

Supplementary Materials

Gigantic Tellegen responses in metamaterials

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1. The relationship between Tellegen material and axion

For the constitutive relations in Eq. (3) of the main text, according to Ampère's law: $\nabla \times \mathbf{H} = \partial_t \mathbf{D} + \mathbf{j}^{(e)}$, we have:

$$\nabla \times \left(\frac{\mathbf{B}}{\mu_0 \mu} - \alpha \mathbf{\vartheta} \mathbf{E} \right) = \partial_t (\varepsilon_0 \varepsilon' \mathbf{E} + \alpha \mathbf{\vartheta} \mathbf{B}) + \mathbf{j}^{(e)} \quad (\text{S1})$$

Then Eq. (S1) can be rewritten as:

$$\nabla \times \frac{\mathbf{B}}{\mu_0 \mu} = \varepsilon_0 \varepsilon' \partial_t \mathbf{E} + \mathbf{j}^{(e)} + \alpha (\mathbf{\vartheta} \partial_t \mathbf{B} + \nabla \times \mathbf{\vartheta} \mathbf{E}) \quad (\text{S2})$$

According to the Faraday's law: $\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$ and $\nabla \mathbf{\vartheta} \times \mathbf{E} = \nabla \times \mathbf{\vartheta} \mathbf{E} - \mathbf{\vartheta} \nabla \times \mathbf{E}$, Eq. (S2)

can be rewritten as:

$$\nabla \times \frac{\mathbf{B}}{\mu_0 \mu} = \varepsilon_0 \varepsilon' \partial_t \mathbf{E} + \mathbf{j}^{(e)} + \alpha (\nabla \mathbf{\vartheta} \times \mathbf{E}) \quad (\text{S3})$$

presenting a modified Maxwell equation that incorporates a time independent axion-coupling term $\mathbf{\vartheta}$. In addition to the electric current density $\mathbf{j}^{(e)}$, there is an extra current density term associated with the axion-coupling term $\mathbf{\vartheta}$. Consequently, the electromagnetic properties of the Tellegen metamaterial described by the constitutive relations in Eq. (3) of the main text, are precisely captured by the equations of axion electrodynamics, i.e., the Tellegen metamaterial features an effective axion field. The axion coupling term introduces an additional term in Lagrangian density: $\Delta \mathcal{L} = \alpha \mathbf{\vartheta} \mathbf{E} \cdot \mathbf{B}$. It is worth noting that a constant axion-coupling has no impact on the electromagnetic response. However, at the interface between the air and the Tellegen metamaterial slab, the effective axion field has a profound impact, giving rise to a Kerr rotation of the polarization plane of the reflected signal.

2. The magnetic and electric dipole moments of Tellegen metamaterials

We provide a microscopic perspective of the Tellegen response by investigating the magnetic and electric dipole moments. As schematically shown in the upper panel of Fig. S2(a), the metamaterial slab is normally illuminated by a standing wave with the electric field polarized along the y -axis. When the slab is placed at the node of magnetic field ($H_x = 0$), the magnetic dipole induced by the incident magnetic field is expected to be zero ($M_x \propto H_x = 0$), while the Tellegen response gives rise to the in-phase magnetic dipole moments induced by the electric field ($M_y \propto E_y$). For our metamaterial, the magnetic dipole moment can be calculated through the integration of the current density $\mathbf{j}$ of the unit-cell: $\mathbf{M} = \frac{1}{2c} \int (\mathbf{r} \times \mathbf{j}) d^3r$, where $\mathbf{r}$ is the coordinate vector and c is the speed of light. The simulated magnetic dipole is shown in the lower panel of Fig. S2(a), where M_x, M_z are near zero across the whole range as expected, while M_y shows a resonating spectrum with a resonating frequency of 9.5GHz. In addition, M_y nearly has the same phase of the incident wave E_y , verifying the in-phase cross-coupling. Similarly, when the metamaterial slab is placed at the node of electric field ($E_y = 0$), the simulated electric dipole $\mathbf{P} = \frac{1}{i\omega} \int \mathbf{j} d^3r$ in Fig. S2(b) shows a nearly zero $P_y \propto E_y = 0$ and a resonating $P_x \propto H_x$, indicating the in-phase electric dipole moment induced by the magnetic field. Therefore, the microscopic dipole analysis verifies the Tellegen response of the metamaterial.

