nature portfolio

Peer Review File

Photonic axion insulator with non-coplanar chiral hinge transport

Corresponding Author: Professor Cheng He

This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
In the present work by Lai and coauthors, entitled “Photonic axion insulator with non-coplanar chiral hinge transport”, the authors propose and experimentally implement axion topological insulator – a higher order topological insulator with unidirectional hinge states. The structure is realized in microwave spectral range by using ferrites arranged into a three-dimensional lattice. As such, the 3D lattice is formed by stacking planes of oppositely magnetized ferrites.
I find the design of the lattice truly ingenious and experimental results fascinating. For the first time the authors demonstrate 3D axion phase, which is a significant step forward for the field of topological photonics and topological physics in general. Overall, the paper is very nicely written and results are clearly presented. I have only a couple of suggestions/comments which I hope the authors will be able to implement.
1) The system the authors consider is inherently open and as such the modes have finite lifetime. While the results are overall quite convincing, I believe the paper would significantly benefit from looking into the effect of radiative and non-radiative loss on the topological phase. What are implications of such non-Hermiticity? What is the effect on topological invariants. Is Z4 still quantized? I think answering these questions will further improve quality of the work and make it more rigorous.

2) On two occasions the authors mention control of “light” (line 65) and laser “emission” (line 230). I doubt the proposed scheme is realizable in optical domain and therefore these claims should be removed or wording should be softened. I suggest the authors to be clear about challenges of realizing such complex geometry and magnetization pattern at scales needed for optical frequencies. Please revise the manuscript accordingly.

3) I found that the authors did not acknowledge properly some earlier relevant works on higher-order electromagnetic. Specifically, what are implications of long-range interactions, which are known to give rise to some nontrivial physics in HOTIS [Nature Photonics 14 (2), 89-94], for their electromagnetic systems? Another relevant work where unidirectional hinge transport was proposed in [Phys. Rev. Applied 13, 064031]. Please compare and highlight differences between hinge modes of this work and those of your work.

4) Finally, some earlier works relying on axion electrodynamic response (called Tellegen response in photonics) can be cited for completeness as well. See for example Refs.[New J. Phys. 17 125015 (2015) and Opt. Express 30, 47004-47016 (2022)].
Provided the proposed minor revisions are made, I will be more than happy to recommend this overall nice and wonderful work for publication in Nature Communications.

Reviewer #2

(Remarks to the Author)
Higher order axion insulator, being trivial both in Chern number and Z2 but nontrivial in Z4 topological invariant, can support chiral spectral flow at their hinges. However, the experimental observation of them in natural materials remains elusive. The authors Lai et al. propose a photonic design of axion insulator based on a three-dimensional antiferromagnetic-like structure and experimentally realize it in microwave bands. They not only observe the non-reciprocal and non-coplanar transport of chiral hinge states but also experimentally demonstrate the tunability of such states. This work represents the first example

of experimentally observing those effects, which is timely and very interesting. It might have broad impact in the community of photonics and other classical-wave systems. Due to its innovative character and potential impact, in my opinion, it is highly desirable to be published in Nature Communications.

Nevertheless, some technical issues and questions must be specified and corrected before the acceptance.

(i). In the section of “3D AXIs stacked from 2D”, the authors elaborate how the hinge states in x/y hinges are constructed, but it is not clear to me whether the hinge states in the z direction are ensured to exist. Can the authors explain the underlying mechanism of protecting the gapless hinge states along z direction?

(ii). I'm curious how the designs of “T-shirt-like hollow holes” and “circular holes” come across the mind, what are their specific roles in creating the topological properties of axion insulator and why they need to be tilted 225/45 deg in the metal plane?

(iii) In line “126”, the three components of topological index Z2 are not weak Chern numbers, they are weak indices of 3D topological insulators, Chern number belongs to Z class instead of Z2 class.

(iv) In line 138, “On each surface, a half-turn of a closed chiral loop results in the half-integer surface Hall response”, it is not necessary to mention Hall response in their result section if it has no implication or connection to the photonic system.

(v)The language and readability of manuscript need to be improved. I have listed a few (but not all) examples which have grammar errors and/or are hard to interpret.
in lines 91-92, “AXIs may also relate to higher-order phases of 3D topological insulators after breaking time-reversal symmetry to open the native gapless Dirac surface states.”, AXIs themselves cannot break TRS but magnetic field can, and “bandgap of Dirac surface states is open” rather than “the surface states is open”.
In lines 135, “It is because..” does not sound natural, what does “it” refer to?
In line 170, it's better to use “absorptive surface” or “lossy surface” instead of “absorption surface”. In the captions of Fig.4, facets are “absorptive” not “absorption”.
In line 198, “Considering it comes from the top surface modification of the odd-layer case” is hard to understand.
In line 214 “tremendous tunability owes to the unique symmetry operation”, the meaning of symmetry operation based on the context is unclear.
There is a typo in line 87, it should be “top and bottom facets” instead of “top and bottom facts”.

(vi)The authors illustrate the relations between 2D and 3D magnetic Tis in the schematic of fig.1 well, but In line 226, it is a bit exaggerated to claim their results “provide a comprehensive picture of AXIs”.

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
The authors have fully addressed my and other reviewers' comments and criticism. I recommend publication of this work in its present form.

Reviewer #2

(Remarks to the Author)
The author did not answer my first question seriously. They provided a perspective of mass term as their explanation, but this explanation is kind of hand-waving and too vague, and they did not investigate their magnetic photonic crystal at all. I insist on asking this question since the external magnetic field is parallel to z direction. From the picture of orbital bounding, the nonreciprocal hinge states along z direction are not guaranteed to exist. I would expect they at least derive the effective Hamiltonian from their photonic system and analyze its mass terms, which has non-local interaction and is completely different from simple tight-binding model. Therefore, it is still unclear to me why their structure supports topological hinge states along z dimension. This is a critical point which determines whether their photonic system truly belong to the axion insulator. I would not commend its publication until they resolve this question in a persuasive way

Version 2:

Reviewer comments:

Reviewer #2

(Remarks to the Author)

The atuhors address my previous comment in a convincing way,I have no further comment.

Report of Reviewer #1 -- NCOMMS-24-28639

Comments of Reviewer #1:

In the present work by Lai and coauthors, entitled "Photonic axion insulator with non-coplanar chiral hinge transport", the authors propose and experimentally implement axion topological insulator – a higher order topological insulator with unidirectional hinge states. The structure is realized in microwave spectral range by using ferrites arranged into a three-dimensional lattice. As such, the 3D lattice is formed by stacking planes of oppositely magnetized ferrites. I find the design of the lattice truly ingenious and experimental results fascinating. For the first time the authors demonstrate 3D axion phase, which is a significant step forward for the field of topological photonics and topological physics in general. Overall, the paper is very nicely written and results are clearly presented. I have only a couple of suggestions/comments which I hope the authors will be able to implement.

Our Response:

We are very grateful to Reviewer #1 for his/her high praise of our work.

Comments of Reviewer #1:

1) The system the authors consider is inherently open and as such the modes have finite lifetime. While the results are overall quite convincing, I believe the paper would significantly benefit from looking into the effect of radiative and non-radiative loss on the topological phase. What are implications of such non-Hermiticity? What is the effect on topological invariants. Is Z4 still quantized? I think answering these questions will further improve quality of the work and make it more rigorous.

Our Response:

We thank the reviewer for raising this important issue. Regarding to the non-Hermiticity of our model, the non-radiative loss mainly comes from the dielectric YIG rods and foams, whose relative permittivity can be expressed as: $\varepsilon_{YIG} = 15 \times (1 - i\varepsilon'_{YIG})$ and $\varepsilon_{foam} = 1.05 \times (1 - i\varepsilon'_{foam})$. Here, the imaginary terms represent the loss. The radiative loss comes from the circular open holes on the top and bottom metallic planes, which are designed for inserting the source and detector in experiments. Although the non-Hermitian Hamiltonian generically has complex eigenvalues, for qualitative analysis, we simply consider the real part of the eigenvalues in the following calculations.

