

Low-noise frequency synthesis and terahertz wireless communication driven by compact turnkey Kerr combs

Corresponding Author: Professor Zhenda Xie

This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

In this manuscript, the authors study the fundamental problem in the Kerr comb application, for the low noise comb in a compact setup. They develop a different model and show that high Q is the key to access low quantum noise limit when the pump power is fixed. This model is valid even though it is different from people's common understanding. As a result, they successfully generate a Kerr comb with lowest free-running phase noise up to date. Indeed, the noise performance is far better than many previous important works on microcombs. This comb is used to generate a particularly interesting repetition rate covering 283 GHz to 384 GHz that is of special importance for next-generation wireless communication. The manuscript is well-written and logically structured. The presented results are likely to attract considerable interest from researchers in the fields of Kerr combs, microwave photonics, and THz communication. In my opinion, the paper is suitable for publication in Nature Communications after addressing the following comments:

1. During the self-injection of soliton Kerr comb, how much optical power is ideal for stable injection locking?
2. I appreciate that the authors measure the pulse shape of the single soliton state, which is always absent in most papers on Kerr comb. What is the pulse looks like for the multiple soliton crystal states shown in fig. 2b?
3. In fig. 2, the authors demonstrate the turnkey operation of a single soliton. However, it is worth noting that only soliton crystals are utilized in terahertz communication applications. Could the authors clarify whether soliton crystals can also be generated through a similar turnkey operation?
4. How was the pump phase noise presented in fig.1d calculated? Given that the pump phase noise limit is associated with the original laser diode phase noise, as indicated by eq. 3. The results shown in fig. 3d appear to make more sense.
5. In fig. S2, the authors show a "turnkey" zone, which is relatively small in the whole parameter space. How to guarantee the excitation of the single soliton state?
6. A recent work [Xing Jin et al. 2024 PRL] points out that the dispersive waves can be used to further compress the quantum noise limit. Can this method be used in your FFPR geometry?
7. Does the power of the comb line affect the measured THz phase noise? Is the ASE effect of the EDFA significant?

Reviewer #2

(Remarks to the Author)

The paper is dealing with the use of low phase noise synthesizer using Kerr based combs, that are coming with compactness compared to OFCs. Authors show that they can run a THz link without CPE, which is of interest. However, I have some comments to be addressed in the paper to make it more detailed and strong.

On the figure 2, the power level measured versus optical power (panel (f)) is showing saturations for 303 GHz, I believe due

to the LNA used. What about the LNA specifications and impact of this saturation on the overall THz system?

On this figure 2, it would be good to show the filtered optical signals, to assess the side mode suppression ratios. What kind of filter is used (panel (c) in this figure 2).

In the figure 3, comparisons between heterodyne mixing and KCTS/KCTS mixing. However:

- the mixing is not done in the same device (BAM or IHM), what about noise figure of these BAM or IHM, and required LO power? Why not measuring the performances using a single down-converter to avoid introducing testing-system variations?
- we don't know here if the LNA is used in its linear operation range, do the authors expect some effects, what are these effects?

- The KCTS#1 is used as "LO" signal in case of KCTS/KCTS mixing, whereas the KCTS#1 is used as RF. What is the reason for this?

- Is the multiplied LO by 24 spurious-free when used as down-conversion pump signal? It is not mentioned in this study if this was checked or not. It is important to really compare the spectra of the 2 local oscillators used in the experiments to be able to properly compare these 2.

In the THz communication setup (text and figure 4):

- What is the source of the spurious signals in the Figure 4.c spectra?

- For 40 GB, the spectra seems to be cut at low frequencies, can the authors zoom on this? This seems to be not expected as in Figure 4.a the optical signal is symmetric.

- A roll-off factor of 0.01 is used for 40 GB, however it seems to be too small, can the authors check this?

- Can the optical filter being tuned to change the THz carrier frequency? What are the minimal filter specifications required?

- Are the antennas used in their near-field region? It seems to me that this is the case, considering the size of these antennas. The system would have to come with a link budget here, even if the THz link itself seems to be not the core of the paper.

- As authors have the usual electronic down-converter, it would be good to set the reference also with the usual THz receiver which is an electronics-based mixer.

In order to properly compare the power consumption of the 2 approaches, it would be good to consider the overall consumption and complexity. Here, while the performance using KCTS is better than ECL (non-locked lasers), obviously using two Kerr combs seems to be quite complex for future integration. As an example, the optical filtering is required at KCTS output, as well as several EDFAs. What are the minimal specs required for the filter?

The work is interesting and should be detailed and completed.

Reviewer #3

(Remarks to the Author)

The authors proposed and demonstrated a microwave signal generator with low phase noise using a few-mode-fiber-based Fabry-Perot resonator (FFPR) pumped by a DFB laser which is locked by the FFPR. Compared to an integrated Kerr comb, the phase noise performance is notably improved. The use of the FFPR for microwave generation at frequencies as high as 300 GHz was demonstrated. The use of the generated microwave for data transmission was also experimentally demonstrated. This is an interesting work, and I support the publication of the paper in NC. Below are a few comments that the authors may wish to address:

1. More details about the self-locking of the DFB laser should be added including a simulation to compare the simulation result with the experimental result.

2. How can the fundamental linewidth of 72.2 MHz be accurately measured for the DFB laser that is locked to the FFPR? Additional details regarding the measurement setup, techniques, and instrumentation used should be included.

3. If the DFB laser is not self injection locked, what is the phase noise performance of the FFPR for microwave generation?

4. It is generally known that a higher Q factor will lead to a lower quantum noise limit. Why on page 4 line 91, it says that "the low quantum noise limit can be achieved with low Q and large mode area"

5. Some minor grammar corrections: "repetition frequency" may be changed to "frequency" or "repetition rate". "State-of-art" should be changed to "state-of-the-art"

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

I appreciate the authors' efforts in revising the manuscript. They have provided comprehensive responses to all my previous concerns, and the newly added content significantly improves the clarity and robustness of the work. The revised version addresses my reservations adequately, and I find the current manuscript suitable for publication in Nature Communications.

Reviewer #2

(Remarks to the Author)

Authors did the required corrections. This is a interesting paper.

Reviewer #3

(Remarks to the Author)

The authors have adequately addressed the issues I raised in the first round of review and have incorporated the necessary changes into the revised manuscript. I am satisfied with the revisions and recommend the manuscript for publication in NC.

Open Access This Peer Review File is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made.

In cases where reviewers are anonymous, credit should be given to 'Anonymous Referee' and the source.

The images or other third party material in this Peer Review File are included in the article's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.

