Supplementary Information

Magnetic geometry induced quantum geometry and nonlinear transports

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1. Brief introduction to spin space groups

Here we briefly introduce the relevant basics of spin space groups (SSGs), including their symmetry elements, group structure, international notations, and identification. For more details on SSG theory, please refer to Refs. [1,2].

1.1. Symmetry elements in spin space groups

Without the effect of spin-orbit coupling (SOC), the key characteristic of SSGs is the partial separation of spatial and spin operations. The symmetry elements of SSGs are denoted by the generalized Seitz notation $\{u||r|\boldsymbol{\tau}\}$ where u and r are the spin and lattice rotation, respectively, and $\boldsymbol{\tau}$ is the lattice translation. In lattice space, r can be a proper rotation $R_{\boldsymbol{n}}(\theta)$ or improper rotation $R_{\boldsymbol{n}}(\theta)P$ (rotoinversion), where $R_{\boldsymbol{n}}(\theta)$ denotes the spatial rotation along the $\boldsymbol{n}$ axis by angle θ, and P denotes spatial inversion. In spin space, u is either a proper spin rotation $U_{\boldsymbol{m}}(\phi)$ or improper spin rotation $U_{\boldsymbol{m}}(\phi)T$, with $U_{\boldsymbol{m}}(\phi)$ and T are the spin rotation along the $\boldsymbol{m}$ axis by angle ϕ and time-reversal, respectively. Notice that the role of T in spin space is analogous to that of inversion P in lattice space.

1.2. Group structure of spin space groups

The general form of SSGs G_{SS}, can be expressed as a direct product of the nontrivial SSG G_{NS} and the spin-only group G_{SO}, *i.e.*, $G_{SS} = G_{NS} \times G_{SO}$. Spin-only group contains pure spin rotations $\{u||E|\mathbf{0}\}$, and nontrivial SSG contains no pure spin rotation.

The spin-only group characterizes the general type of magnetic geometry, including collinear, coplanar (but not collinear), and noncoplanar types. For collinear magnetic geometry, the spin-only group is $G_{SO}^{L} = SO(2) \rtimes \{E, U_{\perp}(\pi)T\}$, where $U_{\perp}(\pi)$ is a two-fold spin rotation about an axis perpendicular to the Néel vector, and $SO(2) = \{U_{\parallel}(\phi)|\phi \in [0,2\pi)\}$ is generated by spin rotation along the Néel vector. For coplanar magnetic geometry, the spin-only group is $G_{SO}^{P} = \{E, U'_{\perp}(\pi)T\}$, where $U'_{\perp}(\pi)$ is a two-fold spin rotation along the axis normal to all magnetic moments. For noncoplanar magnetic geometry, the spin-only group is a trivial group only containing identity operation.

As for the nontrivial spin space group, it describes the spin and spatial symmetry of any magnetic geometry in more detail. The nontrivial spin space group G_{NS} is constructed through the group extension procedure upon crystallographic space groups (SGs). Given a crystallographic SG G_0, it can be decomposed into cosets with respect to one of its normal subgroups $L_0 \lhd G_0$, as

$$G_0 = L_0 \cup g_1 L_0 \cup \cdots \cup g_{n-1} L_0. \tag{S1}$$

Here L_0 is also a SG, and $\{g_1, \cdots, g_{n-1}\}$ are coset representatives with $g_i = \{r_i|\boldsymbol{\tau}_i\}$. When the quotient group G_0/L_0 is isomorphic to a point group (not necessary a crystallographic one), one can attribute the operations in this point group as spin operations, forming $G^s = \{E, u_1, \cdots, u_{n-1}\}$ ($u_i \in SO(3) \times \{E, T\} \cong O(3)$). Then the nontrivial SSG G_{NS} can be constructed through isomorphism relationship

$$G_{NS} = \{E||L_0\} \cup \{u_1||g_1 L_0\} \cup \cdots \cup \{u_{n-1}||g_{n-1} L_0\}. \tag{S2}$$

Any nontrivial SSG can be further categorized into three types by relation between G_0 and L_0.

1) *t*-type SSG

A subgroup $L_0 < G_0$ is referred to as a *translationengleiche* subgroup or a *t*-subgroup of G_0, if their translation subgroups are identical $T(L_0) = T(G_0)$, while the point group part is reduced $P(L_0) \leq P(G_0)$. An SSG formed in this way is called a ***t*-type SSG**. We use a *t*-index i_t to label the reduction of point group $i_t = |P(G_0)|/|P(L_0)|$, where $|G|$ denote the number of elements of a finite group G.

2) *k*-type SSG

A subgroup $L_0 < G_0$ is referred to as a *klassengleiche* subgroup or a *k*-subgroup of G_0, if their point group parts are identical $P(L_0) = P(G_0)$, while the translation subgroup is reduced $T(L_0) < T(G_0)$. An SSG formed in this way is called a ***k*-type SSG**. We use a *k*-index i_k to label the reduction of translation subgroup $i_k = |T(G_0)|/|T(L_0)|$. The *k*-subgroup implies cell multiplication to accommodate the magnetic geometry, where $T(G_0)/T(L_0) = Z_{n_1} \times Z_{n_2} \times Z_{n_3}$ describes orders of multiplications. While n_i are natural numbers, the group structures of G_0/L_0 are countless. A cut-off of $i_k = n_1 n_2 n_3$ has been applied in Ref. [2] to enumerate the SSGs that require cell expansion.

3) *g*-type SSG

A subgroup $L_0 < G_0$ is named *general* subgroup of G_0, if both the translation group and point group part are reduced, *i.e.*, $T(L_0) < T(G_0)$ and $P(L_0) < P(G_0)$. An SSG formed in this way is called a ***g*-type SSG**. We use both $i_t = |P(G_0)|/|P(L_0)|$ and $i_k = |T(G_0)|/|T(L_0)|$ to label the reduction.

Starting from 230 crystallographic space groups and finite translation groups with a maximum order of eight $i_k \leq 8$ (this can cover the case of cell doubling in three directions $Z_{n_1} \times Z_{n_2} \times Z_{n_3} = Z_2 \times Z_2 \times Z_2$), Chen *et al.* have enumerated 100612 SSGs, including 8505 *t*-type, 6738 *k*-type, and 85369 *g*-type SSGs. Each SSG is label by a four-index (L_0, G_0, i_k, m), where m distinguishes inequivalent mapping between G^s and G_0/L_0 through Eq. (S2).

1.3. International notations for spin space groups

Although each SSG are uniquely labeled by four-index (L_0, G_0, i_k, m), this label is not intuitive and show no information on spin rotation. Therefore, we adopt the newly developed international notation on SSGs in the main text. International notation, also known as Hermann-Mauguin notation, is a widely used nomenclature for (magnetic) point groups and (magnetic) space groups.

Before introducing the international notation for SSGs, we first review the international notation for SG. The international notation of a SG is denoted by $Bg_1g_2g_3$, where the first capital letter B describes the Bravais lattice, including primitive (P), based-centered (A, B, C), body-centered (I), face-centered (F) and rhombohedral (R) lattice. The following three letters g_1, g_2, g_3 describe the representative symmetry operations. Since the translation operations could couple spin rotation operations (*k*- and *g*-type SSG), the notation of SGs should be first be expanded into a more comprehensive one $Bg_1g_2g_3\boldsymbol{t}_a\boldsymbol{t}_b\boldsymbol{t}_c\boldsymbol{b}_1\boldsymbol{b}_2\boldsymbol{b}_3$. Here $\boldsymbol{t}_{a,b,c}$ present the integral translation of cell expansion, while for SGs one simply has $\boldsymbol{t}_a = \boldsymbol{a} = (1,0,0), \boldsymbol{t}_b = \boldsymbol{b} = (0,1,0), \boldsymbol{t}_c = \boldsymbol{c} = (0,0,1)$ in fractional coordinate. $b_{1,2,3}$ stand for the fractional translations for the specific Bravais centering-type. Please refer to Appendix B of Ref. [2] for more details, including the conventions on representative operations and fractional translation vectors for distinct Bravais centering-types.

The international notation of a SSG follows its construction procedure from a SG via Eq. (S2). Hence the notation depends on the type of SSG.

1) *t*-type SSG

For *t*-type SSGs, $T(L_0) = T(G_0)$ and $P(L_0) \leq P(G_0)$, indicating that the translation subgroup remains. Then any pure lattice translation operation in L_0 couples identity operation in spin space. Therefore, the international notation for *t*-type SSGs takes the form $B^{g_{s_1}}g_1{}^{g_{s_2}}g_2{}^{g_{s_3}}g_3$, involving spin group generators $\{g_{s_1}||g_1\}$, $\{g_{s_2}||g_2\}$, and $\{g_{s_3}||g_3\}$.

2) *k*-type SSG

For *k*-type SSGs, $T(L_0) < T(G_0)$ and $P(L_0) = P(G_0)$, indicating that spin rotation operations must couple lattice translation operations. Therefore, the international notation for *k*-type SSGs takes the form $B^1g_1{}^1g_2{}^1g_3{}^{g_{s_1}}\boldsymbol{\tau}_1{}^{g_{s_2}}\boldsymbol{\tau}_2{}^{g_{s_3}}\boldsymbol{\tau}_3$ in L_0 basis. This notation implies that besides three representatives $\{E||g_1\}$, $\{E||g_2\}$, and $\{E||g_3\}$ to generate L_0, up to three operations $\{g_{s_1}||E|\boldsymbol{\tau}_1\}$, $\{g_{s_2}||E|\boldsymbol{\tau}_2\}$, and $\{g_{s_3}||E|\boldsymbol{\tau}_3\}$ are newly added to construct the SSG, where $\boldsymbol{\tau}_i = m_a\boldsymbol{a} + m_b\boldsymbol{b} + m_c\boldsymbol{c} = (m_a, m_b, m_c)$ with $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ the lattice vectors of L_0.

3) *g*-type SSG

For *g*-type SSGs, $T(L_0) < T(G_0)$ and $P(L_0) < P(G_0)$, indicating that spin rotation operators can couple to both lattice rotation (combined with fractional transition) g_i, integer transition $\boldsymbol{t}_j$, and fractional transition $\boldsymbol{b}_i$. Hence, the international notation for *g*-type SSGs takes the form $B^{g_{s_1}}g_1{}^{g_{s_2}}g_2{}^{g_{s_3}}g_3|(g_{s_4}, g_{s_5}, g_{s_6}; g_{s_7}, g_{s_8}, g_{s_9})$ in G_0 basis. Notice that in G_0 basis, the capital letter B reflects the in information of Bravais lattice of the magnetic cell. Here $g_{s_4}, g_{s_5}, g_{s_6}$ denote the spin rotations associated with $\boldsymbol{t}_a, \boldsymbol{t}_b, \boldsymbol{t}_c$, while $g_{s_7}, g_{s_8}, g_{s_9}$ denote the spin rotation associated with $\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3$. Particularly, for P Bravais lattice, $g_{s_7}, g_{s_8}, g_{s_9}$ can be omitted because all centering-type fractional translations $\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3$ are zero.

We note that for collinear and coplanar magnetic configurations, the spin-only group should also be described in the international notation. For collinear magnetic configurations, the spin-only group $G_{SO}^L = SO(2) \rtimes \{E, U_\perp(\pi)T\}$ is denoted by ${}^{\infty m}1$, where ∞m is the Hermann-Mauguin notation for the infinite point group $G_{SO}^L \cong D_{\infty h}$ in spin space. For coplanar magnetic configurations, the spin-only group $G_{SO}^P = \{E, U_\perp'(\pi)T\}$ is denoted by ${}^{m}1$, where m denotes the "reflection" $U_\perp'(\pi)T$ in spin space.

1.4. Identification of spin space groups

Due to the complexity of SSGs, we have established an online program FINDSPINGROUP (https://findspingroup.com/) to determine the SSG of a provided magnetic material in `mcif` format. Here we chose one material example VNb_3S_6 collected by the MAGNDATA database for demonstration.

First, we upload the `0.712_VNb3S6.mcif` file (downloaded from the MAGNDATA database at https://www.cryst.ehu.es/magndata/index.php?index=0.712) into the program and then submit it, as shown in Fig. S1.

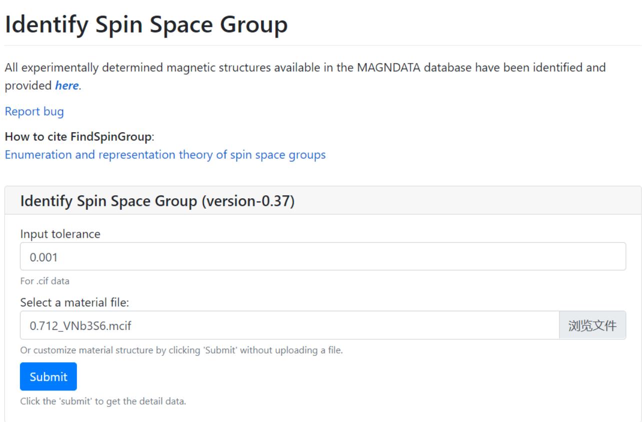

Fig. S1. The interface of the online program. One can upload a file in `mcif` format and submit it for further analysis.

Then the program will load the atomic and magnetic structures and leads the user to the second interface as shown in Fig. S2. In this interface, the user can modify the tolerances, unit cell, and magnetization (Mx, My, Mz) of each atom. In general, it is fine to use the default parameters. Click submit for further analysis.

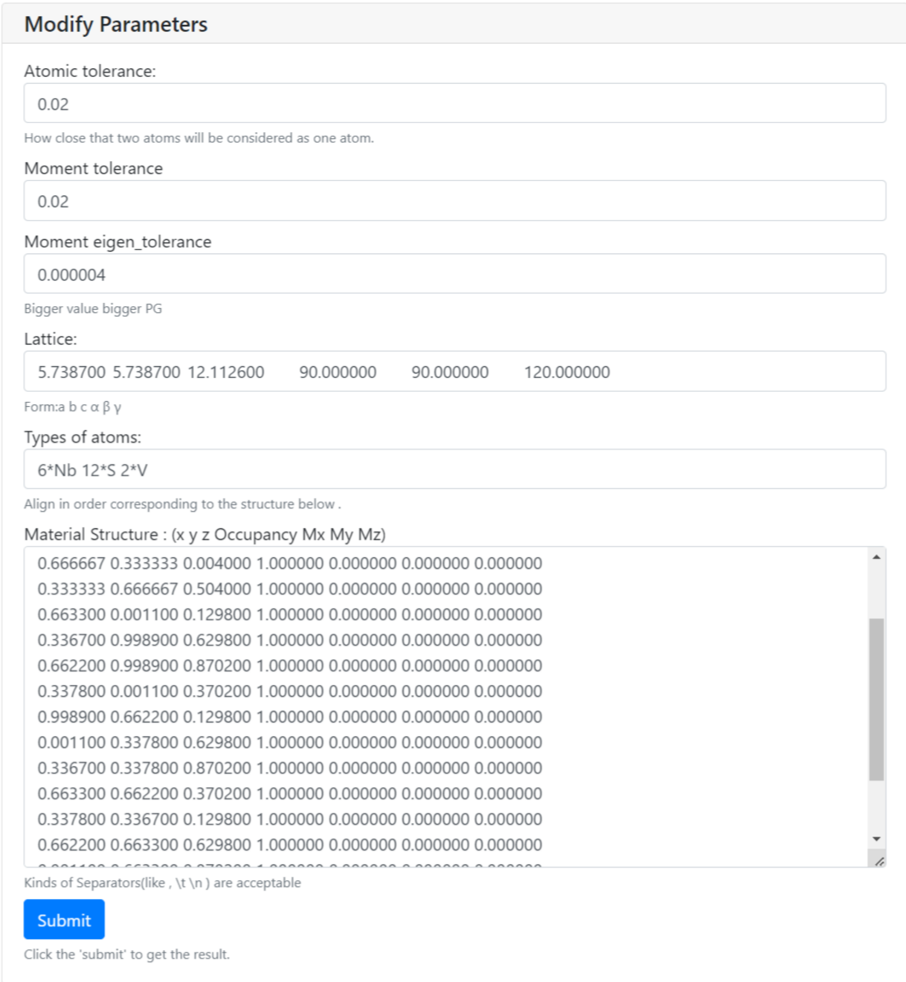

Fig. S2. The second interface after loading the atomic and magnetic structures. The user can modify the default parameters in this interface.

After some time (from few seconds to few minutes, depending on the complexity of the structure uploaded), the SSG as well as the related properties of the uploaded structure (or the manually modified structure) will be identified as the results, as shown in Fig. S3. In this example, the SSG of VNb_3S_6 with collinear antiferromagnetic configuration, encoded in the `0.712_VNb3S6.mcif` file, is identified as

$P^{-1}6_3{}^{-1}2^{1}2^{\infty m}1$. For the meaning of the international notation, please refer to Section 1.3 or Ref. [2].

File Name	Spin Space Group	G0 Symbol	L0 Symbol	it	ik
$0.712/VNb3S6$	$P^{-1}6_3{}^{-1}2^{1}2^{\infty m}1$	$P6_322(182)$	$P312(149)$	2	1

Space Group	Magnetic Space Group	Spin Part PG	Configuration	Magnetic Phase
$P6_322(182)$	$C2'2'2_1(20.33)$ $Type\ III$	$Ci\times C_{\infty}v\ = D_{\infty}h$ $-1\times\ \infty m\ = \infty/mm$	$Collinear$	$Canting\ AFM$

Little Cogroup at General Position		Spin Splitting	
without SOC	with SOC	without SOC	with SOC
$^{1}1^{\infty}1$	1	$k-dependent$	Yes

Effective Magnetic Point Group	Anomalous Hall Effect	
	without SOC	with SOC
$6221'$	No	Yes

L0-std Unit Magnetic Cell:

a	b	c	alpha	beta	gamma
5.7387	5.7387	12.1126	90.0	90.0	120.0

Fractional Coordinates & Moments (in L0-std Unit Magnetic Cell)

No.	Atom	Site	Moment (in lattice)
1	V	[0.3333, 0.6667, 0.0]	[1.5, 0.0, 0.0]
2	V	[0.6667, 0.3333, 0.5]	[-1.5, 0.0, 0.0]

Fig. S3. Results of the identification program, where the full contents are not shown here due to the length limit.

2. Nonlinear transports under quantum kinetic theory

In this section we derive the conductivity tensors of nonlinear transports (NLTs), or more explicitly, the second-order transports. The current density $\boldsymbol{J}$ driven by the second order of external electric field $\boldsymbol{E}$ is given by $J^{\alpha} = \sigma^{\alpha\beta\gamma}E^{\beta}E^{\gamma}$. Here we provide the derivations of the nonlinear conductivity tensors $\sigma^{\alpha\beta\gamma}$, *i.e.*, Eqs. (4)-(6) in Methods within the framework of quantum kinetic theory [3], which completely captures the geometric contributions of Bloch states. Under the length gauge [4], the total Hamiltonian reads $H = H_0 + e\boldsymbol{r}\cdot\boldsymbol{E}$, where $H' = e\boldsymbol{r}\cdot\boldsymbol{E}$ is the field correction with $-e$ the electronic charge $(e > 0)$ and $\boldsymbol{r}$ the position operator. Taking the electric field perturbatively, we expand the density matrix as a polynomial of $E = |\boldsymbol{E}|$ as $\rho = \rho^{(0)} + \rho^{(1)} + \rho^{(2)} + \mathcal{O}(E^3)$, with $\rho^{(N)} \propto E^N$. Under the relaxation time approximation (ignoring disorder effect [5]), the quantum Liouville-von Neumann equation of the stead density matrix ($\partial\rho^{(N)}/\partial t = 0$) reads

$$\frac{i}{\hbar}\left[H_0, \rho^{(N)}\right]_{ln} + \frac{\rho_{ln}^{(N)}}{\tau_r/N} = -i\frac{e\boldsymbol{E}}{\hbar}\cdot\left[\boldsymbol{r}, \rho^{(N-1)}\right]_{ln}, \tag{S3}$$

where τ_r is the relaxation time, $\hbar$ is the reduced Planck constant, and l, n are the band indices. It has been suggested that the relaxation time for $\rho^{(N)}$ should decrease as τ_r/N to avoiding the unphysical results [3]. To circumvent the position operator $\boldsymbol{r}$ that is ill defined in periodic crystal, the commutator $\left[\boldsymbol{r}, \rho^{(N-1)}\right]_{ln}$ should be replaced by $i\left[\mathcal{D}_{\boldsymbol{k}}\rho^{(N-1)}\right]_{ln}$ satisfying

$$\begin{aligned}\left[\mathcal{D}_{\boldsymbol{k}}\rho^{(N-1)}\right]_{ln} &= \nabla_{\boldsymbol{k}}\rho_{ln}^{(N-1)} - i\left[\boldsymbol{A}, \rho^{(N-1)}\right]_{ln} \\ &= \nabla_{\boldsymbol{k}}\rho_{ln}^{(N-1)} - i\sum_{p}\left(\boldsymbol{A}_{lp}\rho_{pn}^{(N-1)} - \rho_{lp}^{(N-1)}\boldsymbol{A}_{pn}\right),\end{aligned} \tag{S4}$$

where $\nabla_{\boldsymbol{k}} = \left(\partial/\partial k_x, \partial/\partial k_y, \partial/\partial k_z\right) \equiv \left(\partial_x, \partial_y, \partial_z\right)$, and $\boldsymbol{A} = (A^x, A^y, A^z)$ is the interband Berry connection $A_{lp}^{\alpha} = i\langle u_l(\boldsymbol{k})|\partial_\alpha u_p(\boldsymbol{k})\rangle$ with $|u_p(\boldsymbol{k})\rangle$ the eigenstate of $H_0(\boldsymbol{k}) = e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}H_0e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}$ with dispersion $\varepsilon_p(\boldsymbol{k})$. In the following, the $\boldsymbol{k}$-dependences of eigenstates and eigenenergies will not be written specifically.

Now we solve the field-corrected density matrix. The unperturbed density matrix reads $\rho_{ln}^{(0)} = f_l \delta_{ln}$ where $f_l = \{1 + \exp[(\varepsilon_l - \mu)/k_B T]\}^{-1}$ is the Fermi-Dirac distribution with k_B and T are Boltzmann constant and temperature, respectively. Starting from $\rho_{ln}^{(0)}$, we then obtain the first- and second-order corrections $\rho^{(1)}$ and $\rho^{(2)}$ by using Eqs. (S3) and (S4). For $\rho^{(1)}$, one solves that

$$\rho_{ll}^{(1),d} = \tau_r \frac{e\boldsymbol{E}}{\hbar} \cdot \nabla_{\boldsymbol{k}} f_l, \tag{S5}$$

as the diagonal components, and

$$\rho_{ln}^{(1),o} = \frac{e\boldsymbol{E} \cdot \boldsymbol{A}_{ln}}{\varepsilon_{ln} - i\hbar/\tau_r} f_{ln}, \tag{S6}$$

as the off-diagonal components ($l \neq n$), respectively, where $\varepsilon_{ln} \equiv \varepsilon_l - \varepsilon_n$ and $f_{ln} \equiv f_l - f_n$. With $\rho^{(1)}$ at hand, we then solve $\rho^{(2)}$, including the diagonal components $\rho_{ll}^{(2),d}$ and off-diagonal components $\rho_{ln}^{(2),o}$. Notice that the diagonal components $\rho_{ll}^{(2),d}$ can be contributed by both the diagonal ($\rho_{ll}^{(1),d}$) and off-diagonal ($\rho_{ln}^{(1),o}$) components of $\rho^{(1)}$. Hence, we split the diagonal components $\rho_{ll}^{(2),d}$ into $\rho_{ll}^{(2),dd}$ and $\rho_{ll}^{(2),do}$, where the first "d" in superscript denote the diagonal components of the second-order density matrix, and the latter "d" or "o" in superscript traces its origin from the diagonal or off-diagonal components of the first-order density matrix. More explicitly, $\rho_{ll}^{(2),dd}$ and $\rho_{ll}^{(2),do}$ are obtained as

$$\rho_{ll}^{(2),dd} = \tau_r^2 \frac{e^2}{2\hbar^2} E_\beta E_\gamma \partial_\beta \partial_\gamma f_l, \tag{S7}$$

$$\rho_{ll}^{(2),do} = -\frac{e^2}{2} E_\beta E_\gamma \sum_{p(\neq l)} \frac{A_{lp}^\beta A_{pl}^\gamma + A_{lp}^\gamma A_{pl}^\beta}{\varepsilon_{pl}^2} f_{lp}. \tag{S8}$$

To derivate Eq. (S8), an expansion is applied in the clean limit ($1/\tau_r \to 0$), as

$$\lim_{1/\tau_r \to 0} \frac{1}{i\hbar N/\tau_r} \frac{1}{\varepsilon_{pl} - i\hbar N/\tau_r} \approx -i \frac{\tau_r}{N\hbar\varepsilon_{pl}} + \frac{1}{\varepsilon_{pl}^2}. \tag{S9}$$

Similarly, the off-diagonal components of the second-order density matrix $\rho_{ll}^{(2),o}$ can also be separated as $\rho_{ln}^{(2),od}$ and $\rho_{ln}^{(2),oo}$ ($l \neq n$), where

$$\rho_{ln}^{(2),od} = \tau_r \frac{e^2}{\hbar} E_\beta E_\gamma \frac{A_{ln}^\beta \partial_\gamma f_{ln}}{\varepsilon_{ln}} + 2ie^2 E_\beta E_\gamma \frac{A_{ln}^\beta \partial_\gamma f_{ln}}{\varepsilon_{ln}^2}, \tag{S10}$$

$$\rho_{ln}^{(2),oo} = \frac{-ie^2 E_\beta E_\gamma}{\varepsilon_{ln}} \left[\frac{\mathcal{D}_{ln}^\beta A_{ln}^\gamma f_{ln}}{\varepsilon_{ln}} - i \sum_{p(\neq l,n)} \left(\frac{A_{lp}^\beta A_{pn}^\gamma f_{pn}}{\varepsilon_{pn}} - \frac{A_{lp}^\gamma A_{pn}^\beta f_{lp}}{\varepsilon_{lp}} \right) \right], \tag{S11}$$

where $\mathcal{D}_{ln}^\beta = \partial_\beta - i\left(A_{ll}^\beta - A_{nn}^\beta\right)$.

After obtaining the second-order density matrix $\rho^{(2)}$, the current driven by second order of electric field is then calculated by

$$J^\alpha = -e\mathrm{Tr}\left[v^\alpha \rho^{(2)}\right] = -e \sum_{\boldsymbol{k},l} \left[v_{ll}^\alpha \rho_{ll}^{(2),d} + \sum_{n(\neq l)} v_{ln}^\alpha \rho_{nl}^{(2),o} \right] \tag{S12}$$

where $v^\alpha = \hbar^{-1}\partial_\alpha H(\boldsymbol{k})$ is the velocity operator, and the trace is taken over both the band and $\boldsymbol{k}$ indices. Note that different components of $\rho^{(2)}$ have distinct dependences on relaxation time τ_r, as can be seen from Eqs. (S7), (S8), (S10), and (S11). Therefore, we separate the current into three parts according to their τ_r-dependences.

2.1. Inversed mass dipole contribution $\propto \tau_r^2$

As shown by Eq. (S7), $\rho_{ll}^{(2),dd}$ depends on the square of τ_r. This term contributes to the so-called second-order Drude conductivity [6], as

$$\sigma_{\mathrm{Drude}}^{\alpha\beta\gamma} = -\tau_r^2 \frac{e^3}{2\hbar^2} \sum_{\boldsymbol{k},l} v_l^\alpha \partial_\beta \partial_\gamma f_l\,, \tag{S13}$$

where $v_l^\alpha = \hbar^{-1}\partial_\alpha \varepsilon_l$ is the group velocity of l-th band. Using integrating by parts, we obtain an alternative form of Eq. (S13), as

$$\sigma_{\mathrm{Drude}}^{\alpha\beta\gamma} = \tau_r^2 \frac{e^3}{2\hbar^3} \sum_{\boldsymbol{k},l} \left(\partial_\beta \partial_\alpha \varepsilon_l\right) \partial_\gamma f_l\,. \tag{S14}$$

Historical, $\hbar^{-2}\partial_\alpha\partial_\alpha\varepsilon_l$ is defined as the inverse effective mass $1/m_{\mathrm{eff}}$ along k_α. Therefore, it is reasonable to consider $w_l^{\alpha\beta} = \hbar^{-2}\partial_\alpha\partial_\beta\varepsilon_l$ as the inverse mass tensor. Mathematically, $w_l^{\alpha\beta}$ is a Hessian tensor of the energy band manifold ε_l, describing the local curvature of the manifold. By this mean, we henceforward use the term inverse mass dipole (IMD) to label $\sigma_{\mathrm{Drude}}^{\alpha\beta\gamma}$, which highlights its geometric origin. Hence, the IMD contribution to NLT tensor reads

$$\boxed{\sigma_{\mathrm{IMD}}^{\alpha\beta\gamma} = -\tau_r^2 \frac{e^3}{2\hbar} \sum_{\boldsymbol{k},l} \left(\partial_\alpha w_l^{\beta\gamma}\right) f_l = \tau_r^2 \frac{e^3}{2} \sum_{\boldsymbol{k},l} v_l^\alpha w_l^{\beta\gamma} \frac{\partial f_l}{\partial \varepsilon_l},} \tag{S15}$$

where we have used the fact that $\sigma_{\mathrm{IMD}}^{\alpha\beta\gamma}$ is symmetric under any permutation of all three indices as $\sigma_{\mathrm{IMD}}^{\alpha\beta\gamma} \propto \sum_l \partial_\alpha \partial_\beta \partial_\gamma \varepsilon_l$. In zero temperature limit, $\sigma_{\mathrm{IMD}}^{\alpha\beta\gamma}$ is obtained by summing over all the IMD $\partial_\alpha w_l^{\beta\gamma}$ of occupied states (first equal sign), or by tracing the inverse mass current $v_l^\alpha w_l^{\beta\gamma}$ at the fermi surface (second equal sign).