Firstly, in our realistic system, both non-radiative loss ($\varepsilon'_{YIG} = 0.0002$ and $\varepsilon'_{foam} = 0.0025$) and radiative loss ($d$=3 mm, diameter of open holes) are very small, and their influence on the hinge states is negligible in our model, as shown in Fig. R1 and Fig. 3b (in the main text), using the nontrivial C2 hinge state on the top surface as an example. This conclusion can also apply to other nontrivial hinge states.

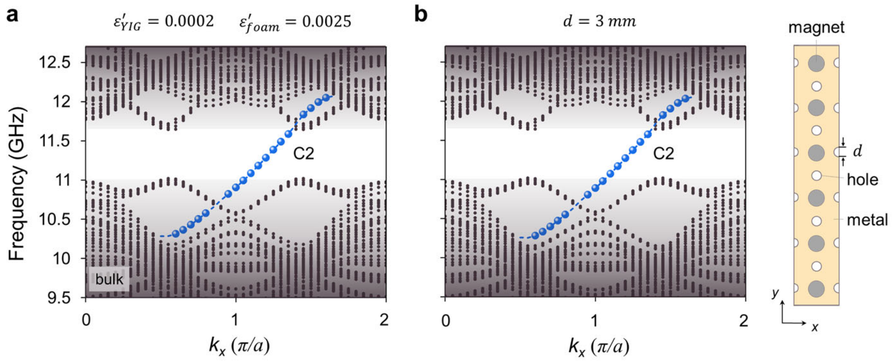


Fig. R1 | Influence of realistic losses on C2 hinge state. a, Non-radiative loss case with $\varepsilon'_{YIG} = 0.0002$ and $\varepsilon'_{foam} = 0.0025$. The relative permittivity of YIG and foam are: $\varepsilon_{YIG} = 15 \times (1 - i\varepsilon'_{YIG})$ and $\varepsilon_{foam} = 1.05 \times (1 - i\varepsilon'_{foam})$, where imaginary terms represent the loss. **b,** Radiative loss case with *d*=3 mm. The radiative loss is introduced through the design of the circular open holes (white areas) on the top metallic plane, as shown in the right panel.

Secondly, although we increase the loss to be a relatively high level, e.g., $\varepsilon'_{YIG} = 0.01$ and $\varepsilon'_{foam} = 0.25$, or *d*=8 mm, the gapless C2 hinge state still exists within the bandgap (Fig. R2), indicating the robustness of our axion insulator phase. The increased non-radiative loss narrows the bandgap of both bulk and surface states, while the increased radiative loss only narrows the bandgap of surface states.

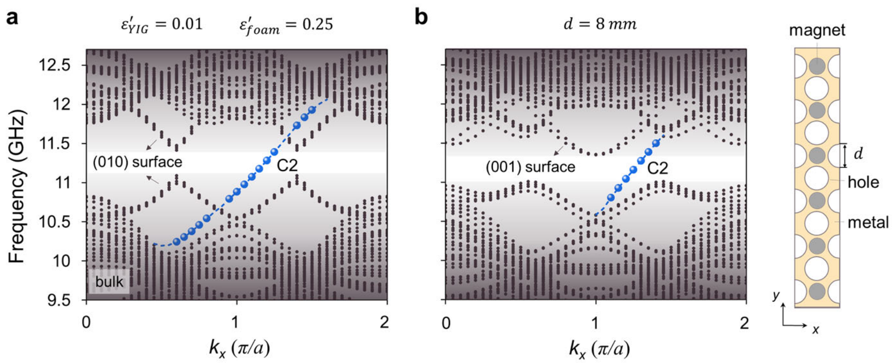


Fig. R2 | Influence of relatively large losses on C2 hinge state. a, The non-radiative loss case with $\varepsilon'_{YIG} = 0.01$ and $\varepsilon'_{foam} = 0.25$. **b,** The radiative loss case with *d*=8 mm. In both cases, the gapless C2 hinge state exists, indicating the robustness of axion insulator phase.

Thirdly, a further increase in the loss will trivialize the C2 hinge state, as expected by the reviewer. Interestingly, the radiative and non-radiative losses could play quite different roles in this case.

(i) The radiative loss can be considered as the surface decoration, which does not alter the bulk band structures. Therefore, the bulk topological invariant remains unchanged, but the huge radiative loss can lead to the surface band inversion (flipping the sign of surface Dirac mass). For example, if we increase the loss to d=12.5 mm, the originally nontrivial C2 hinge state becomes trivial T2, while the trivial T8 hinge state now becomes nontrivial C8. Similarly, we also have C1→T1, T7→C7, and vice versa (for the hinges on the bottom surface), as shown in Fig. R3. It should be noticed that the non-coplanar chiral hinge transport still exists, but in a different route.

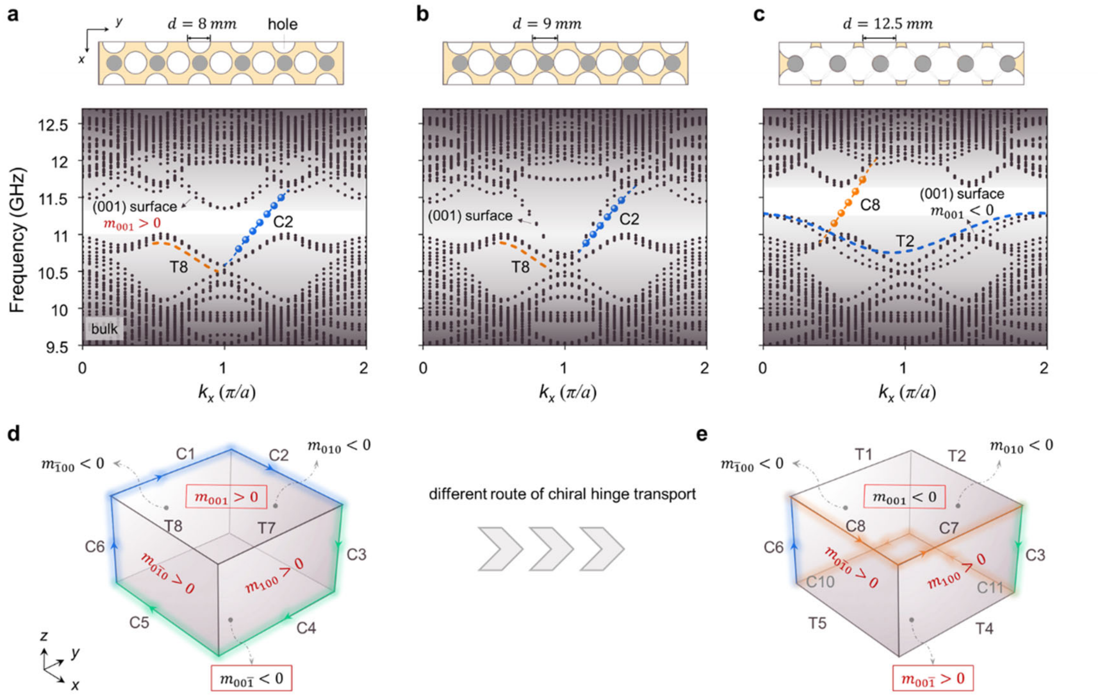


Fig. R3 | Radiative loss-induced tunable chiral hinge transport. a-c, Hinge states under continuously increasing the radiative loss with d=8 mm, d=9 mm, and d=12.5 mm, respectively. **d-e,** Chiral hinge transport for case **a** and case **c**. Here, both top and bottom surfaces are applied radiative loss, m represents the surface Dirac mass.