To view a copy of this license, visit <https://creativecommons.org/licenses/by/4.0/>

Responses to Reviewers' comments:

Reviewer #1: (Remarks to the Author)

In this manuscript, the authors study the fundamental problem in the Kerr comb application, for the low noise comb in a compact setup. They develop a different model and show that high Q is the key to access low quantum noise limit when the pump power is fixed. This model is valid even though it is different from people's common understanding. As a result, they successfully generate a Kerr comb with lowest free-running phase noise up to date. Indeed, the noise performance is far better than many previous important works on microcombs. This comb is used to generate a particularly interesting repetition rate covering 283 GHz to 384 GHz that is of special importance for next-generation wireless communication. The manuscript is well-written and logically structured. The presented results are likely to attract considerable interest from researchers in the fields of Kerr combs, microwave photonics, and THz communication. In my opinion, the paper is suitable for publication in Nature Communications after addressing the following comments:

Our response:

Thanks for your positive comments and suggestion for the publication in Nature Communications.

1. During the self-injection of soliton Kerr comb, how much optical power is ideal for stable injection locking?

Our response:

In our self-injection locking setup, the feedback strength, i. e., the proportion of optical power injected back into laser diode in total output power of laser diode, is one of the key parameters. It determines the laser linewidth narrowing factor and influences the system stability. Increasing feedback strength is beneficial for reaching high linewidth narrowing factor according to Eq. (2) in the manuscript, while too strong feedback will break the stable operation and result in relaxation oscillation of the diode laser. For the packaged device used in experiment, the power of the feedback light for injection locking is set to be ~1 mW, for stable self-injection locking operation as well as highly-reduced laser linewidth.

2. I appreciate that the authors measure the pulse shape of the single soliton state, which is always absent in most papers on Kerr comb. What is the pulse looks like for the multiple soliton crystal states shown in fig. 2b?

Our response:

Thanks for your comments. The pulse shape of the single soliton state is measured by

using an autocorrelator. In this measurement, the single soliton must be power-amplified by an EDFA to ensure sufficient high pulse energy. For multiple soliton crystal states, however, they feature much higher repetition rate than that of single soliton state, making it much more challenging to achieve high pulse energy as required for effective measurement using the autocorrelator. We can refer to the results of the simulation. Fig. R1 shows the simulated spectrum and corresponding time-domain pulse trace of 30-soliton crystal state. In time domain, 30 pulses are equally spaced with consistent peak intensity in a roundtrip time of 100 ps for 10 GHz FFPR. These results have been added in our Supplementary Information (S4.2. Soliton crystal).

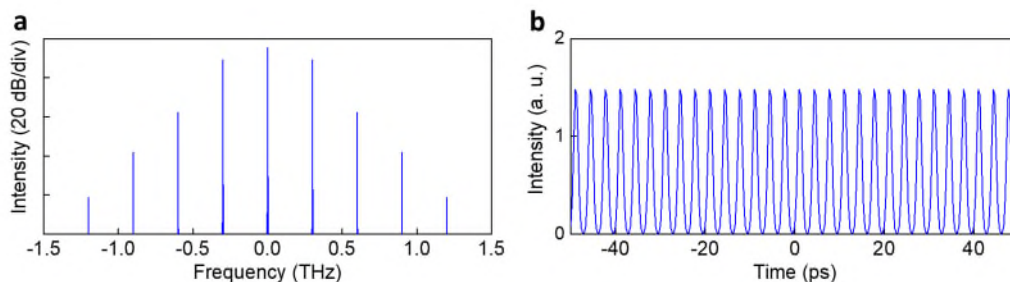


Fig. R1. Simulated 30-soliton crystal state. **a**, Frequency domain. **b**, Time domain.

Following statements are added on page 7, lines 197-200 in the revised manuscript:

“However, it is much more challenging for soliton crystal states with high repetition rate to achieve high pulse energy as required for effective measurement using the autocorrelator. We show its simulated time-domain pulse trace in Supplementary Information.”

3. In fig. 2, the authors demonstrate the turnkey operation of a single soliton. However, it is worth noting that only soliton crystals are utilized in terahertz communication applications. Could the authors clarify whether soliton crystals can also be generated through a similar turnkey operation?

Our response:

Yes, the soliton crystal states can also be generated through turnkey operation. Fig. R2a shows the measured comb power as a function of time in 8-time consecutive switches test for turnkey operation, which indicates good stability and reliability. Fig. R2b is the corresponding spectrum of the soliton crystal state.

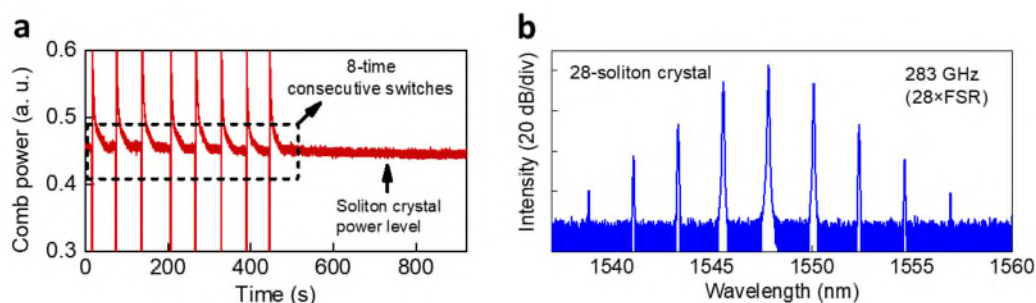


Fig. R2. **a**, Measured comb power as a function of time in 8-time consecutive switches test for turnkey operation of soliton crystal state. **b**, Optical spectrum of soliton crystal state with 283 GHz comb spacing.

Following statements are added on page 7, lines 216-218 in the revised manuscript:

“Note that the soliton crystal states can also be generated through such turnkey operation (see details in Supplementary Information), which is utilized for THz frequency synthesis in the following.”

4. How was the pump phase noise presented in fig.1d calculated? Given that the pump phase noise limit is associated with the original laser diode phase noise, as indicated by eq. 3. The results shown in fig. 3d appear to make more sense.

Our response:

The Fig. 1d is plotted for comparing the quantum noise limit, pump noise limit and thermorefractive noise limit for our FFPR in simulation. For pump noise limit, it can be calculated by subtracting noise reduction factor of self-injection locking and frequency division factor from the phase noise of the unlocked free-running DFB laser. We firstly measure the phase noise of the unlocked free-running DFB laser as shown in Fig. R3, which can also be derived from the red curve of Fig. 1e in the manuscript. As marked in Fig. R3, we get typical phase noise values for different offset frequencies, i. e., 80, 40, 5, -30 dBc/Hz at 10, 100, 1k, 10k Hz offset frequency, respectively. Eq. (2) in the manuscript gives the noise reduction factor, which can be calculated to be about 51 dB with the simulation parameters presented in the manuscript. The frequency division factor can be calculated to be about 86 dB considering the 10 GHz comb repetition rate and 194 THz optical carrier. As a result, we get the pump noise limit in Fig. 1d as -57, -97, -132, -167 dBc/Hz at 10, 100, 1k, 10k Hz offset frequency, respectively.

