"largest GHZ state with genuine entanglement*", and *"the authors go a step further to protect and manipulate the fragile GHZ state."* We also greatly appreciate the Reviewer's valuable suggestions, and have addressed all of them carefully, which helps us improve the manuscript significantly and further sharpen our presentation. Major modifications include three aspects:

1. In the revised Supplementary Information, we present the diagonal terms of the density matrices for the GHZ states;
2. Motivations for our model and a detailed explanation of the parameter choices are included in the revised main text;
3. Experimental data and discussions for the GHZ state under the free decay are included to compare with Rabi and DTC cases.

With these modifications, we believe our manuscript is now ready for publication in Nature Communications. Detailed responses to the Reviewer's comments are provided below.

Comment 1.0: *The manuscript "Creating and controlling global Greenberger-Horne-Zeilinger entanglement on quantum processors" by Bao et al. presents a series of experiments on generating, preserving, and controlling the GHZ entanglement. The authors create GHZ states up to 60 qubits with fidelities no less than 0.59. As far as I know, this is the largest GHZ state with genuine entanglement generated among various quantum platforms, which sets the current record. This achievement is realized using an efficient generation protocol based on high-fidelity single- and two-qubit quantum gates. What is more impressive is that the authors go a step further to protect and manipulate the fragile GHZ state. They design a novel strategy to stabilize the GHZ states against noises within a many-body scarred discrete time crystal (DTC). This DTC is constructed based on a 36-qubit Ising ring, in which two special Floquet eigenstates are nonthermal, forming the time-crystalline order. They experimentally realize it with digital quantum circuits on a 36-qubit quantum processor, and observe an enhanced lifetime of GHZ state compared with that of a noninteracting dynamical decoupling scheme. They also show the dynamic switching between protecting and destroying the target GHZ state.*

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Response 1.0: We thank the Reviewer for the positive evaluation of our manuscript.

Comment 1.1: *I have some comments that the authors should address.*
a. Benchmarking of GHZ states. To obtain the fidelity of a GHZ state, one needs to measure two diagonal terms and two off-diagonal terms of the density matrix. The authors estimate the off-diagonal terms with MQC protocol and show their values in Fig. S7 and Fig. S8. However, the data for diagonal terms are absent. Showing them in the supplementary material would make the benchmarking results more convincing.

Response 1.1: The Reviewer raised a good point. Yes, it will be more convincing to also show the diagonal elements of the density matrices. These data have been visualized in the Figure S6 of the updated Supplementary Information now. We also append this figure below for the convenience of the Reviewer to check.

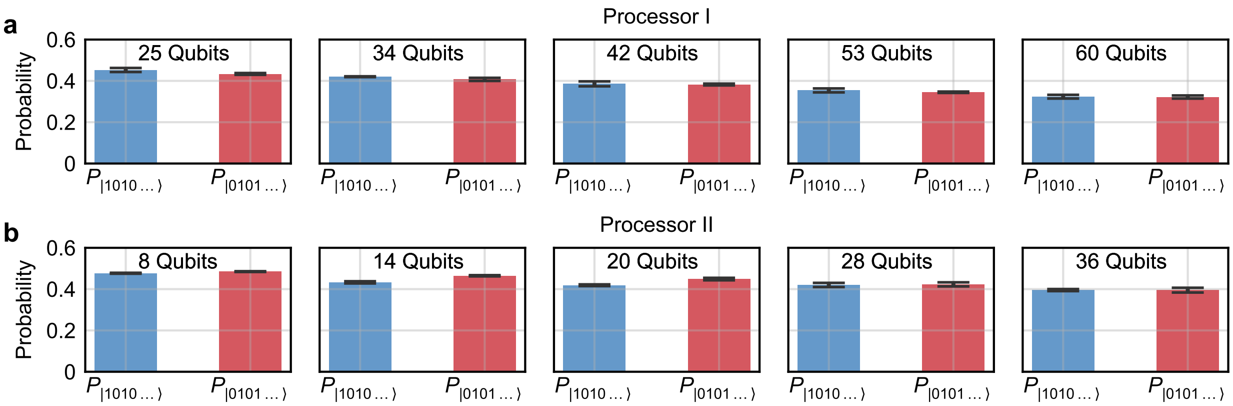


Figure 1: [For Response 1.1] Diagonal terms of GHZ states for different system sizes. **a** and **b** are the measured diagonal terms $P_{|1010...\rangle}$ and $P_{|0101...\rangle}$ on processor I and II, respectively. Data with error bars are repeatedly measured five times and error bars stem from their standard deviations.

Comment 1.2: *b. Description of the model. Hamiltonian (2) is a perturbed Ising model under periodical drives and forms a cat scar DTC in the Fock space localized subspace. In addition to the necessary parameters for constructing the Ising ring, many other parameters also appear in this model, such as ϕ_1, ϕ_2, λ_1, and λ_2. The authors should explicitly explain why these parameters are introduced in the model.*
Comment 1.3: *c. Choice of parameters. In the experimental realization, the authors implement the Hamiltonian (2) with $\phi_1 = -\pi/2$, $\phi_2 = \pi/2 - 0.6$, $\lambda_1 = \lambda_2 = 0.05$. The authors should explain more about how they choose the values of these parameters.*

Response 1.2 and 1.3: We agree with the Reviewer that it will be useful to explain the model and parameters more concretely. This could offer a clear road map for readers to understand the roles of each terms in our model. In the revised Supplementary Information, a new Supplementary Table S2 was added to clarify this.

As the Reviewer mentioned, the two necessary terms required to construct a kicked Ising interaction are the Ising interaction $J_j\sigma_j^z\sigma_{j+1}^z$, and the periodic π-pulses $e^{-i\pi\sum_{j=1}^{N}\sigma_j^x/2}$. All other terms just serve as strong evidence to demonstrate the robustness of the DTC facing more generic perturbations. For clarity, the effects of these perturbing terms are summarized in Table 1 and explained in detail as follows.

Perturbing terms	Purpose of introducing such terms	Parameter range
Single-spin perturbation $e^{i\lambda_1\sigma_j^y/2}$	Check whether DTC is robust against spin-flips trying to delocalize cat eigenstates and detune the period-$2T$ oscillation	$\lambda_1 \ll J \sim 1$
Two-spin perturbation $e^{-i(\sim\sigma_j^z\sigma_{j+1}^x,\sigma_j^x\sigma_{j+1}^z,\sigma_j^x\sigma_{j+1}^x)}$	Transverse interactions to accelerate relaxation and make sure the observed DTC effects represent a steady state	$\lambda_2 \ll J \sim 1$
Longitudinal fields $e^{-i\varphi_1\sigma_j^z/2}$, $e^{-i\varphi_2\sigma_j^z/2}$	Introduce two such terms to solve the dilemma that longitudinal fields should be strong to break integrability, but not to accidentally induce fine-tuned spin echos that interfere with checking DTC robustness by other terms	Strong $\varphi_1, \varphi_2 \sim 1$ to break integrability and avoid echo $\lvert\varphi_1 - \varphi_2\rvert = 0$

Table 1: Motivation of introducing each term and the associated parameter choices.