3. Symmetry analysis for realizing pure and isotropic in-plane Tellegen response

The designed metamaterial structure exhibits Mx, PT, C2y, and C4z symmetry, where Mx, PT, C2y, and C4z denote the combination of mirror operation respect to x-plane and time-reversal, combination of inversion and time-reversal, 2-fold rotation around the y-axis, and 4-fold rotation around the z-axis, respectively. To illustrate that the metamaterial generates a pure and isotropic in-plane Tellegen response, we provide a symmetry analysis based on the Hamiltonian of the metamaterial. Considering the constitutive relations $\mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E} + \frac{1}{c} \xi \mathbf{H}$, $\mathbf{B} = \mu_0 \mu \mathbf{H} + \frac{1}{c} \xi^\dagger \mathbf{E}$, Maxwell's equations $\nabla \times \mathbf{E} = -i\omega \mathbf{B}$, $\nabla \times \mathbf{H} = -i\omega \mathbf{D}$ can be rewritten as a matrix form with the fields organized into a six-vector $\mathbf{G} = (\mathbf{E}, \eta_0 \mathbf{H})^T$, where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the free-space impedance. Then we can obtain $H(\mathbf{r})\mathbf{G} = k_0 \mathbf{G}$, with $H(\mathbf{r}) = \begin{pmatrix} \varepsilon & \xi \\ \xi^\dagger & \mu \end{pmatrix}^{-1} \begin{pmatrix} 0 & i\nabla_r \times \\ -i\nabla_r \times & 0 \end{pmatrix}$ representing the Hamiltonian of the metamaterial. According to the group theory of the Hamiltonian, when the symmetry operation $\hat{D}_i$ applies to the structure, the Hamiltonian must satisfy: $M(\hat{D}_i)H(\mathbf{r})[M(\hat{D}_i)]^{-1} = H(\hat{D}_i \mathbf{r})$, where $M(\hat{D}_i)$ denotes its matrix representation at $\mathbf{G}$ vector space.

Since the C4z symmetry leads to the in-plane isotropic response of the structure, we consider following material parameters

$$\varepsilon = \begin{pmatrix} \varepsilon & -i\varphi & 0 \\ i\varphi & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}, \quad \xi = \begin{pmatrix} i\kappa + \tau & i\Omega - V & \xi_{13} \\ -i\Omega + V & i\kappa + \tau & \xi_{23} \\ \xi_{31} & \xi_{32} & 0 \end{pmatrix}, \quad \mu = \begin{pmatrix} \mu & -i\gamma & 0 \\ i\gamma & \mu & 0 \\ 0 & 0 & \mu \end{pmatrix} \quad (\text{S4})$$

When Mx symmetry applies to the Hamiltonian, the corresponding operation is $M(D_i) = \text{diag}(-1, 1, 1, 1, -1, -1)K$, $\hat{D}_i = \text{diag}(-1, 1, 1)$ and K is the complex conjugation operation. It is straightforward to show that the Mx symmetry enforces $\xi_{13}, \xi_{23}, \xi_{31}, \xi_{32} = 0$. When PT symmetry applies to the Hamiltonian, the corresponding operation is $M(D_i) = \text{diag}(-1, -1, -1, -1, -1, -1)K$, $\hat{D}_i = \text{diag}(-1, -1, -1)$. It is straightforward to show that the PT symmetry enforces $\Omega = 0, \kappa = 0, \gamma = 0, \varphi = 0$. When C2y symmetry applies to the Hamiltonian, $M(D_i) = \text{diag}(-1, 1, -1, -1, 1, -1)$, $\hat{D}_i = \text{diag}(-1, 1, -1)$. Similar with the PT symmetry, it eliminates the moving bianisotropic response V , the final constitute equation is the same as shown in Eq. (1), which indicates pure Tellegen response cancelling out unwanted response.