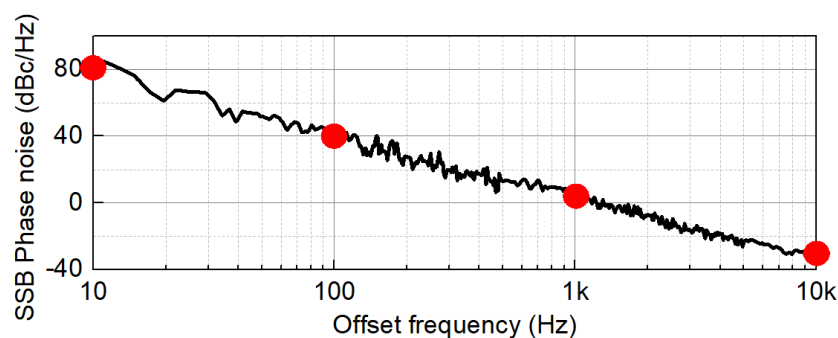


Fig. R3. Measured phase noise of the unlocked free-running DFB laser.

Following statements are added on page 5, lines 145-147 in the revised manuscript:

“The pump noise limit is calculated from the noise of the unlocked free-running DFB laser as shown in Fig. 1e, and taking into account the self-injection locking linewidth narrowing and frequency division effects.”

5. In fig. S2, the authors show a "turnkey" zone, which is relatively small in the whole parameter space. How to guarantee the excitation of the single soliton state?

Our response:

It is true that the microcomb generation based on self-injection locking is feedback phase-sensitive, which impacts the pump laser detuning at the locking point. In experiment, we adjust the feedback phase in self-injection locking loop by the resistive heating of the silicon sheet as marked in Fig. 1e in the manuscript. The voltage-controlled phase tuning efficiency is measured to be about $1 \pi/V$, and the voltage tuning resolution is about 0.001 V, which is sufficient to resolve the small space of the turnkey single soliton region.

Following statements are added on page 7, lines 207-210 in the revised manuscript:

“Even though it is sensitive to the feedback phase, the silicon sheet used for tuning feedback phase via resistive heating features high efficiency of $1 \pi/V$ and high resolution of 0.001V, which is sufficient to guarantee the excitation of the single soliton state.”

6. A recent work [Xing Jin et al. 2024 PRL] points out that the dispersive waves can be used to further compress the quantum noise limit. Can this method be used in your FFPR geometry?

Our response:

Thank you for your comment. It is possible to use this method in our FFPR geometry. X. Jin, et al. achieved noise control through generating the dispersion wave by mode coupling in the silica microresonator. It is an innovative strategy to further compress the quantum noise limit of microcombs. The basic requirement for this realization is utilizing the coupling of different mode families in microresonator to engineer dispersion wave generation.

For our FFPR, it is possible to use this method by using multi-mode fiber for fabrication. For example, the few-mode fiber used in this work naturally has multiple transversal mode families. For dispersion engineering, further design of the FFPR can be expected to achieve intermodal coupling. Thanks to the current micro-processing technology, the geometry of the FFPR can be changed using fiber polishing and tapering. Such change may lead to specific mode coupling or local anomalous structural dispersion points. In addition to changing the geometric structure, other ways like coating chirped mirror can also engineer the FFPR dispersion and result in phase matching between different modes.

7. Does the power of the comb line affect the measured THz phase noise? Is the ASE effect of the EDFA significant?

Our response:

To meet the optimal input power requirement of the UTC-PD, the filtered comb lines are firstly amplified using low-noise EDFA before being sent to UTC-PD for generation THz frequency. The power of comb lines is on the order of tens of micro watts, which is enough for low-noise amplification in EDFA. The spectrum of amplified comb lines is shown in Fig. R4. The optical signal features high signal-to-noise ratio and the ASE noise is not significant.

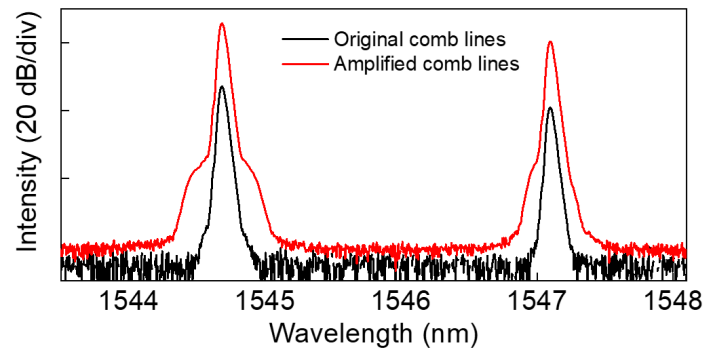


Fig. R4. Spectra of original and amplified comb lines.

Reviewer #2: (Remarks to the Author)

The paper is dealing with the use of low phase noise synthesizer using Kerr based combs, that are coming with compactness compared to OFCs. Authors show that they can run a THz link without CPE, which is of interest. However, I have some comments to be addressed in the paper to make it more detailed and strong.

Our response:

Thank you for your positive comments and we appreciate your invaluable suggestions for helping us to improve our manuscript.

On the figure 2, the power level measured versus optical power (panel (f)) is showing saturations for 303 GHz, I believe due to the LNA used. What about the LNA specifications and impact of this saturation on the overall THz system?

Our response:

Indeed, a nonlinear power tuning curve is shown in Fig. 2f of the manuscript, which is because that our THz LNA (RPG, H-LNA 250-350) works in a saturated region. This THz LNA we used has a typical gain of 22 dB and its nominal P1dB corresponds to -5 dBm. Thus as long as the input power on the UTC-PD exceeds 10 dBm, the LNA enters the saturated working region. In this state, its output power no longer increases linearly with the input power, which introduces a certain power degradation, as compared to the expected value. Additionally, the distortion of signal amplitude will occur if an information-bearing signal is input. As a result, the performance of the overall THz system will reduce. However, in this paper, since this LNA is used to amplify the THz frequency as local oscillator but not the THz signal, the distortion problem is avoided. The minor power loss does not affect the phase noise performance evaluation.

Following statements are added to better clarify the LNA work state and the corresponding impact on page 8, lines 250-251 in the revised manuscript:

“Although the LNA has entered the saturated operating region at this output power, a minor power loss for the terahertz tone does not affect the phase noise performance evaluation.”

On this figure 2, it would be good to show the filtered optical signals, to assess the side mode suppression ratios. What kind of filter is used (panel (c) in this figure 2).

Our response:

Thanks for your valuable suggestion. In our experiment, the waveshaper (WS 4000A) is used as the filter. The spectrum of the filtered and amplified comb line is shown in Fig. R5. The side mode suppression ratio can reach 54.5 dB. These results have been added in our Supplementary Information (S4.2. Soliton crystal).