Checking more varieties of perturbations is inspired by the DTC research over the past several years. Specifically, in our Floquet operator $U_F = U_2 U_1$,

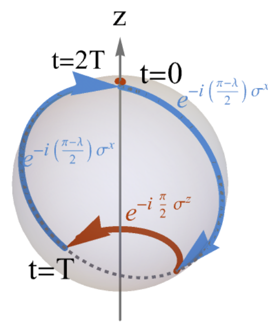
$$U_1 = \left(\prod_{j=1}^{N} e^{-i\varphi_1 \sigma_j^z/2} e^{i\lambda_1 \sigma_j^y/2} \sigma^{-i\varphi_2 \sigma_j^z/2} \right) e^{-i\pi \sum_{j=1}^{N} \sigma_j^x/2}, \quad U_2 = e^{-i\sum_{j=1}^{N} J_j \left(\cos^2(\lambda_2)\sigma_j^z\sigma_{j+1}^z + \sin(\lambda_2)\cos(\lambda_2)(\sigma_j^z\sigma_{j+1}^x + \sigma_j^x\sigma_{j+1}^z) + \sin^2(\lambda_2)\sigma_j^x\sigma_{j+1}^x\right)}.$$

(1) Single-spin perturbation $e^{i\lambda_1\sigma_j^y/2}$, with small $\lambda_1 \ll 1$, checks the stability of a DTC against spin flipping perturbations. This is usually the only perturbation term in most of previous works, which is often chosen to be along x axis and incorporated into the π-pulse $e^{-i(\pi-\lambda_1)\sigma_j^x/2}$.

(2) Two-spin perturbation parameterized by $\lambda_2 \ll 1$ is to accelerate the relaxations and make sure that the DTC oscillation observed within the experimental time scale genuinely represents the behavior of late-time steady state rather than the trivial initial slow relaxation. Specifically, it is shown numerically by Sec. IV B of *Phys. Rev. B 108, 104309 (2023)* that even a weak two-spin term can offer a clear distinction within the first 100 periods between persisting DTC dynamics and slow relaxations that eventually give way to thermalization.

(3) Two longitudinal magnetic field terms, φ_1 and φ_2 ($|\varphi_{1,2}| \sim \pi/2$, $|\varphi_1 - \varphi_2| \neq 0 \mod 2\pi$) , break the integrability of the Hamiltonian, and ensure that our DTC is not a simple fine-tuned single-spin-echo model but an exotic many-body effect. Our choice $\varphi_1 = -\pi/2, \varphi_2 = \pi/2 - 0.6$ fulfills such conditions. Specifically, we introduce a large strong longitudinal field $\varphi_1 = -\pi/2$ to break the integrability of the transverse field Ising model. On the other hand, a single parameter $\varphi_1 = -\pi/2$ may induce trivial spin-echo effects in every two periods,

$$U_F^2 \sim e^{-i\pi\sigma^z/2} e^{-i(\pi-\lambda)\sigma^x/2} e^{-i\pi\sigma^z/2} e^{-i(\pi-\lambda)\sigma^x/2} = 1, \tag{1}$$



as intuitively illustrated by the figure on the right. To fully avoid such a situation, one more longitudinal field $\varphi_2 = \pi/2 - 0.6$ is involved to break the echo condition. See more details in Sec. 5D of the Supplementary Information.

Comment 1.4: *d. Free-decay data. The authors claim that the GHZ state phase oscillation of the free-decay case without any protection drops three times more quickly than that of the DTC case. However, the data for the free-decay case is not shown in the main text and supplementary material. I suggest the authors provide these data in the supplementary material.*

Response 1.4: We thank the Reviewer for this suggestion, which led us to enhance the completeness of our data presentation. In the revised Supplementary Information, a new Supplementary Figure S22 has been added to show the free-decay result and its comparison with Rabi driving case and DTC case.

As reported in previous works [e.g., Phys. Rev. Lett. 106, 130506 (2011)], a general system usually experiences correlated phase noise and tends to accelerate the dephasing of the GHZ state with a rate $\sim e^{-N^2 t}$, which is N times faster than the usual $\sim e^{-Nt}$. Such a "superdecoherence" effect quickly destroys a free-decay GHZ state and dominates its lifetime. Therefore, we use the macroscopic quantum coherence between the two components of GHZ state to quantify its lifetime in this case.

As shown in Figure 2, we plot the rescaled Fourier peak $\mathcal{K}'_f(2N)/\sum_q \mathcal{K}'_f(q)$ for different cases to capture the macroscopic quantum coherence in GHZ subspace. The contrast is obvious. Compared with the DTC case (red) where oscillations persist throughout 30 periods, the free-decay case (green) exhibits the fastest drop of quantum coherence, approaching 0 around $t = 10T$ already, three times more quickly than that in DTC.

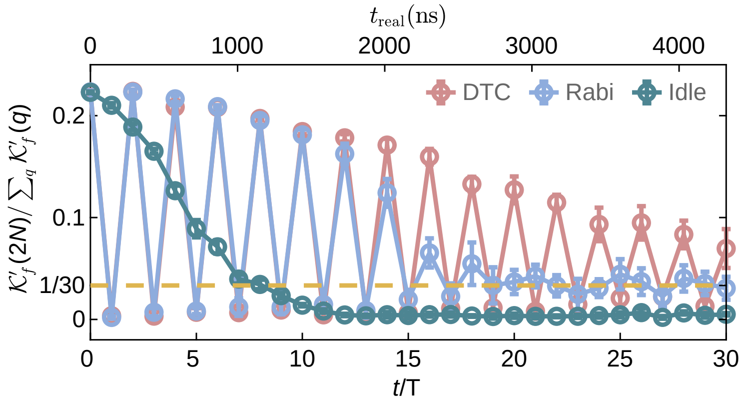


Figure 2: [For Response 1.4] Comparison of quantum coherence under different conditions including the free-decay case. Since the idle case does not involve any unitary perturbation, to enable a fair comparison, we rescale the amplitude $\mathcal{K}'_f(2N)$ with overall amplitudes for all components $\sum_q \mathcal{K}'_f(q)$ within the Fock subspace of the target GHZ state.

In addition, we can see that the Rabi driving case shows a longer coherence time than the free-decay but still much shorter than the DTC case. It begins to drop faster at around $t/T \approx \pi/4\lambda_1 \approx 15$, similar to the result in Fig. 3 of the main text without rescaling. The late-time equilibrium value $1/30$ represents an equal distribution of density matrix in all (totally 30) Fourier components, as without the protection of Ising interaction, the GHZ state tends to be taken into local product state spanning exponentially many ($\sim 2^N$) Fock basis.

For clarity, we briefly summarize the comparison of three cases in the following table. In addition to a newly added Supplementary Figure S22, we also added a new Supplementary Section 5D3 for extended discussions. We also slightly modify the statement of free-decay case to make it more accurate.

Protocol	Idle	Rabi	DTC
Resist correlated dephasing noise?	✗ external superdecoherence $\sim e^{-N^2 t}$ decay	✓ π-pulse for dynamical decoupling	✓ π-pulse for dynamical decoupling
Localized in Fock space?	✓ Fine-tuned	✗ internal unitary perturbation $\sim e^{-Nt^2}$ decay	✓ Ising interaction for robust localization

Table 2: Fate of GHZ states under different driving protocols.

Response to Reviewer # 2 (Remarks to the Author):
We thank the Reviewer for carefully reviewing our manuscript. In the report, the Reviewer pointed out that "large scale GHz state generation is nice to achieve" and "the GHZ protection by evolving with DCT-like Floquet unitary is interesting to see". Meanwhile, the Reviewer also expressed his/her concerns on technical improvement, physical implication, and the advantage of our protocol, and encourages an improved form of our manuscript. All these concerns of the Reviewer can be addressed and we summarize the main points in the following.

1. Compared with the Fibonaccci work, this work investigates totally different physics. Moreover, the qubit number, circuit length, and parallel two-qubit gate numbers exploited here are $2 \sim 3$ times more, which shows our experimental improvement.

2. From aspect of the physical implication, we particularly appreciate the second comment about the physical insight of observing DTCs using a GHZ state. It has inspired a more concrete numerical study showing that evolving a GHZ state in DTCs proves the pairwise cat eigenstates with spectral pairing up to exponential accuracy. This is a defining feature of long-range-entangled DTCs that has been extremely challenging for conventional few-body probes to access.

