nature portfolio

Peer Review File

Dual origin of effective axion response

Corresponding Author: Professor Maxim Gorlach

This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
The manuscript investigates the nature of effective axion fields in condensed matter and photonic systems, with the aim of distinguishing between conventional axion electrodynamics and a newly proposed dual-symmetric version. The authors derive theoretical predictions and propose experimentally observable distinctions between these two axion-like responses. The paper focuses on predicting the behaviour of these axion responses in various media, although I note that I am not an expert in experimental condensed matter or photonic systems and thus cannot fully assess those aspects.

In my understanding, the key finding of this work is the prediction of a novel electromagnetic response, termed the dual axion field, which shares symmetry properties with the conventional axion field but produces distinct electromagnetic behaviour. Both axion fields, according to the authors, manifest primarily at the boundaries of materials, which could influence the design and interpretation of experiments in these systems.

The theoretical predictions are derived from modified Maxwell's equations, incorporating the dual axion field. The distinctions between conventional and dual-symmetric axion responses are well-articulated. While the main assumptions imply the observability of the dual axion field, I suggest the authors emphasize how non-ideal experimental conditions (that may invalidate some of these assumptions) could affect the predicted phenomena. The study would also benefit from more computational simulations of specific experimental outcomes to reinforce the theoretical predictions. While the analysis is rigorous, the absence of experimental data limits the assessment of reproducibility in practical settings.

The introduction of the dual axion field is a notable theoretical advancement. To my knowledge, this dual-symmetric version has not been extensively explored in prior literature. The distinction between conventional and dual axion fields provides a fresh perspective on electromagnetic responses in structured media which, in my opinion, may be valuable contribution to both condensed matter physics and photonics. The predictions offer new directions for experimental investigations aimed at observing the dual axion response in various materials. Furthermore, the dual-symmetric framework may inspire modifications of other physical theories, broadening its theoretical relevance.

The work is potentially of broad interest, with implications extending beyond axion electrodynamics to experimental photonics and materials science. The ability to distinguish between different axion-like responses could impact both theoretical understanding and practical applications. This interdisciplinary relevance aligns with the high-impact focus of Nature Communications.

The manuscript is well-structured and clearly written, but it could benefit from further clarity in explaining the distinction between conventional and dual axion fields. Additionally, incorporating computational simulations would provide quantitative support for the theoretical predictions. The figures are informative, but expanding on the experimental implications and setups for testing the dual axion field could enhance the paper's impact.

Based on the strengths of the theoretical framework and the potential implications of the findings, I recommend revisions. While the theoretical results are robust, the manuscript would benefit from additional clarity, computational simulations, and experimental guidance to fully substantiate the claims.

Suggestions for Improvement

1. The authors should provide a more comprehensive background on axion electrodynamics and its relevance in condensed

matter and photonic systems to assist readers unfamiliar with the topic.

2. I suggest elaborating on the theoretical basis and physical implications of the dual axion field to clarify its distinction from the conventional axion field.

3. Including computational simulations would quantitatively substantiate the theoretical predictions.

4. The authors should discuss how the potential breakdown of their assumptions impact on the results to improve transparency.

5. It would enhance the manuscript if the authors could suggest potential experimental setups to observe the dual axion field, offering concrete examples of how the theoretical predictions can be tested.

Reviewer #3

(Remarks to the Author)
This is a very interesting paper postulating a new class of effects in photonic and condensed matter systems, described by a dual axion field. This is potentially a very significant theoretical result in condensed matter physics and also for the study of fundamental axions. The paper is well written and I appreciate the level of detail given for the calculations in the supplemental material.

The authors demonstrate a duality between axion and dual axion materials, described by equation (7). Given this, it is crucial to demonstrate that there are in fact observable differences between axion and dual axion materials. Otherwise, the authors have merely discovered a different mathematical description for a known phenomenon. To address this, the authors suggest an experiment in which an electric or magnetic dipole is placed inside a sphere of an axion or dual axion material. They claim that the induced magnetic dipole, and therefore the magnetic field measured outside the sphere, is structurally different for the two types of material. The feasibility and results of this proposed experiment are crucial for the paper, and I believe they need more discussion and justification, which could go mostly in the supplemental material. In particular, I have the following questions:

1. How feasible is such an experimental set up? Has a similar experiment been performed with a meta-material before? If not, is it in principle possible?