2.2. Berry curvature dipole contribution $\propto \tau_r$

The first term of $\rho_{ln}^{(2),od}$ in Eq. (S10) is linearly dependent on the relaxation time τ_r, which contributes to a second-order current

$$J_1^\alpha = -\tau_r \frac{e^3}{\hbar} E_\beta E_\gamma \sum_{\boldsymbol{k},l} \sum_{n(\neq l)} \frac{v_{ln}^\alpha A_{nl}^\beta \partial_\gamma f_{nl}}{\varepsilon_{nl}}. \tag{S16}$$

Using the relation $\partial_\alpha(\langle u_l|H(\boldsymbol{k})|u_n\rangle) = 0$ $(l \neq n)$, we find that $A_{ln}^\alpha = i\hbar v_{ln}^\alpha/\varepsilon_{nl}$ and thus

$$J_1^\alpha = -\tau_r \frac{e^3}{\hbar^2} E_\beta E_\gamma \sum_{\boldsymbol{k},l} \sum_{n(\neq l)} i\left(A_{ln}^\alpha A_{nl}^\beta - A_{ln}^\beta A_{nl}^\alpha\right) \partial_\gamma f_l. \tag{S17}$$

Notice that $\sum_{n(\neq l)} i\left(A_{ln}^\alpha A_{nl}^\beta - A_{ln}^\beta A_{nl}^\alpha\right) = \Omega_l^{\alpha\beta}$ is the Berry curvature, since that

$$\begin{aligned}\sum_{n(\neq l)} i\left(A_{ln}^\alpha A_{nl}^\beta - A_{ln}^\beta A_{nl}^\alpha\right) &= i\left(\langle \partial_\alpha u_l | \partial_\beta u_l\rangle - \langle \partial_\beta u_l | \partial_\alpha u_l\rangle\right) \\ &= \partial_\alpha A_l^\beta - \partial_\beta A_l^\alpha = \Omega_l^{\alpha\beta}.\end{aligned} \tag{S18}$$

Then we arrive

$$J_1^\alpha = -\tau_r \frac{e^3}{\hbar^2} E_\beta E_\gamma \sum_{\boldsymbol{k},l} \Omega_l^\alpha \partial_\gamma f_l = \tau_r \frac{e^3}{\hbar^2} E_\beta E_\gamma \sum_{\boldsymbol{k},l} \left(\partial_\gamma \Omega_l^{\alpha\beta}\right) f_l, \tag{S19}$$

bringing us the so-called Berry curvature dipole (BCD) $\partial_\gamma \Omega_l^{\alpha\beta}$ contribution. As the transport tensor should be symmetric under swapping $\beta \leftrightarrow \gamma$ (electric field indices), the transport tensor contributed by BCD reads

$$\boxed{\sigma_{\mathrm{BCD}}^{\alpha\beta\gamma} = \tau_r \frac{e^3}{2\hbar^2}\sum_{\boldsymbol{k},l}\left(\partial_\gamma \Omega_l^{\alpha\beta} + \partial_\beta \Omega_l^{\alpha\gamma}\right) f_l = -\tau_r \frac{e^3}{2\hbar}\sum_{\boldsymbol{k},l}\left(v_l^\gamma \Omega_l^{\alpha\beta} + v_l^\beta \Omega_l^{\alpha\gamma}\right)\frac{\partial f_l}{\partial \varepsilon_l},} \tag{S20}$$

which is the Eq. (5) in the Methods.

Berry curvature $\Omega_l^{\alpha\beta}$ is an anti-symmetric tensor, i.e., $\Omega_l^{\beta\alpha} = -\Omega_l^{\alpha\beta}$, of which only three components $\Omega_l^{xy}, \Omega_l^{yz}, \Omega_l^{zx}$ are independent. Hence, one can define a pseudo vector $\boldsymbol{\Omega}_l$ by these three components with the relation

$$\Omega_l^\eta \equiv \frac{1}{2}\Omega_l^{\alpha\beta}\epsilon^{\alpha\beta\eta}, \tag{S21}$$

where $\epsilon^{\alpha\beta\eta}$ is the Levi-Civita symbol. Notice that the summation over repeated indices is implied. In this notation, the dipole moment of Berry curvature of the occupied states is defined as a rank-two pseudotensor [7]

$$D^{\alpha\beta} = \sum_{\boldsymbol{k},l}\left(\partial_\alpha \Omega_l^\beta\right) f_l\,. \tag{S22}$$

Owing to the anti-symmetric nature, one finds

$$\begin{aligned}
\partial_\gamma \Omega_l^{\alpha\beta} &= \frac{1}{2}\partial_\gamma\left(\Omega_l^{\alpha\beta} - \Omega_l^{\beta\alpha}\right) \\
&= \frac{1}{2}\partial_\gamma \Omega_l^{\mu\nu}\left(\delta^{\alpha\mu}\delta^{\beta\nu} - \delta^{\beta\mu}\delta^{\alpha\nu}\right) \\
&= \frac{1}{2}\partial_\gamma \Omega_l^{\mu\nu}\epsilon^{\alpha\beta\xi}\epsilon^{\mu\nu\xi} \\
&= \epsilon^{\alpha\beta\xi}\partial_\gamma \Omega_l^\xi.
\end{aligned} \tag{S23}$$

Therefore, Eq. (20) can be expressed by using the dipole moment of Berry curvature as

$$\sigma_{\mathrm{BCD}}^{\alpha\beta\gamma} = \tau_r \frac{e^3}{2\hbar^2}\left(\epsilon^{\alpha\beta\xi}D^{\gamma\xi} + \epsilon^{\alpha\gamma\xi}D^{\beta\xi}\right), \tag{S24}$$

recovering the symmetrized results obtained by Sodemann and Fu [7].

2.3. Quantum metric dipole contribution $\propto \boldsymbol{\tau_r^0}$

The remaining parts of the density matrix are independent of the relaxation time, including $\rho_{ll}^{(2),do}$, the second term of $\rho_{ln}^{(2),od}$, and $\rho_{ln}^{(2),oo}$. For $\rho_{ll}^{(2),do}$, we find that

$$J^{\alpha}_{0,do}=\frac{e^3}{2\hbar}E_\beta E_\gamma\sum_{\boldsymbol{k},l}\sum_{n(\neq l)}(\partial_\alpha\varepsilon_l)\frac{A^{\beta}_{ln}A^{\gamma}_{nl}+A^{\gamma}_{ln}A^{\beta}_{nl}}{\varepsilon^2_{nl}}f_{ln}$$

$$=\frac{e^3}{2\hbar}E_\beta E_\gamma\sum_{\boldsymbol{k},l}\sum_{n(\neq l)}(\partial_\alpha\varepsilon_{ln})\frac{A^{\beta}_{ln}A^{\gamma}_{nl}+A^{\gamma}_{ln}A^{\beta}_{nl}}{\varepsilon^2_{ln}}f_l$$

$$=-\frac{e^3}{2\hbar}E_\beta E_\gamma\sum_{\boldsymbol{k},l}\sum_{n(\neq l)}\left(A^{\beta}_{ln}A^{\gamma}_{nl}+A^{\gamma}_{ln}A^{\beta}_{nl}\right)\left(\partial_\alpha\frac{1}{\varepsilon_{ln}}\right)f_l\,. \tag{S25}$$

As for the second term of $\rho^{(2),od}_{ln}$, we have

$$J^{\alpha}_{0,od}=-2e^3E_\beta E_\gamma\sum_{\boldsymbol{k},l}\sum_{n(\neq l)}\frac{iv^{\alpha}_{ln}A^{\beta}_{nl}}{\varepsilon^2_{nl}}\partial_\gamma f_{nl}$$

$$=-\frac{2e^3}{\hbar}E_\beta E_\gamma\sum_{\boldsymbol{k},l}\sum_{n(\neq l)}\frac{A^{\alpha}_{ln}A^{\beta}_{nl}+A^{\alpha}_{nl}A^{\beta}_{ln}}{\varepsilon_{ln}}\partial_\gamma f_l\,. \tag{S26}$$

The current involved $\rho^{(2),oo}_{ln}$ is much trickier. Through tedious derivation following Ref. [3], however, the result is quite simple, where the current reads

$$J^{\alpha}_{0,oo}=-\frac{e^3}{2\hbar}E_\beta E_\gamma\sum_{\boldsymbol{k},l}\sum_{n(\neq l)}\frac{\partial_\alpha\left(A^{\beta}_{ln}A^{\gamma}_{nl}+A^{\gamma}_{nl}A^{\beta}_{ln}\right)}{\varepsilon_{ln}}f_l\,. \tag{S27}$$

We find that

$$J^{\alpha}_{0,do}+J^{\alpha}_{0,oo}=-\frac{e^3}{2\hbar}E_\beta E_\gamma\sum_{\boldsymbol{k},l}\sum_{n(\neq l)}\partial_\alpha\left(\frac{A^{\beta}_{ln}A^{\gamma}_{nl}+A^{\gamma}_{nl}A^{\beta}_{ln}}{\varepsilon_{ln}}\right)f_l$$

$$=-\frac{e^3}{\hbar}E_\beta E_\gamma\sum_{\boldsymbol{k},l}\partial_\alpha\mathcal{G}^{\beta\gamma}_l f_l\,, \tag{S28}$$

where $\mathcal{G}^{\beta\gamma}_l\equiv\sum_{n(\neq l)}\left(A^{\beta}_{ln}A^{\gamma}_{nl}+A^{\gamma}_{nl}A^{\beta}_{ln}\right)/(2\varepsilon_{ln})$ is the band-renormalized quantum metric tensor. Therefore, the total transport tensor independent to τ_r is obtained by

$$\boxed{\begin{aligned}\sigma^{\alpha\beta\gamma}_{\mathrm{QMD}}&=-\frac{e^3}{\hbar}\sum_{\boldsymbol{k},l}\left[\partial_\alpha\mathcal{G}^{\beta\gamma}_l-2\left(\partial_\gamma\mathcal{G}^{\alpha\beta}_l+\partial_\beta\mathcal{G}^{\alpha\gamma}_l\right)\right]f_l\\&=e^3\sum_{\boldsymbol{k},l}\left[v^{\alpha}_l\mathcal{G}^{\beta\gamma}_l-2\left(v^{\gamma}_l\mathcal{G}^{\alpha\beta}_l+v^{\beta}_l\mathcal{G}^{\alpha\gamma}_l\right)\right]\frac{\partial f_l}{\partial\varepsilon_l},\end{aligned}} \tag{S29}$$

which is the Eq. (6) in Methods. Such a dipole $\partial_\alpha\mathcal{G}^{\beta\gamma}_l$ is called quantum metric dipole (QMD). It has been found that the QMD-contributions can be further separated into the

Hall-type (transversal) $\sigma_{\mathrm{BCP}}^{\alpha\beta\gamma}$ and non-Hall-type (transversal and longitudinal) $\sigma_{\mathrm{NHT}}^{\alpha\beta\gamma}$ contributions, with

$$\sigma_{\mathrm{BCP}}^{\alpha\beta\gamma} = -\frac{e^3}{\hbar}\sum_{\boldsymbol{k},l}\left[2\partial_\alpha \mathcal{G}_l^{\beta\gamma} - \left(\partial_\gamma \mathcal{G}_l^{\alpha\beta} + \partial_\beta \mathcal{G}_l^{\alpha\gamma}\right)\right] f_l\,, \tag{S30}$$

and

$$\sigma_{\mathrm{NHT}}^{\alpha\beta\gamma} = \frac{e^3}{\hbar}\sum_{\boldsymbol{k},l}\left(\partial_\alpha \mathcal{G}_l^{\beta\gamma} + \partial_\gamma \mathcal{G}_l^{\alpha\beta} + \partial_\beta \mathcal{G}_l^{\alpha\gamma}\right) f_l\,. \tag{S31}$$

It was proposed by Gao *et al.* that the Hall-type originates from the electric-field-induced Berry connection [8], leading to a terminology of Berry connection polarizability (BCP).

We should emphasize that $\mathcal{G}_l^{\alpha\beta}$ is not exact the quantum metric of quantum state but is a band-renormalized one. The quantum geometry tensor is defined via the interband Berry connection A_{ln}^{α} by $Q_l^{\alpha\beta} = \sum_{n(\neq l)} A_{ln}^{\alpha} A_{nl}^{\beta}$, of which the real and imaginary parts are defined as the quantum metric $G_l^{\alpha\beta}$ and Berry curvature $\Omega_l^{\alpha\beta}$, respectively, as

$$G_l^{\alpha\beta} = \mathrm{Re}Q_l^{\alpha\beta} = \frac{1}{2}\sum_{n(\neq l)}\left(A_{ln}^{\alpha}A_{nl}^{\beta} + A_{ln}^{\beta}A_{nl}^{\alpha}\right), \tag{S32}$$

$$\Omega_l^{\alpha\beta} = -2\mathrm{Im}Q_l^{\alpha\beta} = \sum_{n(\neq l)} i\left(A_{ln}^{\alpha}A_{nl}^{\beta} - A_{ln}^{\beta}A_{nl}^{\alpha}\right). \tag{S33}$$

Quantum metric $G_l^{\alpha\beta}$ characters the differential distance between two neighboring states $|u(\boldsymbol{k} + d\boldsymbol{k})\rangle$ and $|u(\boldsymbol{k})\rangle$, while Berry curvature $\Omega_l^{\alpha\beta}$ describes the differential phase difference between them [9].

3. Symmetry properties and constraints on second-order transport tensors

3.1. Symmetry constraints by time-reversal and inversion symmetry

Here we analysis the symmetry constraints on second-order transport tensors contributed by IMD, BCD, and QMD, respectively. Time-reversal T and inversion P transform the geometry tensor as:

	Inverse mass $w_l^{\alpha\beta}(\boldsymbol{k})$	Quantum metric $\mathcal{G}_l^{\alpha\beta}(\boldsymbol{k})$	Berry curvature $\Omega_l^{\alpha\beta}(\boldsymbol{k})$
T	$w_l^{\alpha\beta}(-\boldsymbol{k})$	$\mathcal{G}_l^{\beta\alpha}(-\boldsymbol{k}) = \mathcal{G}_l^{\alpha\beta}(-\boldsymbol{k})$	$\Omega_l^{\beta\alpha}(-\boldsymbol{k}) = -\Omega_l^{\alpha\beta}(-\boldsymbol{k})$
P	$w_l^{\alpha\beta}(-\boldsymbol{k})$	$\mathcal{G}_l^{\alpha\beta}(-\boldsymbol{k})$	$\Omega_l^{\alpha\beta}(-\boldsymbol{k})$
PT	$w_l^{\alpha\beta}(\boldsymbol{k})$	$\mathcal{G}_l^{\alpha\beta}(\boldsymbol{k})$	$-\Omega_l^{\alpha\beta}(\boldsymbol{k})$

Here we have used the fact that Berry curvature is an anti-symmetric tensor $\Omega_l^{\alpha\beta}(\boldsymbol{k}) = -\Omega_l^{\beta\alpha}(\boldsymbol{k})$, and quantum metric is a symmetric tensor $\mathcal{G}_l^{\beta\alpha}(\boldsymbol{k}) = \mathcal{G}_l^{\alpha\beta}(\boldsymbol{k})$. Hence, the dipoles of the geometry quantities are transformed as:

	IMD $\partial_\gamma w_l^{\alpha\beta}(\boldsymbol{k})$	QMD $\partial_\gamma \mathcal{G}_l^{\alpha\beta}(\boldsymbol{k})$	BCD $\partial_\gamma \Omega_l^{\alpha\beta}(\boldsymbol{k})$
T	$-\partial_\gamma w_l^{\alpha\beta}(-\boldsymbol{k})$	$-\partial_\gamma \mathcal{G}_l^{\alpha\beta}(-\boldsymbol{k})$	$\partial_\gamma \Omega_l^{\alpha\beta}(-\boldsymbol{k})$
P	$-\partial_\gamma w_l^{\alpha\beta}(-\boldsymbol{k})$	$-\partial_\gamma \mathcal{G}_l^{\alpha\beta}(-\boldsymbol{k})$	$-\partial_\gamma \Omega_l^{\alpha\beta}(-\boldsymbol{k})$
PT	$\partial_\gamma w_l^{\alpha\beta}(-\boldsymbol{k})$	$\partial_\gamma \mathcal{G}_l^{\alpha\beta}(-\boldsymbol{k})$	$-\partial_\gamma \Omega_l^{\alpha\beta}(-\boldsymbol{k})$

This implies that: (i) If the system is invariant under time-reversal T (or T_{eff}), IMD $\partial_\gamma w_l^{\alpha\beta}(\boldsymbol{k})$ and QMD $\partial_\gamma \mathcal{G}_l^{\alpha\beta}(\boldsymbol{k})$ are odd function of $\boldsymbol{k}$, with their integral over the Brillouin zone vanished; (ii) If the system is invariant under PT (or PT_{eff}), Berry curvature vanishes and so does the BCD contribution, where the same conclusion can be drown as BCD is an PT-odd function.

We can see that the distinct symmetry constraints of different geometry quantities can also be deduced by the dependences on relaxation time τ_r of the corresponding conductivity tensors [Eqs. (S15), (S20), (S29)].

We recall the definition of the second-order conductivity $J^{\alpha} = \sigma^{\alpha\beta\gamma} E^{\beta} E^{\gamma}$. Since the current density J^{α} is an odd function under time-reversal symmetry T, the tensor is also an odd function of it where the electric field E^{β} is even. Then one can decompose the tensor with respect to the relaxation time τ_r, as

$$\sigma^{\alpha\beta\gamma} = \chi_0^{\alpha\beta\gamma} + \tau_r \chi_1^{\alpha\beta\gamma} + \tau_r^2 \chi_2^{\alpha\beta\gamma} + \tau_r^3 \chi_3^{\alpha\beta\gamma} + \cdots. \tag{S34}$$

As T filps $\tau_r \rightarrow -\tau_r$, $\chi_i^{\alpha\beta\gamma}$ need to be an odd (even) function under time-reversal if i is even (odd) integer. Similar analysis can be performed by inversion symmetry P, leading that all $\chi_i^{\alpha\beta\gamma}$ are odd function under P. These results can be tabulated as:

	$\chi_{2n}^{\alpha\beta\gamma}$	$\chi_{2n+1}^{\alpha\beta\gamma}$
T	$-\chi_{2n}^{\alpha\beta\gamma}$	$\chi_{2n+1}^{\alpha\beta\gamma}$
P	$-\chi_{2n}^{\alpha\beta\gamma}$	$-\chi_{2n+1}^{\alpha\beta\gamma}$
PT	$\chi_{2n}^{\alpha\beta\gamma}$	$-\chi_{2n+1}^{\alpha\beta\gamma}$

Hence, we deduce that $\chi_{2n}^{\alpha\beta\gamma}$ is T-odd while $\chi_{2n+1}^{\alpha\beta\gamma}$ is PT-odd. Recalling

$$\sigma_{\text{IMD}} \sim \tau_r^2 \chi_2, \qquad \sigma_{\text{QMD}} \sim \tau_r^0 \chi_0, \qquad \sigma_{\text{BCD}} \sim \tau_r^1 \chi_1, \tag{S35}$$

we see that both IMD and QMD contribute to χ_{2n} and BCD contribute to χ_{2n+1}, implying IMD and QMD contributions are T-odd and BCD contribution is PT-odd. The analysis is also valid if we replace T by the effective one T_{eff}. Such a phenomenological analysis is consistent with the symmetry analysis on geometric quantities.

3.2. General procedure on symmetry constraints

As mentioned in the main text, in magnets without SOC, the above analysis is not limited to the exact T and PT symmetries but can be extended to the effective symmetries T_{eff} and PT_{eff}. Any symmetry operation in SSG takes the form $\{u||r|\boldsymbol{\tau}\}$ with u and r are the spin and lattice rotation, respectively, and $\boldsymbol{\tau}$ the lattice translation. The T_{eff} symmetry may be the $T\boldsymbol{\tau}$ or the pure spin improper rotation $u = U_n(\theta)T$, where further combining the spatial inversion P brings us the PT_{eff}.

Now we show how the spin and lattice symmetries constrain the second-order transport tensors, by the spirit of the Neumann's principle. Given any spin group symmetry $\{u||r|\boldsymbol{\tau}\}$, the BCD and IMD/QMD tensors are transformed by

$$\sigma_{\text{BCD}}^{\alpha\beta\gamma} = \mathcal{R}^{\alpha\mu}\mathcal{R}^{\beta\nu}\mathcal{R}^{\gamma\eta}\sigma_{\text{BCD}}^{\mu\nu\eta}, \tag{S36}$$

$$\sigma_{\text{IMD/QMD}}^{\alpha\beta\gamma} = \det(\mathcal{U})\,\mathcal{R}^{\alpha\mu}\mathcal{R}^{\beta\nu}\mathcal{R}^{\gamma\eta}\sigma_{\text{IMD/QMD}}^{\mu\nu\eta}, \tag{S37}$$

where $\mathcal{R}$ and $\mathcal{U}$ are the representation matrices of r and u under Cartesian coordinate. Note that the summation over repeated indices is implied. In Eq. (S37), the T-odd nature of the IMD and QMD contributions are encoded by the factor of $\det(\mathcal{U}) = \pm 1$ with $+1(-1)$ for proper (improper) spin rotation.

Next, we demonstrate how to solve the symmetry-allowed transport tensors, by using the BCD contributed tensors as an example. Considering the BCD contributed tensors as a 27-dimensional vector $\{\sigma_{\text{BCD}}^{xxx}, \cdots, \sigma_{\text{BCD}}^{zzz}\}$, Eq. (S36) provides the linear transformation of it constrained by a given spin group symmetry $g_i = \{u_i||r_i|\boldsymbol{\tau}_i\}$. To solve this linear algebra problem, we recode the 27 components with single index $1 \equiv (xxx)$, $2 \equiv (xxy)$, $3 \equiv (xxz)$, …, $26 \equiv (zzy)$, $27 \equiv (zzz)$. Then Eq. (S36) can be rewritten as

$$\sigma^M = \mathbf{T}_i^{MN}\sigma^N \Leftrightarrow (\mathbf{I} - \mathbf{T}_i)\boldsymbol{\sigma} = \mathbf{0} \tag{S38}$$

where $M, N = 1,2,\cdots,27$, and the label of BCD is not written explicitly as the procedure is uniform for IMD and QMD contributions. $\mathbf{T}_i$ is a 27×27 matrix given by g_i, and $\mathbf{I}$ is the identity matrix of order 27. By defining $\widetilde{\mathbf{T}}_i = \mathbf{I} - \mathbf{T}_i$, the nonzero solutions of g_i-allowed tensor components form the kernel of $\widetilde{\mathbf{T}}_i$, known as $\ker\widetilde{\mathbf{T}}_i$. For the last step, we consider a SSG generated by L generators $\{g_1, g_2, \cdots, g_L\}$, where each generator is represented by one kernel $\widetilde{\mathbf{T}}_i$. The tensor components allowed by the SSG is finally obtained by choosing the basis vector in the intersection of kernels $\bigcap_{i=1}^{L}\ker\widetilde{\mathbf{T}}_i$. For IMD and QMD contribution, the procedure is also valid while the construction of $\widetilde{\mathbf{T}}_i$ need to be modified based on Eq. (S37).

3.3. Intrinsic symmetries of second-order transport tensors

As the spin group symmetry will constrain the transport tensors, the intrinsic symmetries of the tensors also limit their nonzero components. First of all, by the definition of the second-order transport tensor $J^{\alpha} = \sigma^{\alpha\beta\gamma} E^{\beta} E^{\gamma}$, one directly conclude that all the second-order transport tensor need to be symmetric under the permutation of the last two indices $\beta \leftrightarrow \gamma$. In fact, this is the only intrinsic symmetries of the QMD contributions, and hence at most 18 components out of 27 are independent in $\sigma_{\mathrm{QMD}}^{\alpha\beta\gamma}$. For IMD and BCD contributions, other intrinsic symmetries involve.

Referring to Eq. (S15), we can see that $\partial_{\alpha} w_{l}^{\beta\gamma} \propto \partial_{\alpha}\partial_{\beta}\partial_{\gamma}\varepsilon_{l}$, indicating that $\sigma_{\mathrm{IMD}}^{\alpha\beta\gamma}$ is symmetric under any permutation of all three indices α, β, and γ. This leads us at most 10 components are independent in $\sigma_{\mathrm{IMD}}^{\alpha\beta\gamma}$. On the other hand, from Eq. (S20), we find that

$$\sigma_{\mathrm{BCD}}^{\alpha\beta\gamma} + \sigma_{\mathrm{BCD}}^{\beta\gamma\alpha} = \tau_{r} \frac{e^{3}}{2\hbar^{2}} \sum_{\boldsymbol{k},l} \left(\partial_{\gamma}\Omega_{l}^{\alpha\beta} + \partial_{\beta}\Omega_{l}^{\alpha\gamma} + \partial_{\alpha}\Omega_{l}^{\beta\gamma} + \partial_{\gamma}\Omega_{l}^{\beta\alpha} \right) f_{l}. \qquad \text{(S39)}$$

As the Berry curvature is anti-symmetric $\Omega_{l}^{\alpha\beta} = -\Omega_{l}^{\beta\alpha}$, equation (S39) can be simplified into an important equality of $\sigma_{\mathrm{BCD}}^{\alpha\beta\gamma}$, as

$$\sigma_{\mathrm{BCD}}^{\alpha\beta\gamma} + \sigma_{\mathrm{BCD}}^{\beta\gamma\alpha} + \sigma_{\mathrm{BCD}}^{\gamma\alpha\beta} = 0. \qquad \text{(S40)}$$

This equality implies the rest intrinsic symmetries of the BCD contribution:

(i) If $\alpha \neq \beta \neq \gamma \neq \alpha$, it turns out that [10]

$$\sigma_{\mathrm{BCD}}^{xyz} + \sigma_{\mathrm{BCD}}^{yzx} + \sigma_{\mathrm{BCD}}^{zxy} = 0. \qquad \text{(S40)}$$

(ii) If $\alpha = \beta \neq \gamma$, it becomes

$$\sigma_{\mathrm{BCD}}^{\alpha\alpha\gamma} = \sigma_{\mathrm{BCD}}^{\alpha\gamma\alpha} = -\frac{1}{2}\sigma_{\mathrm{BCD}}^{\gamma\alpha\alpha}. \qquad \text{(S41)}$$

(iii) If $\alpha = \beta = \gamma$, it leads to

$$\sigma_{\mathrm{BCD}}^{\alpha\alpha\alpha} = 0, \qquad \text{(S42)}$$

which indicates the pure Hall type nature of the BCD contribution. Following Eqs. (S40)-(S42), we settle at most 8 independent components in $\sigma_{\mathrm{BCD}}^{\alpha\beta\gamma}$.

In our procedure on obtaining the symmetry-allowed tensors, the above-mentioned intrinsic symmetry constraints can be included by adding special matrices $\mathbf{T}_{s_j}$ and $\widetilde{\mathbf{T}}_{s_j}$ before solving the intersection of kernels. Taking the IMD contribution as example, the intrinsic symmetry of $\sigma_{\mathrm{IMD}}^{\alpha\beta\gamma}$ is represented by $\mathbf{T}_{\mathrm{IMD},s}$, of which the nonzero elements are $\mathbf{T}_{\mathrm{IMD},s}^{1,1}$, $\mathbf{T}_{\mathrm{IMD},s}^{2,4} = \mathbf{T}_{\mathrm{IMD},s}^{4,10} = \mathbf{T}_{\mathrm{IMD},s}^{10,2}$, $\mathbf{T}_{\mathrm{IMD},s}^{3,7} = \mathbf{T}_{\mathrm{IMD},s}^{7,19} = \mathbf{T}_{\mathrm{IMD},s}^{19,3}$, $\mathbf{T}_{\mathrm{IMD},s}^{5,11} = \mathbf{T}_{\mathrm{IMD},s}^{11,13} = \mathbf{T}_{\mathrm{IMD},s}^{13,5}$, $\mathbf{T}_{\mathrm{IMD},s}^{6,16} = \mathbf{T}_{\mathrm{IMD},s}^{16,20} = \mathbf{T}_{\mathrm{IMD},s}^{20,22} = \mathbf{T}_{\mathrm{IMD},s}^{22,12} = \mathbf{T}_{\mathrm{IMD},s}^{12,8} = \mathbf{T}_{\mathrm{IMD},s}^{8,6}$, $\mathbf{T}_{\mathrm{IMD},s}^{9,21} = \mathbf{T}_{\mathrm{IMD},s}^{21,25} = \mathbf{T}_{\mathrm{IMD},s}^{25,9}$, $\mathbf{T}_{\mathrm{IMD},s}^{14,14}$, $\mathbf{T}_{\mathrm{IMD},s}^{15,17} = \mathbf{T}_{\mathrm{IMD},s}^{17,23} = \mathbf{T}_{\mathrm{IMD},s}^{23,15}$, $\mathbf{T}_{\mathrm{IMD},s}^{18,24} = \mathbf{T}_{\mathrm{IMD},s}^{24,26} = \mathbf{T}_{\mathrm{IMD},s}^{26,18}$, $\mathbf{T}_{\mathrm{IMD},s}^{17,17}$, and all these elements equal one. Provided with this $\widetilde{\mathbf{T}}_{\mathrm{IMD},s} = \mathbf{I} - \mathbf{T}_{\mathrm{IMD},s}$ as preset, the nonzero components of $\sigma_{\mathrm{IMD}}^{\alpha\beta\gamma}$ are obtained by solving basis vectors of $\left(\bigcap_{i=1}^{L} \ker \widetilde{\mathbf{T}}_i\right) \cap \ker \widetilde{\mathbf{T}}_{\mathrm{IMD},s}$. The intrinsic symmetries of the BCD and QMD contributions can be considered likewise.

For convenience, the procedure of identifying finite second-order transport tensors allowed by SSG symmetries has now been equipped in our online program FINDSPINGROUP (https://findspingroup.com/).

4. Database of antiferromagnets with second-order transports

In this section we construct a database of antiferromagnets (AFMs) with second-order conductivity tensors allowed by SSG symmetry, where all the AFMs are experimentally validated. There are six steps in our procedure as shown in Fig. S4.

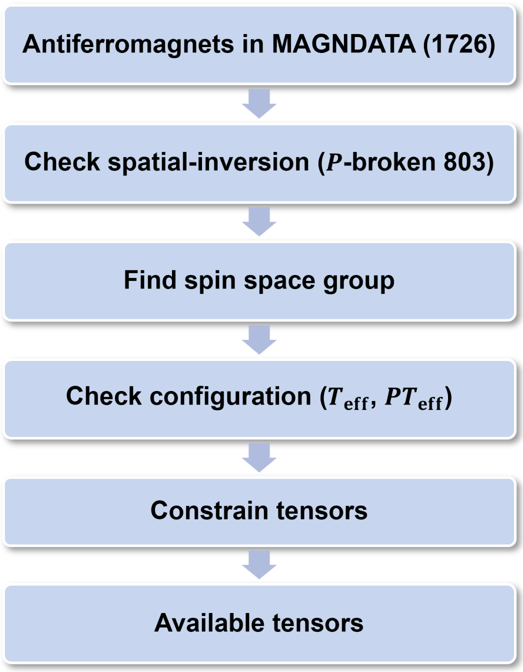

Fig. S4. Procedure on constructing antiferromagnet database within the spin space group theory.

For the first step, all the experimentally verified magnets are collected in the MAGNDATA database [11,12] on the Bilbao Crystallographic Server (BCS; http://www.cryst.ehu.es). In this work, we concentrate on the AFMs, i.e., magnets without net magnetization insured by certain symmetries. Thus, 1726 AFMs selected from MAGNDATA form the starting material pool.

In the second step, to materialize the second-order transport effects, we filtered out the centrosymmetric AFMs whose magnetic space group contain the spatial inversion P or the combined symmetry of inversion and translation $\{P|\boldsymbol{\tau}\}$. In this step, 803 noncentrosymmetric AFMs are left for further analysis.

In the third step, the SSG of each noncentrosymmetric AFM was identified by our online program FINDSPINGROUP (Section 1.4). Thanks to the nomenclature of international notation, the magnetic configuration of each noncentrosymmetric AFM can be directly read from the spin-only part, labeling 495 collinear, 227 coplanar, and 81 noncoplanar AFMs.