3. Spin echo sequence suggested by the third comment could suppress *single-spin* perturbations, but fall short of handling the important type of two-spin interaction perturbations, which has been observed in a recent experiment [Phys. Rev. X 10, 031003 (2020)]. Consequently, a scheme with 60 driving steps per cycle has to be introduced there to partially mitigate interaction errors. In contrast, our DTC scheme with 2 driving steps here is able to suppress generic perturbations. We also performed numerical simulation to verify this argument.

4. Cat scar DTCs do not rely on either dimensionality or boundary conditions, which can be directly checked in numerics. Thus, the restriction to 1D ring suspected in the fourth comment appears invalid.

In summary, the updated manuscript has incorporated a more physical discussion on evolving GHZ states in DTCs, pointing out that it offers unique experimental evidence of the cat eigenstate pair, a long-sought feature in many-body theories. Such methods are new and generalizable to arbitrary dimensions. Thus, with these updates, we believe our manuscript is now in a form suitable for publication in Nature Communications.

In the following, we provide detailed responses to the Reviewer's comments.

Comment 2.0: *This manuscript by Bao et al. presents an experiment where the authors first create large-scale GHZ states with up to 60 qubits and demonstrate their fidelities are above the threshold of 0.5 for multiqubit entanglement. The authors then proceed to evolve the GHZ states under a family of Floquet unitaries which host a few scar states with far-detuned quasienergies. The large overlap between the GHz states and the scar eigenstates allow signatures of the GHz states to be preserved for some 30 cycles of the Floquet drive. The authors believe the techniques demonstrated here are a promising path toward preserving large scale entanglement.*

While the technical aspects of the paper mostly appear sound, I cannot recommend the manuscript for publication in Nat Comm, at least not in its present form.

Response 2.0: We thank the Reviewer for the positive comment that "the technical aspects of the paper mostly appear sound". We are also grateful for his/her critical but very constructive remarks, which provides us with more perspectives to further improve our work. We believe the revised manuscript is now suitable for publication in Nature Communications.

My main criticisms of the work are as follows:

Comment 2.1: *1) While large scale GHz state generation is nice to achieve here, there is no surprise to see this capability given the single- and two-qubit fidelities reported by the authors. If this is the first work where such fidelities have been reported, some credit may be given to the impressive hardware advancements. However, similar processors with similar gate fidelities have been used by the same group in recent publications (e.g. Fibonacci anyon work), and a repeated appreciation of the same technology using another text-book demonstration is somewhat lessened in impact.*

We are happy that the Reviewer is interested in the research progress of our group and pays attention to other works from us. We also appreciate the Reviewer's assessment that "large scale GHZ state generation is nice to achieve".

From the perspective of quantum simulation, the ability to emulate various quantum systems is a critical advantage of the gate-based digital quantum simulator. Its programmability enables us to handle a wide range of problems on a single device if high-fidelity parallel single- and two-qubit gates were realized, which is also what the experimentalists are pursuing. Therefore, the general paradigm pushing this field forward is that experimental team makes great efforts to develop a generation of quantum device, which often possesses a certain amount of qubits and a well-calibrated universal gate set, and then theoretical and experimental researchers work together to fully utilize its programmamibilty to explore various scientific problems. For example, Google's Sycamore processor [Nature 574, 505 (2019)] and USTC's *Zuchongzhi* processor [Phys. Rev. Lett. 127, 180501 (2021)] are still in service for researchers to explore problems they are interested in [Science 384, 48 (2024), Nat. Phys. (2024). https://doi.org/10.1038/s41567-024-02530-z].

For the specific case of our manuscript, it investigates totally different physics compared with the Fibonaccci work [Nat. Phys. (2024). https://doi.org/10.1038/s41567-024-02529-6]. Moreover, technically, it is not a "repetition". In this work, we use ~2 times more qubits, ~3 times longer circuit depths, ~3 times more parallel two-qubit gates, and more importantly, we achieve all this even with a lower gate error. In Table 3, we summarize the key parameters for this manuscript and the Fibonacci work for clarity.

	the Fibonaccci work	This work
Number of qubits	27	60
Maximum circuit length	115	315
Two-qubit gate type	CZ	CPHASE
Maximum two-qubit gates run in parallel	6	18
Two-qubit gate Pauli error (%)	0.66	0.58

Table 3: Comparison between this manuscript and our previous Fibonacci work.

Comment 2.2: *2) The GHZ protection by evolving with DCT-like Floquet unitary is interesting to see, but I fail to appreciate the impact of this demonstration. From the physics perspective, it is guaranteed that an initial state with large overlap with a scar state would survive for quite some time. What is the insight here?*

Response 2.2: The Reviewer raised a good point here, which inspires a more thorough discussion on the impact of our work and its physical implications.

The impact of our work involves two aspects.
Technologically, protection of the GHZ state itself is invaluable to realize quantum-enhanced metrology, which has the potential to provide precision measurement beyond classical limits. Such maximamly en-

tangled states are extremely fragile. Consequently, generating, preserving, and manipulating GHZ states are notoriously challenging, which hinders their practical applications. In the "Outlook" section of *Rev. Mod. Phys. 90, 035005 (2018)*, the authors explicitly pointed out that

"Current active research trends are centered on quantum-enhanced metrology taking into account relevant experimental imperfections and searching for protocols where the fragile entanglement is protected against noise sources."

Our experiment exactly tackles these problems by experimentally realizing a protocol to protect the fragile GHZ states against experimental imperfections, including external (e.g., phase noise and spin-flip noise) and internal ones (e.g., perturbations in our model). Such an experimental demonstration addresses central concerns in the community dedicated to quantum metrology with entangled states, and thus tends to have a substantial impact in this field. Meanwhile, as was pointed out in the "Outlook and Future Directions" section of *Rev. Mod. Phys. 95, 031001 (2023)*, a compelling future direction for DTC physics is the potential application in enhanced quantum metrology, as ferromagnetic cat eigenstates hosted by certain DTCs discussed there are speculated to protect homogeneous GHZ states against interactions. Here we employ an unconventional mechanism, Fock-space localized scars, to shield GHZ states of *arbitrary* spin patterns from generic perturbations. Therefore, our experiment offers a timely demonstration in the community interested in DTC physics, which serves to open the door toward the new direction of DTC-enhanced quantum metrology.

Physically, evolving a GHZ state in DTCs provides long-sought direct experimental evidence of spectral-paired cat eigenstates, which is the defining property of long-range-entangled DTCs. Extensive numerics, including the original theory [e.g., Phys. Rev. Lett. 117, 090402 (2016)], has cited the unique exponential lifetime scaling $\sim e^N$ to report the discovery of DTCs with system size N, which stems exactly from the global entanglement of the underlying eigenstates with spectral pairing [Phys. Rev. B 94, 085112 (2016)]. That is, the eigenstates are pairwise GHZ-like cat eigenstates, such that under generic perturbation, their quasienergy can only shift together, but the quasienergy *difference* is fixed to be π up to exponential accuracy e^{-N}. Direct observation of such a theoretical feature has remained prohibitive. Our experiment is the first one offering a solution, that is, by preparing a GHZ state as sensitive entanglement witnesses to unveil the globally entangled and spectral-paired cat eigenstates — the cat scars. Let us explain in more detail below.