4. Details of the Sample

The size of the fabricated sample is around 50cm*50cm*3mm. The radius of the YIG cylinder is 2mm with height 2mm and the radius of permanent magnets is 2mm with height 1mm. The saddle-shaped connective metallic coil is fabricated by the traditional Printed Circuit Board (PCB) fabrication technology in the Teflon. The inner radius and outside radius of the saddle-like coil are 2.2mm and 3.2mm and the height of the bar of coil is 3mm with radius 0.25mm.

5. Details of the transmission and reflection of the Tellegen slab

● The transmission and reflection coefficient of Fresnel Equation

Consider the wave propagates along the z direction which is shown in Eq. (2), the two eigenmodes of the wave has the same effective index is $n_{eff} = \sqrt{\epsilon\mu - \xi^2}$. With the corresponding forward and backward eigenmode $\{E_x, E_y, H_x, H_y\}$ as following:

$$\left\{ \frac{\sqrt{\epsilon\mu - \xi^2}}{\epsilon}, -\frac{\xi}{\epsilon}, 0, 1 \right\}, \left\{ -\frac{\xi}{\epsilon}, -\frac{\sqrt{\epsilon\mu - \xi^2}}{\epsilon}, 1, 0 \right\}, \left\{ -\frac{\sqrt{\epsilon\mu - \xi^2}}{\epsilon}, -\frac{\xi}{\epsilon}, 0, 1 \right\}, \left\{ -\frac{\xi}{\epsilon}, \frac{\sqrt{\epsilon\mu - \xi^2}}{\epsilon}, 1, 0 \right\} \quad (S5)$$

Using the continuous boundary conditions of both sides of the slab, we could get the transmission and reflection when the incident wave is E_y polarization as:

$$\begin{aligned} R_{xy} &= \frac{2\xi \sin \phi}{2i\sqrt{\epsilon\mu - \xi^2} \cos \phi + (\epsilon + \mu) \sin \phi}, \\ R_{yy} &= \frac{(-\epsilon + \mu) \sin \phi}{2i\sqrt{\epsilon\mu - \xi^2} \cos \phi + (\epsilon + \mu) \sin \phi}, \\ T_{yy} &= \frac{2i\sqrt{\epsilon\mu - \xi^2}}{2i\sqrt{\epsilon\mu - \xi^2} \cos \phi + (\epsilon + \mu) \sin \phi}, \end{aligned} \quad (S6)$$

$$T_{xy} = 0$$

where ϕ is the propagating phase inside the slab, $\phi = k * n_{eff} * d$, k is the vacuum wave number, d is the effective thickness of the slab.

According to Eq. (6), we can obtain the effective parameters ϵ, μ and ξ :

$$\begin{aligned}\epsilon &= \frac{i(1-R_{yy}-T_{yy}q_1)q_2}{kdT_{yy}}, \\ \mu &= \frac{i(1+R_{yy}-T_{yy}q_1)q_2}{kdT_{yy}}, \\ \xi &= \frac{iR_{xy}q_2}{kdT_{yy}}\end{aligned}\tag{S7}$$

where $q_1 = \cos \phi = \frac{1-R_{xy}^2-R_{yy}^2+T_{yy}^2}{2T_{yy}}$ and $q_2 = \phi \csc \phi$. Consequently, utilizing the scattered parameters allow us to extract the effective parameters of the metamaterial. In addition to the focal Tellegen parameters ξ shown in Fig. 3e in the main text, the extracted effective permittivity and permeability are displayed in Fig. S3(a) and Fig. S3(b). The resonances observed in the permittivity and permeability spectra are aligned with the resonance of the Tellegen coefficient. In Fig. S3(a), a slightly negative imaginary part of the extracted permittivity is observed, which is attributed to the finite spatial periodicity of the metamaterial.^{1,2} Note that although the effective permittivity exist slightly negative imaginary part, the complex response of permeability and Tellegen terms ensure that it is overall a lossy medium without gain.