For the fourth step, we recognized the effective symmetries of time-reversal T_{eff} and combination of spatial inversion and time-reversal PT_{eff} that are essential for second-order transports as shown in Tab. I in the main text. As mentioned in the main text, all the AFMs of collinear and coplanar magnetic configurations endure T_{eff} eliminating the IMD and QMD contributions. If no PT_{eff} emerged in collinear and coplanar AFMs, BCD contribution on second-order transports are allowed. As for noncoplanar AFMs, all three contributions from IMD, BCD, and QMD would be symmetrically allowed once the T_{eff} and PT_{eff} are absent.

For the last step, provided the allowed geometry second-order transports in an AFM, we further predicted the nonzero components of the transport tensors by the strategy stated in Section 3. For an AFM, if all the components of transport tensors vanish, it produces no second-order transports even the P, T_{eff}, and PT_{eff} are both absent.

Finally, we established a database of 260 AFMs with geometric second-order transport tensors induced by magnetic geometry. In this database, 120 collinear and 71 coplanar AFMs allow BCD-contributed NLT. We also found 69 noncoplanar AFMs with SSG-allowed NLT, 21 among which feature all NLT contributed by both IMD, BCD, and QMD. All the information of AFMs, SSGs, nonzero tensor components are tabulated in Tabs. S1-S3, where noncentrosymmetric AFMs without geometric second-order transports are also tabulated in Tab. S4. Note that we can also apply the same strategy within the framework of magnetic space group to obtain the allowed tensors, where the results are included in Tabs. S1-S4. The pure SOC contributions on second-order transports lie directly on the differences between the columns from SSG (without SOC) and magnetic space group (with SOC).

We should note that though we start from the simple condition of inversion-breaking, our framework is complete for symmetry analysis on distinct geometry contributions.

For instance, the results for BCD recover previous knowledge that BCD term vanishes unless the point groups of the nonmagnetic materials are gyrotropic point groups [13], i.e., point groups that allow nonzero components of the rank-two pseudo-tensors. One step more, we find that two gyrotropic point groups: tetrahedral group T (23) and octahedral group O (432) force the BCD-contribution $\sigma_{\mathrm{BCD}}^{\alpha\beta\gamma}$ to be vanished. Recall from Eq. (S24) that $\sigma_{\mathrm{BCD}}^{\alpha\beta\gamma} \propto \epsilon^{\alpha\beta\xi}D^{\gamma\xi} + \epsilon^{\alpha\gamma\xi}D^{\beta\xi}$ with $D^{\gamma\xi} = \sum_{\boldsymbol{k},l}\left(\partial_\alpha\Omega_l^\beta\right)f_l$ the rank-two pseudo-tensor known as BCD, and $\epsilon^{\alpha\beta\xi}$ is the Levi-Civita symbol. We found that both gyrotropic point groups T and O allow finite diagonal components of BCD tensor $D^{xx} = D^{yy} = D^{zz}$ where $D^{\alpha\beta} = 0$ if $\alpha \neq \beta$. Nevertheless, the component of $\sigma_{\mathrm{BCD}}^{\alpha\beta\gamma}$ contributed by the diagonal parts of BCD vanish: $\sigma_{\mathrm{BCD}}^{xyz} = \sigma_{\mathrm{BCD}}^{xzy} \propto -D^{yy} + D^{zz} = 0$, $\sigma_{\mathrm{BCD}}^{xyz} = \sigma_{\mathrm{BCD}}^{xzy} \propto -D^{yy} + D^{zz} = 0$, and $\sigma_{\mathrm{BCD}}^{zxy} = \sigma_{\mathrm{BCD}}^{zyx} \propto -D^{xx} + D^{yy} = 0$. Hence, gyrotropic point groups still not guarantee the existence of the second-order conductivity contributed by BCD. Since the BCD contribution is even under time-reversal symmetry, the conclusion is also valid for magnetic (crystallographic) point groups of polyhedral type (23.1’, 432.1’, 4’32’). Our framework of symmetry analysis correctly predicts the vanishment of BCD contribution of these cases that the spin space group and spin point group are homomorphic to either of T (23), O (432), 23.1’, 432.1’, and 4’32’. These cases can be easily found in our database, e.g., row 11-15 in Table S3 for Mn-based noncoplanar antiferromagnets Mn_3XY (X = Ir, Co, Y = Ge, Si), where the spin space group $P^{2_{001}}2_1{}^{3^1_{111}}3$ is homomorphic to T (23) and then no BCD contribution.

Table S1: Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry. Note that without SOC, IMD- and QMD-contributions are forbidden.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
1	$ZnFe_2O_4$ (1.761)	$I^1\text{-}4^12^1\text{d}^{-1}(1/2\ \ 1/2\ \ 0)^{\infty\text{m}}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
2	YMn_2 (1.746)	$P^14_3{}^12^12^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty\text{m}}1$	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
3	$GeNi_2O_4$ (1.563)	$R^13^1\text{m}^{-1}(1/3\ \ 2/3\ \ 1/6)^{\infty\text{m}}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$
4	$GeNi_2O_4$ (1.561)	$R^13^1\text{m}^{-1}(1/3\ \ 2/3\ \ 1/6)^{\infty\text{m}}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$
5	Ce_4Ge_3 (0.448)	$I^1\text{-}4^{-1}2^{-1}\text{d}^{\infty\text{m}}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
6	Ce_4Sb_3 (0.681)	$I^{-1}\text{-}4^{-1}2^1\text{d}^{\infty\text{m}}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
7	Tb_2C_3 (0.345)	$F^{\text{m}_{001}}\text{d}^{\text{m}_{001}}\text{d}^12^{\infty\text{m}}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
8	$GdInCu_4$ (1.699)	$I^1\text{-}4^12^1\text{m}^{-1}(1/2\ \ 1/2\ \ 0)^{\infty\text{m}}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
9	$HoInCu_4$ (1.700)	I^1-4^12^1m^{-1}(1/2 1/2 0)$^{\infty m}$1	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×	×	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
10	NdBiPt (1.574)	P^1-4^1m^{1}2^{-1}(1/2 1/2 1/2)$^{\infty m}$1	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×	×	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$
11	GeV_4S_8 (1.86)	P^1m^{1}n^{1}2$_1{}^{-1}$(0 1/2 0)$^{\infty m}$1	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
12	GdBiPt (1.111)	$R^1$3^{1}m^{-1}(1/3 2/3 1/6)$^{\infty m}$1	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
13	CuMnSb (1.232)	$R^1$3^{1}m^{-1}(1/3 2/3 1/6)$^{\infty m}$1	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
14	CuMnSb (1.233)	$R^1$3^{1}m^{-1}(1/3 2/3 1/6)$^{\infty m}$1	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	×	×	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
15	CuMnSb (1.265)	$R^1$3^{1}m^{-1}(1/3 2/3 1/6)$^{\infty m}$1	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	×	×	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
16	UCu_5 (1.424)	$R^1$3^{1}m^{-1}(1/3 2/3 1/6)$^{\infty m}$1	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	×	×	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
17	$HoCdCu_4$ (1.701)	$R^{1}3^{1}m^{-1}(1/3\ \ 2/3\ \ 1/6)^{\infty m}1$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
18	UCu_5 (1.721)	$R^{1}3^{1}m^{-1}(1/3\ \ 2/3\ \ 1/6)^{\infty m}1$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	×	×	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
19	MnS_2 (1.18)	$P^{1}n^{1}a^{1}2_1{}^{-1}(0\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
20	$MnSe_2$ (1.0.48)	$P^{-1}c^{-1}a^{1}2_1{}^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
21	$Na_2Ni_2TeO_6$ (1.646)	$I^{1}m^{1}a^{1}2^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
22	TbNiAl (2.99)	$C^{1}m^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
23	TbNiAl (1.738)	$I^{1}m^{1}m^{1}2^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
24	$Co_2Mo_3O_8$ (0.332)	$P^{-1}6_3{}^{1}m^{-1}c^{\infty m}1$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
25	$Fe_2Mo_3O_8$ (0.331)	$P^{-1}6_3{}^{1}m^{-1}c^{\infty m}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	$\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	$\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$
26	$Co_2Mo_3O_8$ (0.338)	$P^{-1}6_3{}^{1}m^{-1}c^{\infty m}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	$\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	$\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$
27	ErAuGe (1.33)	$P^{1}m^{1}c^{1}2_1{}^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
28	GdCuSn (1.504)	$P^{1}m^{1}c^{1}2_1{}^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
29	GdAgSn (1.505)	$P^{1}m^{1}c^{1}2_1{}^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
30	CaOFeS (1.472)	$P^{1}m^{1}n^{1}2_1{}^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
31	GdAuSn (1.506)	$P^{1}m^{1}c^{1}2_1{}^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
32	$Na_2Co_2TeO_6$ (1.184)	$P^{1}2_1{}^{1}2_1{}^{1}2^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
33	$AgNiO_2$ (1.50)	$P^{1}2_1{}^{1}2_1{}^{1}2^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
34	$CoNb_3S_6$ (1.349)	$P^{1}2^{1}2^{1}2_1{}^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
35	$Fe_{0.32}NbS_2$ (1.676)	$P^{1}2^{1}2^{1}2_1{}^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
36	VNb_3S_6 (0.712)	$P^{-1}6_3{}^{-1}2^{1}2^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
37	$Fe_{0.967}Nb_3S_6$ (1.589)	$P^{1}2_1{}^{1}2_1{}^{1}2_1{}^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
38	$Fe_{0.35}NbS_2$ (1.677)	$P^{1}2_1{}^{1}2_1{}^{1}2_1{}^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
39	$Ca_3Co_{2-x}Mn_xO_6$ (0.13)	$R^{1}3^{-1}c^{\infty m}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	σ_{zzz}, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	σ_{zzz}, $\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$
40	$PbNiO_3$ (0.21)	$R^{1}3^{-1}c^{\infty m}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	σ_{zzz}, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	σ_{zzz}, $\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$
41	$GaFeO_3$ (0.306)	$R^{1}3^{-1}c^{\infty m}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
42	$MnTiO_3$ (0.50)	$R^{1}3^{-1}c^{\infty m}1$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
43	$Ho_{0.1}Bi_{0.9}FeO_3$ (0.556)	$R^{1}3^{-1}c^{\infty m}1$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
44	$Ho_{0.05}Bi_{0.95}FeO_3$ (0.555)	$R^{1}3^{-1}c^{\infty m}1$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
45	$ScFeO_3$ (0.57)	$R^{1}3^{-1}c^{\infty m}1$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
46	$AgCrS_2$ (1.136)	$C^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
47	$CeFe_3(BO_3)_4$ (1.459)	$R^{1}3^{1}2^{-1}(1/3\ \ 2/3\ \ 1/6)^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
48	$NdFe_3B_4O_{12}$ (1.7)	$R^{1}3^{1}2^{-1}(1/3\ \ 2/3\ \ 1/6)^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
49	$YFe_3(BO_3)_4$ (1.90)	$P^{1}3_{2}{}^{1}2^{1}1^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
50	$TbFe_3(BO_3)_4$ (1.91)	$P^{1}3_{2}{}^{1}2^{1}1^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×	×	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$
51	Ni_3TeO_6 (1.165)	$R^{1}3^{-1}(1/3\ \ 2/3\ \ 1/6)^{\infty m}1$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×	×	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$
52	$NiCr_2O_4$ (1.685)	$C^{1}2^{1}2^{1}2_{1}{}^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×	×	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
53	ZnV_2O_4 (1.24)	$P^{1}4_{3}{}^{1}2^{1}2^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×	×	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
54	$La_2O_2Fe_2OSe_2$ (1.58)	$C^1m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
55	$NiTa_2O_6$ (1.172)	$A^1m^1a^12^{-1}(0\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
56	$CeAuSb_2$ (1.740)	$A^1m^1m^12^{-1}(0\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
57	$CuFeS_2$ (0.802)	$I^1\text{-}4^{-1}2^{-1}d^{\infty m}1$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$
58	Cu_2MnSnS_4 (1.100)	$C^12^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
59	Cu_2MnSnS_4 (1.732)	$C^12^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
60	Cu_2FeGeS_4 (1.734)	$C^{1}2^{-1}(0\ \ 0\ \ 1/2)^{\infty\mathrm{m}}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$, $\sigma_{xyz}=\sigma_{xzy}$
61	$CsCoF_4$ (0.405)	$I^{-1}\text{-}4^{\infty\mathrm{m}}1$	$\sigma_{xxz}=\sigma_{xzx}=-\sigma_{yyz}=-\sigma_{yzy}=-\sigma_{zxx}/2=\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	σ_{zzz}, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=\sigma_{zxx}=\sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	$\sigma_{xxz}=\sigma_{xzx}=-\sigma_{yyz}=-\sigma_{yzy}=-\sigma_{zxx}/2=\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
62	$Ba_2FeSi_2O_7$ (1.641)	$P^{1}\text{-}4^{1}2_1{}^{1}\mathrm{c}^{-1}(0\ \ 0\ \ 1/2)^{\infty\mathrm{m}}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
63	$Ba_2MnSi_2O_7$ (0.229)	$P^{1}\text{-}4^{-1}2_1{}^{-1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
64	$Ba_2CoGe_2O_7$ (0.56)	$P^{1}\text{-}4^{-1}2_1{}^{-1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
65	$SrMn_2V_2O_8$ (0.62)	$I^{-1}4_1{}^{1}\mathrm{c}^{-1}\mathrm{d}^{\infty\mathrm{m}}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
66	$BaMn_2V_2O_8$ (0.967)	$I^{-1}4_1{}^{1}\mathrm{c}^{-1}\mathrm{d}^{\infty\mathrm{m}}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
67	$SrCo_2V_2O_8$ (1.71)	$P^1c^1a^12_1{}^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
68	$CeRhGe_3$ (1.743)	$P^14^1m^1m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	×	×	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$
69	$YBaCuFeO_5$ (1.281)	$I^14^1m^1m^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
70	PrFeAsO (1.586)	$P^1m^1a^12^{-1}(1/2\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
71	Fe_4O_5 (0.999)	$C^1m^{-1}c^{-1}2_1{}^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
72	$SmNiO_3$ (1.353)	$C^1m^1c^12_1{}^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
73	$EuNiO_3$ (1.354)	$C^1m^1c^12_1{}^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
74	$PrNiO_3$ (1.43)	$C^1m^1c^12_1{}^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
75	$NdNiO_3$ (1.45)	$C^1m^1c^12_1{}^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
76	$BaFe_2Se_3$ (1.120)	$C^1\mathrm{c}^{-1}(1/2\ \ 0\ \ 0)^{\infty \mathrm{m}}1$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$
77	$NaMn_2O_4$ (1.723)	$C^1\mathrm{c}^{-1}(1/2\ \ 0\ \ 0)^{\infty \mathrm{m}}1$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$
78	$BaFe_2Se_3$ (1.429)	$C^1\mathrm{m}^{-1}(1/2\ \ 0\ \ 0)^{\infty \mathrm{m}}1$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$
79	$Pb_2Mn_{0.6}Co_{0.4}WO_6$ (2.17)	$P^1\mathrm{m}^{-1}\mathrm{c}^{-1}2_1{}^{\infty \mathrm{m}}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
80	$Na_2CuSO_4Cl_2$ (1.682)	$P^1\mathrm{m}^1\mathrm{n}^12_1{}^{-1}(0\ \ 1/2\ \ 0)^{\infty \mathrm{m}}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
81	$LuMnO_3$ (1.101)	$P^1\mathrm{n}^1\mathrm{a}^12_1{}^{-1}(1/2\ \ 0\ \ 0)^{\infty \mathrm{m}}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
82	$HoMnO_3$ (1.20)	$P^1\mathrm{n}^1\mathrm{a}^12_1{}^{-1}(1/2\ \ 0\ \ 0)^{\infty \mathrm{m}}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
83	$TmMnO_3$ (1.341)	$P^1\mathrm{n}^1\mathrm{a}^1\mathrm{2_1}^{-1}(1/2\ \ 0\ \ 0)^{\infty\mathrm{m}}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
84	HoNiGe (1.374)	$P^1\mathrm{m}^{-1}(0\ \ 1/2\ \ 0)^{\infty\mathrm{m}}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
85	$BaCdVO(PO_4)_2$ (1.298)	$P^1\mathrm{c}^1\mathrm{a}^1\mathrm{2_1}^{-1}(0\ \ 1/2\ \ 0)^{\infty\mathrm{m}}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
86	$FeNb_2O_6$ (1.655)	$P^1\mathrm{2_1}^1\mathrm{2_1}^1\mathrm{2}^{-1}(0\ \ 0\ \ 1/2)^{\infty\mathrm{m}}1$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×	×	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
87	$Sr_2Fe_3Se_2O_3$ (1.463)	$C^1\mathrm{m}^1\mathrm{c}^1\mathrm{2_1}^{-1}(1/2\ \ 0\ \ 0)^{\infty\mathrm{m}}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
88	$Sr_2Fe_3Se_2O_3$ (1.626)	$C^1\mathrm{m}^1\mathrm{c}^1\mathrm{2_1}^{-1}(1/2\ \ 0\ \ 0)^{\infty\mathrm{m}}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
89	$SmMn_2O_5$ (1.192)	$P^1\mathrm{m}^1\mathrm{c}^1\mathrm{2_1}^{-1}(1/2\ \ 0\ \ 0)^{\infty\mathrm{m}}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
90	$Tb_5Pd_2In_4$ (1.697)	$P^1\mathrm{m}^1\mathrm{c}^1\mathrm{2_1}^{-1}(1/2\ \ 0\ \ 0)^{\infty\mathrm{m}}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
91	$PrMn_2O_5$ (1.325)	$P^{1}\mathrm{m}^{-1}(0\ \ 1/2\ \ 0)^{\infty\mathrm{m}}1$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$
92	$BiFe_{0.5}Sc_{0.5}O_3$ (0.67)	$I^{-1}\mathrm{m}^{-1}\mathrm{a}^{1}2^{\infty\mathrm{m}}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
93	Sr_2MnGaO_5 (0.823)	$I^{-1}\mathrm{m}^{-1}\mathrm{a}^{1}2^{\infty\mathrm{m}}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
94	$Sr_2Co_2O_5$ (0.799)	$P^{1}\mathrm{m}^{1}\mathrm{a}^{1}2^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty\mathrm{m}}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
95	$MnCl_2(CO(NH_2)_2)_2$ (1.659)	$P^{1}\mathrm{b}^{1}\mathrm{a}^{1}2^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty\mathrm{m}}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
96	$ZnFeF_5(H_2O)_2$ (0.575)	$I^{-1}\mathrm{m}^{1}\mathrm{m}^{-1}2^{\infty\mathrm{m}}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
97	$Cu_2V_2O_7$ (0.137)	$F^{-1}\mathrm{d}^{1}\mathrm{d}^{-1}2^{\infty\mathrm{m}}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
98	$ErGe_{1.83}$ (0.344)	$C^{1}\mathrm{m}^{-1}\mathrm{c}^{-1}2_1{}^{\infty\mathrm{m}}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
99	$Ca_3Mn_2O_7$ (0.23)	$C^{-1}\mathrm{m}^{-1}\mathrm{c}^{1}2_1{}^{\infty\mathrm{m}}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
100	$Ca_3Ru_2O_7$ (1.263)	$P^{1}m^{1}c^{1}2_1{}^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
101	$BaCoF_4$ (1.439)	$P^{1}n^{1}a^{1}2_1{}^{-1}(0\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
102	$BaFe_2O_4$ (1.754)	$P^{1}n^{1}a^{1}2_1{}^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
103	$BaCoF_4$ (1.438)	$P^{1}2_1{}^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
104	$BaNiF_4$ (1.64)	$P^{1}2_1{}^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
105	$La_{1.5}Ca_{0.5}CoO_4$ (1.583)	$A^{1}m^{1}a^{1}2^{-1}(0\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
106	Li_2CoSiO_4 (1.79)	$C^{1}\mathrm{c}^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	$\times$	$\times$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
107	$Y_2Cu_2O_5$ (0.241)	$P^{-1}\mathrm{n}^{1}\mathrm{a}^{-1}2_1{}^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxz} = \sigma_{xzx}$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
108	$[C(ND_2)_3]Cu(DCOO)_3$ (0.254)	$P^{-1}\mathrm{n}^{1}\mathrm{a}^{-1}2_1{}^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxz} = \sigma_{xzx}$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
109	$[C(ND_2)_3]Cu(DCOO)_3$ (0.255)	$P^{-1}\mathrm{n}^{1}\mathrm{a}^{-1}2_1{}^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
110	$BaCrF_5$ (0.303)	$P^{-1}2_1{}^{-1}2_1{}^{1}2_1{}^{\infty m}1$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxz} = \sigma_{xzx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
111	Lu_2MnCoO_6 (1.32)	$P^{1}2_1{}^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	$\times$	$\times$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
112	Na_2MnF_5 (1.55)	$P^{1}\mathrm{c}^{-1}(0\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	$\times$	$\times$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
113	Pb_2CoOsO_6 (1.565)	$P^1c^{-1}(0\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
114	Pb_2NiOsO_6 (1.592)	$P^1c^{-1}(0\ \ 1/2\ \ 0)^{\infty m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
115	$Mn_4Nb_2O_9$ (0.722)	$C^{-1}c^{\infty m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	$\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz} = \sigma_{zzy}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy} = \sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy} = \sigma_{xyx}$, σ_{xxx}	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
116	Li_2FeGeS_4 (1.735)	$C^1c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$

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Table S1 : (continued) Collinear antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
117	$LuFe_2O_4$ (0.965)	$P^{1}1^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$, $\sigma_{xyz}=\sigma_{xzy}$
118	$CrPS_4$ (1.440)	$C^{1}2^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
119	$CrPS_4$ (1.708)	$C^{1}2^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
120	$LiFeP_2O_7$ (0.83)	$P^{-1}2_1{}^{\infty m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$

Table S2: Coplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry. Note that without SOC, IMD- and QMD-contributions are forbidden.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
1	$ZnFe_2O_4$ (1.760)	$I^1\text{-}4^{m_{110}}m^{m_{110}}2\|(2_{001},2_{001},2_{001};4^3_{001})^m1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
2	$CsCrF_4$ (1.709)	$C^1m^{2_{001}}(1/2\ \ 0\ \ 0)^m1$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$
3	$CsCr_{0.98}Al_{0.02}F_4$ (1.712)	$C^1m^{2_{001}}(1/2\ \ 0\ \ 0)^m1$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$
4	$CsCr_{0.98}Al_{0.02}F_4$ (1.713)	$C^1m^{2_{001}}(1/2\ \ 0\ \ 0)^m1$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$
5	TmPtIn (1.67)	$P^{2_{001}}m^1m^{2_{001}}2\|(2_{010},2_{010},1)^m1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
6	$Ni_2Mo_3O_8$ (1.768)	$C^1m^{m_{100}}c^{m_{100}}2_1\|(1,1,1;2_{001})^m1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$

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Table S2 : (continued) Coplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
7	HoAuGe (1.34)	$P^{1}\mathrm{m}^{2_{001}}(1/2\ \ 1/2\ \ 0)^{m}1$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
8	$InMnO_3$ (1.525)	$P^{3^{1}_{001}}3^{1}1^{\mathrm{m}_{\frac{5}{6}\pi}}\mathrm{m}\vert(1,1,2_{001})^{m}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	×	×	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$
9	$InMnO_3$ (1.524)	$P^{3^{1}_{001}}3^{1}1^{\mathrm{m}_{\frac{5}{6}\pi}}\mathrm{m}\vert(1,1,2_{001})^{m}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	×	×	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$
10	$YbMnO_3$ (0.30)	$P^{3^{2}_{001}}6_{3}{}^{\mathrm{m}_{\frac{1}{6}\pi}}\mathrm{c}^{\mathrm{m}_{\frac{5}{6}\pi}}\mathrm{m}^{m}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	$\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	$\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$
11	$YMnO_3$ (0.44)	$P^{3^{2}_{001}}6_{3}{}^{\mathrm{m}_{\frac{1}{6}\pi}}\mathrm{c}^{\mathrm{m}_{\frac{5}{6}\pi}}\mathrm{m}^{m}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	$\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}=-\sigma_{yyy}$, $\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	$\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}=-\sigma_{yyy}$, $\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$, $\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$
12	$HoMnO_3$ (0.32)	$P^{6^{1}_{001}}6_{3}{}^{\mathrm{m}_{\frac{1}{6}\pi}}\mathrm{c}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{m}^{m}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	σ_{zzz}, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=\sigma_{zxx}=\sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$
13	$YMnO_3$ (0.6)	$P^{6^{1}_{001}}6_{3}{}^{\mathrm{m}_{\frac{1}{6}\pi}}\mathrm{c}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{m}^{m}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	σ_{zzz}, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=\sigma_{zxx}=\sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$
14	$LuFeO_3$ (0.117)	$P^{6^{1}_{001}}6_{3}{}^{\mathrm{m}_{\frac{1}{6}\pi}}\mathrm{c}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{m}^{m}1$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$	×	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$

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Table S2 : (continued) Coplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
15	$HoMnO_3$ (0.31)	$P^{6^1_{001}}6_3{}^{\mathrm{m}_{\frac{1}{6}\pi}}\mathrm{c}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{m}^{m}1$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	$\times$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
16	$ScMnO_3$ (0.7)	$P^{6^1_{001}}6_3{}^{\mathrm{m}_{\frac{1}{6}\pi}}\mathrm{c}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{m}^{m}1$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	$\times$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
17	$ScMnO_3$ (0.8)	$P^{6^1_{001}}6_3{}^{\mathrm{m}_{\frac{1}{6}\pi}}\mathrm{c}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{m}^{m}1$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$
18	$Na_2Co_2TeO_6$ (1.645)	$C^{\mathrm{m}_{100}}2^{1}2^{\mathrm{m}_{100}}2_1\|(1,1,1;2_{001})^{m}1$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	$\times$	$\times$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
19	$BaCoSiO_4$ (0.724)	$P^{6^1_{001}}6_3{}^{m}1$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$
20	$BaCoSiO_4$ (1.0.49)	$P^{6^1_{001}}6_3{}^{m}1$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$

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Table S2 : (continued) Coplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
21	$SmFe_3(BO_3)_4$ (1.266)	$R^{1}3^{1}2^{2_{001}}(1/3\ \ 2/3\ \ 1/6)^{m}1$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$, $\sigma_{xyz} = \sigma_{xzy}$
22	$HoFe_3(BO_3)_4$ (1.93)	$P^{1}3_2{}^{1}2^{1}1^{2_{001}}(0\ \ 0\ \ 1/2)^{m}1$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$, $\sigma_{xyz} = \sigma_{xzy}$
23	$BaCu_3V_2O_8(OD)_2$ (3.17)	$P^{3^{1}_{001}}3_1{}^{\mathrm{m}\frac{1}{6}\pi}2^{1}1^{m}1$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$

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Table S2 : (continued) Coplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
24	$RuCl_3$ (1.726)	$P^{1}1^{2_{001}}(1/2\ \ 0\ \ 0)^{m}1$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$, $\sigma_{xyz}=\sigma_{xzy}$
25	Cu_4O_3 (1.418)	$I^{1}\text{-}4^{\mathrm{m}_{110}}\mathrm{m}^{\mathrm{m}_{110}}2\|(2_{001},2_{001},2_{001};4^{3}_{001})^{m}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
26	$NiCr_2O_4$ (1.688)	$I^{\mathrm{m}_{010}}2_1{}^{2_{001}}2_1{}^{\mathrm{m}_{100}}2_1\|(1,1,1;2_{001})^{m}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
27	$GeCu_2O_4$ (1.185)	$I^{1}\text{-}4^{\mathrm{m}_{110}}\mathrm{m}^{\mathrm{m}_{110}}2\|(2_{001},2_{001},2_{001};4^{3}_{001})^{m}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
28	$DyFe_4Ge_2$ (1.98)	$P^{1}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{m}^{\mathrm{m}_{100}}2\|(1,1,2_{001})^{m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
29	NdSbTe (1.764)	$P^{1}\mathrm{m}^{2_{001}}(0\ \ 1/2\ \ 0)^{m}1$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$
30	Pb_2MnO_4 (0.552)	$P^{4^{1}_{001}}\text{-}4^{\mathrm{m}_{-110}}2_1{}^{\mathrm{m}_{100}}\mathrm{c}^{m}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
31	$Pr_2PdAl_7Ge_4$ (1.757)	$P^{1}\text{-}4^{\mathrm{m}_{100}}2_1{}^{\mathrm{m}_{100}}\mathrm{m}\|(1,1,2_{001})^{m}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$

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Table S2 : (continued) Coplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
32	$Pr_2PdAl_7Ge_4$ (1.772)	$P^{1}\text{-}4^{\mathrm{m}_{100}}2_1{}^{\mathrm{m}_{100}}\mathrm{m}\|(1,1,2_{001})^{m}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	$\times$	$\times$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
33	$Ho_2Ge_2O_7$ (0.107)	$P^{4^{1}_{001}}4_1{}^{\mathrm{m}_{\text{-}110}}2_1{}^{\mathrm{m}_{100}}2^{m}1$	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	$\times$	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$
34	Er_2Pt (1.444)	$P^{1}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{n}^{\mathrm{m}_{100}}2_1\|(1,2_{001},1)^{m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\times$	$\times$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
35	Gd_2BaCuO_5 (1.443)	$P^{1}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{c}^{\mathrm{m}_{100}}2_1\|(1,2_{001},1)^{m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\times$	$\times$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
36	$DyFeO_3$ (0.10)	$P^{\mathrm{m}_{100}}2_1{}^{\mathrm{m}_{010}}2_1{}^{2_{001}}2_1{}^{m}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
37	$Cu_3Mo_2O_9$ (0.129)	$P^{\mathrm{m}_{100}}2_1{}^{\mathrm{m}_{010}}2_1{}^{2_{001}}2_1{}^{m}1$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
38	$La_{0.333}Ca_{0.667}MnO_3$ (1.175)	$P^{2_{001}}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{n}^{\mathrm{m}_{010}}2_1\|(1,2_{001},1)^{m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\times$	$\times$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
39	$La_{0.333}Ca_{0.667}MnO_3$ (1.174)	$P^{2_{001}}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{c}^{\mathrm{m}_{010}}2_1\|(1,2_{001},1)^{m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\times$	$\times$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
40	$La_{0.375}Ca_{0.625}MnO_3$ (1.173)	$P^{2_{001}}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{c}^{\mathrm{m}_{010}}2_1\|(1,2_{001},1)^{m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\times$	$\times$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
41	$LuMnO_3$ (1.340)	$P^{\mathrm{m}_{100}}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{n}^{1}2_1\|(1,2_{001},1)^{m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\times$	$\times$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$