Historically, DTC has been widely known for its hallmark signature of period-doubled oscillation of local magnetic order, which represents a spontaneous breaking of time translation symmetry. But recently, it has attracted increasing attention to the question of what are the *unique* signatures of DTCs, as period-doubled oscillations of observables, with certain stabilities, may exist in many scenarios. One peculiar feature for a DTC, which was emphasized since the original sets of studies in 2016 [e.g., Phys. Rev. Lett. 117, 090402 (2016)], is that the lifetime of *local* spin oscillation prolongs *exponentially* with system size N due to the long-range-entangled eigenstates. From the viewpoint of local dynamics, this is surprising: it means that even if we fix the local environment around a spin at j_0 unchanged, and only add a few spins at sites far away, the lifetime of oscillation at j_0 would increase by several orders of magnitude. This is to be contrasted with short-range entangled systems where far-away spins can at most induce some late-time fluctuations. However, in practice, it is a conundrum to distinguish these two cases by local probes in experiments.

For illustrations, we model the local dynamics at a certain site j_0 under the two scenarios in Figure 3. The long-range-entangled case corresponds to the cat scar DTC presented in our main text, where we see that adding a few more spins to the system, while keeping all local parameters unchanged, increases the *local* oscillation at j_0 by orders of magnitudes (i.e. from $\sim 10^6 T$ for $N = 8$ to $\sim 10^9 T$ for $N = 12$). This is to be contrasted against the short-range-entangled case. By setting all distant Ising bonds $J_{|j-j_0|>2} = 0.1$ and increasing local perturbation $\lambda_1 = 1.28$ for the distant sites $|j - j_0| > 2$, the site j_0 becomes short-range-entangled with only the neighboring 2 sites. In this case, we see that adding more spins at the disentangled regions essentially does not

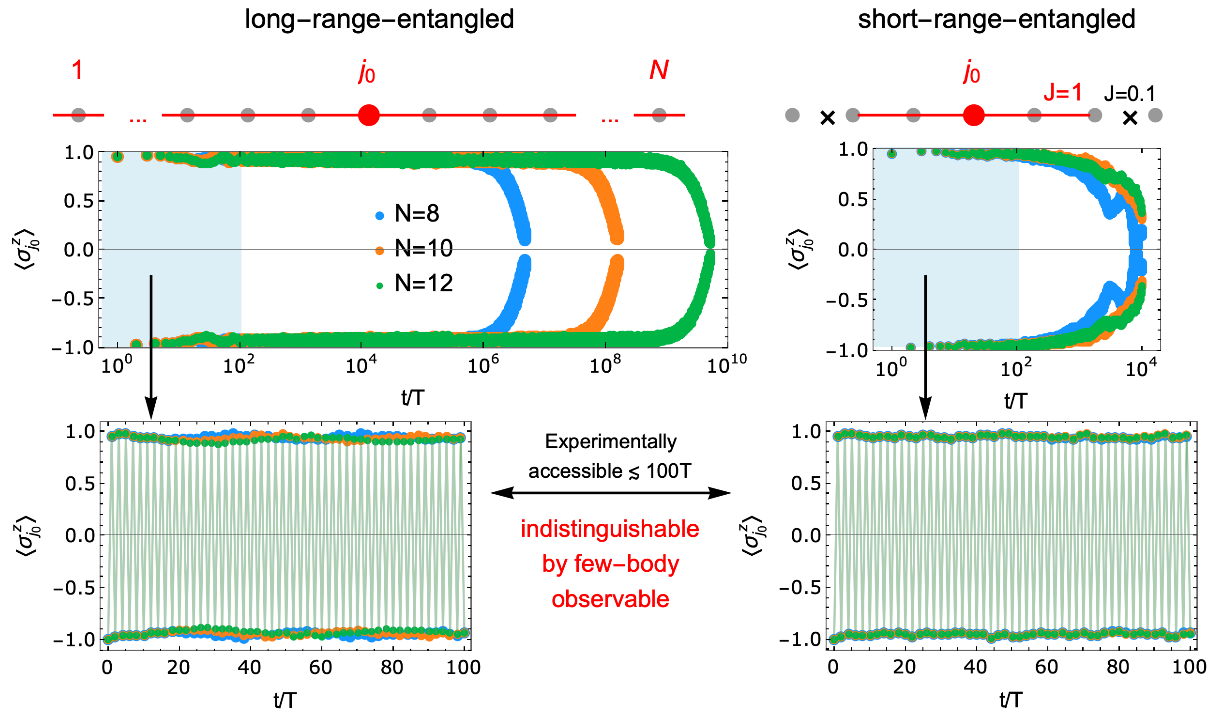


Figure 3: [**For Response 2.2**] Challenge to prove long-range-entangled cat eigenstates in a DTC using conventional few-body observables. (Top left) A long-range entangled DTC in our case with pairwise cat eigenstates gives rise to exponentially long lifetime $\sim e^N$ for the oscillation of $\langle \sigma_j^z \rangle$. (Top right) In contrast, short-range-entangled systems exhibit dynamics dictated by local physics with little relevance to global system size. (Bottom panels) However, experiments across various platforms can only access early time, so it is hard to benchmark the cat eigenstates using local probes unambiguously. Here the long-range-entangled case is numerically simulated using the main text model, where all parameters are the same as in Fig. 3 of the main text except for larger $\lambda_1 = 0.25 < \lambda_c = 0.36$ to observe DTC decay. The short-range-entangled one is further modeled by tuning Ising interaction $J = 0.1$ and perturbation $\lambda_1 = 1.28$ for sites $|j - j_0| \geqslant 3$, such that j_0 is fully disentangled from the system 3 sites away. Initial states are all local product states $|1010\ldots\rangle$.

affect the local oscillation at j_0, as the system sizes $N = 8, 10, 12$ all give the similar $\sim 10^4$ oscillation periods.

However, although different scaling behaviors have been seen in lots of numerical works, it is unpractical for any current experiment to access the exponentially late time of $10^{6\sim12}$ periods. In other words, focusing on the accessible first $\sim 100T$, the long-range and short-range entangled eigenstates render essentially indistinguishable subharmonic oscillations under few-body probes for evolved local product states, as illustrated in the lower panels of Figure 3.

From the above discussion, we identify the crucial experimental objective: accurately determining whether all spins are entangled in the eigenstates. Only when all N spins are entangled would a system yield the characteristic DTC lifetime $\sim e^N$. Otherwise, the system is decomposed into several mutually disentangled regions, and increasing N cannot extend the overall oscillation lifetime exponentially. For such purposes, it can be expected by general arguments that GHZ states overwhelm the local product states in accuracy. For local product states, mutually disentangled spins in the eigenstates do not affect their respective local dynamics. Thus, the signal of disentangled spins can fairly be washed out by the overall oscillations under local probes. In contrast, a single disentangled spin can take a GHZ state entirely out of its original Fock subspace. Thus, the N-body observable $\mathcal{K}'_f(2N, t)$, which measures the macroscopic quantum coherence *in the original Fock subspace*, can be brought all the way to 0 by a single disentangled spin in the eigenstate.

For concreteness, we use an idealized example to illuminate the above arguments. We will compare the effectiveness of two methods for such purposes in the following, namely, the conventional few-body probe (i.e. local magnetic order) associated with local product states, and the N-body probe we adopt for the GHZ state.