(ii) As the non-radiative loss continuously increases, the bulk bandgap will close near the M point to form a semimetal phase ($\varepsilon'_{YIG} = 0.01$ and $\varepsilon'_{foam} = 1$), then reopen to form a trivial insulator phase ($\varepsilon'_{YIG} = 0.01$ and $\varepsilon'_{foam} = 1.5$), as shown in Fig. R4. According to the even/odd parity of Bloch field distributions at eight inversion-invariant momenta, the $\mathbb{Z}_4$ topological index changes from 2 to 0, indicating the phase transition.

Regarding to the quantized or not topological index in non-Hermitian systems, we think the method of symmetry indicator still works in the current stage because of the intact inversion symmetry despite the presence of loss. Thus, analogous to Hermitian systems, we can get a quantized $\mathbb{Z}_4$ topological index. We also demonstrate that such non-Hermiticity can give rise to a novel phase transition (Fig. R4), agreeing well with their topological indices: axion insulator phase ($\mathbb{Z}_4$=2), Weyl semimetal phase ($\mathbb{Z}_4$=1), and trivial insulator phase ($\mathbb{Z}_4$=0), as shown in Fig. R4. We agree with the reviewer that the detailed interplay between topology and non-Hermiticity deserves further investigation.

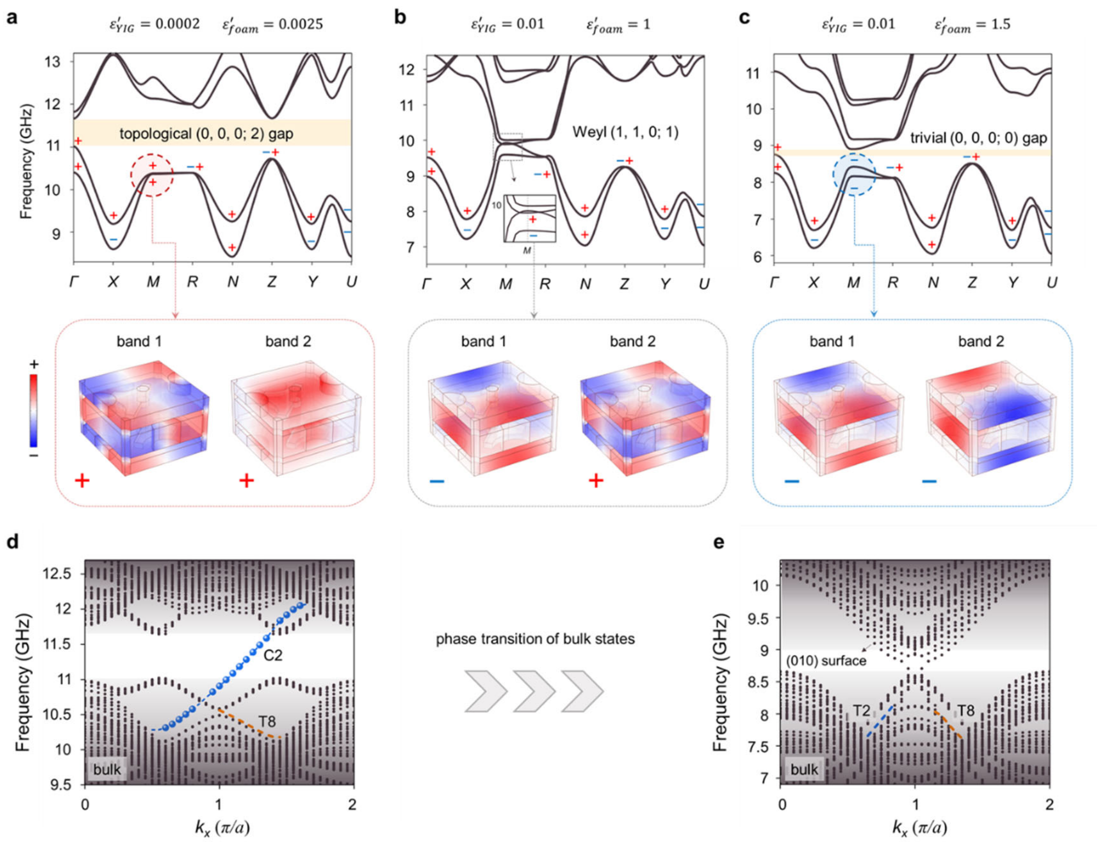


Fig. R4 | Non-radiative loss-induced topological phase transition. a-c, Bulk band structures under continuously increasing the non-radiative loss. The lower panels show the Bloch field distributions (real part of z-component electric field) of the two lower bands at M point. Symbol +/− represent the even/odd parity. **d-e,** Projected band structures for case **a** and case **c**.

Action:

We have added several sentences in the Methods section to illustrate the effect of the loss on the topological phases.

See line 312: "*In the realistic system, both non-radiative and radiative losses are very small, and their influence on the hinge states is negligible (Supplementary Fig. 12). We also simulated that the continuous increase of loss could tune the chiral route (radiative loss) or give rise to the phase transition (non-radiative loss), as shown in Supplementary Figs. 13-15.*"

We have added the above figures in Supplementary Materials (Supplementary Figs. 12-15).

Comments of Reviewer #1:

2) On two occasions the authors mention control of "light" (line 65) and laser "emission" (line 230). I doubt the proposed scheme is realizable in optical domain and therefore these claims should be removed or wording should be softened. I suggest the authors to be clear about challenges of realizing such complex geometry and magnetization pattern at scales needed for optical frequencies. Please revise the manuscript accordingly.

Our Response:

We thank the reviewer for pointing out our inappropriate expression. We agree that the current scheme is difficult to implement in the optical domain. The main challenge is the fabrication of three-dimensional micro- or nano-structured photonic crystals, especially with the anti-ferromagnetic-like pattern for optical frequencies.

Action:

In the revised version, we have softened the relative description in the main text.

See line 66: "*..., offering an unprecedented way to control electromagnetic waves.*"

Also see line 238: "*The observed non-coplanar chiral hinge transport may facilitate photonic communications, microwave isolators and circulators.*"

Comments of Reviewer #1:

3) I found that the authors did not acknowledge properly some earlier relevant works on higher-order electromagnetic. Specifically, what are implications of long-range interactions, which are known to give rise to some nontrivial physics in HOTIS [Nature Photonics 14 (2), 89-94], for their electromagnetic systems? Another relevant work where unidirectional hinge transport was proposed in [Phys. Rev. Applied 13, 064031]. Please compare and highlight differences between hinge modes of this work and those of your work.

Our Response:

We thank the reviewer for raising this issue. The long-range interactions among non-neighboring unit cells may be inevitable in electromagnetic systems. The work mentioned above [*Nat. Photon.* **14**, 89-94 (2020)] provides insight into the interplay between long-range interactions and higher-order topological phases, leading to a new kind of higher-order corner states. In our photonic axion insulator system, the long-range interactions could not lead to new nontrivial physics but may give rise to the novel windings of gapless hinge states.

Here, we consider the long-range onsite interaction (t_{NN}) in the x-y plane using the tight-binding model (Fig. R5a). Even under a relatively large long-range interaction (e.g., t_{NN}=0.5), the chiral hinge states maintain gapless, though it winds twice along the entire Brillouin zone [taking the z-axis hinge state as an example shown in Fig. R5(b-c)]. Such winding comes from the coupling between the nontrivial hinge states and the two additional hinge states (one per layer) induced by the long-range interaction. In fact, in our realistic model, the long-range interaction is very weak, resulting in no winding feature of the hinge state (Fig. 3b in the main text).