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Table S2 : (continued) Coplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
42	$TbFeO_3$ (0.353)	$P^{\mathrm{m}_{100}}2_1{}^{\mathrm{m}_{010}}2_1{}^{2_{001}}2_1{}^{m}1$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxz} = \sigma_{xzx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
43	$NdNiO_3$ (1.44)	$P^{\mathrm{m}_{100}}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{c}^{1}2_1\vert(2_{001},1,1)^{m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
44	$CoNb_2O_6$ (1.224)	$P^{\mathrm{m}_{100}}2_1{}^{\mathrm{m}_{100}}2_1{}^{1}2\vert(1,1,2_{001})^{m}1$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×	×	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
45	$Tm_5Ni_2In_4$ (1.170)	$C^{1}\mathrm{m}^{2_{001}}(1/2\ \ 0\ \ 0)^{m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
46	$HoMn_2O_5$ (1.109)	$C^{1}\mathrm{m}^{2_{001}}(1/2\ \ 0\ \ 0)^{m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
47	$TbMn_2O_5$ (1.108)	$C^{1}\mathrm{m}^{2_{001}}(1/2\ \ 0\ \ 0)^{m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
48	$PrMn_2O_5$ (1.19)	$P^{1}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{c}^{\mathrm{m}_{100}}2_1\vert(1,2_{001},1)^{m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$

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Table S2 : (continued) Coplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
49	$DyMn_2O_5$ (1.324)	$P^1\mathrm{m}^{\mathrm{m}_{100}}\mathrm{c}^{\mathrm{m}_{100}}2_1\|(1,2_{001},1)^m1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
50	$GdMn_2O_5$ (1.54)	$P^1\mathrm{m}^{\mathrm{m}_{100}}\mathrm{c}^{\mathrm{m}_{100}}2_1\|(1,2_{001},1)^m1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
51	$DyMn_2O_5$ (1.76)	$P^1\mathrm{m}^{\mathrm{m}_{100}}\mathrm{c}^{\mathrm{m}_{100}}2_1\|(1,2_{001},1)^m1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
52	$DyMn_2O_5$ (1.599)	$P^1\mathrm{m}^{2_{001}}(0\ \ 1/2\ \ 0)^m1$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$
53	$BiMn_2O_5$ (1.74)	$P^1\mathrm{m}^{\mathrm{m}_{100}}\mathrm{c}^{\mathrm{m}_{100}}2_1\|(2_{001},2_{001},1)^m1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
54	$Ca_2Cr_2O_5$ (1.227)	$P^1\mathrm{m}^1\mathrm{a}^12^{2_{001}}(1/2\ \ 1/2\ \ 1/2)^m1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
55	$BaNiTe_2O_7$ (1.763)	$C^{2_{001}}2\|(1,1,1;2_{010})^m1$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$

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Table S2 : (continued) Coplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
56	$HoCrWO_6$ (0.716)	$P^{2_{001}}n^{m_{100}}a^{m_{010}}2_1{}^{m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxz} = \sigma_{xzx}$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
57	$Na_2CoP_2O_7$ (0.425)	$P^{m_{010}}n^{2_{001}}a^{m_{100}}2_1{}^{m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
58	$Yb_2Cu_2O_5$ (1.280)	$P^{m_{100}}c\|(2_{001}, 1, 1)^{m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
59	Cu_2MnSiS_4 (1.730)	$P^{m_{100}}c\|(2_{001}, 1, 1)^{m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
60	Li_2MnGeO_4 (1.484)	$P^{m_{100}}c\|(2_{001}, 1, 1)^{m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
61	Cu_2MnGeS_4 (1.733)	$P^{m_{100}}c\|(2_{001}, 1, 1)^{m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$

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Table S2 : (continued) Coplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
62	Cu_2FeSiS_4 (1.731)	$P^{m_{100}}c\|(2_{001},1,1)^{m}1$	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$
63	$Mn_3B_7O_{13}I$ (0.134)	$P^{m_{010}}c^{2_{001}}a^{m_{100}}2_1{}^{m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
64	$YbLuCoMnO_6$ (1.329)	$P^{1}2_1{}^{2_{001}}(0\ \ 0\ \ 1/2)^{m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
65	Yb_2CoMnO_6 (1.328)	$P^{1}2_1{}^{-1}(0\ \ 0\ \ 1/2)^{m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
66	Lu_2CoMnO_6 (1.330)	$P^{1}2_1{}^{2_{001}}(0\ \ 0\ \ 1/2)^{m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
67	$HoNiO_3$ (1.48)	$P^{1}2_1{}^{2_{001}}(0\ \ 0\ \ 1/2)^{m}1$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$

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Table S2 : (continued) Coplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC	with SOC		
			BCD	IMD	QMD	BCD
68	$LuNiO_3$ (1.657)	$P^{1}2_{1}{}^{2_{001}}(0\ \ 0\ \ 1/2)^{m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
69	$NiPS_3$ (1.231)	$P^{1}2^{2_{001}}(1/2\ \ 1/2\ \ 0)^{m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$, $\sigma_{xyz} = \sigma_{xzy}$
70	$NaNdFeWO_6$ (1.68)	$P^{1}1^{2_{001}}(1/2\ \ 0\ \ 0)^{m}1$	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$, $\sigma_{xyz} = \sigma_{xzy}$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$, $\sigma_{xyz} = \sigma_{xzy}$
71	$YBaFe_4O_7$ (1.124)	$P^{\mathrm{m}_{100}}2_{1}\vert(2_{001},1,1)^{m}1$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$

Table S3: Noncoplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC			with SOC		
			IMD	QMD	BCD	IMD	QMD	BCD
1	$Ca_2LaZr_2Fe_3O_{12}$ (0.754)	$R^{-3^{1}_{001}}\text{-}3^{2^{\frac{1}{6}\pi}}\mathrm{c}$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	$\times$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	$\times$
2	$Ca_2YZr_2Fe_3O_{12}$ (0.752)	$R^{-3^{1}_{001}}\text{-}3^{2^{\frac{1}{6}\pi}}\mathrm{c}$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	$\times$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	$\times$
3	$Ca_2YZr_2Fe_3O_{12}$ (0.751)	$R^{-3^{1}_{001}}\text{-}3^{2^{\frac{1}{6}\pi}}\mathrm{c}$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	$\times$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	$\times$
4	$Ca_2LaZr_2Fe_3O_{12}$ (0.753)	$R^{-3^{1}_{001}}\text{-}3^{2^{\frac{1}{6}\pi}}\mathrm{c}$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	$\times$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	$\times$
5	$Co_3Al_2Si_3O_{12}$ (0.388)	$I^{4^{1}_{001}}4_1/^{\mathrm{m}_{001}}\mathrm{a}^{2_{100}}\mathrm{c}^{2_{110}}\mathrm{d}$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$	$\times$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$	$\times$
6	$MgCr_2O_4$ (3.4)	$I^{2_{\text{-}110}}\text{-}4^{\mathrm{m}_{\text{-}110}}\mathrm{m}^{\text{-}1}2\|(2_{001}, 2_{001}, 1; 2_{100})$	$\sigma_{xxz} = \sigma_{xzx} = -\sigma_{yyz} = -\sigma_{yzy} = \sigma_{zxx} = -\sigma_{zyy}$	$\sigma_{zxx} = -\sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{yyz} = -\sigma_{yzy}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	$\sigma_{xxz} = \sigma_{xzx} = -\sigma_{yyz} = -\sigma_{yzy} = \sigma_{zxx} = -\sigma_{zyy}$	$\sigma_{zxx} = -\sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{yyz} = -\sigma_{yzy}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$

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Table S3 : (continued) Noncoplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC			with SOC		
			IMD	QMD	BCD	IMD	QMD	BCD
7	$Gd_2Ti_2O_7$ (3.16)	$P^{4^{3}_{100}}\text{-}4^{3^{1}_{-11-1}}3^{2_{1-10}}\mathrm{m}\vert(-1,-1,-1)$	×	×	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	×	×	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$
8	$ZnFe_2O_4$ (1.759)	$P^{4^{1}_{001}}\text{-}4^{\mathrm{m}_{100}}\mathrm{n}^{\mathrm{m}_{110}}2\vert(-1,-1,-1)$	×	×	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×	×	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$
9	Mn (1.85)	$I^{4^{1}_{001}}\text{-}4^{\mathrm{m}_{100}}2^{\mathrm{m}_{110}}\mathrm{m}\vert(1,1,1;-1)$	×	×	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×	×	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$
10	$BaCuTe_2O_6$ (0.658)	$P^{-4^{3}_{010}}4_1{}^{3^{2}_{-11-1}}3^{\mathrm{m}_{1-10}}2$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	×	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	×
11	Mn_3RhGe (0.1005)	$P^{2_{100}}2_1{}^{3^{1}_{111}}3$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	×	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	×
12	Mn_3IrGe (0.1006)	$P^{2_{100}}2_1{}^{3^{1}_{111}}3$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	×	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	×
13	Mn_3IrSi (0.898)	$P^{2_{100}}2_1{}^{3^{1}_{111}}3$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	×	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	×
14	Mn_3IrGe (0.899)	$P^{2_{100}}2_1{}^{3^{1}_{111}}3$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	×	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	×
15	Mn_3CoGe (0.900)	$P^{2_{100}}2_1{}^{3^{1}_{111}}3$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	×	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	×
16	CrSe (2.35)	$P^{2_{010}}6_3/^{-1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{m}^{-1}\mathrm{c}\vert(3^{2}_{001}, 3^{2}_{001}, 1)$	×	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×	$\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$

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Table S3 : (continued) Noncoplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC			with SOC		
			IMD	QMD	BCD	IMD	QMD	BCD
17	Mn_5Si_3 (1.307)	$P^{1}\mathrm{m}^{1}\mathrm{c}^{1}2_1{}^{\text{-}1}(0\ \ 1/2\ \ 1/2)$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$, $\sigma_{xyz} = \sigma_{xzy}$
18	CaCoSO (1.595)	$P^{1}3^{1}1^{2_{001}}\mathrm{m}\vert(1,1,\text{-}1)$	×	×	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
19	$Co_6(OH)_3(TeO_3)_4$ $(OH)_{0.9}(H_{20})$ (0.381)	$P^{\text{-}3^{2}_{001}}6_3{}^{2_{\frac{2}{3}\pi}}\mathrm{m}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{c}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
20	$HoMnO_3$ (0.33)	$P^{6^{1}_{001}}6_3{}^{2_{\frac{2}{3}\pi}}\mathrm{c}^{2_{\frac{5}{6}\pi}}\mathrm{m}$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$

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Table S3 : (continued) Noncoplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC			with SOC		
			IMD	QMD	BCD	IMD	QMD	BCD
21	$HoMnO_3$ (0.43)	$P^{\text{-}3^{2}_{001}}6_3{}^{2_{\frac{2}{3}\pi}}\mathrm{c}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{m}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
22	$HoMn_{0.95}Fe_{0.05}O_3$ (0.654)	$P^{\text{-}3^{2}_{001}}6_3{}^{2_{\frac{2}{3}\pi}}\mathrm{c}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{m}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
23	$HoMnO_3$ (0.652)	$P^{\text{-}3^{2}_{001}}6_3{}^{2_{\frac{2}{3}\pi}}\mathrm{c}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{m}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
24	$HoMn_{0.99}Fe_{0.01}O_3$ (0.653)	$P^{\text{-}3^{2}_{001}}6_3{}^{2_{\frac{2}{3}\pi}}\mathrm{c}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{m}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
25	$HoMn_{0.9}Fe_{0.1}O_3$ (0.655)	$P^{\text{-}3^{2}_{001}}6_3{}^{2_{\frac{2}{3}\pi}}\mathrm{c}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{m}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
26	$HoMnO_3$ (0.42)	$P^{\text{-}3^{2}_{001}}6_3{}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{c}^{2_{100}}\mathrm{m}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
27	$YbMnO_3$ (0.488)	$P^{\text{-}3^{2}_{001}}6_3{}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{c}^{2_{100}}\mathrm{m}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
28	$YbMnO_3$ (0.489)	$P^{\text{-}3^{2}_{001}}6_3{}^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{c}^{2_{100}}\mathrm{m}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$

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Table S3 : (continued) Noncoplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC			with SOC		
			IMD	QMD	BCD	IMD	QMD	BCD
29	$Tb_{14}Ag_{51}$ (1.0.52)	$P^{-6^{5}_{001}}{-6}$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$	×	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$	×
30	$La_{0.33}Sr_{0.67}FeO_3$ (1.0.15)	$R^{1}3^{2_{100}}2\vert(1,1,1;3^{1}_{001},3^{2}_{001})$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$
31	$PrFe_3(BO_3)_4$ (1.161)	$R^{3^{1}_{001}}3^{m_{\frac{1}{6}\pi}}2\vert(1,1,\text{-}1;\text{-}1,1)$	×	×	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×	×	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$
32	$DyFe_3(BO_3)_4$ (1.89)	$P^{-3^{1}_{001}}3_1{}^{m_{\frac{1}{6}\pi}}2^{1}1\vert(1,1,\text{-}1)$	×	×	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×	×	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$
33	$HoFe_3(BO_3)_4$ (1.92)	$P^{-3^{1}_{001}}3_1{}^{m_{\frac{1}{6}\pi}}2^{1}1\vert(1,1,\text{-}1)$	×	×	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×	×	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$
34	Yb_3Pt_4 (0.430)	$R^{-3^{1}_{001}}{-3}$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	×	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	×

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Table S3 : (continued) Noncoplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC			with SOC		
			IMD	QMD	BCD	IMD	QMD	BCD
35	TbSbTe (2.102)	$P^{m_{001}}2\|(-1,1,1)$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
36	TbSbTe (2.101)	$P^{m_{001}}2\|(-1,1,1)$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
37	CuB_2O_4 (0.431)	C^{-1}c	$\sigma_{xxy} = \sigma_{xyx} = \sigma_{xyy} = \sigma_{yxx} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxx} = \sigma_{yyy}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{yyz} = -\sigma_{yzy} = \sigma_{zxx} = -\sigma_{zyy}$, $\sigma_{xzz} = \sigma_{yzz} = \sigma_{zxz} = \sigma_{zyz} = \sigma_{zzx} = \sigma_{zzy}$	$\sigma_{xyy} = \sigma_{yxx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxx} = \sigma_{yyy}$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{yyz} = -\sigma_{yzy}$, $\sigma_{xzz} = \sigma_{yzz}$, $\sigma_{zxx} = -\sigma_{zyy}$, $\sigma_{zxz} = \sigma_{zyz} = \sigma_{zzx} = \sigma_{zzy}$	$\sigma_{xxy} = \sigma_{xyx} = \sigma_{xyy}/2 = -\sigma_{yxx}/2 = -\sigma_{yxy} = -\sigma_{yyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$, $\sigma_{xzz} = -\sigma_{yzz} = -2\sigma_{zxz} = 2\sigma_{zyz} = -2\sigma_{zzx} = 2\sigma_{zzy}$, $\sigma_{zxy} = \sigma_{zyx}$	σ_{xxx}, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, σ_{yyy}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, σ_{zzz}	σ_{xxx}, $\sigma_{xxy} = \sigma_{xyx}$, $\sigma_{xxz} = \sigma_{xzx}$, σ_{xyy}, $\sigma_{xyz} = \sigma_{xzy}$, σ_{xzz}, σ_{yxx}, $\sigma_{yxy} = \sigma_{yyx}$, $\sigma_{yxz} = \sigma_{yzx}$, σ_{yyy}, $\sigma_{yyz} = \sigma_{yzy}$, σ_{yzz}, σ_{zxx}, $\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{zyy}, $\sigma_{zyz} = \sigma_{zzy}$, σ_{zzz}	$\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{zxy} = \sigma_{zyx}$
38	MgV_2O_4 (1.138)	$F^{m_{100}}2^{2_{001}}2^{m_{010}}2\|(1,1,1;-1,-1,1)$	×	×	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×	×	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$

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Table S3 : (continued) Noncoplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC			with SOC		
			IMD	QMD	BCD	IMD	QMD	BCD
39	$Er_2Ge_2O_7$ (0.419)	$P^{-4^3_{001}}4_1{}^{2_{110}}2_1{}^{m_{100}}2$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$
40	$Er_2Ge_2O_7$ (0.942)	$P^{-4^3_{001}}4_1{}^{2_{110}}2_1{}^{m_{100}}2$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$
41	$Ba(TiO)Cu_4(PO_4)_4$ (1.235)	$P^{-4^3_{001}}4^{m_{100}}2_1{}^{2_{-110}}2\vert(1,1,\text{-}1)$	×	×	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×	×	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$
42	$Dy_2Co_3Al_9$ (1.267)	$A^{m_{100}}m^{2_{001}}m^{m_{010}}2\vert(\text{-}1,1,1;1)$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
43	Er_2ReC_2 (0.347)	$P^{2_{001}}n^{1}m^{m_{001}}a$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×	$\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz} = \sigma_{zzy}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy} = \sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy} = \sigma_{xyx}$, σ_{xxx}	×
44	Tb_5Ge_4 (0.141)	$P^{2_{010}}n^{m_{010}}m^{2_{100}}a$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
45	Tb_5Ge_4 (0.412)	$P^{2_{010}}n^{m_{010}}m^{2_{100}}a$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
46	Tb_5Ge_4 (0.411)	$P^{2_{010}}n^{m_{010}}m^{2_{100}}a$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×

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Table S3 : (continued) Noncoplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC			with SOC		
			IMD	QMD	BCD	IMD	QMD	BCD
47	$TbCrO_3$ (2.62)	$P^{m_{010}}m^{m_{001}}n^{2_{100}}2_1\vert(1,2_{010},1)$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
48	$CoGeO_3$ (0.311)	$P^{2_{100}}b^{m_{001}}c^{2_{010}}a$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
49	$Fe_2Se_2O_7$ (0.807)	$P^{2_{100}}c^{m_{001}}c^{2_{010}}n$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
50	$Fe_2Se_2O_7$ (0.808)	$P^{2_{100}}c^{m_{001}}c^{2_{010}}n$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
51	$Fe_2Se_2O_7$ (0.806)	$P^{2_{100}}c^{m_{001}}c^{2_{010}}n$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
52	$Sr_2Fe_3Se_2O_3$ (2.55)	$C^{1}m^{-1}(0\ \ 0\ \ 1/2)^{m_{001}}(1/2\ \ 0\ \ 0)$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$

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Table S3 : (continued) Noncoplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC			with SOC		
			IMD	QMD	BCD	IMD	QMD	BCD
53	$Sr_2Fe_3Se_2O_3$ (2.76)	$C^1m^{-1}(0\ \ 0\ \ 1/2)^{m_{001}}(1/2\ \ 0\ \ 0)$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
54	$BiMn_2O_5$ (1.75)	$P^{m_{001}}m\vert(-1, 1, 1)$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$
55	$NdCrTiO_5$ (0.162)	$P^{2_{001}}b^{2_{010}}a^{m_{100}}m$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
56	$GdMn_2O_5$ (1.299)	$P^{m_{100}}m^{2_{001}}c^{m_{010}}2_1\vert(1, -1, 1)$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
57	$GdMn_2O_5$ (1.300)	$P^{m_{100}}m^{2_{001}}c^{m_{010}}2_1\vert(1, -1, 1)$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
58	Tb_3Ge_5 (0.342)	$F^{2_{100}}d^{2_{010}}d^{2_{001}}2$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxz} = \sigma_{xzx}$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxz} = \sigma_{xzx}$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$

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Table S3 : (continued) Noncoplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC			with SOC		
			IMD	QMD	BCD	IMD	QMD	BCD
59	$Ho_2Cu_2O_5$ (1.279)	$P^{\mathrm{m}_{001}}2_1\|(\text{-}1,1,1)$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
60	$DyFeWO_6$ (1.274)	$P^{\mathrm{m}_{001}}\mathrm{c}\|(1,\text{-}1,1)$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$	×	×	$\sigma_{yzz}=-2\sigma_{zyz}=-2\sigma_{zzy}$, $\sigma_{xzz}=-2\sigma_{zxz}=-2\sigma_{zzx}$, $\sigma_{xyy}=-2\sigma_{yxy}=-2\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=-\sigma_{yxx}/2$
61	$DyCrWO_6$ (0.316)	$P^{\text{-}1}\mathrm{n}^{\mathrm{m}_{001}}\mathrm{a}^{2_{001}}2_1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
62	$Er_2Cu_2O_5$ (0.240)	$P^{2_{100}}\mathrm{n}^{2_{010}}\mathrm{a}^{2_{001}}2_1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
63	$HoCrWO_6$ (0.715)	$P^{2_{100}}\mathrm{n}^{2_{010}}\mathrm{a}^{2_{001}}2_1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$

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Table S3 : (continued) Noncoplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC			with SOC		
			IMD	QMD	BCD	IMD	QMD	BCD
64	Pb_2MnWO_6 (2.38)	$P^{1}m^{2_{001}}c^{2_{001}}2_1\|(2_{010},1,1)$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
65	Co_3TeO_6 (0.145)	$C^{m_{001}}2/^{2_{001}}c$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
66	$Co_2V_2O_7$ (0.281)	$P^{2_{001}}2_1/^{m_{001}}c$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×
67	$Cu_2CdB_2O_6$ (0.394)	$P^{2_{001}}2_1/^{m_{001}}c$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×
68	Fe_2WO_6 (0.809)	$P^{2_{001}}2_1/^{m_{001}}c$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×

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Table S3 : (continued) Noncoplanar antiferromagnets in MAGNDATA with second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Spin space group	without SOC			with SOC		
			IMD	QMD	BCD	IMD	QMD	BCD
69	$BaFe_2Se_3$ (1.710)	$P^{\mathrm{m}_{001}}\mathrm{m}\|(\text{-}1,1,1)$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$	×	×	$\sigma_{yzz} = -2\sigma_{zyz} = -2\sigma_{zzy}$, $\sigma_{xzz} = -2\sigma_{zxz} = -2\sigma_{zzx}$, $\sigma_{xyy} = -2\sigma_{yxy} = -2\sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = -\sigma_{yxx}/2$

Table S4: Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
1	$NpCo_2$ (0.126)	Collinear	$F^{-1}d^{-1}\text{-}3^{1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
2	$MnAl_2O_4$ (0.462)	Collinear	$F^{-1}d^{-1}\text{-}3^{1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
3	$CoRh_2O_4$ (0.461)	Collinear	$F^{-1}d^{-1}\text{-}3^{1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
4	Co_3O_4 (0.463)	Collinear	$F^{-1}d^{-1}\text{-}3^{1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
5	$CsFeO_2$ (0.458)	Collinear	$F^{-1}d^{-1}\text{-}3^{1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
6	$RbFeO_2$ (0.456)	Collinear	$F^{-1}d^{-1}\text{-}3^{1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
7	$CoAl_2O_4$ (0.58)	Collinear	$F^{-1}d^{-1}\text{-}3^{1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
8	Bi_2RuMnO_7 (0.153)	Collinear	$I^{-1}4_1/^{1}a^{1}m^{-1}d^{\infty m}1$	×	×	×
9	CoO (1.69)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
10	$Mn_3Ni_{20}P_6$ (1.145)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
11	UP (1.160)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
12	UAs (1.208)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
13	UN (1.428)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
14	Ba_2YRuO_6 (1.433)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
15	Ba_2LuRuO_6 (1.432)	Collinear	$P^14/^1m^1m^1m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
16	Ba_2TmRuO_6 (1.567)	Collinear	$P^14/^1m^1m^1m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
17	Ba_2YbRuO_6 (1.566)	Collinear	$P^14/^1m^1m^1m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
18	Ba_2MnTeO_6 (1.706)	Collinear	$P^14/^1m^1m^1m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
19	$Mn_6Ni_{16}Si_7$ (1.454)	Collinear	$P^14_2/^1m^1n^1m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
20	CrN (1.28)	Collinear	$P^1m^1m^1n^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
21	CrN (1.678)	Collinear	$P^1m^1m^1n^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
22	MnO (1.31)	Collinear	$R^1\text{-}3^1m^{-1}(1/3\ \ 2/3\ \ 1/6)^{\infty m}1$	×	×	×
23	CoO (1.618)	Collinear	$R^1\text{-}3^1m^{-1}(1/3\ \ 2/3\ \ 1/6)^{\infty m}1$	×	×	×
24	NiO (1.6)	Collinear	$R^1\text{-}3^1m^{-1}(1/3\ \ 2/3\ \ 1/6)^{\infty m}1$	×	×	×
25	Ba_2MnWO_6 (1.707)	Collinear	$R^1\text{-}3^1m^{-1}(1/3\ \ 2/3\ \ 1/6)^{\infty m}1$	×	×	×
26	HoBi (1.753)	Collinear	$R^1\text{-}3^1m^{-1}(1/3\ \ 2/3\ \ 1/6)^{\infty m}1$	×	×	×
27	$KNiF_3$ (1.250)	Collinear	$F^1m^1\text{-}3^1m^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
28	$Pb_{0.8}Bi_{0.2}Fe_{0.728}W_{0.264}O_3$ (1.590)	Collinear	$F^1m^1\text{-}3^1m^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
29	$Pb_{0.7}Bi_{0.3}Fe_{0.762}W_{0.231}O_3$ (1.591)	Collinear	$F^1m^1\text{-}3^1m^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
30	$SrFeO_2F$ (1.84)	Collinear	$F^1m^1\text{-}3^1m^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
31	NdMg (1.162)	Collinear	$P^14/^1m^1m^1m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
32	UPb_3 (1.423)	Collinear	$P^14/^1m^1m^1m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
33	$LaMn_3Cr_4O_{12}$ (1.156)	Collinear	$P^12^13^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
34	$CaCo_3V_4O_{12}$ (2.106)	Collinear	$P^{1}\mathrm{m}^{1}\mathrm{m}^{-1}\mathrm{n}^{\infty\mathrm{m}}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×
35	UNiGa (2.88)	Collinear	$P^{-1}\text{-}6^{-1}2^{1}\mathrm{m}^{\infty\mathrm{m}}1$	×	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×
36	$CeRh_{0.25}Pd_{0.75}Sn$ (1.737)	Collinear	$P^{1}\text{-}6^{1}2^{1}\mathrm{m}^{-1}(0\ \ 0\ \ 1/2)^{\infty\mathrm{m}}1$	×	×	×
37	$Ba_6Co_6ClO_{15.5}$ (1.275)	Collinear	$P^{1}\text{-}6^{1}\mathrm{m}^{1}2^{-1}(0\ \ 0\ \ 1/2)^{\infty\mathrm{m}}1$	×	×	×
38	$PbMn_2Ni_6Te_3O_{18}$ (0.1001)	Collinear	$P^{1}6_3/^{-1}\mathrm{m}^{\infty\mathrm{m}}1$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$	×
39	$K_2Mn_3(VO_4)_2CO_3$ (1.0.21)	Collinear	$P^{-1}6_3/^{1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	×
40	$U_{14}Au_{51}$ (0.282)	Collinear	$P^{-1}6/^{1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx} = -\sigma_{yyy}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	×
41	Cr_2O_3 (0.110)	Collinear	$R^{-1}\text{-}3^{1}\mathrm{c}^{\infty\mathrm{m}}1$	$\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz} = \sigma_{zzy}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy} = \sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy} = \sigma_{xyx}$, σ_{xxx}	×
42	Cr_2O_3 (0.59)	Collinear	$R^{-1}\text{-}3^{1}\mathrm{c}^{\infty\mathrm{m}}1$	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
43	$AgRuO_3$ (0.733)	Collinear	R^{-1}-3^{1}c$^{\infty m}$1	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	×
44	MoP_3SiO_{11} (0.728)	Collinear	R^{-1}-3^{1}c$^{\infty m}$1	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×
45	MoP_3SiO_{11} (0.804)	Collinear	R^{-1}-3^{1}c$^{\infty m}$1	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×
46	$CuFeO_2$ (1.348)	Collinear	$C^{1}2/^{1}$m^{-1}(0 0 1/2)$^{\infty m}$1	×	×	×
47	$KCeS_2$ (1.627)	Collinear	$C^{1}2/^{1}$m^{-1}(0 0 1/2)$^{\infty m}$1	×	×	×
48	$CoCl_2$ (1.246)	Collinear	R^{1}-3^{1}m^{-1}(1/3 2/3 1/6)$^{\infty m}$1	×	×	×
49	$NiBr_2$ (1.248)	Collinear	R^{1}-3^{1}m^{-1}(1/3 2/3 1/6)$^{\infty m}$1	×	×	×
50	$NiCl_2$ (1.247)	Collinear	R^{1}-3^{1}m^{-1}(1/3 2/3 1/6)$^{\infty m}$1	×	×	×
51	$Fe_2Co_2Nb_2O_9$ (0.770)	Collinear	$C^{-1}2/^{1}$c$^{\infty m}$1	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
52	$Co_4Nb_2O_9$ (0.111)	Collinear	P^{-1}-3^{1}c^{1}1$^{\infty m}$1	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	×
53	$Fe_4Nb_2O_9$ (0.443)	Collinear	P^{-1}-3^{1}c^{1}1$^{\infty m}$1	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×
54	$Mn_4Ta_2O_9$ (0.477)	Collinear	P^{-1}-3^{1}c^{1}1$^{\infty m}$1	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	×
55	$Mn_4Nb_2O_9$ (0.507)	Collinear	P^{-1}-3^{1}c^{1}1$^{\infty m}$1	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	×
56	$Mn_4Ta_2O_9$ (0.526)	Collinear	P^{-1}-3^{1}c^{1}1$^{\infty m}$1	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	×
57	FeI_2 (1.209)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
58	Dy_2O_2S (1.211)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
59	Dy_2O_2Se (1.212)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
60	Pu_2O_3 (1.367)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
61	Tb_2O_2Se (1.417)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
62	Tb_2O_2S (1.416)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
63	$SrMn_2As_2$ (0.482)	Collinear	P^{-1}-3^1m^1$1^{\infty m}$1	$\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz} = \sigma_{zzy}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy} = \sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy} = \sigma_{xyx}$, σ_{xxx}	×
64	$YbMn_2Sb_2$ (0.483)	Collinear	P^{-1}-3^1m^1$1^{\infty m}$1	σ_{zzz}, $\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$, σ_{yyy}, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, σ_{xxx}	σ_{zzz}, $\sigma_{zyz} = \sigma_{zzy}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, σ_{yzz}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, σ_{yyy}, $\sigma_{yxy} = \sigma_{yyx}$, σ_{yxx}, σ_{xzz}, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$, σ_{xyy}, $\sigma_{xxy} = \sigma_{xyx}$, σ_{xxx}	×
65	U_2N_2S (0.484)	Collinear	P^{-1}-3^1m^1$1^{\infty m}$1	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	×
66	U_2N_2Se (0.485)	Collinear	P^{-1}-3^1m^1$1^{\infty m}$1	$\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
67	$CaMn_2Sb_2$ (0.523)	Collinear	$P^{-1}\text{-}3^{1}\mathrm{m}^{1}1^{\infty \mathrm{m}}1$	σ_{zzz}, $\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$, σ_{yyy}, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, σ_{xxx}	σ_{zzz}, $\sigma_{zyz} = \sigma_{zzy}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, σ_{yzz}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, σ_{yyy}, $\sigma_{yxy} = \sigma_{yyx}$, σ_{yxx}, σ_{xzz}, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$, σ_{xyy}, $\sigma_{xxy} = \sigma_{xyx}$, σ_{xxx}	×
68	$CaMn_2Sb_2$ (0.92)	Collinear	$P^{-1}\text{-}3^{1}\mathrm{m}^{1}1^{\infty \mathrm{m}}1$	$\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz} = \sigma_{zzy}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy} = \sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy} = \sigma_{xyx}$, σ_{xxx}	×
69	Yb_2O_2Se (1.214)	Collinear	$P^{1}\text{-}3^{1}\mathrm{m}^{1}1^{-1}(0\ \ 0\ \ 1/2)^{\infty \mathrm{m}}1$	×	×	×
70	$CoBr_2$ (1.245)	Collinear	$P^{1}\text{-}3^{1}\mathrm{m}^{1}1^{-1}(0\ \ 0\ \ 1/2)^{\infty \mathrm{m}}1$	×	×	×
71	Ag_2CrO_2 (1.0.1)	Collinear	$C^{1}2/^{\mathrm{m}_{001}}\mathrm{m}^{\infty \mathrm{m}}1$	$\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz} = \sigma_{zzy}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy} = \sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy} = \sigma_{xyx}$, σ_{xxx}	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
72	$MnGeO_3$ (0.125)	Collinear	R^{-1}-$3^{\infty m}1$	σ_{zzz}, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}=-\sigma_{yyy}$, $\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	σ_{zzz}, $\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}=-\sigma_{yyy}$, $\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	×
73	$MnPSe_3$ (0.180)	Collinear	R^{-1}-$3^{\infty m}1$	σ_{zzz}, $\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	σ_{zzz}, $\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, σ_{yzz}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
74	$MnTiO_3$ (0.19)	Collinear	R^{-1}-$3^{\infty m}1$	σ_{zzz}, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}=-\sigma_{yyy}$, $\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	σ_{zzz}, $\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}=-\sigma_{yyy}$, $\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
75	$BaNi_2P_2O_8$ (0.215)	Collinear	R^{-1}-$3^{\infty m}1$	σ_{zzz}, $\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	σ_{zzz}, $\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, σ_{yzz}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
76	$MgMnO_3$ (0.277)	Collinear	R^{-1}-$3^{\infty m}1$	σ_{zzz}, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}=-\sigma_{yyy}$, $\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	σ_{zzz}, $\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}=-\sigma_{yyy}$, $\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
77	$MnPSe_3$ (0.524)	Collinear	R^{-1}-$3^{\infty \mathrm{m}}1$	σ_{zzz}, $\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$, σ_{yyy}, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, σ_{xxx}	σ_{zzz}, $\sigma_{zyz} = \sigma_{zzy}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, σ_{yzz}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, σ_{yyy}, $\sigma_{yxy} = \sigma_{yyx}$, σ_{yxx}, σ_{xzz}, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$, σ_{xyy}, $\sigma_{xxy} = \sigma_{xyx}$, σ_{xxx}	×
78	$FeTiO_3$ (1.581)	Collinear	R^{1}-$3^{-1}(1/3\ \ 2/3\ \ 1/6)^{\infty \mathrm{m}}1$	×	×	×
79	Ca_2MnO_4 (0.211)	Collinear	$I^{-1}4_1/^{-1}\mathrm{a}^{-1}\mathrm{c}^{1}\mathrm{d}^{\infty \mathrm{m}}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×
80	$GdVO_4$ (0.198)	Collinear	$I^{-1}4_1/^{-1}\mathrm{a}^{1}\mathrm{m}^{-1}\mathrm{d}^{\infty \mathrm{m}}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×
81	$TbPO_4$ (0.467)	Collinear	$I^{-1}4_1/^{-1}\mathrm{a}^{1}\mathrm{m}^{-1}\mathrm{d}^{\infty \mathrm{m}}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×
82	$NaCeO_2$ (0.525)	Collinear	$I^{-1}4_1/^{-1}\mathrm{a}^{1}\mathrm{m}^{-1}\mathrm{d}^{\infty \mathrm{m}}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×
83	$MnSn_2$ (1.558)	Collinear	$C^{1}\mathrm{m}^{1}\mathrm{m}^{1}\mathrm{e}^{-1}(0\ \ 1/2\ \ 1/2)^{\infty \mathrm{m}}1$	×	×	×
84	$EuTiO_3$ (0.16)	Collinear	$I^{-1}4/^{-1}\mathrm{m}^{-1}\mathrm{c}^{1}\mathrm{m}^{\infty \mathrm{m}}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
85	$NdNi_2B_2C$ (1.293)	Collinear	$C^{1}2/^{1}\mathrm{m}^{-1}(0\ \ 0\ \ 1/2)^{\infty \mathrm{m}}1$	×	×	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
86	$BaFe_2As_2$ (1.16)	Collinear	$C^1c^1c^1m^{-1}(1/2\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
87	$KCuMnS_2$ (1.392)	Collinear	$C^1c^1c^1m^{-1}(1/2\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
88	$CaFe_2As_2$ (1.52)	Collinear	$C^1c^1c^1m^{-1}(1/2\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
89	CeMgPb (1.142)	Collinear	$C^1m^1c^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
90	Gd_2CuO_4 (1.104)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
91	Pr_2CuO_4 (1.106)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
92	Sm_2CuO_4 (1.107)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
93	$CeRh_2Si_2$ (1.188)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
94	K_2NiF_4 (1.249)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
95	$CeRh_2Si_2$ (1.290)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
96	$CePd_2Si_2$ (1.288)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
97	$CePd_2Ge_2$ (1.289)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
98	$LaSrFeO_4$ (1.29)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
99	HFe_2Ge_2 (1.369)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
100	Pr_2CuO_4 (1.399)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
101	Pr_2CuO_4 (1.398)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
102	$NdCeBaCu_{0.9}Co_{1.1}O_7$ (1.396)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
103	$TbAg_2$ (1.400)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
104	$NdCeBaCuFeO_7$ (1.395)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
105	$Sr_2CuO_2Cl_2$ (1.404)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
106	Nd_2CuO_4 (1.408)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
107	Nd_2CuO_4 (1.407)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
108	$SrNdFeO_4$ (1.40)	Collinear	$C^1m^1m^1m^{-1}(0\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
109	Nd_2CuO_4 (1.406)	Collinear	$C^1\mathrm{m}^1\mathrm{m}^{-1}\mathrm{m}^{-1}(0\ \ 1/2\ \ 1/2)^{\infty\mathrm{m}}1$	$\times$	$\times$	$\times$
110	$SrNdFeO_4$ (1.41)	Collinear	$C^1\mathrm{m}^1\mathrm{m}^{-1}\mathrm{m}^{-1}(0\ \ 1/2\ \ 1/2)^{\infty\mathrm{m}}1$	$\times$	$\times$	$\times$
111	$TbNi_2Si_2$ (1.511)	Collinear	$C^1\mathrm{m}^1\mathrm{m}^{-1}\mathrm{m}^{-1}(0\ \ 1/2\ \ 1/2)^{\infty\mathrm{m}}1$	$\times$	$\times$	$\times$
112	UGeSe (0.413)	Collinear	$I^14/^{-1}\mathrm{m}^1\mathrm{m}^1\mathrm{m}^{\infty\mathrm{m}}1$	$\times$	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	$\times$
113	Mn_2Au (0.639)	Collinear	$I^14/^{-1}\mathrm{m}^1\mathrm{m}^1\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\times$
114	Mn_2Au (0.640)	Collinear	$I^14/^{-1}\mathrm{m}^1\mathrm{m}^1\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\times$
115	$BaMn_2As_2$ (0.18)	Collinear	$I^{-1}4/^{-1}\mathrm{m}^1\mathrm{m}^{-1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	$\times$
116	$Sr_2Mn_3As_2O_2$ (0.212)	Collinear	$I^{-1}4/^{-1}\mathrm{m}^1\mathrm{m}^{-1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	$\times$
117	$BaCrFeAs_2$ (0.366)	Collinear	$I^{-1}4/^{-1}\mathrm{m}^1\mathrm{m}^{-1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	$\times$
118	$SrCr_2As_2$ (0.364)	Collinear	$I^{-1}4/^{-1}\mathrm{m}^1\mathrm{m}^{-1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	$\times$
119	$BaCr_2As_2$ (0.365)	Collinear	$I^{-1}4/^{-1}\mathrm{m}^1\mathrm{m}^{-1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	$\times$
120	$EuMnBi_2$ (0.426)	Collinear	$I^{-1}4/^{-1}\mathrm{m}^1\mathrm{m}^{-1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	$\times$