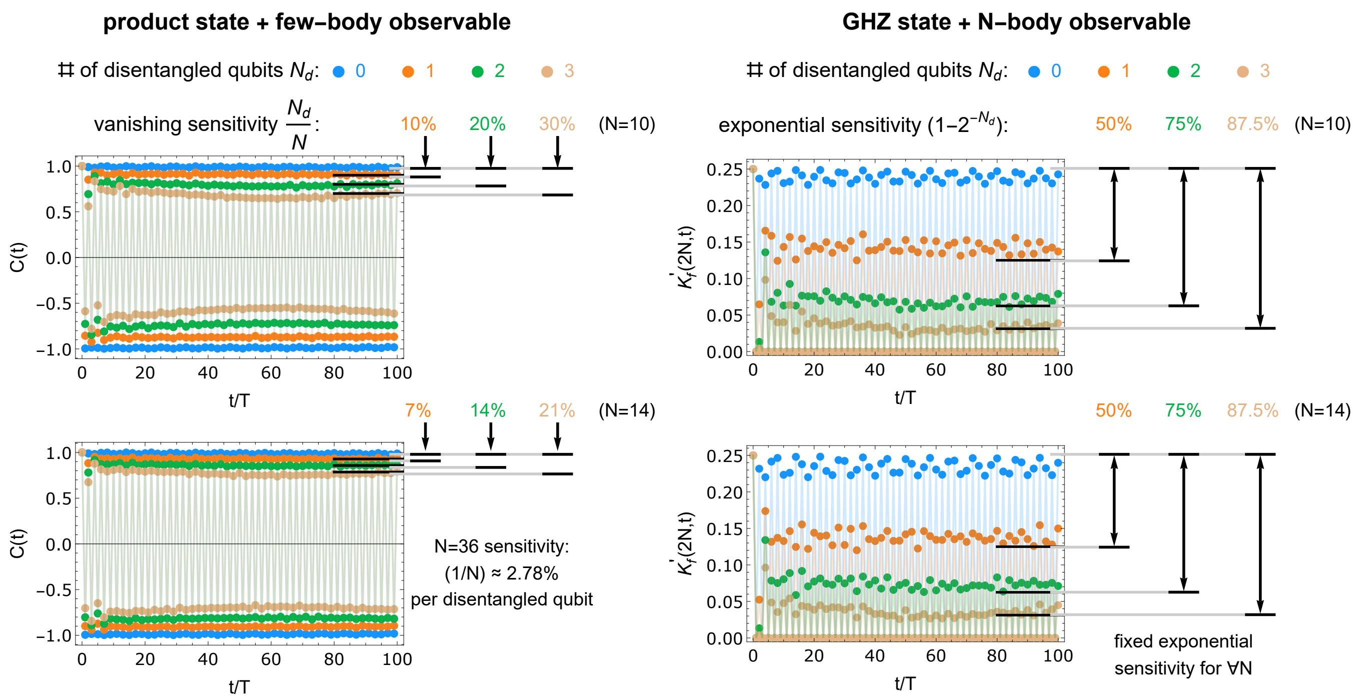


Figure 4: **[For Response 2.2]**Comparison of sensitivity in probing N-qubit entangled cat eigenstates using two methods. Here, we use the same model U_F in both cases, where we start from the main text model with the same parameters as in Fig. 3 of the main text. Then, we disentangle a certain qubit (i.e. at site k) by *locally* weakening its two neighboring Ising bonds $J_{k-1} = J_k = 0.1$ and enhancing the perturbation at site k to $\lambda_1 \in [0.8, 1.2]$, while parameters elsewhere are unchanged. This way, we can controllably disentangle N_d qubits from the system. To extract generic behaviors independent of microscopic details, for each case of $N_\mathrm{d} = 1, 2$, or 3, we average over 100 samples with random locations and random local $\lambda_1 \in [0.8, 1.2]$ for the disentangled qubits. **(Left panels)** Initial states here are local product states $|101010\ldots\rangle$, and we numerically evaluate the normalized autocorrelators $C(t) = (1/L)\sum_j \langle \sigma_j^z(t)\sigma_j^z(0)\rangle$. We see that each disentangled qubit invokes a minor change $(1/N)$, approaching 0 for large system sizes N. **(Right panels)** Initial states are instead GHZ states $(|1010\ldots\rangle + |0101\ldots\rangle)/\sqrt{2}$, and we calculate the macroscopic quantum coherence $\mathcal{K}_f'(2N, t)$ as in the main text. We see a fixed exponential accuracy as N_d disentangled qubit suppresses the oscillation amplitude to $1/2^{N_\mathrm{d}}$, independent of system size N.

In Figure 4, we show the numerical results where we use two methods to benchmark the same model U_F, in which a controllable number of spins are disentangled. Specifically, we start from our main text model with parameters the same as those in Fig. 3 there, such that all N qubits are entangled in the cat scar eigenstates. Then, we controllably disentangle $N_\mathrm{d} = 1$, 2, or 3 spins from the bulk, and see how sensitive the two methods can detect such defective spins. This is achieved by setting the two bonds connecting the disentangled site with weak interaction $J = 0.1$, and endowing large perturbation λ_1 at the disentangled site. In practice, such disentangled spins can be caused by external circuit or qubit quality differences, or internal parameter differences if disorders exist. Thus, for each of the $N_\mathrm{d} = 1, 2, 3$ case in Figure 4, we take average over 100 samples with random locations and random $\lambda_1 \in [0.8, 1.2]$ for disentangled spins, in order to extract universal behaviors independent of accidental microscopic details. Qualitatively different behaviors then emerge between the two methods in Figure 4.

1. Decomposing the N qubits into several mutually disentangled regions is hardly noticeable by the few-body observable (left panel of Figure 3). Here, we start with the product state $|1010\dots\rangle$ of antiferromagnetic pattern, and calculate the autocorrelator $C(t) = (1/L)\sum_j \langle \sigma_j^z(t)\sigma_j^z(0)\rangle$, which is a widely used quantity in previous experiment and numerics. Without disentangled qubits $N_\mathrm{d} = 0$ (blue), we observe an expected period-2T oscillation between $C(t) \sim (-1)^{t/T}$. Then, we see that disentangling each qubit, in the first 100 periods relevant to experiments, only causes an additive decrease of oscillation amplitude by $(1/N)$, which vanishes for larger system sizes. For instance, in our $N = 36$ qubits system, one disentangled qubit will only incur an amplitude drop by $(1/N) \approx 2.78\%$, a vanishingly small quantity that can fairly be washed out by fluctuations.

2. N-body observable for GHZ states are sensitive to whether there are *any* disentangled qubits (right panel of Figure 3). Sharp contrasts are revealed if we start from a GHZ state $(|1010\dots\rangle + |0101\dots\rangle)/\sqrt{2}$, and calculate the macroscopic quantum coherence $\mathcal{K}_f'(2N,t)$ as in Fig. 3 of the main text. Without disentangled qubits ($N_\mathrm{d} = 0$), as in our main text, the oscillation amplitudes of $\mathcal{K}_f(2N,t)$ approach the value 1/4, which represents an ideally protected GHZ state (see Eq. S36 of the Supplementary Information). Then, each disentangled qubit suppresses the oscillation amplitude by a factor of 1/2, leading to the exponential suppression $1/2^{N_\mathrm{d}}$ if N_d qubits are disentangled. Contrary to the few-body probe $C(t)$ that has shrinking sensitivity for larger systems, the N-body observable $\mathcal{K}_f'(2N,t)$ here has a fixed *exponential sensitivity* regardless of system size. Thus, the observation in Fig. 3 of our main text, with oscillation amplitudes predicted by an analytical theory for cat scar DTCs, offers strong evidence of a globally entangled cat eigenstate pair, because just a few defective spins can bring down the amplitude by orders of magnitude.

Such a contrast is a reflection that few-body observables are only affected by local parameters, but the GHZ state with N-body observable would sensitively detect whether *all* spins are *simultaneously* in the GHZ state subspace. Then, in the latter case, each disentangled spin would take random directions, with 1/2 probability of remaining in the original direction. Due to probability multiplication, N_d disentangled spins would lead to a suppression of amplitude to $1/2^{N_\mathrm{d}}$. In applications, the extreme sensitivity of GHZ state has often been exploited, which usually only concerns the amplified *phase* angle as single-qubit phase rotations $e^{i\phi\sigma_j^z}$ will accumulate into a large relative phase $e^{iN\phi}$ between the two components in a GHZ state. Our discussion above, instead, illustrates a more generic sensitivity of GHZ states to arbitrary spin rotations, where any defective spin would be revealed by the amplified exponential amplitude suppression.