With the extracted effective parameters ϵ, μ , and ξ , we can put them back into Comsol model (Electromagnetic Waves, Frequency Domain) to calculate the transmission parameters of an effective medium slab. The Tellegen slab has the same effective thickness $d_0=12\text{mm}$, and is illuminated by a E_y -polarized plane wave (Fig. S4(a)). As shown in Fig. S4(b) and Fig. S4(c), the

calculated transmission parameters of such an effective medium slab (denoted by lines) match well with the simulated scattered parameters of the designed metamaterial (denoted by symbols).

- **Calculation of the polarization of reflected wave.**

Due to the presence of loss in the real structure, the polarization state of the reflected beam becomes elliptically polarized. In the simulation, we define ellipticity as $\chi = 1/2(\arcsin S_3)$, where S_1, S_2, S_3 are the stokes parameters of reflected wave. In the experiment we define $\chi = \arctan(\sqrt{I_{min}/I_{max}})$, where I is the received energy at different polarization angle, i.e. we rotate the polarization angle of receive antenna to measure the energy of every linear state. The ellipticity is plot in Fig. S5. We find that the linearity preserves well in the resonance regime(9.3GHz-9.7GHz) which is consistent with reflection calculated by the Fresnel equations. Some deviation of linearity is caused by the loss the structure. Away from the resonance regime (9.7GHz-10.4GHz), although the ellipticity is relatively high, the reflected energy is very small, especially the cross-polarization part. Thus, we don't focus on the off-resonance regime. When we remove all the loss inside in the metamaterials, we observe the reflected wave is linear polarization across the whole range of the spectrum in Fig.S6.

6. The proof that transmitted wave has the same polarization with incident wave.

We first assume cross polarization is generated in transmission, i.e. the E_y polarization incident wave along $+z$ direction generates the transmission of cross polarization E_x , as illustrated in Fig. S9a. Under the inversion operation, the propagation direction and coil are flipped meanwhile the

external static field B_z remains unchanged due to the nature of pseudovector, as shown in Fig. S9b. In addition, the system obeys the Onsager–Casimir relation which states that under the opposite external static field, a x-polarized incident wave along $-z$ direction would generate a transmitted wave of cross polarization E_y , which is viewed as a time-reversal partner of that shown in Fig. S9c. Rotating the configuration in Fig. S9c along z -axis 90° leads to the configuration shown in Fig. S9d, which has exactly the same metamaterial structure shown in Fig. S9b, but the transmitted wave is out of phase with Fig. S9b. Hence, the two scenarios contradict each other unless $E_x=0$ which means that there is no cross-polarization in transmission, i.e. the transmission must have the same polarization as incident wave. However, lacking any symmetry protection, the polarization of reflected wave can be changed as a result of nonreciprocal interaction of incident wave with the metamaterial.

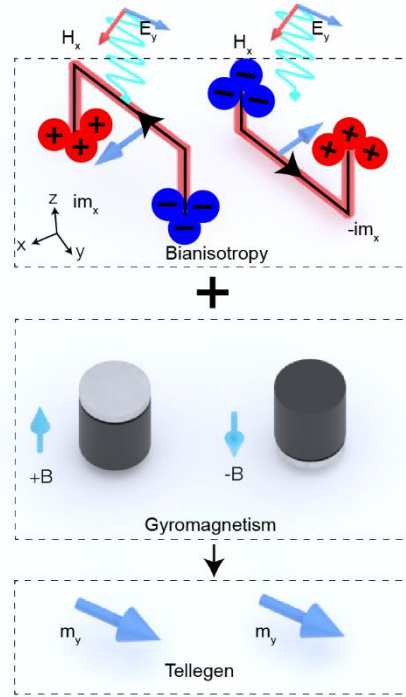


Fig. S1 The process that the y-oriented electrical field E_y induce m_y .