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
121	$BaMn_2P_2$ (0.464)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×
122	$ThCr_2Si_2$ (0.466)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
123	$HoCr_2Si_2$ (0.465)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×
124	$BaMn_2Sb_2$ (0.470)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×
125	$EuMn_2Ge_2$ (0.474)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×
126	$LaMn_2Si_2$ (0.472)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×
127	$Ba_2Mn_3Sb_2O_2$ (0.471)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×
128	$ErCr_2Si_2$ (0.486)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×
129	$LaMn_2Si_2$ (0.498)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×
130	$TbCr_2Si_2$ (0.518)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
131	$HoCr_2Si_2$ (0.519)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
132	$CaMn_2Ge_2$ (0.603)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
133	$CaMn_2Ge_2$ (0.604)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
134	$BaMn_2Ge_2$ (0.605)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
135	$BaMn_2Ge_2$ (0.606)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
136	$BaMnSb_2$ (0.611)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
137	$SrMnBi_2$ (0.73)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
138	$BaMn_2Bi_2$ (0.89)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
139	$EuMnBi_2$ (0.919)	Collinear	$I^{-1}4/^{-1}m^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
140	$TbNi_2Ge_2$ (1.510)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
141	PrScSb (0.454)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
142	$TbRh_2Si_2$ (1.187)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×

Continued on next page

Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
143	$DyCo_2Si_2$ (1.21)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
144	$CeAu_2Si_2$ (1.291)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
145	$HoNi_2B_2C$ (1.292)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
146	$HoNi_2B_2C$ (1.294)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
147	$DyNi_2B_2C$ (1.295)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
148	$PrNi_2B_2C$ (1.296)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
149	$HoNi_2B_2C$ (1.312)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
150	$NdCo_2P_2$ (1.251)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
151	$Sr_2MnO_2Ag_{1.5}Se_2$ (1.372)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
152	$La_{0.25}Pr_{0.75}Co_2P_2$ (1.316)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
153	$NdRh_2Si_2$ (1.421)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
154	$HoCo_2Ge_2$ (1.427)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
155	URu_2Si_2 (1.442)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
156	$TbCo_2Si_2$ (1.512)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
157	$HoCo_2Si_2$ (1.513)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
158	$HoCo_2Si_2$ (1.514)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
159	CeC_2 (1.530)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
160	PrC_2 (1.531)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
161	NdC_2 (1.532)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
162	UPd_2Si_2 (1.536)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
163	URh_2Si_2 (1.537)	Collinear	$P^{1}4/^{1}m^{1}m^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
164	$ErFe_2Si_2$ (1.635)	Collinear	$P^{1}4/^{1}n^{1}m^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
165	$EuMnBi_2$ (2.50)	Collinear	$P^{-1}4_2/^{-1}m^{1}m^{-1}c^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×
166	$EuMnBi_2$ (2.98)	Collinear	$P^{-1}4_2/^{-1}m^{1}m^{-1}c^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx}$	×
167	$CeRh_2Si_2$ (2.30)	Collinear	$P^{1}m^{1}m^{1}a^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	×	×	×
168	$NdPd_5Al_2$ (1.507)	Collinear	$P^{1}m^{1}m^{1}n^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
169	Cr_2TeO_6 (0.143)	Collinear	$P^{1}4_2/^{-1}m^{1}n^{1}m^{\infty m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
170	Fe_2TeO_6 (0.142)	Collinear	$P^{1}4_2/^{-1}m^{1}n^{1}m^{\infty m}1$	×	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×
171	Cr_2TeO_6 (0.76)	Collinear	$P^{1}4_2/^{-1}m^{1}n^{1}m^{\infty m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
172	Cr_2TeO_6 (0.959)	Collinear	$P^{1}4_2/^{-1}m^{1}n^{1}m^{\infty m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
173	Fe_2TeO_6 (0.960)	Collinear	$P^{1}4_2/^{-1}m^{1}n^{1}m^{\infty m}1$	×	$\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×
174	$K_2CoP_2O_7$ (0.230)	Collinear	$P^{-1}4_2/^{-1}m^{1}n^{-1}m^{\infty m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
175	Cr_2WO_6 (0.144)	Collinear	$P^{-1}4_2/^{-1}m^{-1}n^{1}m^{\infty m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
176	Cr_2WO_6 (0.75)	Collinear	$P^{-1}4_2/^{-1}m^{-1}n^{1}m^{\infty m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×

Continued on next page

Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
177	V_2WO_6 (0.966)	Collinear	$P^{-1}4_2/^{-1}m^{-1}n^{1}m^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
178	Ce_2PdGe_3 (0.166)	Collinear	$P^{-1}4_2/^{-1}m^{1}m^{-1}c^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
179	Bi_2CuO_4 (0.348)	Collinear	$P^{1}4/^{-1}n^{1}c^{1}c^{\infty m}1$	×	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	×
180	Bi_2CuO_4 (0.694)	Collinear	$P^{1}4/^{-1}n^{1}c^{1}c^{\infty m}1$	×	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	×
181	Bi_2CuO_4 (0.695)	Collinear	$P^{1}4/^{-1}n^{1}c^{1}c^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
182	UPt_2Si_2 (0.194)	Collinear	$P^{1}4/^{-1}n^{1}m^{1}m^{\infty m}1$	×	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	×
183	CuMnAs (0.222)	Collinear	$P^{1}4/^{-1}n^{1}m^{1}m^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
184	UBi_2 (0.378)	Collinear	$P^{1}4/^{-1}n^{1}m^{1}m^{\infty m}1$	×	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	×
185	TbRuAsO (0.452)	Collinear	$P^{1}4/^{-1}n^{1}m^{1}m^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
186	CuMnAs (0.881)	Collinear	$P^{1}4/^{-1}n^{1}m^{1}m^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
187	CeMnAsO (0.186)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
188	$YbMnBi_2$ (0.267)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
189	CaMnSi (0.599)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
190	CaMnGe (0.601)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×
191	CaMnSi (0.600)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
192	CaMnGe (0.602)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×
193	NdMnAsO (0.623)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
194	KMnBi (0.618)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
195	KMnSb (0.617)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
196	LaMnAsO (0.624)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
197	LaMnAsO (0.619)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
198	NdMnAsO (0.620)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
199	NaMnP (0.628)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
200	NaMnP (0.626)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
201	NaMnAs (0.630)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
202	NaMnP (0.627)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
203	NaMnSb (0.631)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
204	NaMnSb (0.632)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
205	NaMnAs (0.629)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
206	NaMnBi (0.634)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
207	NaMnBi (0.635)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
208	CeMnSbO (0.665)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
209	LaMnSbO (0.667)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
210	$CaMnBi_2$ (0.72)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
211	$YbMnSb_2$ (0.766)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
212	$YbMnBi_2$ (0.769)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
213	ThMnPN (0.920)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
214	ThMnPN (0.921)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
215	ThMnAsN (0.922)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
216	ThMnAsN (0.923)	Collinear	$P^{-1}4/^{-1}n^{1}m^{-1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
217	Fe_2As (1.131)	Collinear	$P^{1}4/^{1}n^{1}m^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
218	Mn_2As (1.132)	Collinear	$P^{1}4/^{1}n^{1}m^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
219	NdCoAsO (1.179)	Collinear	$P^{1}4/^{1}n^{1}m^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
220	DyOCl (1.643)	Collinear	$P^{1}4/^{1}n^{1}m^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
221	DySbTe (1.765)	Collinear	$P^{1}4/^{1}n^{1}m^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
222	$NdNiMg_{15}$ (1.457)	Collinear	$P^{1}m^{1}m^{1}a^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	×	×	×
223	CeMnAsO (0.187)	Collinear	$P^{1}m^{1}m^{-1}n^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
224	NdMnAsO (0.621)	Collinear	$P^{1}m^{1}m^{-1}n^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
225	NdMnAsO (0.622)	Collinear	$P^{1}m^{1}m^{-1}n^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
226	PrMnSbO (0.668)	Collinear	$P^{1}m^{1}m^{-1}n^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
227	CeMnSbO (0.666)	Collinear	$P^{1}m^{1}m^{-1}n^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
228	Cr_2As (1.130)	Collinear	$P^{1}m^{1}m^{1}n^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
229	DyB_4 (0.22)	Collinear	$P^{1}b^{-1}a^{1}m^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
230	TbB_4 (0.469)	Collinear	$P^1b^{-1}a^1m^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×
231	ErB_4 (0.468)	Collinear	$P^1b^{-1}a^1m^{\infty m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
232	Nd_2Pd_2In (1.335)	Collinear	$P^1m^1m^1a^{-1}(1/2\ \ 0\ \ 1/2)^{\infty m}1$	×	×	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
233	$UNiGa_5$ (1.254)	Collinear	$I^14/^1m^1m^1m^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	×	×	×
234	$CeIr(In_{0.97}Cd_{0.03})_5$ (1.598)	Collinear	$I^14/^1m^1m^1m^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	×	×	×
235	$UPdGa_5$ (1.683)	Collinear	$I^14/^1m^1m^1m^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	×	×	×
236	Dy_2CoGa_8 (1.80)	Collinear	$I^14/^1m^1m^1m^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	×	×	×
237	Nd_2RhIn_8 (1.82)	Collinear	$I^14/^1m^1m^1m^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	×	×	×
238	Tb_2CoGa_8 (1.87)	Collinear	$I^14/^1m^1m^1m^{-1}(1/2\ \ 1/2\ \ 0)^{\infty m}1$	×	×	×
239	$UPtGa_5$ (1.255)	Collinear	$P^14/^1m^1m^1m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
240	$NpRhGa_5$ (1.261)	Collinear	$P^14/^1m^1m^1m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
241	$CeRhAl_4Si_2$ (1.486)	Collinear	$P^14/^1m^1m^1m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
242	$CeIrAl_4Si_2$ (1.487)	Collinear	$P^14/^1m^1m^1m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
243	$NpCoGa_5$ (1.671)	Collinear	$P^14/^1m^1m^1m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
244	$YBaCo_2O_5$ (1.703)	Collinear	$P^1m^1m^1a^{-1}(0\ \ 1/2\ \ 0)^{\infty m}1$	×	×	×
245	$TaBaFe_2O_5$ (1.704)	Collinear	$P^1m^1m^1a^{-1}(0\ \ 1/2\ \ 0)^{\infty m}1$	×	×	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
246	$KRuO_4$ (0.285)	Collinear	$I^{-1}4_1/^{-1}a^{\infty m}1$	$\sigma_{xxz}=\sigma_{xzx}=-\sigma_{yyz}=-\sigma_{yzy}=\sigma_{zxx}=-\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxx}=-\sigma_{zyy}$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{yyz}=-\sigma_{yzy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
247	$KOsO_4$ (0.284)	Collinear	$I^{-1}4_1/^{-1}a^{\infty m}1$	$\sigma_{xxz}=\sigma_{xzx}=-\sigma_{yyz}=-\sigma_{yzy}=\sigma_{zxx}=-\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxx}=-\sigma_{zyy}$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{yyz}=-\sigma_{yzy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
248	$DyCrO_4$ (0.372)	Collinear	$I^{-1}4_1/^{-1}a^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
249	$Mn_3Ta_2O_8$ (0.734)	Collinear	$I^{-1}4_1/^{-1}a^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
250	Sr_2CuTeO_6 (1.168)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
251	Sr_2CoOsO_6 (1.72)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
252	$TlFe_{1.6}Se_2$ (0.208)	Collinear	$I^{1}4/^{-1}m^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
253	$TlFe_{1.6}Se_2$ (0.209)	Collinear	$I^{1}4/^{-1}m^{\infty m}1$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×
254	$K_{0.8}Fe_{1.8}Se_2$ (0.418)	Collinear	$I^{1}4/^{-1}m^{\infty m}1$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×
255	$Rb_yFe_{2-x}Se_2$ (0.54)	Collinear	$I^{1}4/^{-1}m^{\infty m}1$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×
256	$K_yFe_{2-x}Se_2$ (0.55)	Collinear	$I^{1}4/^{-1}m^{\infty m}1$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$	×
257	$BaNd_{0.9}Y_{0.1}MoO_6$ (1.12)	Collinear	$P^{1}4/^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
258	Sr_2FeOsO_6 (1.47)	Collinear	$P^{1}4/^{1}m^{-1}(1/2\ \ 1/2\ \ 1/2)^{\infty m}1$	×	×	×
259	$CeCu_2$ (0.290)	Collinear	$I^{1}m^{1}m^{-1}a^{\infty m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×
260	$Tm_3Cu_4Ge_4$ (1.727)	Collinear	$P^{1}m^{1}m^{1}n^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
261	$Eu_{0.5}Ca_{0.5}Fe_2As_2$ (1.483)	Collinear	$C^{1}c^{1}c^{1}m^{-1}(1/2\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
262	PrFeAsO (1.585)	Collinear	$P^{1}c^{1}c^{1}m^{-1}(1/2\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
263	$DyGe_{1.75}$ (0.341)	Collinear	$C^{1}m^{-1}m^{1}m^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
264	$TbGe_2$ (0.343)	Collinear	$C^{1}m^{-1}m^{1}m^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
265	$GdNiSi_3$ (0.406)	Collinear	$C^{1}m^{-1}m^{1}m^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
266	$Sr_2Fe_{1.9}Co_{0.1}O_{5.5}$ (0.400)	Collinear	$C^{1}m^{-1}m^{1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
267	$Sr_4Fe_4O_{11}$ (0.401)	Collinear	$C^{1}m^{-1}m^{1}m^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
268	$Ba_4Ru_3O_{10}$ (0.693)	Collinear	$C^{1}m^{1}c^{-1}e^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
269	$Ba_4Ru_3O_{10}$ (0.692)	Collinear	$C^{1}m^{1}c^{-1}e^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
270	Er_2PtGe_6 (0.909)	Collinear	$C^{1}m^{1}c^{-1}e^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
271	Er_2PtGe_6 (0.932)	Collinear	$C^{1}m^{1}c^{-1}e^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
272	DyGe (1.361)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
273	$ErGe_3$ (0.330)	Collinear	$C^{1}\mathrm{m}^{-1}\mathrm{c}^{1}\mathrm{m}^{\infty\mathrm{m}}1$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×
274	$DyCoSi_2$ (0.453)	Collinear	$C^{1}\mathrm{m}^{-1}\mathrm{c}^{1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
275	$TbNiGe_2$ (0.566)	Collinear	$C^{1}\mathrm{m}^{-1}\mathrm{c}^{1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
276	$HoNi_{0.64}Ge_2$ (0.567)	Collinear	$C^{1}\mathrm{m}^{-1}\mathrm{c}^{1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
277	$TbNi_{0.4}Ge_2$ (0.568)	Collinear	$C^{1}\mathrm{m}^{-1}\mathrm{c}^{1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
278	$TbCu_{0.4}Ge_2$ (0.569)	Collinear	$C^{1}\mathrm{m}^{-1}\mathrm{c}^{1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
279	$TbNiSi_2$ (0.910)	Collinear	$C^{1}\mathrm{m}^{-1}\mathrm{c}^{1}\mathrm{m}^{\infty\mathrm{m}}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
280	ErNiGe (1.379)	Collinear	$P^{1}2_1/^{1}\mathrm{c}^{-1}(1/2\ \ 0\ \ 0)^{\infty\mathrm{m}}1$	×	×	×
281	Y_2BaCuO_5 (1.445)	Collinear	$P^{1}2_1/^{1}\mathrm{c}^{-1}(1/2\ \ 0\ \ 0)^{\infty\mathrm{m}}1$	×	×	×
282	PrPdSn (1.744)	Collinear	$P^{1}2_1/^{1}\mathrm{c}^{-1}(1/2\ \ 0\ \ 0)^{\infty\mathrm{m}}1$	×	×	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
283	$EuZrO_3$ (0.147)	Collinear	$P^{1}n^{1}m^{-1}a^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
284	$EuZrO_3$ (0.146)	Collinear	$P^{1}n^{1}m^{-1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
285	$Cu_{0.95}MnAs$ (0.223)	Collinear	$P^{1}n^{1}m^{-1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
286	Fe_3BO_5 (0.386)	Collinear	$P^{1}n^{1}m^{-1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
287	$GdAlO_3$ (0.410)	Collinear	$P^{1}n^{1}m^{-1}a^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
288	$CaCr_{0.86}Fe_{3.14}As_3$ (0.429)	Collinear	$P^{1}n^{1}m^{-1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
289	$Tl_3Fe_2S_4$ (0.801)	Collinear	$P^{1}n^{-1}m^{1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
290	$YCr_{0.5}Fe_{0.5}O_3$ (0.946)	Collinear	$P^{1}n^{-1}m^{-1}a^{\infty m}1$	×	×	×
291	Gd_5Ge_4 (0.14)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
292	$LiCoPO_4$ (0.193)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
293	$SrEr_2O_4$ (0.216)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
294	$LiMnPO_4$ (0.24)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
295	Tb_2ReC_2 (0.346)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
296	$RbFeCl_5(D_2O)$ (0.362)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
297	$KFeCl_5(D_2O)$ (0.363)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
298	$LiCoPO_4$ (0.383)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
299	$LiMnPO_4$ (0.382)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
300	$LiCoPO_4$ (0.384)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
301	FeOOH (0.399)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
302	$EuMnSb_2$ (0.421)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
303	$EuMnSb_2$ (0.423)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
304	$MnPd_2$ (0.798)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
305	$SrGd_2O_4$ (0.821)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
306	$NaFePO_4$ (0.87)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
307	$KMn_4(PO_4)_3$ (0.86)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
308	$LiFePO_4$ (0.95)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
309	$CaFe_2O_4$ (0.969)	Collinear	$P^{-1}n^{1}m^{1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
310	$NdMnO_3$ (0.609)	Collinear	$P^{-1}n^{-1}m^{1}a^{\infty m}1$	×	×	×
311	$RbFeO_2$ (0.455)	Collinear	$P^{1}b^{1}c^{-1}a^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
312	$CsFeO_2$ (0.457)	Collinear	$P^1b^1c^{-1}a^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
313	$KFeO_2$ (0.459)	Collinear	$P^1b^1c^{-1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
314	$KFeO_2$ (0.460)	Collinear	$P^1b^1c^{-1}a^{\infty m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
315	$Li_{1.5}Fe(SO_4)_2$ (0.245)	Collinear	$P^1b^1c^{-1}a^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
316	$LiFe(SO_4)_2$ (0.246)	Collinear	$P^1b^1c^{-1}a^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
317	$Li_2Co(SO_4)_2$ (0.244)	Collinear	$P^1b^1c^{-1}a^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
318	$Li_2Fe(SO_4)_2$ (0.243)	Collinear	$P^1b^1c^{-1}a^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
319	$Li_2Ni(SO_4)_2$ (0.71)	Collinear	$P^1b^1c^{-1}a^{\infty m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
320	$CoNb_2O_6$ (1.656)	Collinear	$P^12_1/^1c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
321	Fe_2WO_6 (0.814)	Collinear	$P^{1}b^{-1}c^{1}n^{\infty m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
322	$CoSe_2O_5$ (0.161)	Collinear	$P^{-1}b^{-1}c^{-1}n^{\infty m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
323	$MnTa_2O_6$ (0.816)	Collinear	$P^{-1}b^{-1}c^{-1}n^{\infty m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
324	$MnNb_2O_6$ (0.815)	Collinear	$P^{-1}b^{-1}c^{-1}n^{\infty m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
325	DyRuAsO (0.451)	Collinear	$P^{1}m^{1}m^{-1}n^{\infty m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
326	$[C(ND_2)_3]Mn(DCOO)_3$ (0.256)	Collinear	$P^{1}n^{-1}n^{-1}a^{\infty m}1$	×	×	×
327	$CaMnGe_2O_6$ (0.155)	Collinear	$C^{1}2/^{-1}c^{\infty m}1$	σ_{zzz}, $\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$, σ_{yyy}, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, σ_{xxx}	σ_{zzz}, $\sigma_{zyz} = \sigma_{zzy}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, σ_{yzz}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, σ_{yyy}, $\sigma_{yxy} = \sigma_{yyx}$, σ_{yxx}, σ_{xzz}, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$, σ_{xyy}, $\sigma_{xxy} = \sigma_{xyx}$, σ_{xxx}	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
328	$CaMnGe_2O_6$ (0.156)	Collinear	$C^{1}2/^{-1}c^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
329	$MnGeO_3$ (0.312)	Collinear	$C^{1}2/^{-1}c^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
330	$NaCrSi_2O_6$ (0.504)	Collinear	$C^{1}2/^{-1}c^{\infty m}1$	σ_{zzz}, $\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	σ_{zzz}, $\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, σ_{yzz}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
331	$KFeSe_2$ (0.637)	Collinear	$C^{1}2/^{-1}c^{\infty m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
332	$RbFeSe_2$ (0.638)	Collinear	$C^{1}2/^{-1}c^{\infty m}1$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×
333	$KFeS_2$ (0.633)	Collinear	$C^{1}2/^{-1}c^{\infty m}1$	$\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz} = \sigma_{zzy}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy} = \sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy} = \sigma_{xyx}$, σ_{xxx}	×
334	$RbFeS_2$ (0.636)	Collinear	$C^{1}2/^{-1}c^{\infty m}1$	$\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz} = \sigma_{zzy}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy} = \sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy} = \sigma_{xyx}$, σ_{xxx}	×
335	$Cs_2FeCl_{5.}D_2O$ (0.252)	Collinear	$C^{-1}2/^{1}c^{\infty m}1$	$\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz} = \sigma_{zzy}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy} = \sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy} = \sigma_{xyx}$, σ_{xxx}	×
336	$Fe_4Nb_2O_9$ (0.442)	Collinear	$C^{-1}2/^{1}c^{\infty m}1$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
337	$Cs_2[FeCl_5(H_2O)]$ (0.476)	Collinear	$C^{-1}2/^{1}c^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
338	CuO (1.62)	Collinear	$P^{1}2_1/^{1}c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
339	$LiCrGe_2O_6$ (0.217)	Collinear	$P^{1}2_1/^{-1}c^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
340	$Pb_2VO(PO_4)_2$ (0.505)	Collinear	$P^{1}2_1/^{-1}c^{\infty m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×
341	$LiCrGe_2O_6$ (0.961)	Collinear	$P^{1}2_1/^{-1}c^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
342	$LiCrGe_2O_6$ (0.962)	Collinear	$P^{1}2_1/^{-1}c^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
343	$LiCrGe_2O_6$ (0.963)	Collinear	$P^{1}2_1/^{-1}c^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
344	$LiCrGe_2O_6$ (0.964)	Collinear	$P^{1}2_1/^{-1}c^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
345	$Fe_3(PO_4)_2$ (0.264)	Collinear	$P^{-1}2_1/^{1}c^{\infty m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×
346	Na_2MnPO_4F (0.827)	Collinear	$P^{-1}2_1/^{1}c^{\infty m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×
347	Na_2MnPO_4F (0.828)	Collinear	$P^{-1}2_1/^{1}c^{\infty m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
348	Na_2MnPO_4F (0.830)	Collinear	$P^{-1}2_1/^{1}c^{\infty m}1$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×
349	Na_2MnPO_4F (0.829)	Collinear	$P^{-1}2_1/^{1}c^{\infty m}1$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×
350	Na_2RuO_4 (0.933)	Collinear	$P^{-1}2_1/^{1}c^{\infty m}1$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×
351	Sc_2NiMnO_6 (1.199)	Collinear	$P^{1}2_1/^{1}c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
352	$Na_{0.5}Li_{0.5}FeGe_2O_6$ (1.276)	Collinear	$P^{1}2_1/^{1}c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
353	$CuFe_2(P_2O_7)_2$ (1.297)	Collinear	$P^{1}2_1/^{1}c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
354	$Li_{0.31}Na_{0.69}FeGe_2O_6$ (1.331)	Collinear	$P^{1}2_1/^{1}c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
355	La_2CoPtO_6 (1.462)	Collinear	$P^{1}2_1/^{1}c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
356	Mn_3TeO_6 (1.485)	Collinear	$P^{1}2_1/^{1}c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
357	$LiCoF_4$ (1.526)	Collinear	$P^{1}2_1/^{1}c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
358	$MnPb_4Sb_6S_{14}$ (1.63)	Collinear	$P^{1}2_1/^{1}c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×