In summary, the physical insight drawn from observing GHZ states in a DTC is that it offers a unique and convincing benchmark of N-qubit entangled cat eigenstate pair, which is the defining theoretical feature long-sought-after since the beginning of DTC research. The amplitude of $\mathcal{K}_f'(2N,t)$ matching that predicted by defect-free analytical theory offers strong evidence that the underlying cat eigenstate pair is genuinely N-qubit entangled, as any disentangled qubits will suppress this amplitude exponentially. The temporal period-2T oscillation of $\mathcal{K}_f'(2N,t)$ further proves the quasienergy spectral difference π, accomplishing the direct experimental proof of spectral-paired cat eigenstates robust against generic perturbations.

Accordingly, we have followed the suggestion of the Reviewer to add a paragraph at the end of the section "Preserving GHZ state", explaining physical significance of evolving GHZ states in DTCs:

"Physically, the sensitive GHZ state offers long-sought opportunities to directly reveal the spectral-paired cat eigenstates, a defining property of long-range-entangled DTCs [8, 14, 17]. We show in Supplementary Information that disentangling each qubit in the eigenstate reduces the value of $\mathcal{K}_f'$ exponentially, in contrast to a vanishing impact to conventional probes, like magnetic orders for product states. Thus, $\mathcal{K}_f'$ in Fig. 3 opens the door to accurately seeing genuine N-body entanglement in individual eigenstates."

Comment 2.3: *3) From an application perspective, the authors seem to be making the case that the protocol* *offers a way to protect the GHz states against noise. My question here is what if you just do careful spin echo on the state, for example using an XY4 type of sequence which is designed to be also robust against small perturbations in the pulses? Would you get much better protection that way, since it's much simpler and doesn't involve any two-qubit gates at all? Without seeing a clear comparison with state-of-the-art dynamical decoupling sequences, I do not think this convoluted scheme for state preservation has much value beyond an interesting demonstration.*

Response 2.3: Thanks for the comment. XY4 spin echo sequence mentioned by the Reviewer, is effective at mitigating *single-spin* perturbations, but cannot protect against two-spin perturbations arising from qubit interactions. This has been a conundrum across multiple platforms. For instance, in the task of quantum sensing, errors following the standard quantum limit $\sim 1/\sqrt{N}$ or the Heisenberg limit $\sim 1/N$ all require using large numbers of qubits, but perturbation by interactions accumulates as the qubit number ramps up. In a recent experiment on NV centers [PRX 10, 031003 (2020)], it was explicitly measured that XY-8 spin echos barely increase coherence time, because it cannot affect the spin interactions there. We quote the discussions below:

"In our system, however, the XY-8 sequence only provides a small improvement ($T_2 = 1.6\ \mu s$) [Fig. 2(c), blue circles], since the XY-8 T_2 is limited by strong spin-spin interactions, which are not affected by π rotations".

To mitigate interaction errors, a protocol involving 60 pulses per cycle (i.e. in contrast to 4 pulses in XY4) has to be introduced then. Such a 60-pulse scheme does not eliminate the interaction effect but symmetrizes it into the Heisenberg form, such that only ferromagnetic Fock states without any inhomogeneity become an eigenstate. In contrast, our setup with cat scar DTCs involves one π-pulse and one Ising interaction per cycle (note that all other terms are unnecessary, see **Response 1.2 and 1.3**), which could stabilize GHZ states of any spin patterns. Thus, for platforms where quantum gates can be efficiently implemented, our scheme shows a clear advantage in simplicity and flexibility to suppress pulse and interaction errors simultaneously.

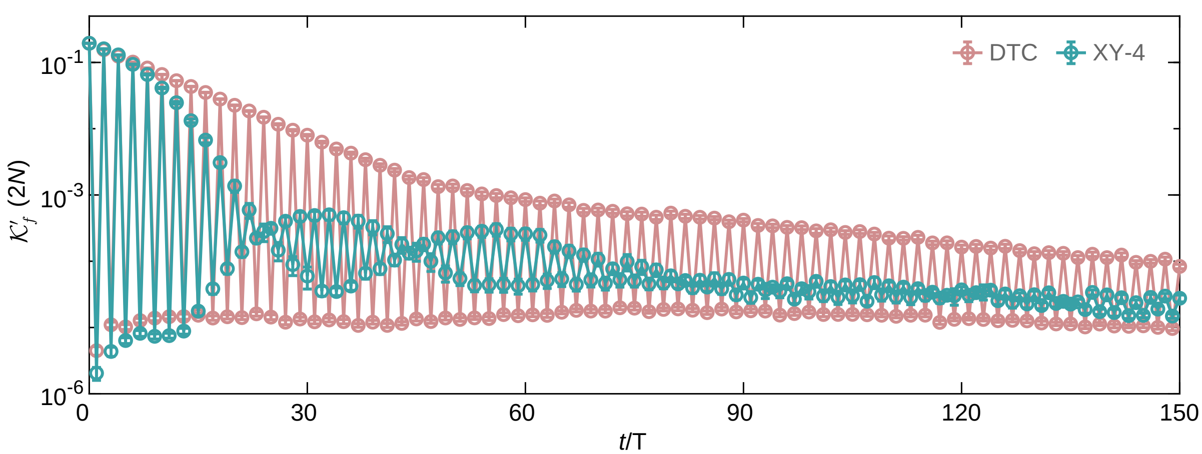


Figure 5: [**For Response 2.3**] Numerically simulated $\mathcal{K}_f'(2N)$ for DTC and XY-4 cases with small perturbation γ randomly sampled in range [0,0.025]. Numerical results are averaged over 3,000 random samples of noise realizations for good convergence, error bars are the standard error of the statistical mean of 100 random γ samples.

For cautiousness, we perform a noisy numerical simulation to benchmark the difference between XY-4 and DTC schemes. Similar to the NV centers mentioned above, uncontrollable interactions also exist in superconducting qubit platforms, in the form of photon swap between nearby qubits. See i.e. [Phys. Rev. Appl. 10, 054062 (2018)] for nearest neighbor leakage and [Nature 614, 676–681 (2023), Nat. Commun 15, 4918

(2024)] for next-nearest-neighbor cases. This effect can be modeled by the Hamiltonian,

$$H' = \gamma \sum_{j=1}^{N} \left(\sigma_j^+ \sigma_{j+1}^- + \sigma_j^- \sigma_{j+1}^+ \right), \qquad \sigma_j^\pm \equiv \sigma_j^x \pm i\sigma_j^y, \tag{2}$$

where $\gamma \ll 1$ parameterizes the strength. To benchmark the XY-4 versus the DTC schemes, we choose γ randomly sampled in range $[0, 0.025]$ as the perturbation term for each cycle in quantum sensing circuits. The noise simulation method and noise parameters are consistent with those outlined in Supplementary Section 3C. As shown in Figure. 5, we plot the Fourier peak $\mathcal{K}_f'(2N)$ on a 10-qubit chain for both cases. The DTC scheme maintains persistent oscillations, whereas the XY-4 scheme initially preserves the entanglement for a few cycles and then experiences a decline with minimal revivals.

Comment 2.4: *4) Lastly, since the Hamiltonian has to be in 1D, the protocol seems to be even further limited in use since generically it is not possible to deform any 2D geometry into a 1D closed ring form as the authors have done.*

Response 2.4: Thanks for the comment. However, with all due respect, the Reviewer's conjecture in this comment is invalid. The cat scar DTC in our manuscript does *not* rely on either dimensionality or boundary conditions, which is also a big advantage of our protocol, in contast to some previous DTC models, such as MBL-based DTC, where the dimensionality is still a debated open question. We are sorry for not explicitly pointing out this point in our manuscript. In the revised version, we clearly address the generality of our method and a new Supplementary Section 5E was added for detailed discussions.