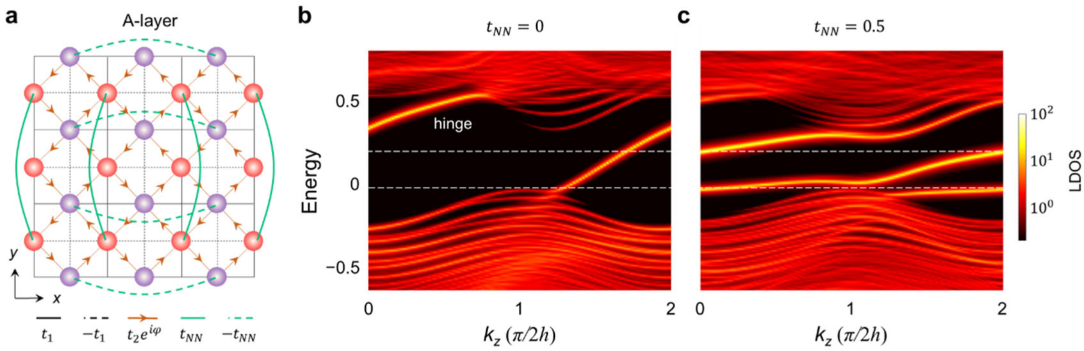


Fig. R5 | Influence of long-range interactions. a, Tight-binding model of the A layer. The B layer is inversion symmetric to the A layer. **b-c,** The local density of state (LDOS) for the z-axis hinge state without (t_{NN}=0) and with (t_{NN}=0.5) long-range interaction. The gapless hinge states wind twice along the entire Brillouin zone when t_{NN}=0.5. The tight-binding parameters are chosen to be $t_1 = 1, t_2 = 0.4, \varphi = \pi/4$.

Regarding to the unidirectional hinge transport, we compare the differences between our work and the higher-order Chern insulator case [*Phys. Rev. Appl.* **13**, 064031 (2020)]. In our axion insulator phase, the hinge states are inversion-symmetry distributed at the x-/y-/z-axis hinges between two surfaces with opposite signs of surface Dirac masses, eventually forming a unique closed and non-coplanar chiral loop of hinge transport C6-C1-C2-C3-C4-C5 (Fig. R6a). These chiral hinge states are protected by $\mathbb{Z}_4$ topological invariant. However, the higher-order Chern insulator case is protected by the Wannier-based Chern number. To the best of our knowledge, the chirality of unidirectional hinge transport is coplanar, exhibiting on the lateral surfaces (Fig. R6b).

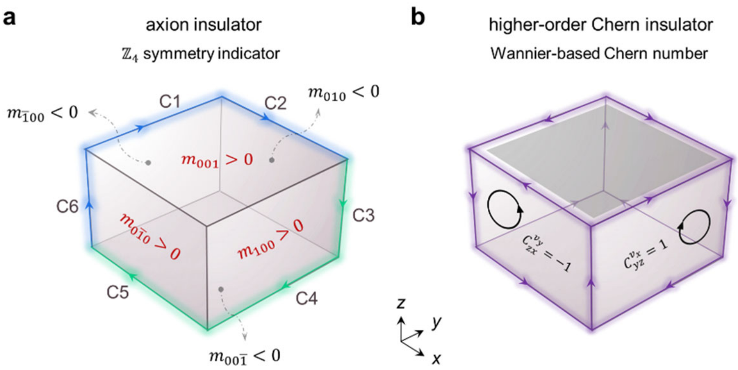


Fig. R6 | Schematic of two kinds of unidirectional hinge transport. a, Non-coplanar chiral hinge transport of our photonic axion insulator. **b,** Higher-order Chern insulator case [*Phys. Rev. Applied* **13**, 064031 (2020)]. Here, *C* represents the Wannier-based Chern number of the lateral surfaces.

Action:

In the Introduction section, we have cited the relevant works on higher-order electromagnetic.

See line 53: "*... non-magnetic topological and higher-order topological phases have successfully found their classical-wave analogues* [Ref. 30: *Nat. Photon.* **14**, 89-94 (2020)] ..."

Also see lines 58: "*..., further uncovering the flexibility of the photonic and electrical circuit platforms in exploring MTPs* [Ref. 40: *Phys. Rev. Applied* **13**, 064031 (2020)], ..."

In the Methods section, we have discussed the influence of long-range interactions.

See line 288: "*In addition to the nearest coupling, long-range interactions between non-neighbouring unit cells are inevitable in electromagnetic systems* [Ref. 30: *Nat. Photon.* **14**, 89-94 (2020)]. *We theoretically investigate the influence of long-range interactions on the axion insulator phase. Even under a relatively large long-range interaction, the* $\mathbb{Z}_4$ *topology associated with gapless hinge states remains intact (Supplementary Fig. 11).*"

In the Discussion and Outlook section, we have compared two kinds of unidirectional hinge transport.

See line 227: "*Such AXI protected by the* $\mathbb{Z}_4$ *topological invariant exhibits a unique non-coplanar closed chiral loop for hinge transport, which is different from the 3D higher-order Chern insulator protected by Wannier-based Chern numbers with coplanar chiral hinge transport on the lateral surfaces* [Ref. 40: *Phys. Rev. Appl.* **13**, 064031 (2020)]."

We have added Fig. R5 in Supplementary Materials (Supplementary Fig. S11).

Comments of Reviewer #1:

4) Finally, some earlier works relying on axion electrodynamic response (called Tellegen response in photonics) can be cited for completeness as well. See for example Refs. [New J. Phys. 17 125015 (2015) and Opt. Express 30, 47004-47016 (2022)].

Our Response and Action:

We have cited the related works on axion electrodynamic response in the Introduction part.

See line 56: "*Moreover, recent progress in the study of axion electrodynamic response* [Refs. 34-36: *New J. Phys.* **17**, 125015 (2015); *Proc. Natl. Acad. Sci.* **113**, 4924 (2016); *Opt. Express* **30**, 47004 (2022)], ..."

Comments of Reviewer #1:

Provided the proposed minor revisions are made, I will be more than happy to recommend this overall nice and wonderful work for publication in Nature Communications.

Our Response

Again, we thank Reviewer #1 for his/her recommendation and detailed comments.

Report of Reviewer #2 -- NCOMMS-24-28639

Comments of Reviewer #2:

Higher order axion insulator, being trivial both in Chern number and Z2 but nontrivial in Z4 topological invariant, can support chiral spectral flow at their hinges. However, the experimental observation of them in natural materials remains elusive. The authors Lai et al. propose a photonic design of axion insulator based on a three-dimensional antiferromagnetic-like structure and experimentally realize it in microwave bands. They not only observe the non-reciprocal and non-coplanar transport of chiral hinge states but also experimentally demonstrate the tunability of such states. This work represents the first example of experimentally observing those effects, which is timely and very interesting. It might have broad impact in the community of photonics and other classical-wave systems. Due to its innovative character and potential impact, in my opinion, it is highly desirable to be published in Nature Communications.

Our Response:

We thank Reviewer #2 for his/her positive comments and detailed review.

Comments of Reviewer #2:

Nevertheless, some technical issues and questions must be specified and corrected before the acceptance.

(i). In the section of "3D AXIs stacked from 2D", the authors elaborate how the hinge states in x/y hinges are constructed, but it is not clear to me whether the hinge states in the z direction are ensured to exist. Can the authors explain the underlying mechanism of protecting the gappless hinge states along z direction?

Our Response:

We thank the reviewer for raising this critical point. As we mentioned in previous version, for an odd-layer scenario, the x-/y-directional hinge states on the top and bottom facets are divided into two symmetric yet open halves. The z-directional hinge states should be present, linking the separate halves to form the eventual closure of the loop. We agree with the review that it needs to be clarified whether the z-directional hinge states are ensured to exist.

To explain the underlying mechanism, we analyze the surface Dirac masses for all six surfaces. The axion insulator relates to higher-order phases of three-dimensional topological insulators, wherein the bandgap of Dirac surface states is opened by adding magnetic fields to break the time-reversal symmetry. Thus, these gapped surface states will acquire finite masses. Guaranteed by the inversion symmetry, the opposite surfaces have opposite signs of surface Dirac masses (m), e.g., $m_{100} > 0, m_{\bar{1}00} < 0$, as shown in Fig. R7. The hinge formed by two surfaces with opposite signs of surface Dirac mass terms possesses the gapless hinge state. The hinge energy flow is clockwise around the positive surface Dirac mass region [Ref: *Phys. Rev. Res.* **2**, 043274

(2020)]. And, the hinge formed by the same signs of two surface Dirac masses is trivial. Consequently, under inversion symmetry, hinge states of C1-C6 eventually form a closed and non-coplanar chiral loop.