Continued on next page

Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
359	Li_2MnSiO_4 (1.78)	Collinear	$P^{1}2_1/^{1}c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
360	$NiWO_4$ (1.194)	Collinear	$P^{1}2/^{1}c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
361	$Mn_{0.81}Cu_{0.19}WO_4$ (1.315)	Collinear	$P^{1}2/^{1}c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
362	$FeWO_4$ (1.653)	Collinear	$P^{1}2/^{1}c^{-1}(1/2\ \ 0\ \ 0)^{\infty m}1$	×	×	×
363	$MnPS_3$ (0.163)	Collinear	$C^{1}2/^{-1}m^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
364	$YbCl_3$ (0.444)	Collinear	$C^{1}2/^{-1}m^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
365	$Er_2Si_2O_7$ (0.527)	Collinear	$C^{1}2/^{-1}m^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
366	$YbCl_3$ (0.585)	Collinear	$C^{1}2/^{-1}m^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
367	$Er_2Si_2O_7$ (0.650)	Collinear	$C^{1}2/^{-1}m^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
368	$YbCl_3$ (0.723)	Collinear	$C^{1}2/^{-1}m^{\infty m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
369	$Fe_{0.48}TiSe_2$ (1.268)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
370	$Fe_{0.48}TiSe_2$ (1.269)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
371	Ag_2NiO_2 (1.49)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
372	$CoV_2O_{6-alpha}$ (1.17)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
373	$Fe_{0.25}TiSe_2$ (1.270)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
374	UCr_2Si_2 (1.470)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
375	CoV_2O_6 (1.70)	Collinear	$C^{1}2/^{1}m^{-1}(0\ \ 0\ \ 1/2)^{\infty m}1$	×	×	×
376	$GeCo_2O_4$ (1.564)	Coplanar	$R^{m_{100}}\text{-}3^{1}m\vert(1,1,2_{001};2_{001},1)^{m}1$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
377	$GeNi_2O_4$ (1.562)	Coplanar	$R^{\mathrm{m}_{100}}\text{-}3^{1}\mathrm{m}\vert(1,1,2_{001};2_{001},1)^{m}1$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
378	$DyBe_{13}$ (1.517)	Coplanar	$I^{1}4/^{\mathrm{m}_{100}}\mathrm{m}^{2_{001}}\mathrm{c}^{2_{001}}\mathrm{m}\vert(1,1,1;2_{001})^{m}1$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
379	$TbBe_{13}$ (1.518)	Coplanar	$I^{1}4/^{\mathrm{m}_{100}}\mathrm{m}^{2_{001}}\mathrm{c}^{2_{001}}\mathrm{m}\vert(1,1,1;2_{001})^{m}1$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
380	$CaMn_3V_4O_{12}$ (1.758)	Coplanar	$R^{3^{1}_{001}}\text{-}3\vert(1,1,2_{001};2_{001},1)^{m}1$	×	×	×
381	$LaMn_3V_4O_{12}$ (1.119)	Coplanar	$I^{1}\mathrm{m}^{3^{1}_{001}}\text{-}3\vert(1,1,1;2_{001})^{m}1$	×	×	×
382	$U_3Ru_4Al_{12}$ (0.12)	Coplanar	$C^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{c}^{1}\mathrm{m}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
383	$EuIn_2As_2$ (1.0.31)	Coplanar	$P^{2_{001}}6_3/^{\mathrm{m}_{100}}\mathrm{m}^{1}\mathrm{m}^{2_{001}}\mathrm{c}^{m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
384	$RbFeCl_3$ (1.0.40)	Coplanar	$P^{\mathrm{m}_{010}}6_3/^{1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{m}^{1}\mathrm{c}\vert(3^{1}_{001},3^{1}_{001},1)^{m}1$	×	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	×
385	$Ba_3CoSb_2O_9$ (1.0.44)	Coplanar	$P^{\mathrm{m}_{100}}6_3/^{2_{001}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{m}^{2_{001}}\mathrm{c}\vert(3^{2}_{001},3^{2}_{001},1)^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
386	$CsMnI_3$ (1.0.36)	Coplanar	$P^{\mathrm{m}_{010}}6_3/^{2_{001}}\mathrm{m}^{2_{001}}\mathrm{c}^{\mathrm{m}_{100}}\mathrm{m}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
387	$RbNiCl_3$ (1.0.34)	Coplanar	$P^{\mathrm{m}_{010}}6_3/^{2_{001}}\mathrm{m}^{2_{001}}\mathrm{c}^{\mathrm{m}_{100}}\mathrm{m}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
388	$CsMnI_3$ (1.0.37)	Coplanar	$P^{\mathrm{m}_{010}}6_3/^{2_{001}}\mathrm{m}^{2_{001}}\mathrm{c}^{\mathrm{m}_{100}}\mathrm{m}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
389	$CsCoBr_3$ (1.0.3)	Coplanar	$P^{\mathrm{m}_{010}}6_3/^{2_{001}}\mathrm{m}^{2_{001}}\mathrm{c}^{\mathrm{m}_{100}}\mathrm{m}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
390	$ThMn_2$ (1.0.24)	Coplanar	$P^{\mathrm{m}_{010}}6_3/^{1}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{m}^{2_{001}}\mathrm{c}\|(3^2_{001},3^2_{001},1)^{m}1$	σ_{zzz}, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=\sigma_{zxx}=\sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}$	×
391	$CsFeCl_3$ (1.0.14)	Coplanar	$P^{\mathrm{m}_{010}}6_3/^{1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{m}^{1}\mathrm{c}\|(3^1_{001},3^1_{001},1)^{m}1$	σ_{zzz}, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=\sigma_{zxx}=\sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}$	×
392	$CsNiCl_3$ (1.0.4)	Coplanar	$P^{\mathrm{m}_{010}}6_3/^{2_{001}}\mathrm{m}^{2_{001}}\mathrm{c}^{\mathrm{m}_{100}}\mathrm{m}^{m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
393	$CsMnBr_3$ (1.0.35)	Coplanar	$P^{\mathrm{m}_{100}}6_3/^{2_{001}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{m}^{2_{001}}\mathrm{c}\|(3^2_{001},3^2_{001},1)^{m}1$	$\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}=-\sigma_{yyy}$	$\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}=-\sigma_{yyy}$	×
394	$Ba_3CoSb_2O_9$ (1.0.45)	Coplanar	$P^{\mathrm{m}_{100}}6_3/^{2_{001}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{m}^{2_{001}}\mathrm{c}\|(3^2_{001},3^2_{001},1)^{m}1$	$\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	$\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	×
395	$EuIn_2As_2$ (1.0.32)	Coplanar	$P^{6^5_{001}}6_3/^{\mathrm{m}_{\frac{1}{3}\pi}}\mathrm{m}^{1}\mathrm{m}^{6^5_{001}}\mathrm{c}\|(1,1,3^2_{001})^{m}1$	σ_{zzz}, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=\sigma_{zxx}=\sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}$	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
396	$RbNiCl_3$ (1.0.41)	Coplanar	$P^{\mathrm{m}_{100}}6_3/^{2_{001}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{m}^{2_{001}}\mathrm{c}\lvert(3^2_{001},3^2_{001},1)^m1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
397	$CsNiCl_3$ (1.0.42)	Coplanar	$P^{\mathrm{m}_{100}}6_3/^{2_{001}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{m}^{2_{001}}\mathrm{c}\lvert(3^2_{001},3^2_{001},1)^m1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
398	TmAgGe (3.1)	Coplanar	$P^{3^2_{001}}\text{-}6^{\mathrm{m}_{\frac{1}{6}\pi}}2^{\mathrm{m}_{\frac{5}{6}\pi}}\mathrm{m}^m1$	×	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	×
399	$CsCr_{0.94}Fe_{0.06}F_4$ (1.711)	Coplanar	$P^{3^2_{001}}\text{-}6^{\mathrm{m}_{\frac{1}{6}\pi}}2^{\mathrm{m}_{\frac{5}{6}\pi}}\mathrm{m}\lvert(1,1,2_{001})^m1$	×	×	×
400	ErAuIn (1.747)	Coplanar	$P^{3^2_{001}}\text{-}6^{\mathrm{m}_{\frac{1}{6}\pi}}2^{\mathrm{m}_{\frac{5}{6}\pi}}\mathrm{m}\lvert(1,1,2_{001})^m1$	×	×	×
401	TbAuIn (1.748)	Coplanar	$P^{3^2_{001}}\text{-}6^{\mathrm{m}_{\frac{1}{6}\pi}}2^{\mathrm{m}_{\frac{5}{6}\pi}}\mathrm{m}\lvert(1,1,2_{001})^m1$	×	×	×
402	TmPdIn (1.163)	Coplanar	$P^{3^2_{001}}\text{-}6\lvert(1,1,2_{001})^m1$	×	×	×
403	$U_{14}Au_{51}$ (0.283)	Coplanar	$P^{6^1_{001}}6/^1\mathrm{m}^m1$	σ_{zzz}, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=\sigma_{zxx}=\sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}$	×
404	$Cu_{0.82}Mn_{1.18}As$ (0.278)	Coplanar	$P^{3^2_{001}}\text{-}6^m1$	σ_{zzz}, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=\sigma_{zxx}=\sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}$	×
405	$Ba_2Co_9O_{14}$ (1.343)	Coplanar	$R^1\text{-}3^1\mathrm{m}^{2_{001}}(1/3\ \ 2/3\ \ 1/6)^m1$	×	×	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
406	$Co_4Nb_2O_9$ (0.196)	Coplanar	$P^{2_{001}}$-3^{1}c^{1}1^{m}1	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×
407	$Co_4Ta_2O_9$ (0.511)	Coplanar	$P^{2_{001}}$-$3^{m_{010}}$c^{1}1^{m}1	$\sigma_{yzz} = \sigma_{zyz} = \sigma_{zzy}$, $\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, $\sigma_{xxy} = \sigma_{xyx} = \sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz} = \sigma_{zzy}$, $\sigma_{zxz} = \sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy} = \sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy} = \sigma_{xyx}$, σ_{xxx}	×
408	$Co_4Nb_2O_9$ (0.197)	Coplanar	$P^{2_{001}}$-$3^{m_{010}}$c^{1}1^{m}1	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×
409	$Fe_4Nb_2O_9$ (0.441)	Coplanar	$P^{2_{001}}$-$3^{m_{010}}$c^{1}1^{m}1	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
410	$Co_4Nb_2O_9$ (0.529)	Coplanar	$P^{2_{001}}\text{-}3^{\mathrm{m}_{010}}\mathrm{c}^{1}1^{m}1$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×
411	$Ba_3MnNb_2O_9$ (1.0.8)	Coplanar	$P^{\mathrm{m}_{010}}\text{-}3^{\mathrm{m}_{010}}\mathrm{m}^{1}1\|(3^{1}_{001}, 3^{1}_{001}, 1)^{m}1$	σ_{zzz}, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = \sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	σ_{zzz}, $\sigma_{zxx} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxx} = -\sigma_{xyy} = -\sigma_{yxy} = -\sigma_{yyx}$	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
412	$Ba_3Nb_2NiO_9$ (1.13)	Coplanar	$P^{\mathrm{m}_{010}}\text{-}3^{\mathrm{m}_{010}}\mathrm{m}^{1}1\|(3^{2}_{001}, 3^{2}_{001}, 2_{001})^{m}1$	×	×	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
413	$Ba_3NiTa_2O_9$ (1.725)	Coplanar	$P^{\mathrm{m}_{010}}\text{-}3^{\mathrm{m}_{010}}\mathrm{m}^{1}1\|(3^{2}_{001}, 3^{2}_{001}, 2_{001})^{m}1$	×	×	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
414	VCl_2 (1.237)	Coplanar	$P^{\mathrm{m}_{010}}\text{-}3^{\mathrm{m}_{010}}\mathrm{m}^{1}1\|(3^{2}_{001}, 3^{2}_{001}, 2_{001})^{m}1$	×	×	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$
415	VBr_2 (1.238)	Coplanar	$P^{\mathrm{m}_{010}}\text{-}3^{\mathrm{m}_{010}}\mathrm{m}^{1}1\|(3^{2}_{001}, 3^{2}_{001}, 2_{001})^{m}1$	×	×	$\sigma_{xxz} = \sigma_{xzx} = \sigma_{yyz} = \sigma_{yzy} = -\sigma_{zxx}/2 = -\sigma_{zyy}/2$

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
416	$Ba_3CoNb_2O_9$ (1.665)	Coplanar	$P^{\mathrm{m}_{010}}\text{-}3^{\mathrm{m}_{010}}\mathrm{m}^{1}1\|(3^{2}_{001},3^{2}_{001},2_{001})^{m}1$	$\times$	$\times$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$
417	Na_2MnTeO_6 (1.0.51)	Coplanar	$R^{\text{-}3^{1}_{001}}\text{-}3^{\mathrm{m}\frac{2}{3}\pi}\mathrm{c}^{m}1$	$\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$, $\sigma_{xxx}=-\sigma_{xyy}=-\sigma_{yxy}=-\sigma_{yyx}$	$\times$
418	$CsFe(MoO_4)_2$ (1.499)	Coplanar	$P^{\mathrm{m}_{010}}\text{-}3\|(3^{2}_{001},3^{2}_{001},2_{001})^{m}1$	$\times$	$\times$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$, $\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$
419	$KFe(PO_3F)_2$ (1.669)	Coplanar	$P^{\mathrm{m}\frac{1}{3}\pi}\text{-}3\|(3^{1}_{001},3^{1}_{001},4^{1}_{001})^{m}1$	$\times$	$\times$	$\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zxx}/2=-\sigma_{zyy}/2$, $\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$
420	$FeSn_2$ (2.66)	Coplanar	$I^{1}4/^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{c}^{\mathrm{m}_{010}}\mathrm{m}\|(1,1,1;m_{100})^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\times$
421	$FeGe_2$ (2.68)	Coplanar	$I^{1}4/^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{c}^{\mathrm{m}_{010}}\mathrm{m}\|(1,1,1;m_{100})^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\times$
422	$Ba_2Mn_3Sb_2O_2$ (2.53)	Coplanar	$F^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{m}\|(1,1,1;m_{100},m_{100},1)^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\times$
423	$Sr_2Mn_3Sb_2O_2$ (2.27)	Coplanar	$F^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{m}\|(1,1,1;m_{100},m_{100},1)^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\times$

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC IMD	with SOC QMD	with SOC BCD
424	$La_{0.73}Tb_{0.27}Mn_2Si_2$ (2.58)	Coplanar	$I^{\mathrm{m}_{100}}4/^{2_{001}}\mathrm{m}^{1}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{m}\vert(1,1,1;m_{010})^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
425	$GdMn_2Si_2$ (2.96)	Coplanar	$I^{\mathrm{m}_{100}}4/^{2_{001}}\mathrm{m}^{1}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{m}\vert(1,1,1;m_{010})^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
426	$Sr_2CuO_2Cu_2S_2$ (1.456)	Coplanar	$I^{\mathrm{m}_{110}}4/^{\mathrm{m}_{\text{-}110}}\mathrm{m}^{\mathrm{m}_{110}}\mathrm{m}^{1}\mathrm{m}\vert(2_{001},2_{001},2_{001};4^{1}_{001})^{m}1$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
427	$La_{0.25}Pr_{0.75}Co_2P_2$ (1.317)	Coplanar	$P^{1}4/^{1}\mathrm{m}^{1}\mathrm{m}^{1}\mathrm{m}^{2_{001}}(0\ \ 0\ \ 1/2)^{m}1$	×	×	×
428	TbC_2 (1.533)	Coplanar	$P^{1}\mathrm{m}^{1}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{m}\vert(1,1,2_{001})^{m}1$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
429	$LuFe_4Ge_2$ (0.140)	Coplanar	$P^{\mathrm{m}_{010}}\mathrm{n}^{\mathrm{m}_{100}}\mathrm{n}^{1}\mathrm{m}^{m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
430	$FePbBiO_4$ (0.214)	Coplanar	$P^{2_{001}}4_2/^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{b}^{\mathrm{m}_{010}}\mathrm{c}^{m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
431	CeMnAsO (0.188)	Coplanar	$P^{1}\mathrm{m}^{1}\mathrm{m}^{2_{001}}\mathrm{n}^{m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
432	CeNiAsO (1.272)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{1}\mathrm{m}\vert(1,1,2_{001})^{m}1$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
433	DySbTe (2.105)	Coplanar	$P^{2_{001}}2_1/^{1}\mathrm{m}\vert(1,1,m_{010})^{m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×
434	Sr_2FeO_3Cl (1.380)	Coplanar	$P^{\mathrm{m}_{110}}4/^{\mathrm{m}_{100}}\mathrm{n}^{\mathrm{m}_{110}}\mathrm{m}^{1}\mathrm{m}\vert(2_{001},2_{001},1)^{m}1$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
435	Sr_2FeO_3Br (1.381)	Coplanar	$P^{\mathrm{m}_{110}}4/^{\mathrm{m}_{100}}\mathrm{n}^{\mathrm{m}_{110}}\mathrm{m}^{1}\mathrm{m}\vert(2_{001},2_{001},1)^{m}1$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
436	Ca_2FeO_3Br (1.383)	Coplanar	$P^{\mathrm{m}_{110}}4/^{\mathrm{m}_{100}}\mathrm{n}^{\mathrm{m}_{110}}\mathrm{m}^{1}\mathrm{m}\vert(2_{001},2_{001},1)^{m}1$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
437	Ca_2FeO_3Cl (1.382)	Coplanar	$P^{\mathrm{m}_{110}}4/^{\mathrm{m}_{100}}\mathrm{n}^{\mathrm{m}_{110}}\mathrm{m}^{1}\mathrm{m}\vert(2_{001},2_{001},1)^{m}1$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
438	Sr_2FeO_3F (1.385)	Coplanar	$P^{\mathrm{m}_{110}}4/^{\mathrm{m}_{100}}\mathrm{n}^{\mathrm{m}_{110}}\mathrm{m}^{1}\mathrm{m}\vert(2_{001},2_{001},1)^{m}1$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
439	Sr_2FeO_3F (1.387)	Coplanar	$P^{\mathrm{m}_{110}}4/^{\mathrm{m}_{100}}\mathrm{n}^{\mathrm{m}_{110}}\mathrm{m}^{1}\mathrm{m}\vert(2_{001},2_{001},1)^{m}1$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
440	Sr_2FeO_3F (1.386)	Coplanar	$P^{\mathrm{m}_{110}}4/^{\mathrm{m}_{100}}\mathrm{n}^{\mathrm{m}_{110}}\mathrm{m}^{1}\mathrm{m}\vert(2_{001},2_{001},2_{001})^{m}1$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
441	NdB_4 (0.491)	Coplanar	$P^{4^1_{001}}4/^{1}\mathrm{m}^{\mathrm{m}_{110}}\mathrm{b}^{\mathrm{m}_{010}}\mathrm{m}^{m}1$	σ_{zzz}, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}=\sigma_{zxx}=\sigma_{zyy}$	σ_{zzz}, $\sigma_{zxx}=\sigma_{zyy}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	×
442	GdB_4 (0.9)	Coplanar	$P^{4^1_{001}}4/^{1}\mathrm{m}^{\mathrm{m}_{110}}\mathrm{b}^{\mathrm{m}_{010}}\mathrm{m}^{m}1$	×	$\sigma_{xyz}=\sigma_{xzy}=-\sigma_{yxz}=-\sigma_{yzx}$	×
443	U_2Pd_2In (0.320)	Coplanar	$P^{4^1_{001}}4/^{1}\mathrm{m}^{\mathrm{m}_{110}}\mathrm{b}^{\mathrm{m}_{010}}\mathrm{m}^{m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
444	U_2Pd_2Sn (0.321)	Coplanar	$P^{4^1_{001}}4/^{1}\mathrm{m}^{\mathrm{m}_{110}}\mathrm{b}^{\mathrm{m}_{010}}\mathrm{m}^{m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
445	U_2Pd_2In (0.625)	Coplanar	$P^{4^1_{001}}4/^{1}\mathrm{m}^{\mathrm{m}_{110}}\mathrm{b}^{\mathrm{m}_{010}}\mathrm{m}^{m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
446	U_2Pd_2In (0.80)	Coplanar	$P^{4^1_{001}}4/^{1}\mathrm{m}^{\mathrm{m}_{110}}\mathrm{b}^{\mathrm{m}_{010}}\mathrm{m}^{m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
447	U_2Pd_2Sn (0.81)	Coplanar	$P^{4^1_{001}}4/^{1}\mathrm{m}^{\mathrm{m}_{110}}\mathrm{b}^{\mathrm{m}_{010}}\mathrm{m}^{m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$	×
448	NdB_4 (0.492)	Coplanar	$P^{\mathrm{m}_{010}}\mathrm{b}^{\mathrm{m}_{100}}\mathrm{a}^{1}\mathrm{m}^{m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
449	$Pb_2BaCuFeO_5Br$ (2.45)	Coplanar	$P^{1}4/^{1}\mathrm{n}^{1}\mathrm{m}^{1}\mathrm{m}^{2_{001}}(1/2\ \ 1/2\ \ 0)^{m}1$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
450	$Pb_2BaCuFeO_5Cl$ (2.46)	Coplanar	$P^{1}4/^{1}\mathrm{n}^{1}\mathrm{m}^{1}\mathrm{m}^{2_{001}}(1/2\ \ 1/2\ \ 0)^{m}1$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
451	Ho_2BaNiO_5 (1.14)	Coplanar	$C^{1}2/^{1}\mathrm{m}^{2_{001}}(0\ \ 0\ \ 1/2)^{m}1$	×	×	×
452	Er_2BaNiO_5 (1.15)	Coplanar	$C^{1}2/^{1}\mathrm{m}^{2_{001}}(0\ \ 0\ \ 1/2)^{m}1$	×	×	×
453	Nd_2BaNiO_5 (1.216)	Coplanar	$C^{1}2/^{1}\mathrm{m}^{2_{001}}(0\ \ 0\ \ 1/2)^{m}1$	×	×	×
454	Tb_2BaNiO_5 (1.217)	Coplanar	$C^{1}2/^{1}\mathrm{m}^{2_{001}}(0\ \ 0\ \ 1/2)^{m}1$	×	×	×
455	Nd_2BaCoO_5 (1.350)	Coplanar	$C^{1}2/^{1}\mathrm{m}^{2_{001}}(0\ \ 0\ \ 1/2)^{m}1$	×	×	×
456	Dy_2BaNiO_5 (1.36)	Coplanar	$C^{1}2/^{1}\mathrm{m}^{2_{001}}(0\ \ 0\ \ 1/2)^{m}1$	×	×	×
457	Er_2BaNiO_5 (1.53)	Coplanar	$C^{1}2/^{1}\mathrm{m}^{2_{001}}(0\ \ 0\ \ 1/2)^{m}1$	×	×	×
458	PrFeAsO (1.584)	Coplanar	$C^{\mathrm{m}_{100}}\mathrm{m}^{1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{e}\vert(1,1,1;2_{001})^{m}1$	×	×	×
459	Dy_2PdGe_6 (0.906)	Coplanar	$C^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{c}^{2_{001}}\mathrm{e}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
460	Tb_2PdGe_6 (0.905)	Coplanar	$C^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{c}^{2_{001}}\mathrm{e}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
461	Ho_2PdGe_6 (0.907)	Coplanar	$C^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{c}^{2_{001}}\mathrm{e}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
462	Tb_2PtGe_6 (0.908)	Coplanar	$C^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{c}^{2_{001}}\mathrm{e}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
463	Tb_2PdGe_6 (0.929)	Coplanar	$C^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{c}^{2_{001}}\mathrm{e}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
464	Dy_2PdGe_6 (0.928)	Coplanar	$C^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{c}^{2_{001}}\mathrm{e}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
465	Tb_2PtGe_6 (0.931)	Coplanar	$C^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{c}^{2_{001}}\mathrm{e}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
466	Ho_2PdGe_6 (0.930)	Coplanar	$C^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{c}^{2_{001}}\mathrm{e}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
467	Cs_2CoCl_4 (1.51)	Coplanar	$P^{1}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
468	$LiFePO_4$ (0.152)	Coplanar	$P^{\mathrm{m}_{100}}\mathrm{n}^{1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{a}^{m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
469	$LaCa_2Fe_3O_9$ (1.0.30)	Coplanar	$P^{2_{010}}\mathrm{n}^{2_{100}}\mathrm{m}^{1}\mathrm{a}^{m}1$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxz} = \sigma_{xzx}$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
470	$EuMnSb_2$ (0.422)	Coplanar	$P^{2_{001}}\mathrm{n}^{1}\mathrm{m}^{1}\mathrm{a}^{m}1$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×
471	$FePO_4$ (0.17)	Coplanar	$P^{2_{001}}\mathrm{n}^{1}\mathrm{m}^{\mathrm{m}_{100}}\mathrm{a}^{m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$
472	$BaNd_2O_4$ (1.96)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	×
473	$BaNd_2O_4$ (1.95)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	×
474	Dy_2TiO_5 (1.698)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	×
475	$SrHo_2O_4$ (2.8)	Coplanar	$P^{2_{001}}2_1/^{1}\mathrm{c}\vert(m_{010},1,1)^{m}1$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×
476	NbMnP (0.803)	Coplanar	$P^{\mathrm{m}_{010}}\mathrm{n}^{1}\mathrm{m}^{2_{001}}\mathrm{a}^{m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
477	CsO_2 (0.1004)	Coplanar	$P^{\mathrm{m}_{100}}\mathrm{n}^{1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{a}^{m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
478	$DyCoO_3$ (0.159)	Coplanar	$P^{m_{100}}n^{1}m^{m_{010}}a^{m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×
479	$TbCoO_3$ (0.160)	Coplanar	$P^{m_{100}}n^{1}m^{m_{010}}a^{m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
480	$DyScO_3$ (0.171)	Coplanar	$P^{m_{100}}n^{1}m^{m_{010}}a^{m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×
481	$TbAlO_3$ (0.350)	Coplanar	$P^{m_{100}}n^{1}m^{m_{010}}a^{m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×
482	$LiCoPO_4$ (0.385)	Coplanar	$P^{m_{100}}n^{1}m^{m_{010}}a^{m}1$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy} = \sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$, $\sigma_{xxz} = \sigma_{xzx}$	×
483	$EuMnSb_2$ (0.424)	Coplanar	$P^{m_{100}}n^{1}m^{m_{010}}a^{m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×
484	$TbCoO_3$ (0.520)	Coplanar	$P^{m_{100}}n^{1}m^{m_{010}}a^{m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
485	$DyCoO_3$ (0.521)	Coplanar	$P^{m_{100}}n^{1}m^{m_{010}}a^{m}1$	$\sigma_{xyz} = \sigma_{xzy} = \sigma_{yxz} = \sigma_{yzx} = \sigma_{zxy} = \sigma_{zyx}$	$\sigma_{zxy} = \sigma_{zyx}$, $\sigma_{yxz} = \sigma_{yzx}$, $\sigma_{xyz} = \sigma_{xzy}$	×
486	$NdInO_3$ (0.783)	Coplanar	$P^{m_{100}}n^{1}m^{m_{010}}a^{m}1$	$\sigma_{xzz} = \sigma_{zxz} = \sigma_{zzx}$, $\sigma_{xyy} = \sigma_{yxy} = \sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz} = \sigma_{zzx}$, $\sigma_{yxy} = \sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
487	$NdScO_3$ (0.782)	Coplanar	$P^{\mathrm{m}_{100}}\mathrm{n}^{1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{a}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
488	$DyAlO_3$ (0.842)	Coplanar	$P^{\mathrm{m}_{100}}\mathrm{n}^{1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{a}^{m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
489	$RbRuO_4$ (0.924)	Coplanar	$P^{\mathrm{m}_{100}}\mathrm{n}^{1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{a}^{m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
490	$KCrF_4$ (0.182)	Coplanar	$P^{\mathrm{m}_{010}}\mathrm{n}^{2_{001}}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{a}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
491	$NdCrO_3$ (0.589)	Coplanar	$P^{\mathrm{m}_{100}}\mathrm{n}^{\mathrm{m}_{010}}\mathrm{m}^{2_{001}}\mathrm{a}^{m}1$	×	×	×
492	$ErCrO_3$ (0.590)	Coplanar	$P^{\mathrm{m}_{100}}\mathrm{n}^{\mathrm{m}_{010}}\mathrm{m}^{2_{001}}\mathrm{a}^{m}1$	×	×	×
493	$HoBaCuO_5$ (2.85)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{010}}\mathrm{c}^{m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
494	$SrNd_2O_4$ (1.577)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	×
495	$DyBaCuO_5$ (1.650)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	×
496	$HoBaCuO_5$ (1.651)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	×
497	$DyBaCuO_5$ (0.805)	Coplanar	$P^{\mathrm{m}_{100}}\mathrm{n}^{1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{a}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×
498	$LiNiPO_4$ (0.88)	Coplanar	$P^{\mathrm{m}_{100}}\mathrm{n}^{1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{a}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	×