2. The authors state that any voids must be smaller than the lattice spacing, such that material can be treated as homogeneous. (In contrast to the case of a larger void, in which there is no observable difference between axion and dual axion materials.) Presumably the antenna generating the dipole field must also be smaller than the lattice spacing in order to be considered “inside” the material. Is this feasible?

3. Even in the case that the antenna is smaller than the lattice spacing, it introduces an impurity into the material, which is not taken into account in the authors’ calculations. Can the authors justify that this may be neglected?

4. The authors assume that the fields outside the sphere take the standard forms for the fields generated by an electric or magnetic dipole moment (equations S158 and S159). It is not obvious to me that this is correct. The presence of the meta material may change the form of these fields in a way not simply described by an induced dipole moment. Can the authors show why these ansatz fields are sufficient?

If the authors can give satisfactory answers to these questions, I would be happy to recommend publication in Nature Communications.

Reviewer #4

(Remarks to the Author)
In this paper, the authors explored a dual version of axion dynamics in the medium. In the vacuum, the axion term in the Lagrangian of electrodynamics is self-dual, but the key point the authors make is that it splits into two terms in a medium, described clearly in Eqs. S87 and S93 in the SM (of the arXiv version). The authors argued that these two axion terms lead to distinguishable effects.

Unfortunately, I do not believe this work represents a significant scientific advance that warrants publication in Nat Comm.

1. The duality of E&M is ancient and well-understood. Indeed it is richer in a medium, but fundamentally there is nothing new. As the author mentioned themselves, the "dual axion electrodynamics" only leads to quantitative differences, not qualitatively new results.

2. The key results in the main text, such as effective magnetic monopoles, already exists in the "original" axion electrodynamics, known as the Witten effect

3. The main text of the paper is written in a rather technical fashion, while the essence could have been transparently conveyed by Eqs. S87 and S93 in the SM (of the arXiv version).

I recommend that the authors seek publication in a more specialized journal.

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
The authors have satisfactorily addressed all my comments in their response. All changes are clearly marked, and new figures and simulations convincingly support their claims. I find their response is comprehensive, constructive, and scientifically solid. I believe the manuscript is publishable in its present form.

Reviewer #3

(Remarks to the Author)
I thank the authors for their comprehensive response to my questions and those of the other referees. I believe the paper is now substantially improved and am happy to recommend publication in Nature Communications.

“Dual origin of effective axion response”
by Timur Z. Seidov, Eduardo Barredo-Alamilla, Daniel A. Bobylev, Leon Shaposhnikov, Maxim Mazanov, and Maxim A. Gorlach

Response Letter

First of all, we would like to thank all Reviewers for assessing our paper and providing constructive feedback. Based on the Referee’s suggestions, we have improved and extended our manuscript. In the main text, we have added a new section “Suggested experiments” and new Fig. 4 with the simulation results. For easy tracking, all changed fragments in the main text are shown by the blue font. We have also added Secs. X and XI to the Supplementary Materials and introduced minor edits in the other sections. Our detailed replies to the raised questions appear below.

Response to Reviewer #1

Q1. The authors should provide a more comprehensive background on axion electrodynamics and its relevance in condensed matter and photonic systems to assist readers unfamiliar with the topic.

A1. We thank the Referee for this comment. To provide a background on axion electrodynamics, we have extended the introductory part and mentioned several distinctive physical effects associated with the effective axion response. This includes the rotation of polarization plane for light reflected from the axion medium, emergence of image magnetic charges induced by the point charge above the surface of a topological insulator, while the field of a point electric dipole surrounded by the axion medium acquires a nonzero magnetic dipole component - the phenomenon known as Witten effect. In addition, recent studies suggested that the effective axion field can be strongly enhanced in artificial photonic media - metamaterials which has been confirmed by the recent experiments and opened exciting opportunities to experimentally probe the predictions of axion electrodynamics.