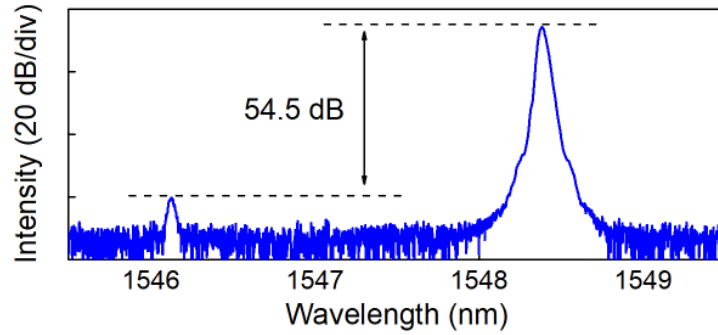


Fig. R5. The spectrum of the filtered comb line with a resolution of 0.03 nm.

Following statements are added on page 8, lines 243-246 in the revised manuscript:

“The terahertz-interval comb lines are filtered out by using a waveshaper (WS 4000A) with high side mode suppression ratio over 54 dB (see details in Supplementary Information) and then amplified, and converted to terahertz wave by a UTC-PD in each KCTS.”

In the figure 3, comparisons between heterodyne mixing and KCTS/KCTS mixing. However:

- the mixing is not done in the same device (BAM or IHM), what about noise figure of these BAM or IHM, and required LO power? Why not measuring the performances using a single down-converter to avoid introducing testing-system variations?

Our response:

Technically, both the BAM-based direct-beating and the IHM-based harmonic-mixing belong to the heterodyne mixing category, for downconverting the THz signal to the IF signal. In our experiment, two mature commercial products (BAM: VDI-WR2.8BAMULP; IHM: OML-C02.8DAS02) were used to compare the phase noise levels of the two cases. Among them, the cascaded frequency multiplier chain ($\times 24$) with stepwise amplification in the integrated IHM is well designed to achieve good spurious suppression effect. This is beneficial for the phase noise measurement of IHM-based harmonic-mixing case. In fact, by comparing the curves of “Harmonic mixing” and “24th harmonics of LO” in Fig. 3d, we can find that the phase noise is mainly dominated by the frequency multiplier chain in IHM-based harmonic-mixing case. Therefore, whether the single and same down-converter is used has no observable influence on the conclusion of this paper.

The required LO powers for BAM and IHM are about -3 dBm and 7 dBm, respectively. Note that the frequencies of the two LO are completely different, the former is THz LO at around 300 GHz, the latter is the RF LO at around 13.5 GHz. Neither BAM nor IHM product discloses the nominal noise figure. However, the used THz LNA with a typical noise figure of 12 dB dominates the overall noise in BAM-based direct-beating case. For the IHM-based harmonic-mixing case, dominant role is the 24-fold frequency multiplier chain with stepwise amplification in the integrated IHM, whose noise figure

is much larger than 12 dB.

Following statements are added to explain the above problem on page 9, lines 278-280 in the revised manuscript:

“Note that although the mixer integrated into IHM receiver may be different from the BAM, this does not affect our conclusion, because the mixing noise is not the dominant role in both two cases.”

- we don't know here if the LNA is used in its linear operation range, do the authors expect some effects, what are these effects?

Our response:

To meet the optimal power requirement of the BAM LO port (around -3 dBm), we set the LNA output power to about -2.5 dBm, which corresponds to operating in the nonlinear region. This saturation effect can be observed by the non-linear increase of the LAN output power curve in Fig. 2f. As mentioned in the above response to the first comment, this will cause a minor power loss on the THz tone, however, it does not affect the phase noise performance evaluation.

- The KCTS#1 is used as “LO” signal in case of KCTS/KCTS mixing, whereas the KCTS#1 is used as RF. What is the reason for this?

Our response:

Sorry for the confusion. In fact, the two KCTSs we used in experiment are almost identical in terms of noise performance, as the same setup for Kerr comb generation as well as FFPRs with similar Q factors are used for building them. Fig. R6 shows the measured 10 GHz repetition rate phase noise of these two Kerr combs, indicating very close performance. As a result, either KCTS#1 or KCTS#2 can be used as RF or LO, and it does not affect our measurement results.

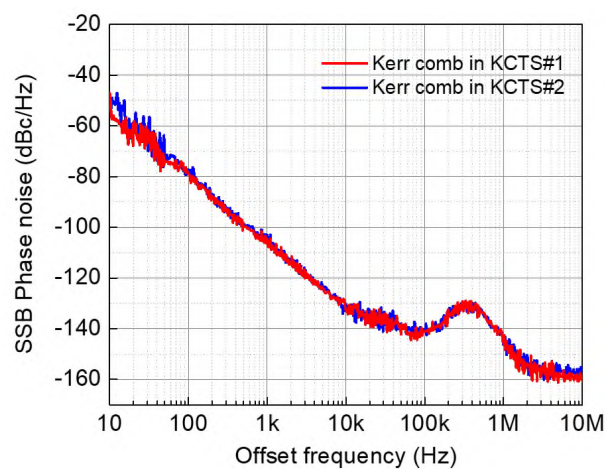


Fig. R6. Measured 10 GHz repetition rate phase noise of Kerr combs in KCTS#1 and KCTS#2.

Following statements are revised to clarify the identical noise performance of two KCTSs on page 8, lines 238-243 in the revised manuscript:

“As shown in Figs. 2c and 2e, two identical Kerr comb-based terahertz sources (KCTSs) are built, based on Comb #1 and Comb #2, which features identical noise performance, for the operation at terahertz frequency from 283 GHz to 384 GHz, occupying the typical terahertz communication spectra allocated by the World Radiocommunication Conferences and the IEEE 802.15.3d standard.”

- Is the multiplied LO by 24 spurious-free when used as down-conversion pump signal? It is not mentioned in this study if this was checked or not. It is important to really compare the spectra of the 2 local oscillators used in the experiments to be able to properly compare these 2.

Our response:

Thanks for your suggestion. As mentioned in the response to the third comment, the integrated IHM shows a good spurious suppression effect. When we set the LO to 13.5 GHz with an input power of 7 dBm, no obvious spurious tone can be observed from the spectra of down-converted IF tone captured by electrical spectrum analyzer (ESA), as is shown in Fig. R7b. Fig. R7a also shows the corresponding spectra for BAM receiver for comparison. These results have been added in our Supplementary Information (S6. Spurious tone analysis before phase noise measurement).

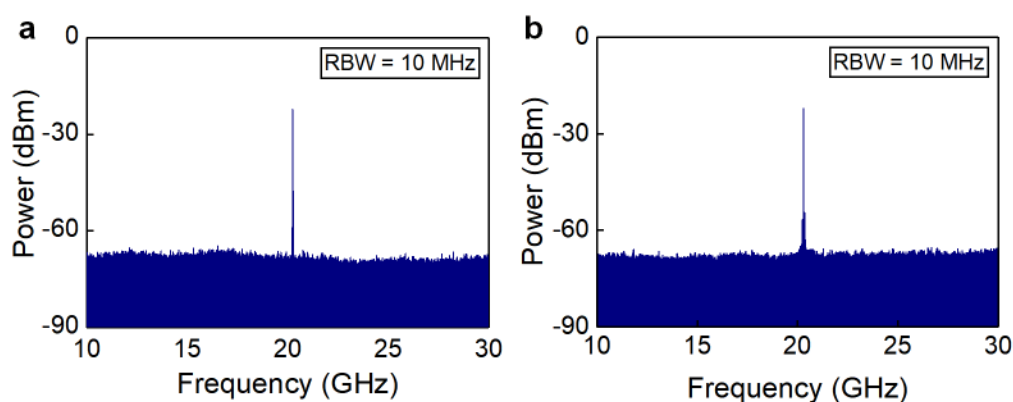


Fig. R7. Spectra of the down-converted IF signal captured by ESA. **a**, BAM scheme. **b**, IHM scheme.