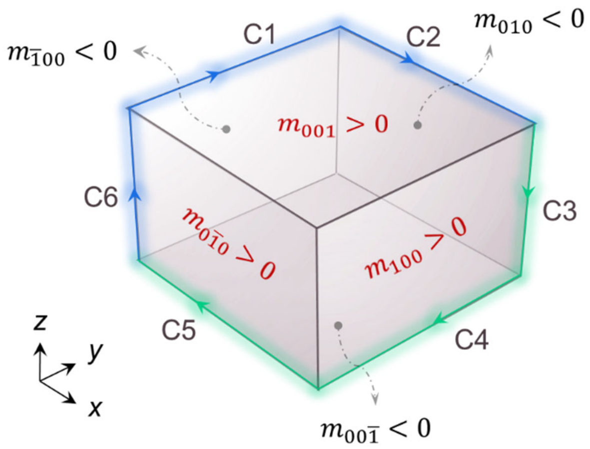


Fig. R7 | Schematic of surface Dirac masses in odd-layer scenario (inversion symmetry). The gapless hinge states exist at the hinge formed by two surfaces with opposite signs of surface Dirac mass terms (*m*).

Action:

We have explained the underlying mechanism of gapless hinge states in the main text.

See line 95: "*These gapped surface states will then acquire finite masses, with opposite signs for opposite surfaces guaranteed by the inversion symmetry. The hinge formed by two surfaces with the opposite signs of surface Dirac mass terms gives rise to gapless hinge states, as shown in Fig. 1c. The energy flow is clockwise around the positive surface Dirac mass region* [Ref. 41: *Phys. Rev. Res.* **2**, 043274 (2020)]."

We added a new reference [Ref. 41: *Phys. Rev. Res.* **2**, 043274 (2020)] in the main text.

We have also slightly revised upper panel of Fig. 1c by adding the surface Dirac masses. See:

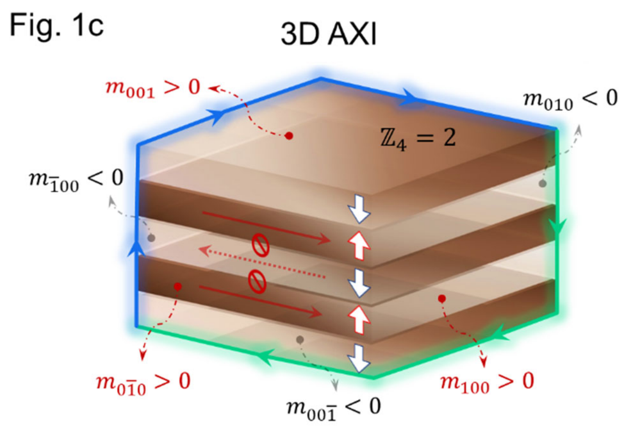


Fig. 1 | From 2D to 3D magnetic structures. … **c**, … The gapless hinge state appears at the hinge formed by two surfaces with opposite signs of surface Dirac mass terms (*m*).

Comments of Reviewer #2:

(ii). I'm curious how the designs of "T-shirt-like hollow holes" and "circular holes" come across the mind, what are their specific roles in creating the topological properties of axion insulator and why they need to be tilted 225/45 deg in the metal plane?

Our Response:

We thank the reviewer's comments. Indeed, the "T-shirt-like hollow holes" or "tilted 225/45 degrees" are not the necessary conditions to realize the axion insulator. Such structure is an optimized result to obtain a relatively large full bandgap and facilitate experiments. Our design strategy is as follows.

(i) Symmetry requirement. To avoid the influence of lateral surface states, we break the M_z symmetry but keep the inversion symmetry intact. So, we drill holes in one half of the AB interlayer and in the other half of the BA interlayer, as shown in Fig. R8. To achieve a relatively large M_z-symmetry-breaking effect, we choose the "T-shirt-like hollow holes", which are easy to fabricate and good-looking.

(ii) Relatively large bandgap. In our model, the larger directional interlayer coupling can bring a larger bulk and surface bandgap. Therefore, we tilt the "T-shirt-like hollow holes" 225/45 degrees to enlarge the directional interlayer coupling between YIG rods of neighbouring layers (along the $[11\bar{1}]$ direction).

(iii) Convenience for measurement. The circular through holes in both the corner and the centre of unit cells are designed to insert the source and detector in experiments. Thus, we can probe the electromagnetic fields inside the sample.

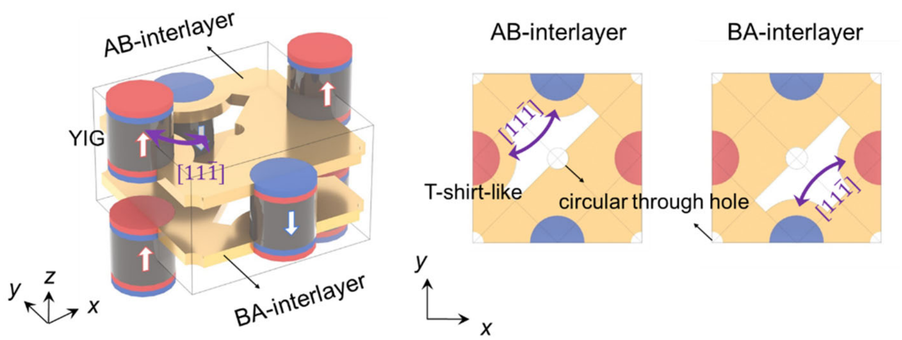


Fig. R8 | Photonic unit cell and top views of the AB and BA interlayers. Here, purple arrows denote the $[11\bar{1}]$ directional interlayer coupling between YIG rods of neighbouring layers.

Action:

We have added a sentence to address this issue.

See line 105: "*..., which is elaborately designed to enlarge the directional interlayer coupling to obtain a relatively large bandgap.*"

We have also added some description in the Methods section.

See line 263: "*In the design of the axion insulator, the T-shirt-like hollow holes in one (the other) half of the AB (BA) interlayer provide a relatively large M_z-symmetry-breaking effect, and they are tilted 225/45 degrees to enlarge the directional interlayer coupling between YIG rods of* *neighbouring layers (along the* $[11\bar{1}]$ *direction).*"

Also see line 295: "*The circular through holes (radius* r_2*) in the sample allow us to insert the detector to measure the internal electric fields of the sample.*"

Comments of Reviewer #2:

(iii). In line "126", the three components of topological index Z2 are not weak Chern numbers, they are weak indices of 3D topological insulators, Chern number belongs to Z class instead of Z2 class.

Our Response:

We agree with the reviewer that the three $\mathbb{Z}_2$ components can be chosen as the three weak indices (v_x, v_y, v_z) of 3D topological insulators. For a 3D inversion-symmetric case, (v_x, v_y, v_z) are equivalent to the parity of the Chern number on three distinct 2D $k_{x,y,z} = \pi$ planes.

Action:

To avoid misunderstanding, we used "*weak indices of 3D topological insulators*" in the current version.

Comments of Reviewer #2:

(iv). In line 138, "On each surface, a half-turn of a closed chiral loop results in the half-integer surface Hall response", it is not necessary to mention Hall response in their result section if it has no implication or connection to the photonic system.

Our Response and Action:

We appreciate the reviewer's suggestion. We have deleted these unnecessary descriptions.

Comments of Reviewer #2:

(v). The language and readability of manuscript need to be improved. I have listed a few (but not all) examples which have grammar errors and/or are hard to interpret.

in lines 91-92, "AXIs may also relate to higher-order phases of 3D topological insulators after breaking time-reversal symmetry to open the native gapless Dirac surface states.", AXIs themselves cannot break TRS but magnetic field can, and "bandgap of Dirac surface states is open" rather than "the surface states is open".