To complement this discussion, we have included the references to the recent experimental works which have just appeared in the literature. The key contributions include:

Q. Yang, et al (Shuang Zhang group), “Gigantic Tellegen responses in metamaterials”, Nature Communications 16, 151 (2025).
G.-G. Liu, et al. (Baile Zhang group), “Photonic axion insulator”, Science 387, 162 (2025).
S. S. Jazi, et al. (Viktar Asadchy group), “Realization of the Tellegen effect in resonant optical metasurfaces”, arXiv preprint 2503.22184, DOI: 10.48550/ARXIV.2503.22184 (2025).
J.-X. Qiu, et al. (Su-Yang Xu group), “Observation of the axion quasiparticle in 2D MnBi2Te4”, Nature, DOI: 434 10.1038/s41586-025-08862-x (2025).

These excellent experimental works highlight a broad research interest in axion electrodynamics in condensed matter and photonics, opening new possibilities and applications.

Q2. I suggest elaborating on the theoretical basis and physical implications of the dual axion field to clarify its distinction from the conventional axion field.

A2. We thank the Referee for the comment. In the revised manuscript, we further clarify both theoretical basis and physical effects originating from the dual axion field.

Theoretical basis. In Sec. II of the Supplementary Materials we derive the theoretical description of the metamaterial possessing dual axion response and explain how one obtains Eqs. (8)-(9) of the main text for the averaged fields. In Sec. III we argue that χ and $\tilde{\chi}$ responses of the material can coexist. Finally, in Sec. IV of the Supplementary Materials we derive the mapping between the two responses showing that the scattering of the arbitrary incident field is identical for χ and $\tilde{\chi}$ cases for the proper choice of the material parameters of the two structures under comparison. However, the interaction with the localized sources is profoundly different. These derivations justify chosen description of the dual axion field.

Physical effects. Similarly to the axion electrodynamics, the dual axion response results in the Kerr rotation of the polarization plane of reflected light. We also observe an analog of the Witten effect when an electric dipole introduced in a finite sample of the axion or dual axion medium produces the fields outside that contain both electric dipole and magnetic dipole components. However, quantitative details of the Witten effect in axion and dual axion media are different, and we harness this to distinguish the two responses.

Quite importantly, the existence of the dual axion field results not only in the quantitative corrections, but also in the qualitatively new phenomena. One of them is the concept of ***nonreciprocal optical decoy*** which we introduce and discuss in the revised manuscript.

To illustrate this, we consider a spherical particle (meta-atom) featuring the combination of axion and dual axion responses such that $\chi = \frac{\varepsilon}{\mu}\tilde{\chi}$. This ratio between the two non-reciprocal responses ensures that the arbitrary excitation of such a meta-atom by the incident wave will not reveal any kind of non-reciprocity. The system will scatter just as a usual dielectric particle with some permittivity and permeability [see Supplementary Materials, Sec. IV, Eq. (S84)].

However, the situation is dramatically different if the meta-atom is excited by the point electric dipole placed inside it. If d_0 is a dipole moment of the source electric dipole, the far field detected outside of the meta-atom will feature both electric and magnetic dipole contributions captured by the formulas

$$d = \frac{3(\mu+2)(\chi\tilde{\chi}+1)}{\varepsilon(\mu\chi\tilde{\chi}+\mu+2\chi\tilde{\chi}^2+2)+2(\mu\chi^2+\mu+2\chi\tilde{\chi}+2)}d_0,$$

$$m = \frac{3(\chi\tilde{\chi}+1)(\mu\chi+2\tilde{\chi})}{\varepsilon(\mu\chi\tilde{\chi}+\mu+2\chi\tilde{\chi}^2+2)+2(\mu\chi^2+\mu+2\chi\tilde{\chi}+2)}d_0.$$

Note that the magnetic dipole contribution is non-zero and describes the non-reciprocity of electromagnetic response.