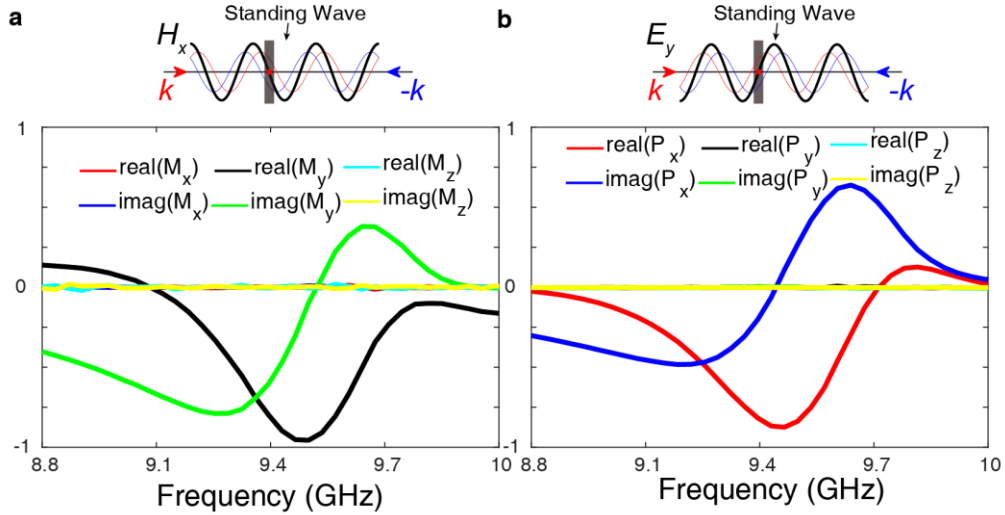


Fig.S2. The dipole moments in the Tellegen metamaterials under E_y polarized wave. (a) The magnetic dipole component at the node of magnetic field and (b) the electric dipole component at the node of electric field. The unit of y-axis is arbitrary. The upper figures show the configurations of the excited standing wave (black), created by the superposition of a right traveling (red) and left traveling (blue) wave.

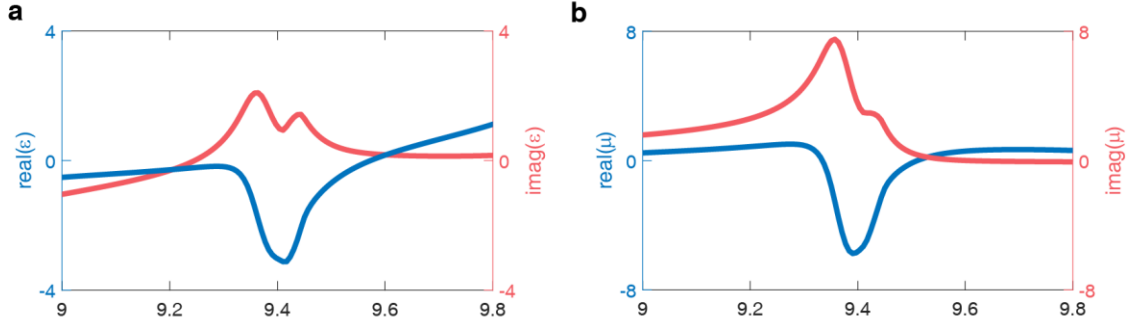


Fig. S3 The extracted effective permittivity ϵ (a) and permeability μ (b) from the metamaterial. Blue and red lines represent the real and imaginary parts, respectively.