In lines 135, "It is because.." does not sound natural, what does "it" refer to?

In line 170, it's better to use "absorptive surface" or "lossy surface" instead of "absorption surface". In the captions of Fig.4, facets are "absorptive" not "absorption".

Our Response and Action:

We sincerely appreciate the reviewer's comments and suggestions about the language and readability of our manuscript. In the revised version, we have thoroughly checked the entire text to correct the grammatical and spelling errors. The language has also been polished.

See line 94: "…, *wherein the bandgap of Dirac surface states is opened by adding magnetic fields to break the time-reversal symmetry.*"

See line 143: "*Such odd-layer configuration perfectly maintains the inversion symmetry, …*"

We have used *"absorptive"* instead of *"absorption"* in the whole manuscript.

Comments of Reviewer #2:

In line 198, "Considering it comes from the top surface modification of the odd-layer case" is hard to understand.

Our Response and Action:

We thank the reviewer's comments. We have revised such ambiguous description.

See line 206: "*Since the even-layer (e.g., 14-layer) configuration can be regarded as the removal of just one top layer from the odd-layer (e.g., 15-layer) case, the lateral and bottom chiral hinge states (C3-C6) remain unchanged as those in Fig. 3, while the surface Dirac mass term of the top facet now becomes negative* $(m_{001} < 0)$."

Comments of Reviewer #2:

In line 214 "tremendous tunability owes to the unique symmetry operation", the meaning of symmetry operation based on the context is unclear.

There is a typo in line 87, it should be "top and bottom facets" instead of "top and bottom facts".

Our Response and Action:

We thank the reviewer for pointing out the unclear expression and typos. In the current version, we have deleted the above ambiguous description and corrected the typos.

Comments of Reviewer #2:

(vi). The authors illustrate the relations between 2D and 3D magnetic TIs in the schematic of Fig.1 well, but In line 226, it is a bit exaggerated to claim their results "provide a comprehensive picture of AXIs".

Our Response and Action:

This inappropriate description has been revised.

See line 235: "*Our results provide a deeper understanding of AXIs, …*"

Summary

As the reviewer can see, all technical issues have been well addressed. We sincerely hope the reviewer can support us.

A List of Changes -- NCOMMS-24-28639

All the revisions are highlighted in the manuscript.

Main Text:

We have added several sentences in the main text to address the reviewers' comments.

We have slightly revised Fig. 1(c) by adding the surface Dirac masses.

We have revised and deleted some ambiguous or inappropriate descriptions according to the reviewers' suggestions.

We have added 6 new references, including Refs. [30, 34-36, 40, 41].

Supplementary Materials:

We have added additional 5 supplementary figures and corresponding descriptions in the Supplementary Materials, including Supplementary Figs. 11-15.

Second Report of Reviewer #1 -- NCOMMS-24-28639A

Comments of Reviewer #1:

The authors have fully addressed my and other reviewers' comments and criticism. I recommend publication of this work in its present form.

Our Response:

We appreciate Reviewer #1 for their recommendation of our work.

Second Report of Reviewer #2 -- NCOMMS-24-28639A

Comments of Reviewer #2:

The author did not answer my first question seriously. They provided a perspective of mass term as their explanation, but this explanation is kind of hand-waving and too vague, and they did not investigate their magnetic photonic crystal at all. I insist on asking this question since the external magnetic field is parallel to z direction. From the picture of orbital bounding, the nonreciprocal hinge states along z direction are not guaranteed to exist.

Our Response:

Thank you very much for your insightful comments. There was a misunderstanding during the first round of review. The surface mass terms were used to explain the non-coplanar fashion of the closed chiral hinge loop in an axion insulator phase. The question regarding how z-directional magnetization induces the same z-directional nonreciprocal hinge states from the picture of orbital bounding is quite thought-provoking. In general (trivial) cases, it is definitely true only the out-of-plane magnetization can influence the in-plane orbit, resulting in nonreciprocal propagation. However, in magnetic topological phases, the situation is significantly different. For instance, recent studies in electronic systems have shown that in-plane magnetizations could also induce in-plane Hall response in certain antiferromagnetic structures [Refs: *Sci. Adv.* **6**, eaaz8009 (2020); *Nat. Mat.* **24**, 63 (2025)].

The underlying physics can be understood as a fictitious magnetic field induced by the nontrivial Berry curvature (or saying an effective magnetic field in the momentum space). This phenomenon arises from the simultaneous breaking of time-reversal symmetry and mirror symmetry in specific types of antiferromagnetic order. In our photonic system, the magnetization breaks time-reversal symmetry ($\mathcal{T}$), and the elaborately designed directional coupling between neighbouring layers breaks mirror symmetry respective to the *z* direction (M_z). Thus, the in-plane (*x*-/*y*-directional) fictitious magnetic field can be induced when the Berry curvature, such as $\int Tr(\Omega_{x,y})dk_z$, is nontrivial [Fig. R1(a-b)] [Refs: *Sci. Adv.* **6**, eaaz8009 (2020); *Nat. Mat.* **24**, 63 (2025)]. Here, $\Omega_{x,y}$ represents the *x*-/*y*-component of Berry curvature.

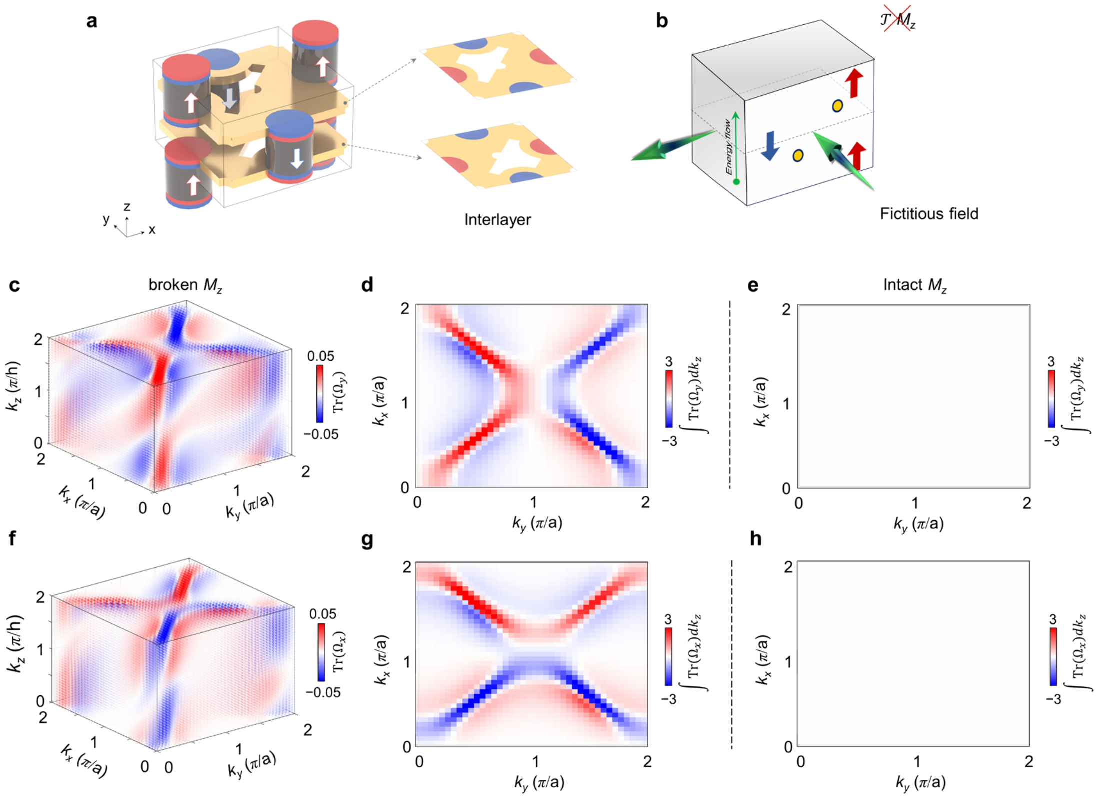


Fig. R1 | Fictitious fields induced by nontrivial Berry curvature. a, Unit cell of our photonic axion insulator. The magnetizations break the time-reversal symmetry ($\mathcal{T}$) and the directional interlayer structures break the mirror symmetry along the *z* axis (M_z). **b,** Schematic of in-plane fictitious fields depending on nontrivial Berry curvature with broken $\mathcal{T}M_z$ symmetry. **c-d,** The *y*-component Berry curvature (Ω_y) and its integral along the k_z axis. **e,** Trivial case with unbroken M_z symmetry. **f-h,** The same as **c-e** except with the *x*-component Berry curvature (Ω_x).