We thus arrive to the concept of nonreciprocal optical decoy. By any scattering experiment, an outside observer cannot retrieve the nonreciprocal response of the meta-atom. Therefore, when observing electric d and magnetic m dipole moments of the structure, the observer concludes that d and m are the actual dipole moments of the source. However, this is not true, as the meta-atom is only excited by the electric dipole d_0.

We believe that this qualitatively new result could initiate a new research avenue in applied electromagnetism, metamaterials and beyond having presumably not only fundamental interest, but also practical applications.

Besides that, this result suggests that the coexisting axion and dual axion responses can suppress each other in the scattering experiments, while showing strong nonreciprocity when excited by the localized sources.

For clarity, we briefly discuss this possibility in the main text and provide a detailed analysis in the Supplementary Materials, Sec. X.

Q3. Including computational simulations would quantitatively substantiate the theoretical predictions.

A3. We thank the Referee for this suggestion. To support our theoretical predictions further, we have included several types of additional numerical simulations to the main text and Supplementary Materials:

1. We explore an experimentally relevant scenario when the metamaterial structure is planar and magnetized layers form an antiferromagnetic pattern. Simulating the excitation of such a system by the dipole antenna, we demonstrate a clear distinction between the axion and dual axion responses, even though the two media feature an identical response to the incident plane wave. In fact, this simulation shows a direct path towards experimental observation of the dual axion response in the microwave metamaterials. These results are included as Fig. 4 of the main text and Fig. S8 of the Supplementary Materials. We have also added a new section “Suggested experiments” in the main text.

2. We analyze an experimentally important situation when the metamaterial is lossy. Simulating the excitation of a spherical structure by the same dipole antenna, we demonstrate that axion and dual axion responses can be distinguished even in the presence of the moderate losses. We have included this in Sec. XI and Fig. S7 of the Supplementary Materials, and we discuss this in more details in our reply A4 below.

3. Finally, we examine the excitation of the spherical metamaterial structure by the antennas of the different sizes. We demonstrate that if the size of the antenna is smaller than the diameter of the cavity, the results for χ and $\tilde{\chi}$ responses coincide. However, once the size of the antenna exceeds the diameter of the cavity, the two responses can be distinguished. These simulations clarify an important and subtle question: when one can assume that the sources are “inside” the material and when they are “outside”. These results are included in Sec. XI and Fig. S6 of the Supplementary Materials.

Taken together, these additional simulations indicate that χ and $\tilde{\chi}$ responses can be distinguished in a realistic situation. Based on the recent advances in the design and fabrication of metasurfaces and metamaterials featuring effective axion response [V.

Asadchy, *et al*. "Realization of the Tellegen Effect in Resonant Optical Metasurfaces", arXiv:2503.22184 (2025); Q. Yang, *et al*, “Gigantic Tellegen responses in metamaterials”, Nature Communications **16**, 151 (2025)], we are optimistic that the predicted dual axion response could be observed experimentally in the near future.

Q4. The authors should discuss how the potential breakdown of their assumptions impact on the results to improve transparency.

A4. We thank the Referee for this question. One of the key assumptions in our analysis is the absence of losses in the metamaterial. Thus, a valid question is whether one can observe any difference between χ and $\tilde{\chi}$ responses in the presence of the material losses. To address that, we have simulated the excitation of a spherical metamaterial structure [Fig. 3d of the main text] by the finite-length dipole antenna in the presence of moderate losses incorporated as imaginary corrections to the permittivity, permeability and gyrotropy of the layers. Inspecting the field outside of the metamaterial and projecting it onto the vector spherical harmonics, we extract electric and magnetic dipole contributions to the field produced by the entire structure. These effective electric and magnetic dipole moments are evaluated for the entire range of the excitation frequencies from 2 to 8 GHz.