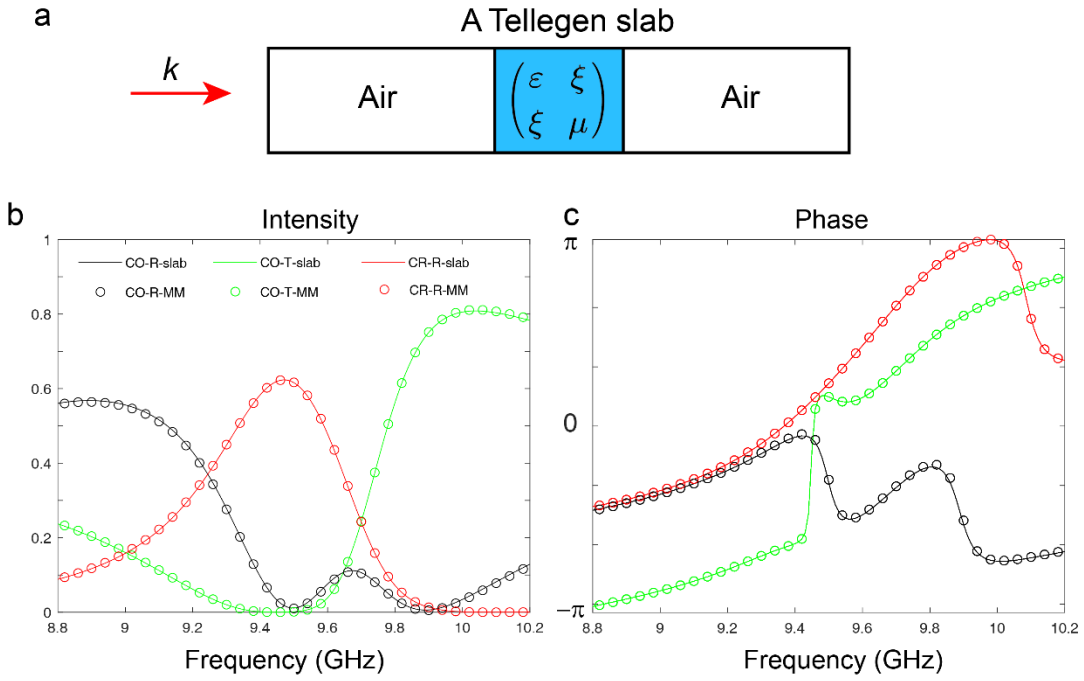


Fig. S4 Validation of the extracted effective parameters. (a) A Tellegen slab has the same effective thickness and effective parameters as the metamaterial, and is illuminated by a Ey-polarized incident wave. The intensity (b) and phase (c) of the transmission and reflection waves. Lines and symbols respectively represent the simulated results from the Tellegen slab and the metamaterial.

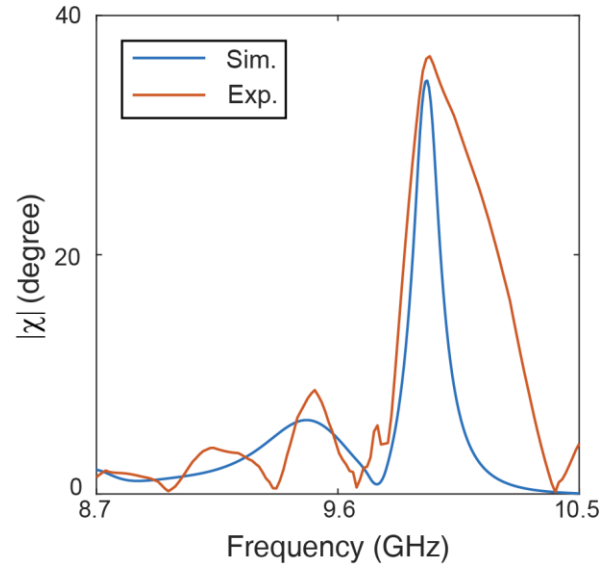


Fig. S5 The ellipticity of reflected wave. The red and blue line represent the simulation and experiment results respectively.