Following statements are added on page 8, lines 262-264 in the revised manuscript:

“We confirm that the down-conversion signals in both measurements are spurious-free by measuring their spectra using ESA (see details in Supplementary Information).”

In the THz communication setup (text and figure 4):

- What is the source of the spurious signals in the Figure 4.c spectra?

Our response:

These spurious signals in the Fig. 4c spectra mainly come from the leakage of sampling

clock of the arbitrary waveform generator (AWG) and digital storage oscilloscope (DSO).

To confirm this, we first connect the output baseband signal (23GBd as an example) from 92 GSa/s AWG directly to the electrical spectrum analyzer (ESA), to observe the corresponding spectrum component. The result is shown in Fig. R8a. A spurious tone located at 23 GHz is observed. Then when we further change the signal baud rate and the AWG sampling rate, we find it is the AWG sampling rate but not the signal baud rate that determines the location of the spurious. Therefore, we conclude this spurious tone comes from the clock leakage of AWG.

Secondly, we build an optical and THz back-to-back links employing the IHM-based harmonic-mixing scheme (refer to the Fig. 3b in our main manuscript). The center frequencies of the generated THz signal and down-converted IF signal are also set to about 303 GHz and 20.25 GHz, respectively, which are consistent with the parameters in our THz communication experiments. The captured spectrum by ESA is shown in Fig. R8b. Obviously, two new spurious tones which are located at 2.75 GHz and 43.25 GHz respectively, can be observed. These two spurious tones can also be found in the spectra of Fig. 4c in the main manuscript. In fact, the right spurious tone is generated by $20.25 \text{ GHz} + 23 \text{ GHz} = 43.25 \text{ GHz}$, and the left one is generated by the flip of $20.25 \text{ GHz} - 23 \text{ GHz} = -2.75 \text{ GHz}$.

Finally, we unload the AWG signal and find that the two spurious tones are still present, as shown in Fig. R8c. This means the clock leakage of AWG is independent of whether the signal is loaded or not, which only depends on the enable state of the channel digital-to-analog converter (DAC) in AWG.

Other than the above two spurious tones resulting from AWG, there is another spurious tone around 40 GHz can be observed from the spectra of Fig. 4c in the main manuscript, which is captured by the DSO. Since this tone does not exist when using ESA, we infer that it originates from the clock leakage of DSO.

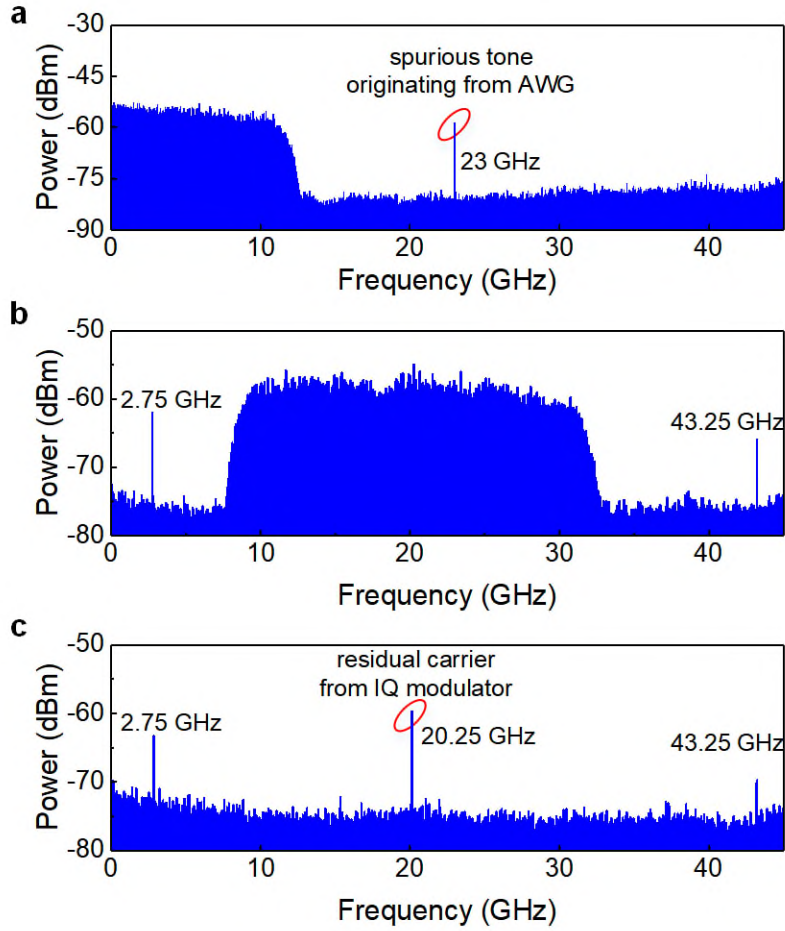


Fig. R8. Single sideband spectra captured by ESA. **a**, Spectra of the 23 GBd baseband signal generated by 92 GSa/s AWG (electrical back-to-back case). **b**, Spectra of the down-converted 20.25 GHz IF signal with 23GBd signal loading (optical and THz back-to-back case). **c**, Spectra of the down-converted 20.25 GHz IF signal without 23GBd signal loading (optical and THz back-to-back case).

In order to make it easier for the reader to understand, following statements are added on page 10, lines 321-323 in the revised manuscript:

“Note that several spurious tones can be observed from the Fig. 4c, mainly due to the clock leakage from the AWG and DSO.”

- For 40 GB, the spectra seems to be cut at low frequencies, can the authors zoom on this? This seems to be not expected as in Figure 4.a the optical signal is symmetric.

Our response:

Sorry for the confusion. The spectra for 40 GBd case do not to be cut, but a guard band (frequency interval with zero frequency) of several tens of MHz is reserved during down conversion process. Firstly, the optical spectra in Fig. 4a are symmetric because it is a baseband signal. However, the dual sideband (DSB) spectra of the IF signal are present in Fig. 4c. For a real-valued IF signal, its DSB spectra also exhibit conjugate

symmetry characteristic. Secondly, we further take the single sideband spectra into consideration. Since the center frequency of the IF signal is set to about 20.25 GHz, and the overall signal bandwidth for 40 GBd case is 40.4 GHz ($40 \text{ GHz} \times 1.01$), thus the guard band can be calculated by $20.25 \text{ GHz} - 0.5 \times 40.4 \text{ GHz} = 50 \text{ MHz}$. Similarly, the guard band for 23 GBd case is $20.25 \text{ GHz} - 0.5 \times 23 \times (1 + 0.1) \text{ GHz} = 7.6 \text{ GHz}$. The former is so small that similar to a cut in the spectra, while the latter is large enough and easy to observe.