While the characteristic peaks in the spectrum become less visible, the two types of response can still be distinguished due to the different values of the magnetic dipole moment. For convenience, we provide the simulation results below. Also, we summarize these results in the Supplementary Materials, Sec. XI and briefly mention in the revised main text.

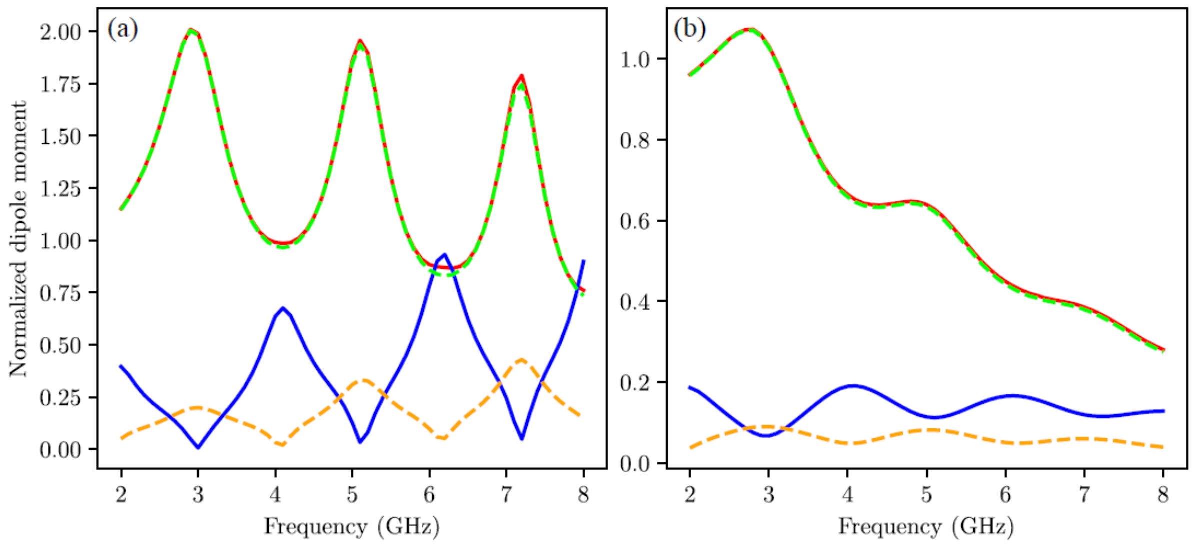

Figure R1. The influence of the Ohmic losses on the distinction between the axion and the dual axion responses. Spherical metamaterial structures have the same geometric parameters as in the main text and no cavity inside. The antenna has a length $L = 4a_0$, a_0 is the period of the lattice (twice the thickness of the layer). The material parameters for the lossless case (a) are the same as in the main text, whereas in the case of lossy structure (b) we assume $\varepsilon = 5 + 0.5\,i$, $\mu = 1 + 0.1\,i$, $g_e = 4 + 0.4\,i$ for the axion case, while the parameters of the dual axion structure are recovered from the mapping. (a,b) Electric dipole moment of the structure versus frequency normalized to the magnitude of the source electric dipole. Cases with no losses (a) and with losses (b). Solid and dashed lines correspond to the axion and dual axion metamaterials, respectively. Red and green lines correspond to the electric dipole moment, blue and orange curves correspond to the magnetic dipole moment multiplied by the factor of 2 for better visibility.

Q5. It would enhance the manuscript if the authors could suggest potential experimental setups to observe the dual axion field, offering concrete examples of how the theoretical predictions can be tested.

A5. We thank the Referee for this important suggestion. Very recently, there has been substantial progress in realizing metasurfaces with effective Tellegen response [Asadchy, *et al*. arXiv:2503.22184 (2025); Q. Yang, *et al,* Nature Communications 16, 151 (2025)]. So far, these realizations were restricted only to a single layer of non-reciprocal meta-atoms. However, fabricating more layers to create a bulk metamaterial structure is certainly feasible, especially in the microwave range.