To clearly illustrate it, we calculate the Ω_y based on the tight-binding model $H(k) = H_1 + H_2$ (as outlined in the Methods Section). In this model, the intralayer coupling H_1 breaks the $\mathcal{T}$ symmetry, and the interlayer coupling term $H_2 = f_5\frac{\sigma_x+i\sigma_y}{2}\tau_x + f_6\frac{\sigma_x-i\sigma_y}{2}\tau_x$ breaks M_z symmetry. As shown in Fig. R1(c-d), the presence of Ω_y and $\int Tr(\Omega_y)dk_z$ with nontrivial values suggest the existence of a fictitious field in the *y* direction. The sign of the field relies on the sign of the integral of Berry curvature over the entire Brillouin zone. For comparison, when the M_z symmetry is preserved ($H_2 = 0$), the system turns into the conventional (trivial) case, where no fictitious field can be found corresponding to vanishing Ω_y values (Fig. R1e). We also calculate the *x*-directional fictitious field case, as shown in Fig. R1(f-h). Consequently, the nonreciprocal *z*-directional hinge states can emerge even with the same *z*-directional magnetization in our magnetic topological phase.

Here, we also employ the Wilson loop approach to verify the existence of *z*-directional gapless chiral hinge states. In this approach, we choose an *x*-directed slab that has a finite size along the *x* direction while being periodic along the other two directions (Fig. R2a). The corresponding band structures are shown in Fig. R2b. Using the finite-element method applied to the actual photonic structure, we numerically calculate the k_y-directed Wilson loop spectrum, as shown in Fig. R2(c-d). The Wilson loop exhibits a +1 winding along the k_z direction, corresponding to a nontrivial slab Chern number (G_x=1). The result implies that our system is topologically equivalent to a Chern insulator, which can possess gapless chiral states in the *z* direction when sliced into an *x*-slab geometry.

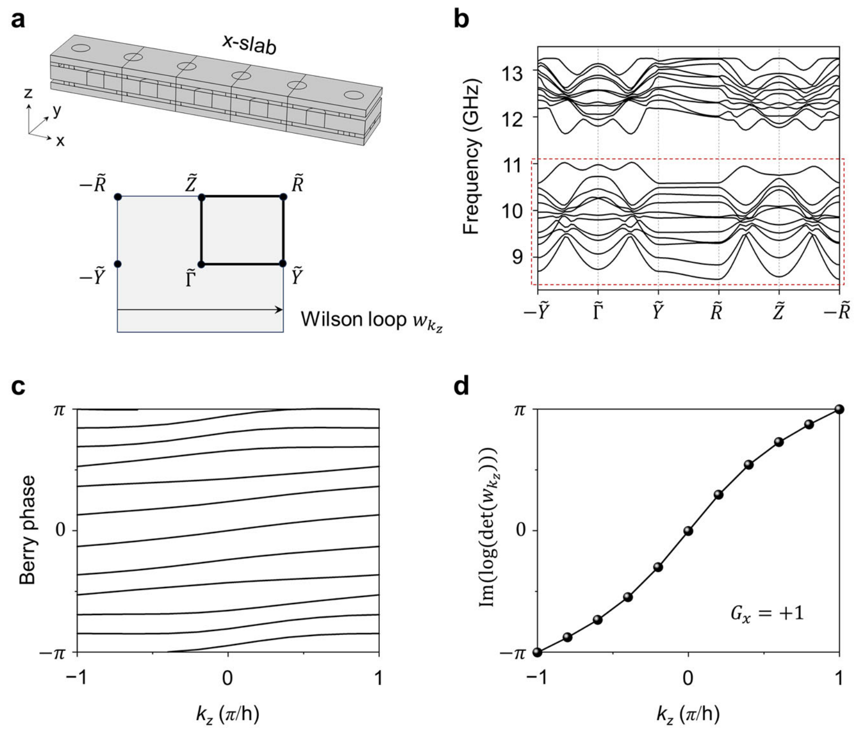


Fig. R2 | Wilson loop spectrum for an *x*-directed slab. a, An *x*-directed slab with 5 unit cells along the *x* axis, while being periodic along the other two directions. **b**, Band structures. **c**, The k_y-directed Wilson loop spectrum for the 11 bands below the bandgap. **d**, Summation of the Wilson loop eigenvalues in **c**, exhibiting +1 winding along the k_z axis, corresponding to the nontrivial slab Chern number (G_x=1) and axion angle ($\theta = \pi$).

Notably, the nontrivial slab Chern number can be connected to the axion angle θ: $G_x = n\nu_x + \theta/\pi$ [Ref: *Phys. Rev. B* **98**, 245117 (2018)]. Here, $\nu_x = 0$ denotes the weak indices of three-dimensional topological insulators, and *n* counts the units of the slab, thereby also confirming the nontrivial axion angle of $\theta = \pi$ of our photonic system.

Action:

We have added several sentences in the main text to illustrate this issue.

See line 233 in the Discussion and Outlook: "*It should be noticed that the nonreciprocal z-directional hinge states under the same z-directional magnetization cannot occur from the traditional picture of orbital bounding in topologically trivial cases. The underlying physics can be explained by the xy-plane fictitious magnetic field induced by the nontrivial Berry curvature, resulting from the simultaneous breaking of time-reversal symmetry and mirror symmetry in our antiferromagnetic-like structure (See Supplementary Figs. 11-12). This phenomenon is similar to the in-plane magnetization-induced Hall effect observed in electronic systems* [Refs. 46-47*: Sci. Adv.* ***6****, eaaz8009 (2020); Nat. Mat.* ***24****, 63 (2025)*]. "

Also see line 146: "*Each surface exhibits a half-turn of a closed chiral loop. The opposing surfaces combine to create a full turn, representing the nontrivial axion angle* $\theta = \pi$ [Ref. 45: *Phys. Rev. B* ***98****, 245117 (2018)*]."

We have also added the above figures in the Supplementary Materials (Supplementary Figs. 11-12).

Comments of Reviewer #2:

I would expect they at least derive the effective Hamiltonian from their photonic system and analyze its mass terms, which has non-local interaction and is completely different from simple tight-binding model. Therefore, it is still unclear to me why their structure supports topological hinge states along z dimension. This is a critical point which determines whether their photonic system truly belong to the axion insulator.

Our Response:

We thank Reviewer #2 for their comments. In our photonic axion insulator, all the surface states are gapped with the broken time-reversal symmetry. These surface states can be analytically described via two-dimensional massive Dirac equations. For instance, considering the *xz*-surface states, the effective Hamiltonian can be written as $H_s(k_x, k_z) = v_x k_x \tau_z + v_z k_z \tau_y + m\tau_x$, where $v_{x,z}$ denotes the group velocity along each direction, $\tau_{x,y,z}$ denotes Pauli matrix, and *m* denotes the surface Dirac mass. Here, the sign of m depends on the sign of the Berry curvature of the lower-energy surface states near the valley, as shown in Fig. R3.