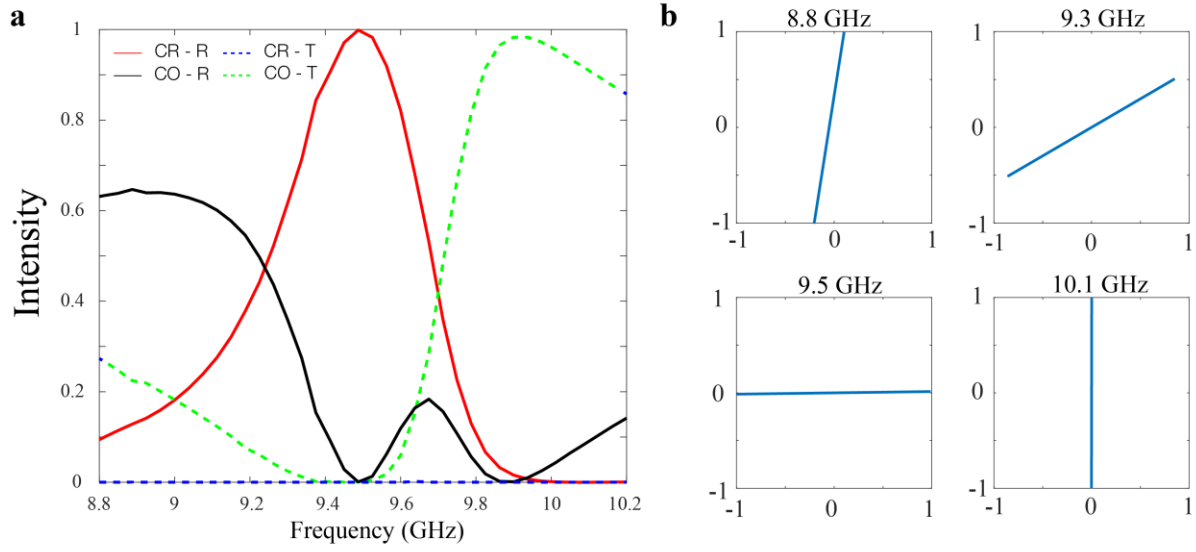


Fig. S6 The Tellegen metamaterials without loss. **a**, simulated reflection and transmission intensity spectra in a lossless metamaterial. The labels “CO-” and “CR-” indicate co-polarization and cross-polarization respectively, and “T” and “R” present transmission and reflection respectively. **b**, the simulated and measured polarization state of the reflected beam at the specific frequencies. We observe the linear polarization across the whole range.