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Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
499	Ag_2RuO_4 (0.918)	Coplanar	$P^{\mathrm{m}_{100}}\mathrm{n}^{1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{a}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\times$
500	$MnGeO_3$ (0.313)	Coplanar	$P^{\mathrm{m}_{010}}\mathrm{b}^{\mathrm{m}_{010}}\mathrm{c}^{2_{001}}\mathrm{a}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\times$
501	$CoSe_2O_5$ (0.119)	Coplanar	$P^{\mathrm{m}_{010}}\mathrm{b}^{2_{001}}\mathrm{c}^{\mathrm{m}_{010}}\mathrm{n}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\times$
502	$Mn(Nb_{0.5}Ta_{0.5})_2O_6$ (0.817)	Coplanar	$P^{\mathrm{m}_{010}}\mathrm{b}^{2_{001}}\mathrm{c}^{\mathrm{m}_{010}}\mathrm{n}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\times$
503	$MnNb_2O_6$ (0.819)	Coplanar	$P^{\mathrm{m}_{010}}\mathrm{b}^{2_{001}}\mathrm{c}^{\mathrm{m}_{010}}\mathrm{n}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\times$
504	$MnTa_2O_6$ (0.818)	Coplanar	$P^{\mathrm{m}_{010}}\mathrm{b}^{2_{001}}\mathrm{c}^{\mathrm{m}_{010}}\mathrm{n}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\times$
505	Fe_2WO_6 (0.812)	Coplanar	$P^{\mathrm{m}_{100}}\mathrm{b}^{\mathrm{m}_{010}}\mathrm{c}^{\mathrm{m}_{100}}\mathrm{n}^{m}1$	$\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, σ_{xxx}	$\sigma_{xzz}=\sigma_{zzx}$, $\sigma_{yxy}=\sigma_{yyx}$, σ_{xzz}, σ_{xyy}, σ_{xxx}	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$
506	MnV_2O_6 (1.196)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	$\times$	$\times$	$\times$
507	$SrFe_2S_2O$ (0.762)	Coplanar	$P^{\mathrm{m}_{100}}\mathrm{m}^{1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{n}^{m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	$\times$
508	$SrFe_2Se_2O$ (0.761)	Coplanar	$P^{\mathrm{m}_{100}}\mathrm{m}^{1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{n}^{m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	$\times$

Continued on next page

Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
509	$BaFe_2S_2O$ (0.987)	Coplanar	$P^{m_{100}}m^{1}m^{m_{010}}n^{m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
510	$BaFe_2Se_2O$ (0.988)	Coplanar	$P^{m_{100}}m^{1}m^{m_{010}}n^{m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
511	YFe_4Ge_2 (0.27)	Coplanar	$P^{m_{010}}n^{m_{100}}n^{1}m^{m}1$	$\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$	×
512	$Ba_3MnSb_2O_9$ (1.0.46)	Coplanar	$C^{m_{100}}2/^{2_{001}}c\vert(3^{1}_{001},1,1;3^{2}_{001})^{m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	$\sigma_{yyz}=\sigma_{yzy}=-\sigma_{zyy}/2$, $\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=-\sigma_{zxx}/2$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$
513	$BaFeO_{2.5}$ (1.83)	Coplanar	$P^{1}2_1/^{1}c^{2_{001}}(1/2\ \ 0\ \ 0)^{m}1$	×	×	×
514	$LiFeGe_2O_6$ (1.39)	Coplanar	$P^{m_{100}}2_1/^{m_{100}}c\vert(2_{001},1,1)^{m}1$	×	×	×
515	Ba_2CoO_4 (1.302)	Coplanar	$P^{m_{100}}2_1/^{m_{100}}c\vert(2_{001},1,1)^{m}1$	×	×	×
516	$LiFeSi_2O_6$ (0.28)	Coplanar	$P^{m_{100}}2_1/^{m_{010}}c^{m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×
517	$NaMnF_4$ (1.345)	Coplanar	$P^{m_{100}}2_1/^{m_{100}}c\vert(2_{001},1,1)^{m}1$	×	×	×
518	$GdPO_4$ (1.118)	Coplanar	$P^{m_{100}}2_1/^{m_{100}}c\vert(2_{001},1,1)^{m}1$	×	×	×
519	$BiNiO(PO_4)$ (1.127)	Coplanar	$P^{m_{100}}2_1/^{m_{100}}c\vert(2_{001},1,1)^{m}1$	×	×	×
520	$BiCoO(PO_4)$ (1.128)	Coplanar	$P^{m_{100}}2_1/^{m_{100}}c\vert(2_{001},1,1)^{m}1$	×	×	×

Continued on next page

Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
521	$BiMnTeO_6$ (1.301)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	×
522	$C_{10}H_6MnN_4O_4$ (0.1010)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{010}}\mathrm{c}^{m}1$	$\sigma_{yzz}=\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{xzz}=\sigma_{zxz}=\sigma_{zzx}$, σ_{yyy}, $\sigma_{xyy}=\sigma_{yxy}=\sigma_{yyx}$, $\sigma_{xxy}=\sigma_{xyx}=\sigma_{yxx}$, σ_{xxx}	$\sigma_{zyz}=\sigma_{zzy}$, $\sigma_{zxz}=\sigma_{zzx}$, σ_{yzz}, σ_{yyy}, $\sigma_{yxy}=\sigma_{yyx}$, σ_{yxx}, σ_{xzz}, σ_{xyy}, $\sigma_{xxy}=\sigma_{xyx}$, σ_{xxx}	×
523	Ba_2CoO_4 (1.476)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	×
524	Ba_2CoO_4 (1.477)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	×
525	$FePb_4Sb_6S_{14}$ (1.660)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	×
526	Sr_2MnMoO_6 (1.716)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	×
527	Sr_2MnWO_6 (1.717)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	×
528	$CuSb_2O_6$ (1.133)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	×
529	$Li_2Fe(SO_4)_2$ (1.147)	Coplanar	$P^{\mathrm{m}_{100}}2_1/^{\mathrm{m}_{100}}\mathrm{c}\vert(2_{001},1,1)^{m}1$	×	×	×
530	$CrReO_4$ (1.202)	Coplanar	$C^{\mathrm{m}_{100}}2/^{\mathrm{m}_{010}}\mathrm{m}\vert(1,1,1;2_{001})^{m}1$	×	×	×
531	TbOOH (2.21)	Coplanar	$P^{2_{001}}2_1/^{1}\mathrm{m}\vert(1,1,m_{010})^{m}1$	σ_{zzz}, $\sigma_{yyz}=\sigma_{yzy}=\sigma_{zyy}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}=\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xxz}=\sigma_{xzx}=\sigma_{zxx}$	σ_{zzz}, σ_{zyy}, $\sigma_{zxy}=\sigma_{zyx}$, σ_{zxx}, $\sigma_{yyz}=\sigma_{yzy}$, $\sigma_{yxz}=\sigma_{yzx}$, $\sigma_{xyz}=\sigma_{xzy}$, $\sigma_{xxz}=\sigma_{xzx}$	×
532	CoO (3.19)	Noncoplanar	$P^{4^{1}_{001}}4_2/^{2_{001}}\mathrm{n}^{\mathrm{m}_{100}}\mathrm{n}^{2_{110}}\mathrm{m}\vert(\text{-}1,\text{-}1,\text{-}1)$	×	×	×
533	Ce_3NIn (1.152)	Noncoplanar	$P^{4^{1}_{001}}4/^{1}\mathrm{m}^{\mathrm{m}_{010}}\mathrm{m}^{\mathrm{m}_{-110}}\mathrm{m}\vert(1,1,\text{-}1)$	×	×	$\sigma_{zxy}=\sigma_{zyx}$, $\sigma_{xyz}=\sigma_{xzy}=\sigma_{yxz}=\sigma_{yzx}$
534	$SrCuTe_2O_6$ (0.440)	Noncoplanar	$P^{4^{1}_{100}}4_1{}^{3^{1}_{111}}3^{2_{110}}2$	×	×	×
535	$SrCuTe_2O_6$ (0.530)	Noncoplanar	$P^{4^{1}_{100}}4_1{}^{3^{1}_{111}}3^{2_{110}}2$	×	×	×

Continued on next page

Table S4 : (continued) Antiferromagnets in MAGNDATA without second-order transport tensors triggered by magnetic geometry.

No.	Materials (ID)	Configuration	Spin space group	with SOC		
				IMD	QMD	BCD
536	Yb_2O_3 (1.720)	Noncoplanar	$I^{2_{100}}\mathrm{a}^{3^2_{-1-11}}\text{-}3\|(1,1,1;\text{-}1)$	×	×	×
537	$CaCu_3Ti_4O_{12}$ (1.775)	Noncoplanar	$I^{1}\mathrm{m}^{3^1_{001}}\text{-}3\|(1,1,1;\text{-}1)$	×	×	×
538	$Dy_3Ru_4Al_{12}$ (1.115)	Noncoplanar	$C^{\mathrm{m}_{001}}2/^{\mathrm{m}_{001}}\mathrm{m}\|(1,1,1;\text{-}1)$	×	×	×
539	$Cu_6(SiO_3)_6(H_2O)_6$ (1.498)	Noncoplanar	$R^{3^1_{001}}\text{-}3\|(1,1,\text{-}1;\text{-}1,1)$	×	×	×
540	Cr_2ReO_6 (1.201)	Noncoplanar	$P^{\mathrm{m}_{001}}2_1/^{\mathrm{m}_{001}}\mathrm{c}\|(\text{-}1,1,1)$	×	×	×
541	$FeSb_2O_4$ (0.97)	Noncoplanar	$P^{2_{001}}4_2/^{2_{010}}\mathrm{m}^{2_{100}}\mathrm{b}^{2_{010}}\mathrm{c}$	σ_{zzz}, $\sigma_{yyz} = \sigma_{yzy} = \sigma_{zyy}$, $\sigma_{xxz} = \sigma_{xzx} = \sigma_{zxx}$	σ_{zzz}, σ_{zyy}, σ_{zxx}, $\sigma_{yyz} = \sigma_{yzy}$, $\sigma_{xxz} = \sigma_{xzx}$	$\sigma_{yyz} = \sigma_{yzy} = -\sigma_{zyy}/2$, $\sigma_{xxz} = \sigma_{xzx} = -\sigma_{zxx}/2$
542	$Co_3(PO_4)_2$ (1.342)	Noncoplanar	$P^{\mathrm{m}_{001}}2_1/^{\mathrm{m}_{001}}\mathrm{c}\|(\text{-}1,1,1)$	×	×	×
543	CaV_2O_4 (1.73)	Noncoplanar	$P^{\mathrm{m}_{001}}2_1/^{\mathrm{m}_{001}}\mathrm{c}\|(\text{-}1,1,1)$	×	×	×

5. Collinear and coplanar antiferromagnets

5.1. Collinear antiferromagnet VNb_3S_6

The AFM VNb_3S_6 is crystallized by 1H-NbS_2 layers in hexagonal lattice with V atoms sandwiched between layers (Fig. S5**a**). The antiferromagnet order has been verified as A-type: the magnetic moments in the same (adjacent) V layer are parallel (anti-parallel) with the Néel vector orientated along a axis [14]. The SSG of VNb_3S_6 is identified as $P^{-1}6_3{}^{-1}2^{1}2^{\infty m}1$. From the notation of SSG, one can directly know that V atoms with opposite magnetic moments are transformed by the spin group symmetry ${}^{-1}6_3 \equiv \{T||R_z(\pi/3)|(1/2)\boldsymbol{\tau}_c\}$ with $\boldsymbol{\tau}_c = (0,0,1)$ instead of PT or $T\boldsymbol{\tau}$. Therefore, VNb_3S_6 is also categorized as a g-wave altermagnet [15,16], where the SOC-free spin splitting is expected off certain high-symmetric lines and planes. To confirm, the band structure without SOC along momenta $\Gamma' - \mathrm{K}' - \mathrm{M}' - \Gamma'$ is calculated and shown as Fig. 3 in the main text, where the momenta are defined as $\Gamma' = \Gamma + 0.1\boldsymbol{b}_3$, $\mathrm{M}' = \mathrm{M} + 0.1\boldsymbol{b}_3$, and $\mathrm{K}' = \mathrm{K} + 0.1\boldsymbol{b}_3$ (Fig. S5**b**).

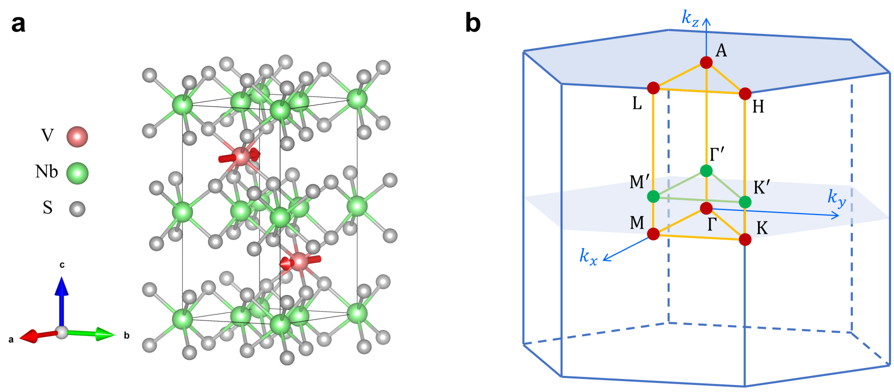

Fig. S5. Crystal structure and Brillouin zone of VNb_3S_6.

Now we consider the NLT tensors allowed by the SSG of VNb_3S_6. Due to the $T_{\mathrm{eff}} = U_z(\pi)T$ emerged in all collinear AFMs, the IMD and QMD contributions are naturally forbidden, leaving the BCD contributions Eq. (S20) to be symmetrically allowed. By referring to the Tab. S1, row 36, we can directly find that the SSG-allowed BCD contributions are

$$\sigma_{\mathrm{BCD}}^{xyz} = \sigma_{\mathrm{BCD}}^{xzy} = -\sigma_{\mathrm{BCD}}^{yxz} = -\sigma_{\mathrm{BCD}}^{yzx}, \tag{S43}$$

where the rest of the components must vanish. Once can check this information from Tab. S1 by using the generators of the nontrivial SSG $P^{-1}6_3{}^{-1}2^12$ to constrain the BCD contributed tensors, where this group is generated by $g_1 = \{T||R_{[100]}(\pi)|(1/2)\boldsymbol{\tau}_c\}$, $g_2 = \{E||R_{[120]}(\pi)|\mathbf{0}\}$, and $g_3 = \{T||R_z(\pi/3)|(1/2)\boldsymbol{\tau}_c\}$. Rewriting $g_i = \{u_i||r_i|\boldsymbol{\tau}_i\}$, we obtain the representation matrices of u_i and r_i, i.e., $\mathcal{U}_i$ and $\mathcal{R}_i$ in Cartesian coordinate as

$$\mathcal{U}_1 = -\mathbf{I}_{3\times3}, \qquad \mathcal{R}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \tag{S43}$$

$$\mathcal{U}_2 = \mathbf{I}_{3\times3}, \qquad \mathcal{R}_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \tag{S44}$$

$$\mathcal{U}_3 = -\mathbf{I}_{3\times3}, \qquad \mathcal{R}_3 = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{S45}$$

where $\mathbf{I}_{3\times3}$ is a 3×3 identity matrix. Following the transformation Eq. (S36), each generator constrains the nonzero BCD-contributed tensor components to be

$$g_1: \sigma_{\mathrm{BCD}}^{yyx} = -\frac{1}{2}\sigma_{\mathrm{BCD}}^{xyy}, \qquad \sigma_{\mathrm{BCD}}^{zzx} = -\frac{1}{2}\sigma_{\mathrm{BCD}}^{xzz}, \qquad \sigma_{\mathrm{BCD}}^{xyz}, \qquad \sigma_{\mathrm{BCD}}^{yzx}, \qquad \sigma_{\mathrm{BCD}}^{zxy}, \tag{S46}$$

$$g_2: \sigma_{\mathrm{BCD}}^{xxy} = -\frac{1}{2}\sigma_{\mathrm{BCD}}^{yxx}, \qquad \sigma_{\mathrm{BCD}}^{zzy} = -\frac{1}{2}\sigma_{\mathrm{BCD}}^{yzz}, \qquad \sigma_{\mathrm{BCD}}^{xyz}, \qquad \sigma_{\mathrm{BCD}}^{yzx}, \qquad \sigma_{\mathrm{BCD}}^{zxy}, \tag{S47}$$

$$g_3: \sigma_{\mathrm{BCD}}^{xxz} = \sigma_{\mathrm{BCD}}^{yyz} = -\frac{1}{2}\sigma_{\mathrm{BCD}}^{zxx} = -\frac{1}{2}\sigma_{\mathrm{BCD}}^{zyy}, \qquad \sigma_{\mathrm{BCD}}^{xyz} = -\sigma_{\mathrm{BCD}}^{yxz}, \tag{S48}$$

where nonzero components related by switching the last two indices are not written explicitly. For example, in Eq. (S46), $\sigma_{\mathrm{BCD}}^{yxy} = \sigma_{\mathrm{BCD}}^{yyx} = -\frac{1}{2}\sigma_{\mathrm{BCD}}^{xyy}$ is also symmetrically allowed by generator g_1. Therefore, the nonzero BCD-contributed tensor components, allowed by both g_1, g_2, g_3 as well as the SSG, are

$$\sigma_{\mathrm{BCD}}^{xyz} = \sigma_{\mathrm{BCD}}^{xzy} = -\sigma_{\mathrm{BCD}}^{yxz} = -\sigma_{\mathrm{BCD}}^{yzx}, \tag{S49}$$

which is consistent with the results in Tab. S1.

The calculated band structures and the transport tensor of VNb_3S_6 without the consideration of SOC are shown in Fig. 3 in the main text. Here we investigate the rule of SOC. The inclusion of SOC lowers the symmetries of the system, splitting the energy degeneracy at high-symmetric momenta. As shown in Fig. S6**a**, without SOC, bands along $\Gamma - A$ are spin degenerate due to the spin group symmetry $g_3 = \{T||R_z(\pi/3)|(1/2)\boldsymbol{\tau}_c\}$, whereas g_3 is broken and the degeneracy is lifted under the inclusion of SOC that fully lock the spin and spatial rotations. Considering SOC lowers the symmetry group of VNb_3S_6 from SSG to magnetic space group $C2'2'2_1$ (#20.33), which allows additional tensors or components to emerging. By referring Tab. S1 again, we find that nonzero IMD and QMD contributions are now triggered by SOC, with the allowed components to be

$$\sigma_{\mathrm{IMD}}^{zzz}, \quad \sigma_{\mathrm{IMD}}^{yyz} = \sigma_{\mathrm{IMD}}^{yzy} = \sigma_{\mathrm{IMD}}^{zyy}, \quad \sigma_{\mathrm{IMD}}^{xxz} = \sigma_{\mathrm{IMD}}^{xzx} = \sigma_{\mathrm{IMD}}^{zxx}, \tag{S50}$$

$$\sigma_{\mathrm{QMD}}^{zzz}, \quad \sigma_{\mathrm{QMD}}^{zyy}, \quad \sigma_{\mathrm{QMD}}^{zxx}, \quad \sigma_{\mathrm{IMD}}^{yyz} = \sigma_{\mathrm{IMD}}^{yzy}, \quad \sigma_{\mathrm{IMD}}^{xxz} = \sigma_{\mathrm{IMD}}^{xzx}. \tag{S51}$$

Moreover, the symmetry-allowed BCD contribution now becomes

$$\sigma_{\mathrm{BCD}}^{xyz} = \sigma_{\mathrm{BCD}}^{xzy}, \quad \sigma_{\mathrm{BCD}}^{yxz} = \sigma_{\mathrm{BCD}}^{yzx}, \quad \sigma_{\mathrm{BCD}}^{zxy} = \sigma_{\mathrm{BCD}}^{zyx}, \tag{S52}$$

indicating that $\sigma_{\mathrm{BCD}}^{zxy}$ and $\sigma_{\mathrm{BCD}}^{xyz} - \sigma_{\mathrm{BCD}}^{yzx}$ are pure SOC effect.

Besides the new components, the value of the $\sigma_{\mathrm{BCD}}^{xyz}$ will change after considering SOC, where the difference in magnitude represents how much of the role SOC plays in nonlinear transports. In Fig. S6**b**, we compare the computed $\sigma_{\mathrm{BCD}}^{yzx}$ with and without SOC as a function of chemical potential. We observe that the value of $\sigma_{\mathrm{BCD}}^{yzx}$ are not significantly affected by SOC in a large energy window ($|E - E_F| \leq 0.3$ eV), where without SOC a peak of $\sigma_{\mathrm{BCD}}^{yzx} = 75\ \mathrm{S}^2/\mathrm{A}$ at $E - E_F \sim 0.25$ eV is only increased by 6% to $80\ \mathrm{S}^2/\mathrm{A}$ with the inclusion of SOC. This results not only indicate the opposite contributions from magnetic geometry and SOC in this system, but, more importantly, showcase that magnetic geometry is the dominate driven force of nonlinear transports in VNb_3S_6 even with SOC counted.

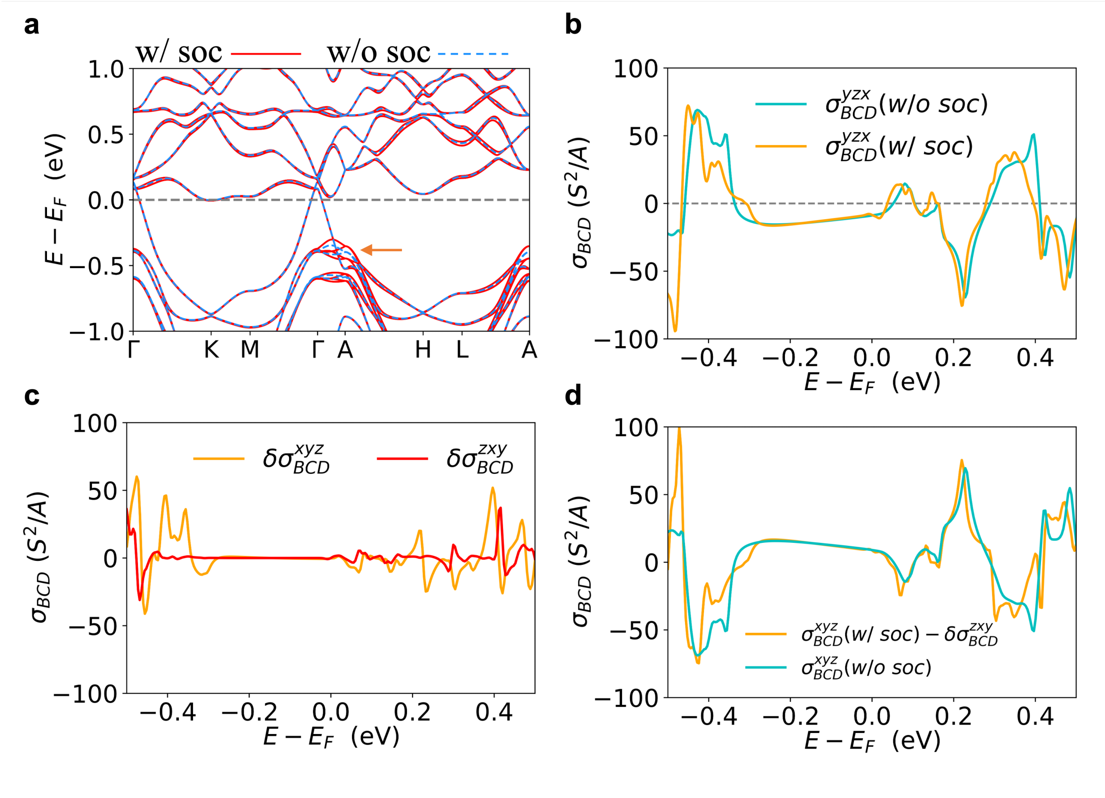

Fig. S6. First-principles calculations of collinear AFM VNb_3S_6. **a**, Band structures with SOC (red line) and without SOC (blue dashed line). The degenerate bands are split after considering SOC, as marked by orange arrow. **b**, Nonlinear conductivity tensor contributed by BCD with SOC (yellow line) and without SOC (cyan line) by setting relaxation time $\tau_r = 10^{-9}\,\mathrm{s} = 1\,\mathrm{ns}$. **c**, Contributions from spin-orbit coupling of distinct tensor components in VNb_3S_6. **d**, Qualitatively extracting the geometric nonlinear conductivity from numerical results with SOC. The number of sampled k points is 1100*1100*1100.

From our database, we found that for VNb_3S_6 the component $\sigma_{\mathrm{BCD}}^{zxy}$ is pure SOC effects, while both $\sigma_{\mathrm{BCD}}^{xyz}$ and $\sigma_{\mathrm{BCD}}^{yxz}$ contain magnetic geometry and SOC contributions. Therefore, we can compare the magnitude of SOC contributions in different components. Defining $\delta\sigma_{\mathrm{BCD}}^{\alpha\beta\gamma} = \sigma_{\mathrm{BCD}}^{\alpha\beta\gamma}(\mathrm{w/\ soc}) - \sigma_{\mathrm{BCD}}^{\alpha\beta\gamma}(\mathrm{w/o\,soc})$ as the SOC contribution, the SOC contributions for different components are presented in Fig. S6**c**. We found that considering SOC does not introduce significant corrections to the conductivity tensor in a large energy window ($|E - E_F| \leq 0.3$ eV). This is because the band structure is only slightly changes within the energy window (Fig. S6**a**). On the contrary, SOC produce considerable contributions only if the degenerate bands near $E - E_F \approx -0.4$ eV are split and crossed after introducing SOC (see orange arrow in Fig. S6**a**). Moreover, as shown in Fig. S6**c**, both SOC induced parts are of the same magnitude over a quite wide range of energy ($|E - E_F| \leq 0.2$ eV). If we try to use the pure SOC signal ($\sigma_{\mathrm{BCD}}^{zxy}$) as reference, we may roughly evaluate the magnitude of the SOC contribution in the entangled signal ($\sigma_{\mathrm{BCD}}^{xyz}$). In other words, one may use the experimentally measurable data $\sigma_{\mathrm{BCD}}^{xyz}(\mathrm{w/\ soc}) - \sigma_{\mathrm{BCD}}^{zxy}$ to qualitatively estimate the geometric nonlinear conductivity $\sigma_{\mathrm{BCD}}^{xyz}(\mathrm{w/o\,soc})$. As presented in Fig. S6**d**, the rough estimation basically recovers the results from SOC-free calculations. The possible rough estimation suggests that one can verify the magnetic geometry contribution by measuring distinct NLT signals in different surface terminations, where $\sigma_{\mathrm{BCD}}^{xyz}$ can be measured in the (0001) surface with $\sigma_{\mathrm{BCD}}^{xyz}$ measured in $(10\bar{1}0)$ surface. This is similar to the verification of spin-splitter torque [17] by comparing the combined contributions [magnetic order + SOC, e.g. RuO_2 (100) surface] and SOC-only contributions [e.g., RuO_2 (110) surface], as adopted by independent experiments [18].

5.2. Coplanar antiferromagnet $Ca_2Cr_2O_5$

The AFM $Ca_2Cr_2O_5$ shares the same structure of brownmillerite $Ca_2(Al, Fe)_2O_5$ but changing all the transition metal atoms to Cr atoms (Fig. S7**a**), whose ligands, O atoms, provide two types of polyhedral crystal fields: one is tetrahedron and the other is octahedron [19]. As for the magnetic configuration, the magnetic moments of Cr atoms inside the adjacent tetrahedron are antiparallel with Néel vector orientated along $\boldsymbol{n}_1 = [0.00, 0.073, 0.241]$, while the magnetic moments inside the adjacent octahedron also antiparallel but with Néel vector $\boldsymbol{n}_2 = [0.00, 0.073, -0.241]$. Therefore, all the magnetic moments of Cr atoms lie on a plane normal to $\boldsymbol{n}_1 \times \boldsymbol{n}_2$, resulting in a coplanar antiferromagnetic order. The SSG of it is identified as $P^1m^1a^12^{2_{001}}(1/2,1/2,1/2)^{m}1$, showing k-type nontrivial SSG and the spin-only part. As mentioned, all coplanar AFMs endure T_{eff} and only BCD contribution on second-order transport is allowed. By referring to Tab. S2, row 54, we predict that the nonzero BCD-contributions are

$$\sigma_{\text{BCD}}^{yyz} = \sigma_{\text{BCD}}^{yzy} = -\frac{1}{2}\sigma_{\text{BCD}}^{zyy}, \qquad \sigma_{\text{BCD}}^{xxz} = \sigma_{\text{BCD}}^{xzx} = -\frac{1}{2}\sigma_{\text{BCD}}^{zxx}. \tag{S53}$$

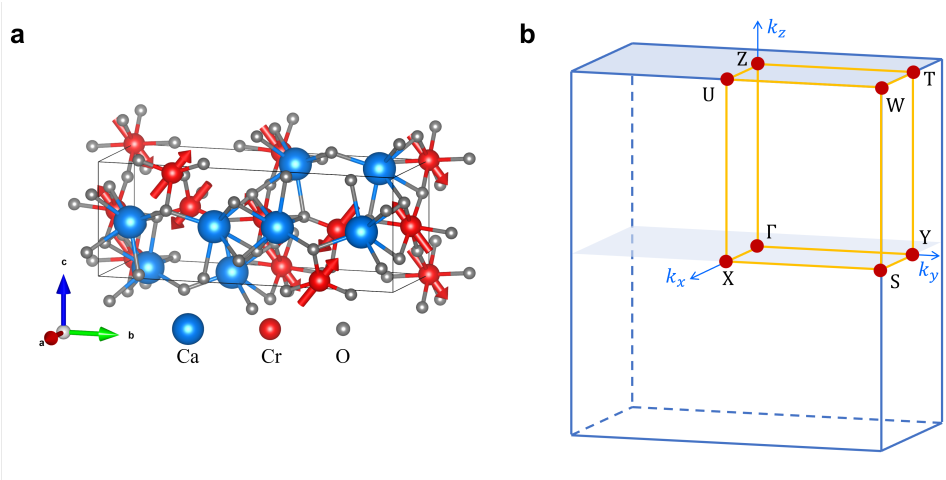

Fig. S7. Crystal structure and Brillouin zone of $Ca_2Cr_2O_5$.

To verify the prediction, we preform the first-principles calculations on it, where the band structures and BCD-contributed transport tensors are shown in Fig. S8. It is shown that without SOC, the Hall-type $\sigma_{\text{BCD}}^{zxx}$ is finite and it behaves a local peak around Fermi energy with maximum $\sim 12\ \text{S}^2/\text{A}$, which is consistent with the information in our database. The inclusion of SOC further breaks the spin symmetry, especially the spin rotation-translation symmetry $\{U_y(\pi)||E|(1/2,1/2,1/2)\}$ and hence band degeneracies along high-symmetric momenta are lifted, as shown in Fig. S8**a**. More interestingly, introducing SOC does not trigger IMD and QMD contributions in the magnetic space group P_C2_1 (#4.12) of $Ca_2Cr_2O_5$ but trigger extra σ_{BCD} components as

$$\sigma_{\text{BCD}}^{xyz} = \sigma_{\text{BCD}}^{xzy}, \qquad \sigma_{\text{BCD}}^{yzx} = \sigma_{\text{BCD}}^{yxz}, \qquad \sigma_{\text{BCD}}^{zxy} = \sigma_{\text{BCD}}^{zyx}, \tag{S54}$$

which is tabulated in Tab. S2. Meanwhile, SOC also modifies the original component $\sigma_{\mathrm{BCD}}^{zxx}$, where the computed results are shown in Fig. S8**b**. It is shown that $\sigma_{\mathrm{BCD}}^{zxx}$ near Fermi energy is nearly quenched by SOC due to the splitting of band degeneracy. However, the magnetic geometry contribution still dominates the SOC effect near $E - E_F = \pm 0.2$ eV where flat bands from d-orbital located.