The description of the guard band for 23 and 40 GBd cases has been given on page 9 in the original manuscript as follows:

“The modulation baud rate is fundamentally limited by twice the IF signal carrier frequency of around 40 GBd, so that we start with 23 GBd (roll-off factor 0.1) and the spectra of captured IF signal shows a guard band of 7.6 GHz between the zero frequency and signal edge, as expected. Then we push the baud rate close to the limit with 40 GBd (roll-off factor 0.01). As shown in Fig. 4c, attributed to the high spectral stability of KCTSs, a small but clear guard band as narrow as 50 MHz is enough for signal demodulation. It helps to take full use of spectrum and reduce the demand for operation bandwidth of the analog-to-digital converter at the receiving end.”

- A roll-off factor of 0.01 is used for 40 GB, however it seems to be too small, can the authors check this?

Our response:

Thank you for your reminding! After re-checking, the roll-off factor of 40 GBd case is indeed 0.01. The main reason for choosing such a small roll-off factor is to combat the high-frequency fading problem in hybrid optical fiber and THz wireless links. In Fig. R9 we also show the spectra of 40 GBd signal with the roll-off factor of 0.01 and 0.1, respectively. They are generated by offline Matlab. It can be found that the former has a total bandwidth of 40.4 GHz with a sharp edge, while the later corresponds to 44 GHz with a smooth edge. When bandwidth limitations are not considered, the latter exhibits better performance. However, in our hybrid optical fiber and THz wireless links, the spectrum ranges exceeding 40 GHz will experience rapid fading due to the bandwidth limitations of RF cables and optoelectronic devices. As a result, a small roll-off factor is a good choice to compress the signal bandwidth as much as possible, so as to combating the high-frequency fading problem.

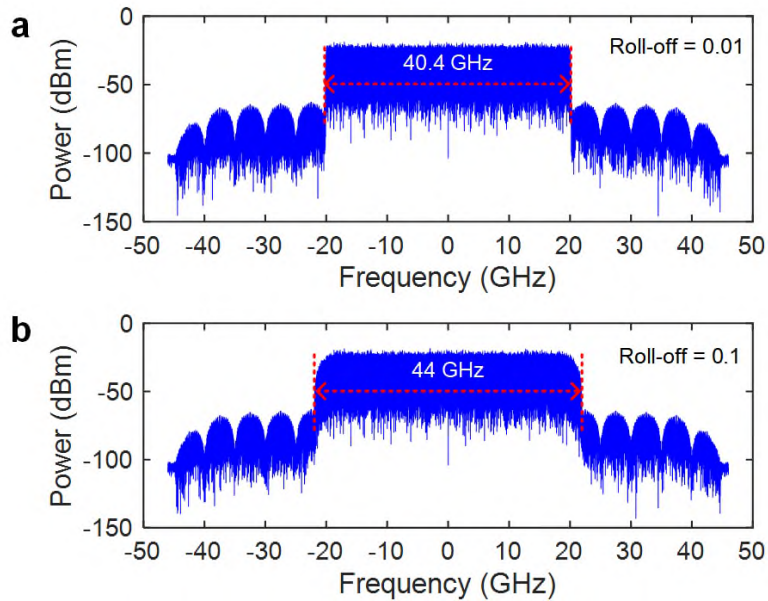


Fig. R9. Spectra of 40 GBd signal with the roll-off factor of **a**, 0.01 and **b**, 0.1 through offline Matlab generation.

- Can the optical filter being tuned to change the THz carrier frequency? What are the minimal filter specifications required?

Our response:

Yes, the THz carrier frequency can be tuned by changing the pass band of optical filter to select different comb line, or by changing the soliton comb state as shown in Figs. 2b and 2c. In this demonstration, in order to flexibly adjust wavelength, we use a table-top waveshaper (WS 4000A) as filter. Indeed, the basic requirement of this experiment is to filter out comb lines with a high optical density contrast (~ 50 dB by using waveshaper in this work). There are many other bandpass filters that meet this requirement, such as grating filter, Fabry–Pérot interferometer, dichroic mirror, and etc., which can also be used for comb line filtering.

- Are the antennas used in their near-field region? It seems to me that this is the case, considering the size of these antennas. The system would have to come with a link budget here, even if the THz link itself seems to be not the core of the paper.

Our response:

Yes, the antennas in our THz experiment operate at radiating near field region (i.e., Fresnel region). A detailed analysis of the Fresnel region and the power budget of our terahertz link have been added to the revised Supplementary Information (S7.1 Power budget for terahertz wireless link).

Following statements are added on page 9, lines 307-308 in the revised manuscript:

“This experiment is conducted under near-field communication condition, which can refer to the detailed analysis in the Supplementary Information.”

- As authors have the usual electronic down-converter, it would be good to set the reference also with the usual THz receiver which is an electronics-based mixer.

Our response:

Thanks for your valuable suggestion. We have added the performance comparison between the BAM-based and IHM-based receiver to S7.2 section in our revised Supplementary Information.

In order to properly compare the power consumption of the 2 approaches, it would be good to consider the overall consumption and complexity. Here, while the performance using KCTS is better than ECL (non-locked lasers), obviously using two Kerr combs seems to be quite complex for future integration. As an example, the optical filtering is required at KCTS output, as well as several EDFAs. What are the minimal specs required for the filter?

Our response:

Thanks for your comments. In this work, we focus on low noise frequency synthesis and exploring the impact of low noise characteristics on consumption and complexity of terahertz wireless communication systems. For the KCTS itself, in fact, its consumption and complexity can be further reduced. For example, FFPR device with higher quality factor is technically feasible to achieve lower electrical pump power consumption and higher output comb power that may enables EDFA-free operation. By optimizing the self-injection locking loop and the mount design, the overall dimensions can be reduced to a standard butterfly laser package, wherein passive filters such like wavelength division multiplexers can also be integrated. Considering that our KCTS already has the turnkey function and high passive stability, no additional electrical control components are required, so that the power consumption and complexity of the device will not be higher than ECL.

Following statements are added on page 12, lines 398-403 in the revised manuscript:

“The improved Q will lead to the access of even lower quantum noise limits, and enable higher fundamental repetition rate for even smaller compact setup size. Also, higher Q helps to lower the pump power consumption and achieve higher conversion efficiency for amplifier-free operation in frequency synthesis applications. By optimizing the self-injection locking loop design and utilizing passive filters, the reduction of overall system complexity can be expected.”

The work is interesting and should be detailed and completed.