Effective surface Hamiltonian: $H_s(k_x, k_z) = v_x k_x \tau_z + v_z k_z \tau_y + m\tau_x$

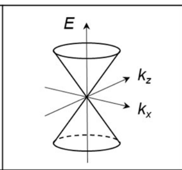
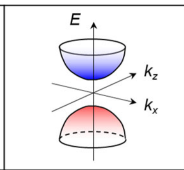
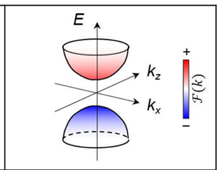
Band structure			
Sign of Dirac mass term (m)	m = 0	m > 0	m < 0
Lower band's Berry curvature [$\mathcal{F}(k)$]	ill-defined	$\mathcal{F}(k) > 0$	$\mathcal{F}(k) < 0$

Fig. R3 | Schematic of surface Dirac masses (*m*) and related surface Berry curvature [$\mathcal{F}(\boldsymbol{k})$] for *xz* surface. Here, $v_{x,z}$ denotes the group velocity, $\tau_{x,y,z}$ denotes Pauli matrix.

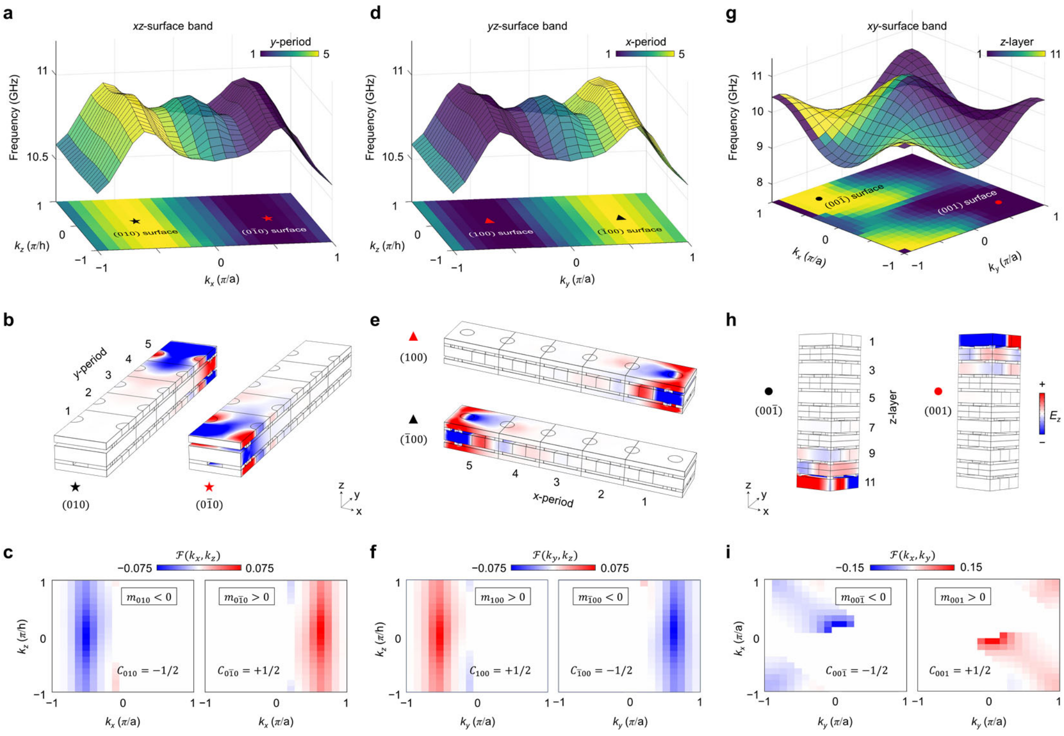


Fig. R4 | Surface Dirac masses of our photonic AXI. a, The *xz*-surface band below the bandgap, including both the (010) and $(0\bar{1}0)$ surface states. Colorscale presents the contribution of all states to the (010) surface (denoted as period 5) and $(0\bar{1}0)$ surface (period 1). **b,** Field distribution of the (010) and $(0\bar{1}0)$ surface modes. **c,** Numerically calculated Berry curvature $\mathcal{F}(k_x, k_z)$. The opposite signs of Berry curvature near the valley indicate the opposite Dirac masses for opposite surfaces. The integration of Berry curvature gives the surface Chern number $C_{\boldsymbol{n}} = \pm 1/2$, equivalent to $sign(m_{\boldsymbol{n}})/2$, where the subscript $\boldsymbol{n}$ represents surface normal. **d-f,** The *yz*-surface case. **g-i,** The *xy*-surface case.

Here, we numerically calculate the Berry curvature for each surface state, as shown in Fig. R4. Taking the *xz* surface as an example, the surface band below the bandgap (Fig. R4a) includes both (010) and $(0\bar{1}0)$ surface states, which are symmetrically distributed in half of the Brillouin zone enforced by inversion symmetry (Fig. R4b). However, their Berry curvatures exhibit opposite values around different valleys due to the breaking of $\mathcal{T}M_z$ symmetry, resulting in opposite signs of Dirac masses for opposite surfaces (Fig. R4c). Similarly, we also calculate the *yz*-surface and *xy*-surface cases [Fig. R4(d-f) and Fig. R4(g-i)], consistent with the mass distributions in Fig. 1c in the main text.

By further integrating the Berry curvature, we can find the surface Dirac mass terms are related to the half-quantized surface Chern numbers via the relation: $C_{\boldsymbol{n}} = \pm 1/2$, equivalent to $sign(m_{\boldsymbol{n}})/2$. Therefore, on the hinge between two neighbouring surfaces with opposite Dirac masses, the differences of surface Chern number results in a topological invariant $\Delta C_{\boldsymbol{n}} = \pm 1$, leading to the existence of topological hinge states (the sign of $\Delta C_{\boldsymbol{n}}$ determines the direction of energy flow).

Action:

We have added several sentences in the main text to illustrate this issue.

See line 95: "*These gapped surface states will then acquire finite Dirac masses, with the signs depending on the Berry curvature of the lower-energy surface band. Due to inversion symmetry, the mass terms for opposite surfaces have opposite signs.*"

Also see line 177: "*We also numerically calculate the surface Dirac masses and related surface Chern numbers for our photonic AXI (See Supplementary Fig. 7), agreeing with those in Fig. 1c.*"

We have also added Fig. R4 in the Supplementary Materials (Supplementary Fig. 7).

Comments of Reviewer #2:

I would not commend its publication until they resolve this question in a persuasive way.

Our Response:

At last, we would like to emphasize that the current work focuses on the experimental realization of a photonic axion insulator. The experimental results presented in Figures 3 and 4 in the main text, as well as Figures 9 and 10 in the Supplementary Materials, unambiguously demonstrate nonreciprocal chiral hinge transport, which agrees well with numerical simulations. Although a derivation of the effective

Hamiltonian directly from Maxwell's equations in such a complex 3D antiferromagnetic structure may provide a new and alternative view, we consider it to be a lengthy process and beyond the scope of our current work. Nonetheless, as the reviewer can see, we have employed various theoretical methods, including symmetry indicator analysis (nontrivial $\mathbb{Z}_4$ topological invariant), surface Dirac mass evaluation (related to surface Chern number $C_{\boldsymbol{n}} = \pm 1/2$), and axion angle calculation ($\theta = \pi$), to confirm that our photonic model truly belongs to the axion insulator.

We believe all the issues have been addressed in detail. We hope the reviewer can support us now.

A list of changes -- NCOMMS-24-28639A

All the revisions are highlighted in the manuscript.

Main Text:

We have added several sentences in the main text to address the reviewers' comments.

We have added 3 new references, including Refs. [45-47].

Supplementary Figures:

We have added additional 3 supplementary figures and corresponding descriptions in the Supplementary Materials, including Supplementary Figs. 7, 11, 12.

Third Report of Reviewer #2 -- NCOMMS-24-28639B

Comments of Reviewer #2:

The atuhors address my previous comment in a convincing way, I have no further comment.

Our Response:

We appreciate Reviewer #2 for their recommendation of our work.