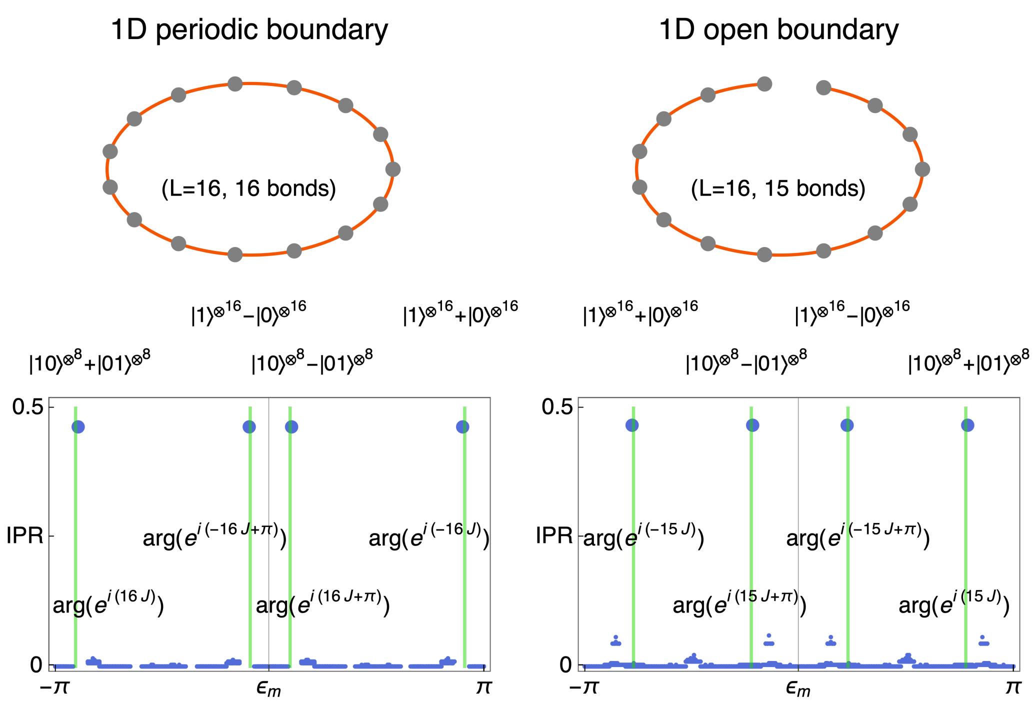


Figure 6: [**For Response 2.4**] Comparison of different boundary conditions in 1D. Changing the boundary condition does not affect the necessary cat scar property, including the high IPR~ 0.5, predictable scar patterns, and π-quasienergy pairing between scars of the same patterns. The only harmless change is a global quasienergy shift by J for pairwise scars, because the open boundary case has one fewer bond on the edge.

Here, we straightforwardly demonstrate the generality of our scheme by considering other boundary conditions and dimensions numerically. In Figure 6, we perform exact diagonalization to obtain eigenstate IPRs for periodic (left panel) and open (right panel) conditions in 1D for system size $L = 16$. All parameters used here are the same as those in Fig. 3 of our main text. It is readily observed that cat scars exist in both cases, with high IPR→ 0.5 and quasienergy π-pairing between each pair with the same spin pattern. The only minor difference without observable effect is that quasienergy for pairwise cat scars is shifted together by J for open boundary conditions ($\pm 15J$) compared with periodic boundary condition ($\pm 16J$), which is attributable to one fewer bond on the edge if open boundary condition is taken. (Recall that Ising quasienergy contribution $\sim Js_j s_{j+1}$).

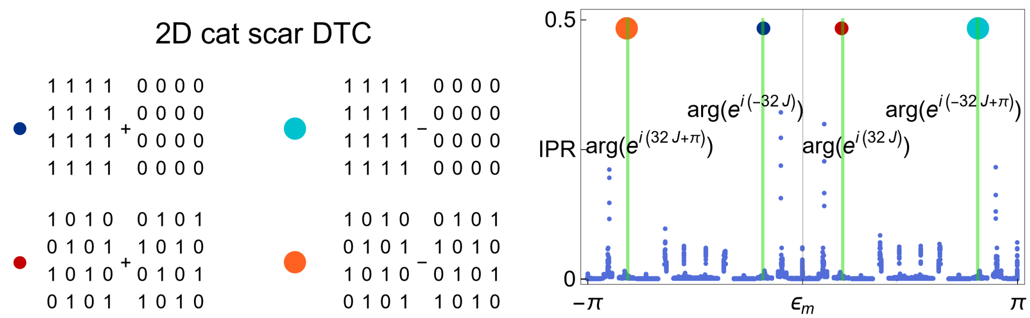


Figure 7: [**For Response 2.4**] Generalization of cat scar DTC models into 2D. We see that the cat scars of 2D FM and AFM patterns show up as expected.

Our model can also be generalized into higher dimensions directly, as shown in Figure 7 for a 2D example. Here, Ising interactions connect different qubits into a square lattice, and parameter choices are again the same as in Fig. 3 of the main text. As expected, cat scars with two-dimensional ferromagnetic and anti-ferromagnetic patterns show up again with IPR$\sim$ 0.5 and quasienergy π pairing.

We feel that this comment by Reviewer #2 is, in some sense, similar to **Comments 1.2 and 1.3** by Reviewer #1, which encourages a clearer clarification of what aspects of our models are essentially required. Such clarification is particularly useful for the current comment because the fact that cat scar DTCs can work in higher dimensions is a significant advantage compared with other DTC schemes, i.e. MBL DTCs that only work in 1D where MBL is believed to be stable, and prethermal DTCs with Landau's symmetry breaking that has to live in 2D or higher for short-range-interacting systems. We have appended Figures 6 and 7 of this reply in Sec 5E of the Supplementary Information.

Finally, to recap, we stress the reasons to work in 1D with periodic boundary conditions.

1. 1D geometry allows for checking longer-distance correlation and entanglement compared with 2D for a given number of qubits, and therefore provides more definitive benchmarks of all-to-all entangled eigenstates for DTCs.

2. Ising model kicked by transverse fields (which can be mapped to driven Kitaev chains) may give rise to edge Majorana modes if an open boundary is taken. Such edge modes produce Bell-state-like long-range entanglement, which is certainly harmless and even helpful for protecting the GHZ states. But these edge modes only concern two edge spins, not the bulk all-to-all entangled cat eigenstates. Also, Majorana edge states are generated by symmetry-protected topological matters that can be destroyed by symmetry-violating perturbations, unlike DTCs withstanding generic perturbations. Thus, for the purpose of unambiguously showing stable bulk entanglement for DTC eigenstates, it is desirable to avoid edge spins in contemporary experiments (see i.e. the text below Eq. (2) in Ref. [34], where two edge spins are neglected in measuring correlations). In our case, we alternatively engineer periodic boundary conditions to erase edge effects entirely, such that any entanglement protection must be provided genuinely by the bulk DTC order without relying on edge modes.

Response to Reviewer # 3 (Remarks to the Author):
We thank the Reviewer for highlighting that our manuscript is "a noteworthy achievement" and recommending publication. We also appreciate his/her constructive comments for further improving our manuscript. We take these comments seriously and have substantially revised the manuscript. We believe the improved manuscript is ready for publishing in Nature Communications. The detailed response to the each of the Reviewer's comments is provided below.