Our response:

Thanks for your positive comments on our work. We have revised our manuscript as you suggested and believe it is now suitable for publication in Nature Communications.

Reviewer #3: (Remarks to the Author)

The authors proposed and demonstrated a microwave signal generator with low phase noise using a few-mode-fiber-based Fabry-Perot resonator (FFPR) pumped by a DFB laser which is locked by the FFPR. Compared to an integrated Kerr comb, the phase noise performance is notably improved. The use of the FFPR for microwave generation at frequencies as high as 300 GHz was demonstrated. The use of the generated microwave for data transmission was also experimentally demonstrated. This is an interesting work, and I support the publication of the paper in NC. Below are a few comments that the authors may wish to address:

Our response:

Thanks for your support for the publication in Nature Communications and invaluable suggestions.

1. More details about the self-locking of the DFB laser should be added including a simulation to compare the simulation result with the experimental result.

Our response:

Thank you for your thorough review and constructive comments. Self-injection locking happens when there is a detuning between the laser cavity and the microresonator, caused by “injection pulling”. This results in a small phase difference between the laser output and the feedback. The tuning curve of the self-injection locking to high-Q microresonator can be described:

$$\xi = \zeta + \frac{K}{2} \frac{2\zeta \cos \bar{\psi} + (1 + \beta^2 - \zeta^2) \sin \bar{\psi}}{(1 + \beta^2 - \zeta^2)^2 + 4\zeta^2}, \quad (\text{R1})$$

here the actual effective detuning $\zeta = 2(\omega_0 - \omega_{\text{eff}}) / \kappa$ $\xi = 2(\omega_0 - \omega_m) / \kappa$ refers to the relative detuning of laser from the microresonator resonance ω_0 . κ is its loaded linewidth. β is the feedback strength. $K_0 = 8\eta \Gamma \kappa_{\text{do}} \sqrt{1 + \alpha_g^2} / (\kappa R_0)$ is the combined coupling coefficient. κ_{do} is the laser diode output mirror coupling rate. α_g is the Henry factor. R_0 is the amplitude reflection coefficient of the laser output face $\bar{\psi} = \omega \tau_s - \arctan \alpha_a + 3/2\pi$ is the phase delay, τ_s is feedback round-trip time. When considered the influence of the microresonator nonlinearity, the mode amplitudes of forward a_μ and backward field b_μ should added into the linear self-injection locking model as the nonlinear terms:

$$\begin{aligned}
\dot{a}_\mu &= (-1 + i\zeta_\mu)a_\mu + i\beta b_\mu + i \sum_{\mu'=v+\eta-\mu} a_v a_\eta a_{\mu'}^* \\
&\quad + 2i\alpha_x a_\mu \sum_\eta |b_\eta|^2 + f\delta_{\mu 0}, \\
\dot{b}_\mu &= (-1 + i\zeta_\mu)b_\mu + i\beta^* a_\mu + i \sum_{\mu'=v+\eta-\mu} b_v b_\eta b_{\mu'}^* \\
&\quad + 2i\alpha_x b_\mu \sum_\eta |a_\eta|^2,
\end{aligned} \tag{R2}$$

where f is normalized pump amplitude, α_x is across-modulation coefficient (here, $\alpha_x = 1$ for the same polarization). $\zeta_\mu = 2(\omega_{\text{eff}} - \omega_\mu + \mu D_1)/\kappa$ is the normalized detuning. For simplicity, Eq. (R2) can be transferred to:

$$\begin{aligned}
(-1 + i\zeta)a + i\beta b + ia(|a|^2 + 2\alpha_x |b|^2) + f &= 0 \\
(-1 + i\zeta)b + i\beta^* a + ib(|b|^2 + 2\alpha_x |a|^2) &= 0
\end{aligned} \tag{R3}$$

the nonlinear detuning shift $\delta\zeta_{nl}$ and nonlinear coupling shift $\delta\beta_{nl}$ are introduced to solve, where $\bar{\zeta} = \zeta + \delta\zeta_{nl}$, $\bar{\beta}^2 = \delta\beta_{nl}^2 + \beta^2$. And we redefine $\bar{\xi} = \xi + \delta\zeta_{nl}$, then the new equation becomes:

$$\bar{\xi} = \bar{\zeta} + \frac{K_0}{2} \frac{2\bar{\zeta} \cos\bar{\psi} + (1 + \bar{\beta}^2 - \bar{\zeta}^2) \sin\bar{\psi}}{(1 + \bar{\beta}^2 - \bar{\zeta}^2)^2 + 4\bar{\zeta}^2} \tag{R4}$$

The nonlinear detuning and coupling can be expressed as:

$$\begin{aligned}
\delta\zeta_{nl} &= \frac{2\alpha_x + 1}{2} f^2 \frac{1 + (\bar{\zeta} + \delta\beta_{nl})^2 + \beta^2}{(1 + \bar{\beta}^2 - \bar{\zeta}^2)^2 + 4\bar{\zeta}^2} \\
\delta\beta_{nl} &= \frac{2\alpha_x - 1}{2} f^2 \frac{1 + (\bar{\zeta} + \delta\beta_{nl})^2 - \beta^2}{(1 + \bar{\beta}^2 - \bar{\zeta}^2)^2 + 4\bar{\zeta}^2}.
\end{aligned} \tag{R5}$$

We show the distinctions between conventional generation of the dissipative Kerr solitons and self-injection locked soliton excitation below. The orange dashed line corresponds to the linear tuning curve of a free-running laser. The blue solid line corresponds to the tuning curve of typical self-injection locking considering the nonlinearity of microresonator.

Table R1 | Parameters used in analytical calculations.

Parameters	Value
f	0.8
β	0.1
K_0	47
$\bar{\psi}$	0.15 π

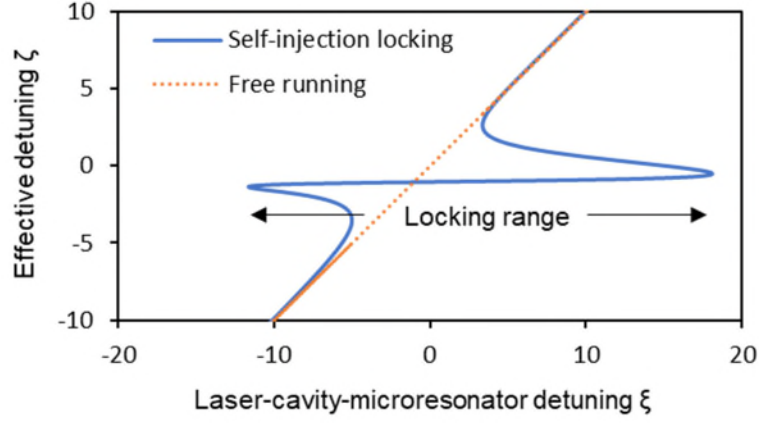


Fig. R10. The self-injection locking tuning curves.