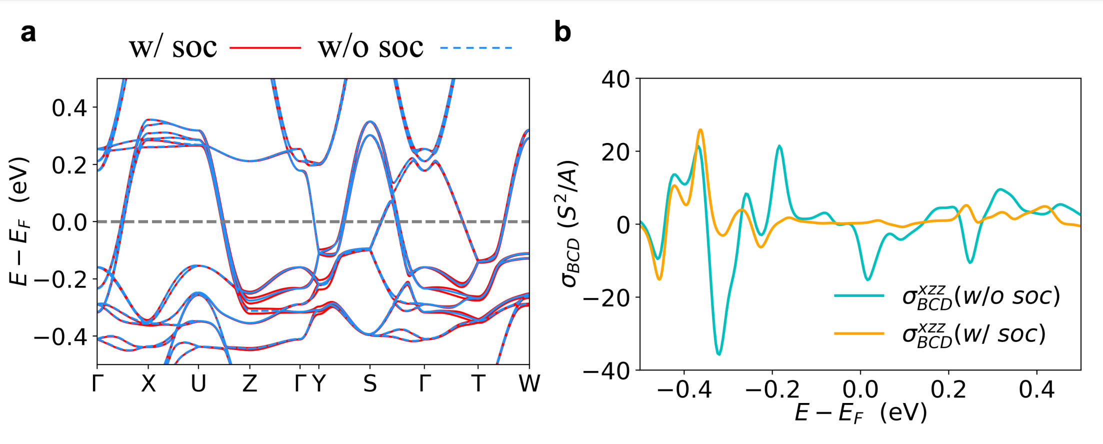

Fig. S8. First-principles calculations of coplanar AFM $Ca_2Cr_2O_5$. **a**, Band structures with SOC (red line) and without SOC (blue dashed line). **b**, Nonlinear conductivity tensor contributed by BCD with SOC (yellow line) and without SOC (cyan line) by setting relaxation time $\tau_r = 10^{-9}\,\mathrm{s} = 1\,\mathrm{ns}$. The number of sampled k points is 1100*1100*1100.

6. Noncoplanar antiferromagnets

6.1. CrSe

CrSe is a room temperature (Néel temperature 290 K) noncoplanar AFM [20,21], shown in Fig. S9**a**. Separated by Se layers, Cr atoms form layers of trigonal sublattice in ab plane, where the in-plane magnetic components inside each Cr layer are related by spin rotation $U_z\left(\frac{2\pi}{3}\right)$ with alternating out-of-plane (along c) magnetic components between neighboring layers. As described in the main text, the SSG of CrSe is $P^{2_{010}}6_3/^{-1}m^{m_{010}}m^{-1}c|(3_{001}^2,3_{001}^2,1)$, where $\{U_{[010]}(\pi)T||P|\mathbf{0}\}$ inside serves as PT_{eff} that forbids the BCD contribution. From Tab. S3, row 16, we obtain that only QMD contribution is allowed by the SSG, of which the finite component would be

$$\sigma_{\text{QMD}}^{xyz}=\sigma_{\text{QMD}}^{xzy}=-\sigma_{\text{QMD}}^{yxz}=-\sigma_{\text{QMD}}^{yzx}. \tag{S55}$$

We note that even IMD and QMD terms follow the same transformation under SSG elements, i.e., Eq. (S37), they behave differently due to the distinct intrinsic symmetry. More explicitly, the IMD contribution is prohibited by its full symmetric property under any permutation between indices.

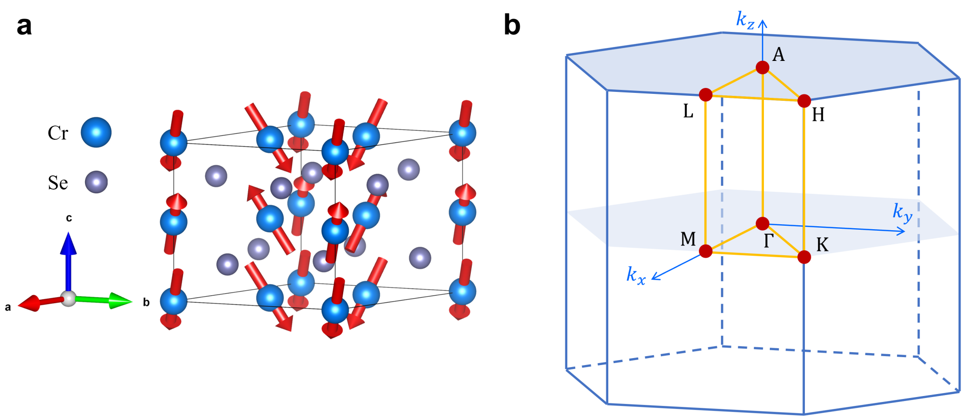

Fig. S9. Crystal structure and Brillouin zone of CrSe.

The results from first-principles calculations are shown in Fig. S10. Without SOC, the complex band structures (Fig. S10**a**) contribute to both significant quantum metric tensor and QMD-contributed tensor (Fig. 4 in the main text). The nonzero value of the component $\sigma_{\mathrm{QMD}}^{xyz}$ also in line with the prediction in Tab. S3. Considering SOC explicitly split the bands across the Fermi energy. This in turn conduct to the way weaker QMD contribution, which is near two order smaller than the pure magnetic geometry effect. However, the total $\sigma_{\mathrm{QMD}}^{xyz}$ with maximum $\sim 0.2\ \mathrm{S}^2/\mathrm{A}$, including the magnetic geometry and SOC effects, is already larger than the reported QMD-contributed tensor $\sim 0.01\ \mathrm{S}^2/\mathrm{A}$ in $MnBi_2Te_4$ thin film [22].

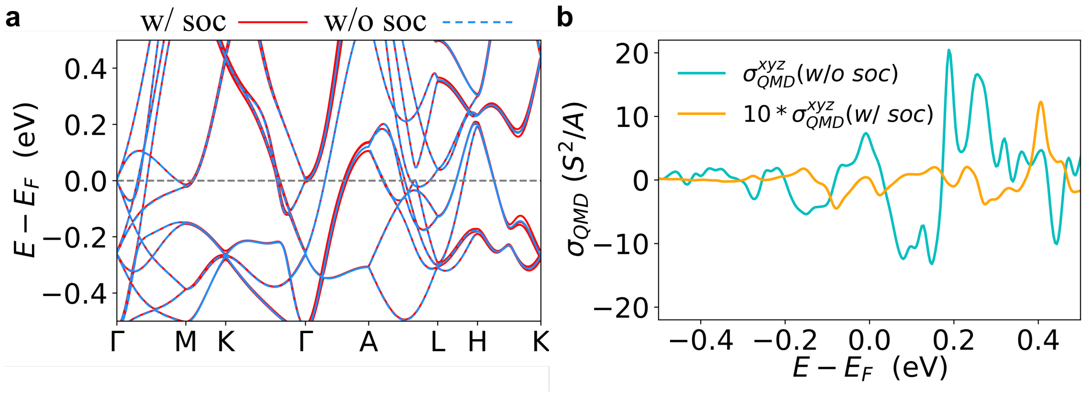

Fig. S10. Results of noncoplanar AFM CrSe. **a**, Band structures with SOC (red line) and without SOC (blue dashed line). **b**, Nonlinear conductivity tensor contributed by QMD with SOC (yellow line) and without SOC (cyan line). The number of sampled k points is 1700*1700*1700.

6.2. CuB_2O_4

The copper metaborate CuB_2O_4 crystallizes in a tetragonal lattice with $a = b = 11.458$ Å and $c = 5.620$ Å with noncoplanar antiferromagnetic order [23] (Fig. S11**a**). The SSG of CuB_2O_4 is identified as $C^{-1}c$, indicating that the opposite magnetic moments on Cu atoms are connected by $\{T||m_{[110]}|\boldsymbol{\tau}_b\}$. From Tab. S3, row 37, we find that both IMD, QMD, and BCD contributions are allowed. If setting $[110]$ direction as the principal axis, the nonzero components are found to be

$$\begin{gathered}
\sigma_{\mathrm{IMD}}^{xxx}=\sigma_{\mathrm{IMD}}^{yyy},\\
\sigma_{\mathrm{IMD}}^{xxz}=\sigma_{\mathrm{IMD}}^{xzx}=\sigma_{\mathrm{IMD}}^{zxx}=-\sigma_{\mathrm{IMD}}^{yyz}=-\sigma_{\mathrm{IMD}}^{yzy}=-\sigma_{\mathrm{IMD}}^{zyy},\\
\sigma_{\mathrm{IMD}}^{xxy}=\sigma_{\mathrm{IMD}}^{xyx}=\sigma_{\mathrm{IMD}}^{yxx}=\sigma_{\mathrm{IMD}}^{xyy}=\sigma_{\mathrm{IMD}}^{yxy}=\sigma_{\mathrm{IMD}}^{yyx},\\
\sigma_{\mathrm{IMD}}^{xzz}=\sigma_{\mathrm{IMD}}^{zxz}=\sigma_{\mathrm{IMD}}^{zzx}=\sigma_{\mathrm{IMD}}^{yzz}=\sigma_{\mathrm{IMD}}^{zyz}=\sigma_{\mathrm{IMD}}^{zzy},
\end{gathered}\tag{S56}$$

$$\begin{gathered}
\sigma_{\mathrm{BCD}}^{yxy}=\sigma_{\mathrm{BCD}}^{yyx}=-\sigma_{\mathrm{BCD}}^{xxy}=-\sigma_{\mathrm{BCD}}^{xyx}=\frac{1}{2}\sigma_{\mathrm{BCD}}^{yxx}=-\frac{1}{2}\sigma_{\mathrm{BCD}}^{xyy},\\
\sigma_{\mathrm{BCD}}^{zyz}=\sigma_{\mathrm{BCD}}^{zzy}=-\sigma_{\mathrm{BCD}}^{zxz}=-\sigma_{\mathrm{BCD}}^{zzx}=\frac{1}{2}\sigma_{\mathrm{BCD}}^{xzz}=-\frac{1}{2}\sigma_{\mathrm{BCD}}^{yzz},\\
\sigma_{\mathrm{BCD}}^{xxz}=\sigma_{\mathrm{BCD}}^{xzx}=-\sigma_{\mathrm{BCD}}^{yyz}=-\sigma_{\mathrm{BCD}}^{yzy}=\frac{1}{2}\sigma_{\mathrm{BCD}}^{zyy}=-\frac{1}{2}\sigma_{\mathrm{BCD}}^{zxx},\\
\sigma_{\mathrm{BCD}}^{xyz}=\sigma_{\mathrm{BCD}}^{xzy}=\sigma_{\mathrm{BCD}}^{yxz}=\sigma_{\mathrm{BCD}}^{yzx}=-\sigma_{\mathrm{BCD}}^{zxy}=-\sigma_{\mathrm{BCD}}^{zyx},
\end{gathered}\tag{S57}$$

$$\begin{gathered}
\sigma_{\mathrm{QMD}}^{xxx}=\sigma_{\mathrm{QMD}}^{yyy},\\
\sigma_{\mathrm{QMD}}^{xyy}=\sigma_{\mathrm{QMD}}^{yxx},\qquad \sigma_{\mathrm{QMD}}^{xzz}=\sigma_{\mathrm{QMD}}^{yzz},\qquad \sigma_{\mathrm{QMD}}^{zxx}=-\sigma_{\mathrm{QMD}}^{zyy},\\
\sigma_{\mathrm{QMD}}^{xxy}=\sigma_{\mathrm{QMD}}^{xyx}=\sigma_{\mathrm{QMD}}^{yxy}=\sigma_{\mathrm{QMD}}^{yyx},\\
\sigma_{\mathrm{QMD}}^{zxz}=\sigma_{\mathrm{QMD}}^{zyz}=\sigma_{\mathrm{QMD}}^{zzx}=\sigma_{\mathrm{QMD}}^{zzy},\\
\sigma_{\mathrm{QMD}}^{xxz}=\sigma_{\mathrm{QMD}}^{xzx}=-\sigma_{\mathrm{QMD}}^{yyz}=-\sigma_{\mathrm{QMD}}^{yzy},\\
\sigma_{\mathrm{QMD}}^{xyz}=\sigma_{\mathrm{QMD}}^{xzy}=-\sigma_{\mathrm{QMD}}^{yxz}=-\sigma_{\mathrm{QMD}}^{yzx}.
\end{gathered}\tag{S58}$$

To verify it, we calculated the band structures and the transport tensor of CuB_2O_4 without the consideration of SOC are shown in Fig. S11. All the numerical results show consistence with the symmetry analysis.

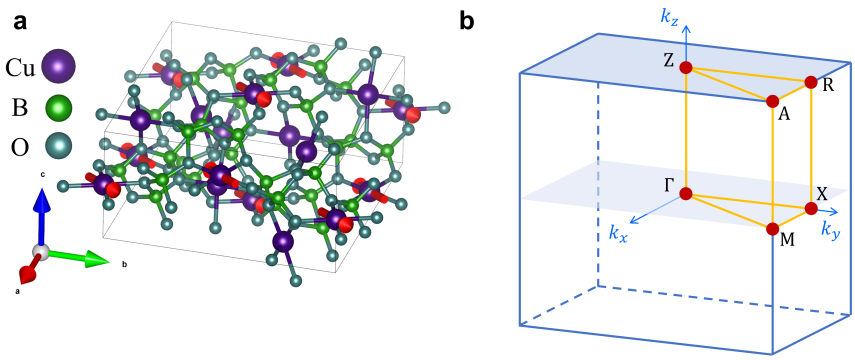

Fig. S11. Crystal structure and Brillouin zone of CuB_2O_4.

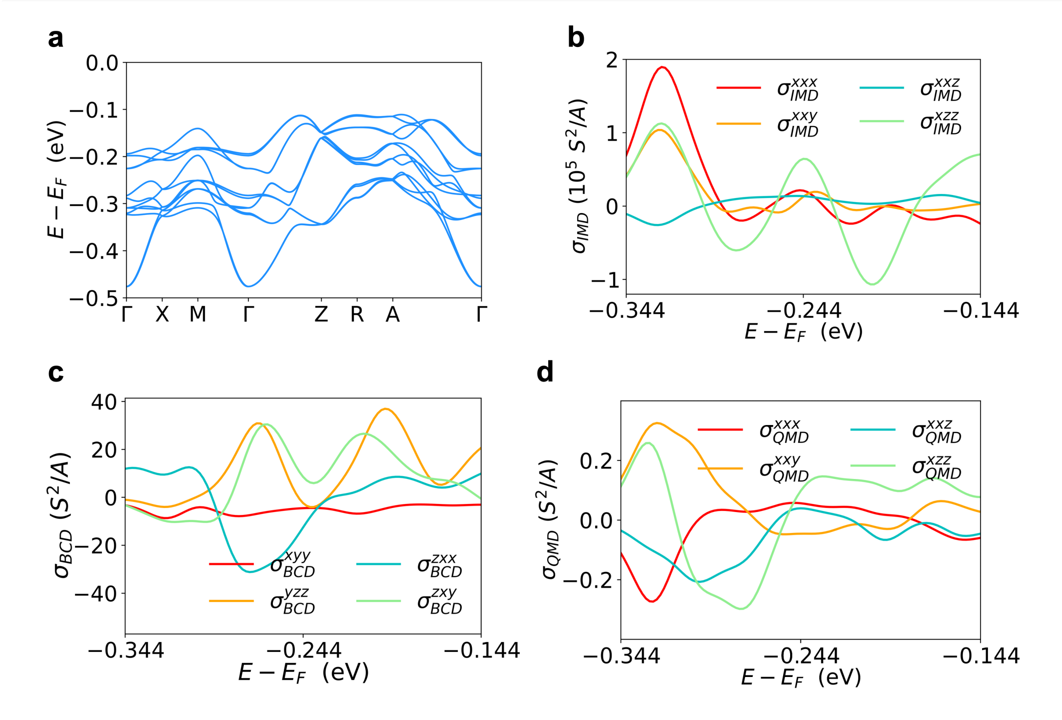

Fig. S12. Results of noncoplanar AFM CuB_2O_4. **a**, Band structures without SOC. **b-d**, Nonlinear conductivity tensor contributed by IMD (**b**), BCD (**c**), and QMD (**d**) without SOC by setting relaxation time $\tau_r = 10^{-9}$ s $= 1$ ns. The number of sampled k points are 990*990*990 for calculating IMD and BCD contributions, and 1700*1700*1700 for calculating QMD contribution.

6.3. Mn_3CoGe

The Mn-based transition compounds Mn_3CoGe, Mn_3IrGe, and Mn_3IrSi are both crystallized in a cubic lattice [24] (Fig. S13**a**). They also have similar noncoplanar antiferromagnetic order that the angles between any two magnetic moments of Mn are 120 degrees. Here we take Mn_3CoGe as an example as the other two compounds share the same symmetries with Mn_3CoGe. The SSG of Mn_3CoGe is $P^{2_{100}}2_1{}^{3_{111}^{1}}3$, implying two generators $\{U_{[100]}(\pi)||R_{[100]}(\pi)|\boldsymbol{\tau}_a\}$ and $\left\{U_{[111]}\left(\frac{2\pi}{3}\right)\middle|\middle|R_{[111]}\left(\frac{2\pi}{3}\right)\middle|\mathbf{0}\right\}$. Interestingly, the axes and angles of spin and spatial rotations in both generators are same, leading to an important fact that the SSG and magnetic space group of Mn_3CoGe (Mn_3IrGe and Mn_3IrSi) are exact the same. This can be directly seen from Tab. S3, row

15, that the nonzero components of second-order transport are same under the SSG and magnetic space group of Mn_3CoGe. More explicitly, only one individual component of both IMD and QMD contributions is allowed, as

$$\sigma^{xyz}_{\mathrm{IMD/QMD}} = \sigma^{xzy}_{\mathrm{IMD/QMD}} = \sigma^{yzx}_{\mathrm{IMD/QMD}} = \sigma^{yxz}_{\mathrm{IMD/QMD}} = \sigma^{zxy}_{\mathrm{IMD/QMD}} = \sigma^{zyx}_{\mathrm{IMD/QMD}}. \quad \text{(S59)}$$

We note that the absence of PT_{eff} seems to allow BCD contribution. However, the three-fold rotational symmetry along $[111]$ direction connects $\sigma^{xyz}_{\mathrm{BCD}} = \sigma^{yzx}_{\mathrm{BCD}} = \sigma^{zxy}_{\mathrm{BCD}}$, combining with the intrinsic constraint Eq. (S40) ($\sigma^{xyz}_{\mathrm{BCD}} + \sigma^{yzx}_{\mathrm{BCD}} + \sigma^{zxy}_{\mathrm{BCD}} = 0$) enforces them to be zero. Therefore, it also hints us that any perturbation destroying the three-fold rotational symmetry significantly will turn on the BCD contributed tensor.

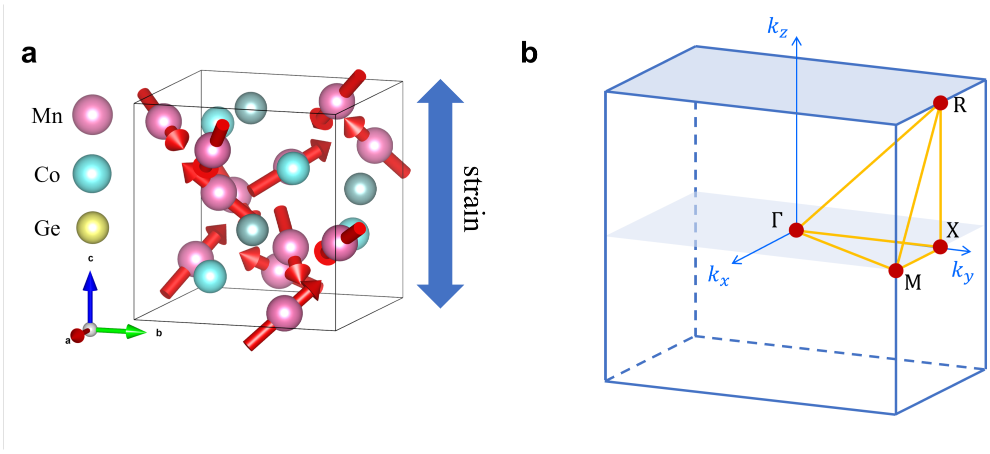

Fig. S13. Crystal structure and Brillouin zone of Mn_3CoGe.

Now we conduct calculations on the realistic Mn_3CoGe crystal, where the results are shown in Fig. S14. The band structures validate the metallic ground state of Mn_3CoGe, where finite $\sigma^{xyz}_{\mathrm{IMD/QMD}}$ component presents in wide energy range. Meanwhile, all the tensor components from the BCD contribution vanish, verifying the correctness of our framework. As mentioned above, any perturbation breaking three-fold rotation typically unloose the constraint on σ_{BCD}, where uniaxial strain is a simple choice. Hence, a uniaxial strain along c axis is introduced (Fig. 13**a**) and its effect is simulated by 3% extension of the lattice constant of $\boldsymbol{c}$. As a result, the band degeneracies at R are basically lifted after strain-engineering, as shown by energy bands in gray of Fig.

S14. More importantly, the SSG is now becomes $P^{2_{100}}2_1{}^{2_{010}}2_1{}^{2_{001}}2_1$, leading to the emerge of two independent components of σ_{BCD}:

$$\sigma_{\mathrm{BCD}}^{xyz} = \sigma_{\mathrm{BCD}}^{xzy}, \qquad \sigma_{\mathrm{BCD}}^{yzx} = \sigma_{\mathrm{BCD}}^{yxz}, \tag{S60}$$

where $\sigma_{\mathrm{BCD}}^{zxy} = \sigma_{\mathrm{BCD}}^{zyx} = -\sigma_{\mathrm{BCD}}^{xyz} - \sigma_{\mathrm{BCD}}^{yzx}$ is not independent. Such an analysis is confirmed by our numerical results in Fig. S14**d**, where an observable value of $\sigma_{\mathrm{BCD}} = 10{\sim}30\ \mathrm{S}^2/\mathrm{A}$ by few percentages of structure deformation under strain-engineering.

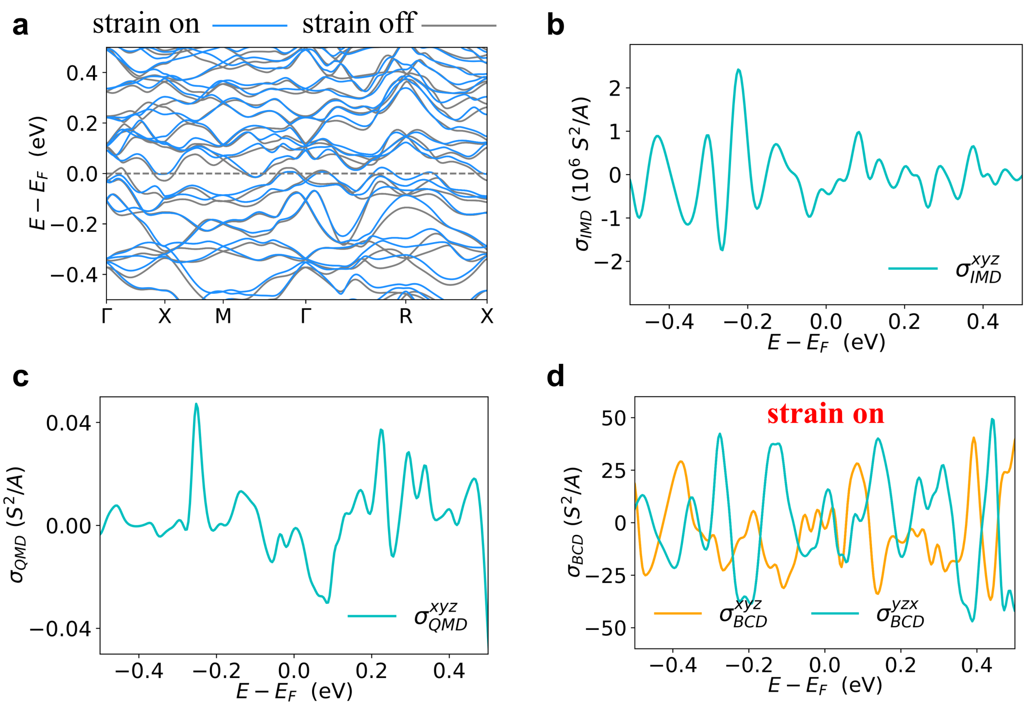

Fig. S14. Results of noncoplanar AFM Mn3CoGe. **a**, Band structures without SOC. **b-c**, Nonlinear conductivity tensor contributed by IMD (**b**) and QMD (**c**) without SOC by setting relaxation time $\tau_r = 10^{-9}\ \mathrm{s} = 1\ \mathrm{ns}$. **d**, QMD contribution induced by uniaxial strain along c axis. The number of sampled *k* points are 550*550*550 for calculating IMD and BCD contributions, and 1500*1500*1500 for calculating QMD contribution.

7. Angle-resolved transport measurement

Here we propose an experimentally realizable method to potentially disentangle the partial contributions of SOC via experimental data. Firstly, our database predicts components of nonlinear conductivities that are nonzero only in the presence of SOC, which is the most straightforward and realizable way to measure the SOC contributions. However, for other components that are nonzero with and without SOC, fully disentangling the SOC effects seems impossible only from experimental data without numerical calculations. To the best of our knowledge, we think the most feasible way is to extract the partial contributions from SOC by angle-resolved transport measurement.

Taking altermagnet VNb_3S_6 as an example, spin group symmetry analysis predicts the SOC-free components from BCD is $\sigma_{xyz} = \sigma_{xzy} = -\sigma_{yxz} = -\sigma_{yzx}$, while both IMD and QMD contributions vanish. Once SOC is precent, IMD and QMD are emerged as purely SOC effect, and now the BCD contributions, predicted by magnetic group symmetry, are $\sigma^{zxy} = \sigma^{zyx}$, $\sigma^{xyz} = \sigma^{xzy}$, and $\sigma^{yxz} = \sigma^{yzx}$. Due to the less symmetry constraints, components σ^{xyz} and σ^{yxz} are now disconnected. As shown in Fig. S15, one can measure the crossed nonlinear transports by applying both out-of-plane electric field $E_z(\omega_1)$ and in-plane electric field $\boldsymbol{E}(\omega_2) = (E_x, E_y)(\omega_2)$ where the electric field are dynamical with small frequencies ω_1 and ω_2, respectively. The current density of frequency $\omega_1 + \omega_2$ is then the measurable second-order signal $J_\alpha = \sigma_{\mathrm{BCD}}^{\alpha\beta z} E_\beta E_z$ with $\alpha, \beta = x, y$, where we concentrate on the BCD contributions that can be extracted by scaling low of relaxation time. In the local (crystalline) coordinate system and with SOC, the nonzero components of the conductivity are $\sigma_{\mathrm{BCD}}^{xyz}$ and $\sigma_{\mathrm{BCD}}^{yxz}$, and $\sigma_{\mathrm{BCD}}^{xyz} \neq \sigma_{\mathrm{BCD}}^{yxz}$ in general. Remine that spin-group-symmetry analysis predict that the SOC-free contribution is $\sigma_{\mathrm{BCD},0}^{xyz} = -\sigma_{\mathrm{BCD},0}^{yxz} = \sigma_0$. Hence, the total conductivity can be expressed as $\sigma_{\mathrm{BCD}}^{xyz} = \sigma_0 + \delta\sigma_x$ and $\sigma_{\mathrm{BCD}}^{yxz} = -\sigma_0 + \delta\sigma_y$, where $\delta\sigma_x$ and $\delta\sigma_y$ account for the SOC corrections. Now one can measure different current density via changing the azimuth angle θ, i.e., choosing distinct leads, and for each angle θ one measures both the longitudinal and transversal current density. This is equivalent to measure the conductivity tensor contributed by BCD in the laboratory coordinate system, i.e., $J_{\alpha'} = \sigma_{\text{BCD}}^{\alpha'\beta' z} E_z E_{\beta'}$ with $\alpha', \beta' = x', y'$. After simple algebra, the conductivity tensor in the rotated laboratory coordinate system reads

$$\sigma_{\text{BCD}}^{x'x'z} = \frac{1}{2}(\delta\sigma_x + \delta\sigma_y)\sin 2\theta = -\sigma_{\text{BCD}}^{y'y'z}, \tag{S61}$$

$$\sigma_{\text{BCD}}^{x'y'z} = \sigma_0 + \frac{1}{2}(\delta\sigma_x - \delta\sigma_y) + \frac{1}{2}(\delta\sigma_x + \delta\sigma_y)\cos 2\theta = -\sigma_{\text{BCD}}^{x'y'z}. \tag{S62}$$

Therefore, by fitting the π-period longitudinal signal $\sigma_{\text{BCD}}^{x'x'z}$ one extracts the average correction from SOC as $\frac{1}{2}(\delta\sigma_x + \delta\sigma_y)$, and obtains the rest part $\sigma_0 + \frac{1}{2}(\delta\sigma_x - \delta\sigma_y)$ through the transversal signal $\sigma_{\text{BCD}}^{x'y'z}$. Even though the geometric contribution σ_0 has not been directly extracted, combining the rest part $\sigma_0 + \frac{1}{2}(\delta\sigma_x - \delta\sigma_y)$ and results from different surfaces $\sigma_0 \approx \sigma_{\text{BCD}}^{xyz} - \sigma_{\text{BCD}}^{zxy}$, one can quantificationally estimate the geometric NLT effects from experiments [$\sigma_{\text{BCD}}^{xyz}$ can be measured in the (0001) surface with $\sigma_{\text{BCD}}^{xyz}$ measured in $(10\bar{1}0)$ surface, see Section 5.1].

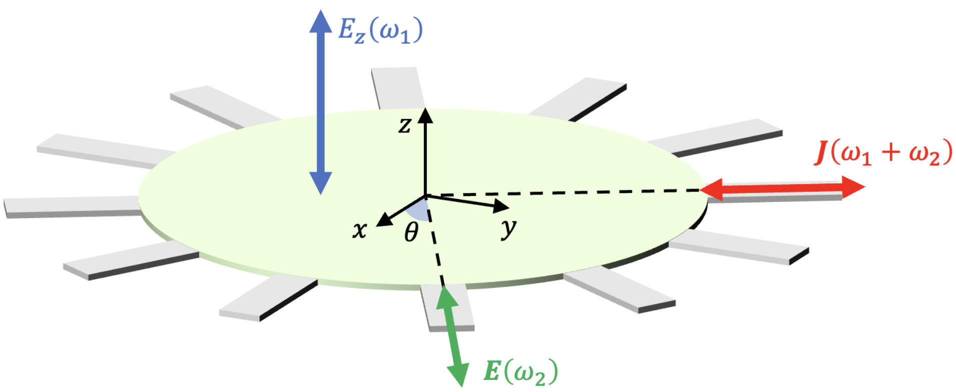

Fig. S15. Sketch of the angle-resolved measurement of nonlinear transports of VNb_3S_6 to partially extract the SOC contributions.

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