Comment 3.0: *Bao et al. experimentally prepare many-qubit (60 or 36) Greenberger-Horne-Zeilinger (GHZ) states in an array of superconducing transmon qubits. They verify that their prepared states have high fidelity with a perfect GHZ state. Further, the authors apply driving to their prepared state such that the GHZ state is a time-crystalline steady state embedded in the spectrum of the evolution operator. They claim this procedure can be used to stabilize the GHZ state, and demonstrate that gates can be performed on the state without interrupting the stabilization procedure.*

This work seems to have the most relevance to the continuing efforts to simulate interesting quantum dynamical systems in synthetic devices. The preparation of the GHZ steady state of a discrete time crystal is a noteworthy achievement, and should be published. The authors convincingly demonstrate that the time-crystalline realization of the GHZ state survives longer than a GHZ state which evolves under non-interacting Rabi drives when subjected to a particular perturbation.

Meanwhile, the claimed relevance to future applications of quantum technologies is insufficiently substantiated. GHZ states are indeed very useful for metrology, and I am somewhat familiar with models of measurement-based quantum computation which use GHZ states as sources of entanglement to replace strong interactions. However, both of these applications seem unrelated to a GHZ state of transmons. Strong interactions are already available in this case, and I am not familiar with any proposal for metrology using highly entangled states of superconducting qubits. The references accompanying the claim that GHZ states "play vital roles... in future quantum technologies such as fault-tolerant quantum computation" are too broad to understand what the authors are alluding to.

Response 3.0: We sincerely thank the Reviewer for carefully reviewing the manuscript, accurate summary of the main results, and the recommendation for publication. We also appreciate the Reviewer's constructive comments on potential applications of the GHZ states, which prompted us to further improve the presentation of our manuscript.

Our original intention to write "...play vital roles... in future quantum technologies such as fault-tolerant quantum computation" as the first sentence of our abstract is to provide a *general* introduction to the GHZ state from the perspective of the whole field, rather than a claim on concrete applications using specific physical platforms. That is why the Reviewer may feel it is "too broad to understand what the authors are alluding to". We are sorry for this possible misleading and have modified this sentence as "Greenberger-Horne-Zeilinger (GHZ) states, also known as two-component Schrodinger cats, play vital roles in the foundation of quantum physics and the potential quantum applications" so that it looks more general as an introduction sentence. We hope it will be more clear for the readers.

As for specific applications of the GHZ states using superconducting qubits, to the best of our knowledge, there has not been experimental demonstration along this line before, which may result in an impression that "these applications seem unrelated to a GHZ state of transmons". However, this situation is largely due to the fact that preparing a large-scale GHZ state in experiments is very hard, and even if being prepared, its short coherence time (caused by the superdecoherence effect) cannot support further manipulations, such as interactions and measurements, for practical applications. Our manuscript overcomes this challenge and demonstrates an unconventional way to stabilize the fragile GHZ state with DTC, which efficiently prolongs its lifetime and enables the subsequent manipulations required for potential applications. In fact, we are now working on this exciting direction to make use of GHZ states and nonequilibrium quantum matter to enhance quantum metrology. Recently, the community also realized this interesting topic and has also proposed relevant proposals suitable for superconducting qubits [arXiv:2405.00328 (2024), PRA 109, L050203 (2024)]. In addition, the protocol in our manuscript is platform-independent and can also be used on other quantum computing platforms, measurement-based or not, to stabilize GHZ states.

Addressing the following questions and comments would improve the paper.

Comment 3.1: *1. Clarify or remove the claims of application to quantum computation, as outlined above.*

Response 3.1: As explained in Response 3.0 above, we have followed the Reviewer's suggestion to modify this sentence as "Greenberger-Horne-Zeilinger (GHZ) states, also known as two-component Schrodinger cats, play vital roles in the foundation of quantum physics and the potential quantum applications" so that it looks more general as an introduction sentence of the abstract.

Comment 3.2: *2. When the authors compare the DTC evolution to Rabi driving, what is meant by the brief comparison to "free decay"? Is this the decay of the qubits with no circuit applied, due only to uncontrolled interactions among the qubits or with the environment, or is this U_1 of equation (2) without the pi pulse?*
2a. "Protection" of the GHZ state in this context somewhat implies that the lifetime of the GHZ state with driving is longer than with no driving. Clarifying whether or not this is actually the case is important.

Response 3.2: We thank the Reviewer for raising this constructive suggestion and have modified the main text accordingly to make the "free-decay" more clear.

Here, the "free decay" refers to the first scenario pointed out by the Reviewer: qubits keep idling with no circuit applied. Experimentally, we replace all the gates in Floquet drivings, including both U_1 and U_2, with the same length of idle time, and our results exactly show the case of "protection" of the GHZ state mentioned by the Reviewer: the lifetime of the GHZ state with driving is longer than that with no driving, even if U_1 and U_2 are composed of imperfect gates. This is mainly because the correlated phase noise tends to accelerate the dephasing of a free-decay GHZ state with a rate $\sim e^{-N^2 t}$, yielding the "superdecoherence" effect [Phys. Rev. Lett. 106, 130506 (2011), Rev. Mod. Phys. 90, 035005 (2018)]. The cat scar DTC in our manuscript, combining periodic π-pulse driving and strong Ising interactions, enables the GHZ state to resist the generic internal perturbations in the unitary evolution and external correlated phase noises. Please also refer to the Response 1.4 (including Figure 1 and Table 2) for a more detailed comparison between free-decay, Rabi, and DTC cases.

In the revised Supplementary Information, we also added a new Supplementary Section 5D3, Supplementary Figures S22, S23 for more experimental data and extended discussions.

Comment 3.3: *3. The x axis in Figure 3 is the number of drive cycles. This is a fair comparison between Rabi and DTC only when the primary source of error is the number of gates, and not the native relaxation and dephasing of the transmons, without gates applied. Can the authors verify that this is the case? Relatedly, how long is $t/T = 30$ in seconds? Is it the same for Rabi and DTC?*

Response 3.3: The Reviewer raised an excellent point here. An absolutely fair comparison between different cases in experiments is hard. In our experiments, our principle is to keep the *real time* the same for different cases. That is to say, we replace U_2 (or U_1 and U_2) in DTC with the same length of idle time for Rabi (or free-decay) case so the full period T is identical for all three cases. Although such a comparison is a little unfavorable to the DTC case due to the imperfect realization of U_1 and U_2, DTC still outperforms other cases.

In our experiment, a single Floquet driving period is $T = 144$ ns and thus $t = 30T = 4320$ ns. In the revised main text, we have explicitly pointed out the real time for a single period T is 144 ns and we always keep the real time the same for comparing different cases. In addition, we also added an upper x-axis of the real time to the Supplementary Figure 22 (also Figure 2 of this response letter) for clarity.

REVIEWERS' COMMENTS

Reviewer #1 (Remarks to the Author):

After careful reading of their revised manuscript, I found that all my concerns were satisfactorily addressed by the authors. I think this revised manuscript can be recommended for publication in Nature Communication.

Reviewer #2 (Remarks to the Author):

I'm happy with the authors' response to my questions and can recommend publication of the revised manuscript.

Reviewer #3 (Remarks to the Author):

The clarifications made by Bao et al. address my concerns from the first report to my satisfaction.

My primary concern was that the utility of a GHZ state of transmon qubits had been misrepresented, but the authors' rebuttal that the lack of useful proposals involving such systems is largely due to lack of motivation is reasonable. As the authors supported this claim with a reference to [arXiv:2405.00328 (2024), PRA 109, L050203 (2024)], it may help to include this reference in the article.

I did not realize from the previous manuscript that the real simulation time was kept constant when comparing different protocols. I agree with the authors that this is probably the best way to compare the protocols, and clarifying that this was done improves my impression of this comparison.