Within the locking range, the noise reduction factor ((low Fourier frequency) has been mentioned in the manuscript,

$$\eta = 1 + \frac{Q}{Q_{LD}} \sqrt{\beta(1 + \alpha_g^2)}, \quad (R6)$$

where $Q = 6 \times 10^8$ and $Q_d = 4 \times 10^3$ are the quality factors of the FFPR and the laser diode cavity, respectively. The laser diode has $\alpha = 2.5$ and $\beta = 0.1$ is the feedback strength. A noise reduction factor of 1.2×10^5 can be calculated. According to the results in Fig. 1e, the frequency noise of the DFB laser shows over 10^5 reduction with injection locking state at the offset frequency 10 Hz, which is consistent with the simulation results.

These results have been added in our Supplementary Information (S2. Numerical simulations of self-injection locking laser).

Following statements are added on page 6, lines 178-180 in the revised manuscript:

“For the noise at 10 Hz offset frequency, the noise reduction is on the order of 50 dB, which agrees well with the theoretical calculation using Eq. (2) and the parameters presented in the caption of Fig. 1d.”

2. How can the fundamental linewidth of 72.2 MHz be accurately measured for the DFB laser that is locked to the FFPR? Additional details regarding the measurement setup, techniques, and instrumentation used should be included.

Our response:

We measure the fundamental linewidth by using the correlated delayed self-heterodyne method, which is a general method for narrow fundamental linewidth measurement based on short optical delay [J. Lightwave Technol. 26, 3048-3055 (2008)]. The setup is shown in Fig. R11, consisting of 1 km delayed fiber. After amplified in an EDFA, the DKS comb is divided into two arms by a 1×2 fiber coupler. The laser in one arm is frequency-shifted in a 40 MHz acoustic optical modulator, while the laser in another arm is delayed by 1 km single-mode fiber. Polarization controllers (PC) are used to

ensure maximum interference intensity. These two arms are recombined by another 1×2 fiber coupler. Their beating signal is then detected by a PD.

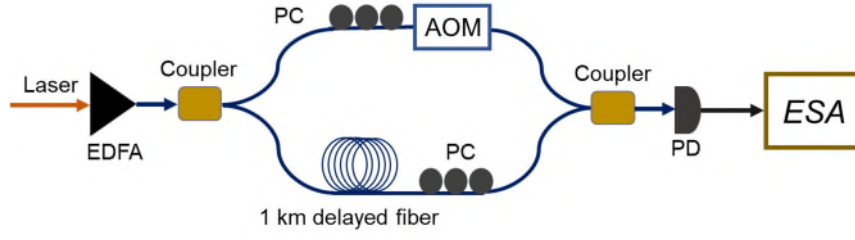


Fig. R11. EDFA: erbium-doped fiber amplifier; PC: polarization controller; AOM: acoustic optical modulator (frequency shifter); PD: photo detector; ESA: electric spectrum analyzer.

The phase noise of the beating signal is measured using an ESA, which is the differential phase noise of the laser. The relationship of differential phase noise and the phase noise is given by [J. Lightwave Technol. 26, 3048-3055 (2008)]:

$$S_{\phi}(f) = \frac{1}{4[\sin(\pi f \tau)]^2} S_{\Delta\phi}(f) \quad (R7)$$

where $S_{\Delta\phi}(f)$ is the differential phase noise, $S_{\phi}(f)$ is the laser phase noise, f is the frequency offset, τ is the delay time, then the phase noise can be converted to frequency noise:

$$S_v(f) = f^2 \times S_{\phi}(f) \quad (R8)$$

where $S_v(f)$ is the frequency noise. The fundamental linewidth of laser can be derived from the white noise floor of the frequency noise curve as shown in Fig. 1e of the manuscript.

$$\Delta\nu = \pi S_w \quad (R9)$$

where $\Delta\nu$ is the fundamental linewidth and S_w is the value of the frequency noise at white noise floor.

These results have been added in our Supplementary Information (S5. Linewidth measurement).

Following statements are added on page 6, lines 169-173 in the revised manuscript: “The frequency noise of the pump is measured in comparison to its non-locked free-running state, using the correlated delayed self-heterodyne method, as shown in Fig. 1e. This is a general method for narrow fundamental linewidth measurement based on short optical delay. The setup consists of 1 km fiber for time delay and a 40 MHz acoustic optical modulator for frequency shift (see details in Supplementary Information).”

3. If the DFB laser is not self injection locked, what is the phase noise performance of the FFPR for microwave generation?

Our response:

As we discussed in the manuscript, the noise performance of a Kerr comb is fundamentally limited by the quantum noise, which is further affected by the pump noise and thermorefractive noise on top of the quantum noise. Fig. 3a in the manuscript shows that current phase noise of generated 10 GHz microwave is actually limited by the pump noise at the offset frequency below 1 kHz. If the DFB laser is not self-injection locked, the laser phase noise will increase by tens of dB (roughly 30 dB to 50 dB in the offset frequency from 10 Hz to 1 kHz) compared to the injection-locked case, as shown in Fig. 1e of the manuscript. As a result, the pump noise limit will also be raised by tens of dB in response, which will eventually be reflected in the phase noise of the generated microwave.

4. It is generally known that a higher Q factor will lead to a lower quantum noise limit. Why on page 4 line 91, it says that “the low quantum noise limit can be achieved with low Q and large mode area”

Our response:

Thanks for your professional comment. It is a different case from ours. For Kerr comb generation in a microresonator **without pump power limit**, the lower quantum noise limit can indeed be achieved with low Q and large mode area, which is theoretically studied by Matsko, A. B. and Maleki, L. [Opt. Express 21, 28862–28876 (2013)]. This theory is consistent with the results of past works on Kerr combs. However, for Kerr comb **with limited pump power**, as we discussed in this work, we find that high Q actually helps to reduce the quantum noise as shown in Eq. (1) in the manuscript.

5. Some minor grammar corrections: “repetition frequency” may be changed to “frequency” or “repetition rate”. “State-of-art” should be changed to “state-of-the-art”

Our response:

Thanks for your corrections. We have double-checked the grammar in the manuscript and revised all the errors.

Responses to Reviewers' comments:

Reviewer #1: (Remarks to the Author)

I appreciate the authors' efforts in revising the manuscript. They have provided comprehensive responses to all my previous concerns, and the newly added content significantly improves the clarity and robustness of the work. The revised version addresses my reservations adequately, and I find the current manuscript suitable for publication in Nature Communications.

Our response:

Thanks for your support for the publication in Nature Communications and invaluable suggestions.

Reviewer #2: (Remarks to the Author)

Authors did the required corrections. This is a interesting paper.

Our response:

Thank you for your support and we appreciate your invaluable suggestions for helping us to improve our manuscript.

Reviewer #3: (Remarks to the Author)

The authors have adequately addressed the issues I raised in the first round of review and have incorporated the necessary changes into the revised manuscript. I am satisfied with the revisions and recommend the manuscript for publication in NC.

Our response:

Thanks for your positive comments and suggestion for the publication in Nature Communications.