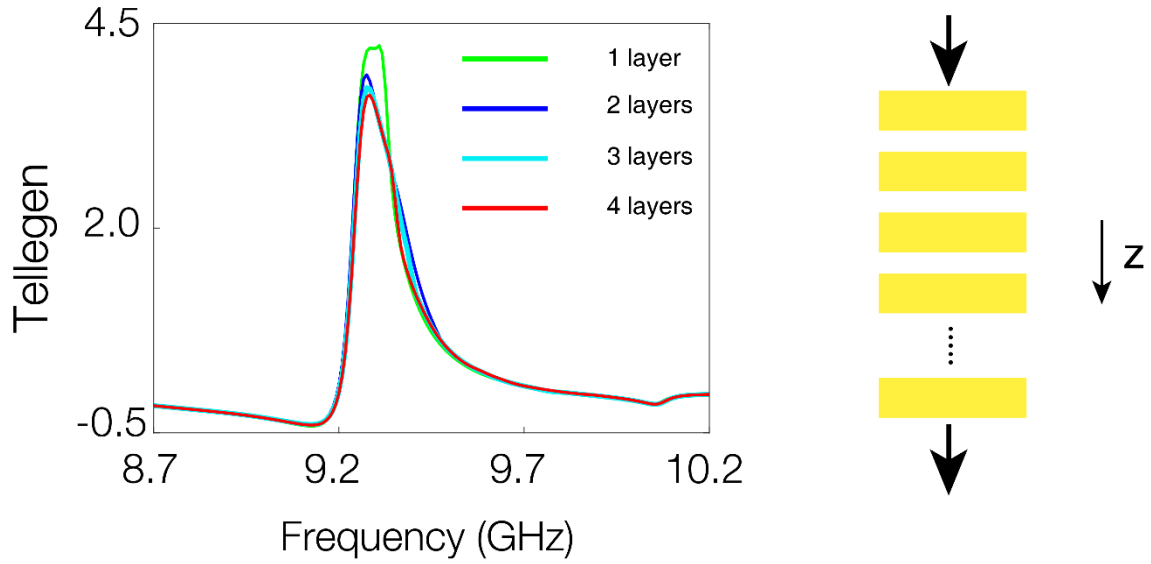


Fig. S7 The simulated Tellegen parameter of multiple layers of slab. By Fresnel equation, the simulated Tellegen parameter for 1-4 layers of slabs is shown. The layers are arranged along the z -axis with a periodicity of 12 mm.

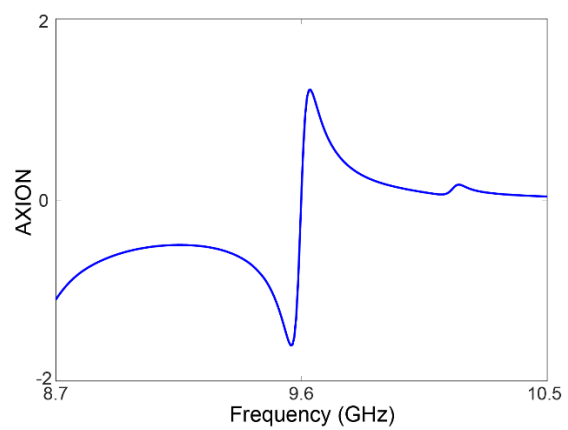


Fig. S8 The effective axion parameter from the simulation results.

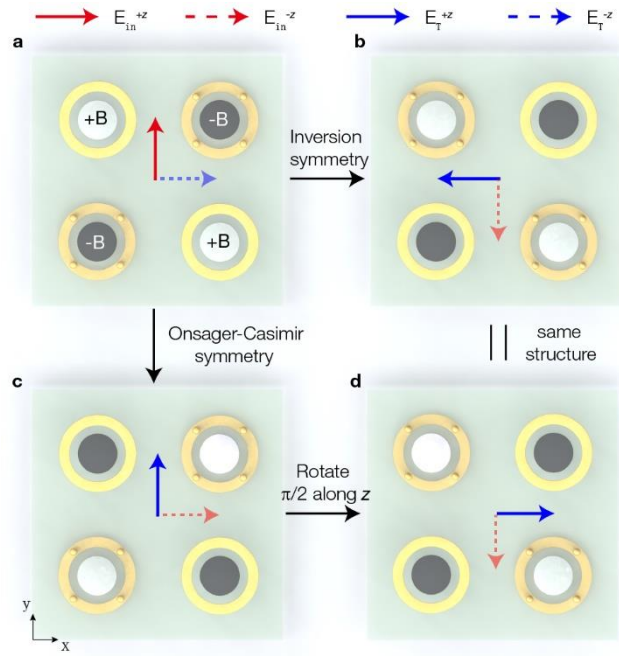


Fig. S9. The prove of same transmission polarization as incident polarization. Two lines (solid and dashed) represent different sides of the structure and two colors (red and blue) represent the incident and transmitted wave. **a** The assumption that E_y incident wave induces the E_x transmitted wave. **(b)** the scenario under the inversion operation and **(c)** the scenario under the Onsager–Casimir relations and **(d)** the scenario **(c)** is rotated 90° along z -axis. **(b)** and **(d)** contradict each other unless $E_x=0$ which means the cross-polarization is forbidden in the transmission.

Reference

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- 2 Koschny, T. *et al.* Impact of inherent periodic structure on effective medium description of left-handed and related metamaterials. *Phys Rev B* **71** (2005).