In light of these advances, we are convinced that the shortest path to observe dual axion response experimentally is to utilize Tellegen meta-atoms already fabricated by the other groups and perform two sorts of measurements:

1. Measurement of cross-polarized reflection (r_{xy}) and transmission (t_{xy}) for the incident plane wave. Observation of the nonzero r_{xy} along with $t_{xy} = 0$ excludes the conventional magneto-optical response providing a smoking gun of the axion or dual axion field.
2. In the second experiment, we suggest exciting the metamaterial structure by the dipole antenna. Measuring cross- and co-polarized fields outside and comparing that with the effective medium prediction or with numerical simulations, one can infer whether the structure features axion or dual axion response.

To support this proposal, we have performed two independent sets of full-wave numerical simulations that implement the same underlying physics in slightly different geometries. Both simulations model planar multilayered metamaterials featuring either χ or $\tilde{\chi}$ response with parameters chosen to satisfy the mapping. In Figure R2, we show the results of a two-dimensional simulation with periodic boundary conditions applied along the transverse direction. In Figure R3, we present a three-dimensional simulation using periodic boundary conditions in both transverse directions.

We first analyze a plane wave excitation, shown in panels (a) of Figs. R2 and R3. As predicted by the mapping, the reflection and transmission coefficients, both co- and cross-polarized, are nearly identical for the χ and $\tilde{\chi}$ cases [Figs. R2(b,c) and R3(b,c); note that Fig. R2(b,c) shows $|R_{xx}|^2$, $|R_{xy}|^2$, etc, while Fig. R3(b,c) shows $|R_{xx}|$, $|R_{xy}|$, etc.] Slight discrepancies between the results should be attributed to the approximate nature of the effective medium description which causes some inaccuracies in the high-frequency region. This confirms that the scattering measurements alone do not allow to discern axion and dual axion responses.

Next, we consider the excitation of the structure by the oscillating sources in both setups. In the 2D case, we simulate an infinite periodic linear array of thin wires, each carrying an oscillating current [Fig. R1(d)]. The lattice of such wires produces a plane wave polarized along the wires and a set of rapidly decaying higher-order Floquet harmonics. Due to the influence of the axion medium, the field outside acquires not only co-polarized, but also the cross-polarized component. The frequency dependence of those components is depicted in Fig. R1(e,f). Quite remarkably, we observe that the cross-polarized fields in χ and $\tilde{\chi}$ cases are profoundly different. We therefore conclude that such experiments allow to clearly distinguish the two versions of the axion-type response. Even though this particular measurement has not been done before, it is certainly feasible for microwave metamaterials.

In the 3D case, we simulate the excitation of the structure by an infinite periodic planar array of electric dipoles localized at the center of the slab [Fig. R2(d)]. Periodic boundary conditions are applied in both transverse directions to exclude the influence of the side surfaces and to emulate an infinite array of identical dipoles, analogous to the wire array in the 2D setup. As in the previous case, the axion-type response induces a cross-polarized electric field component in the far field. The retrieved E_x field along the z-axis shows a pronounced and frequency-dependent difference between the χ and $\tilde{\chi}$ metamaterials [Fig. R2(f)], in close agreement with the 2D results. This further validates the robustness and feasibility of the proposed experimental strategy, independent of the specific excitation scheme. Notably, the suggested scheme utilizes a planar geometry which could be readily implemented in experiments.

These complementary simulations prove that such an experiment is not only theoretically well founded but also practically realizable using existing microwave metamaterials. In light of the recent advances in the design and fabrication of metasurfaces and metamaterials featuring effective axion response [V. Asadchy, *et al*. "Realization of the Tellegen Effect in Resonant Optical Metasurfaces", arXiv:2503.22184 (2025); Q. Yang, *et al*, "Gigantic Tellegen responses in metamaterials", Nature Communications **16**, 151 (2025)], we are convinced that the predicted dual axion response can be observed experimentally in the near future.

To clarify this important aspect to the reader, we have added this discussion and our numerical simulations to the main text as a new section "Suggested experiments", while further details are moved to the Supplementary Materials, Sec. XI.

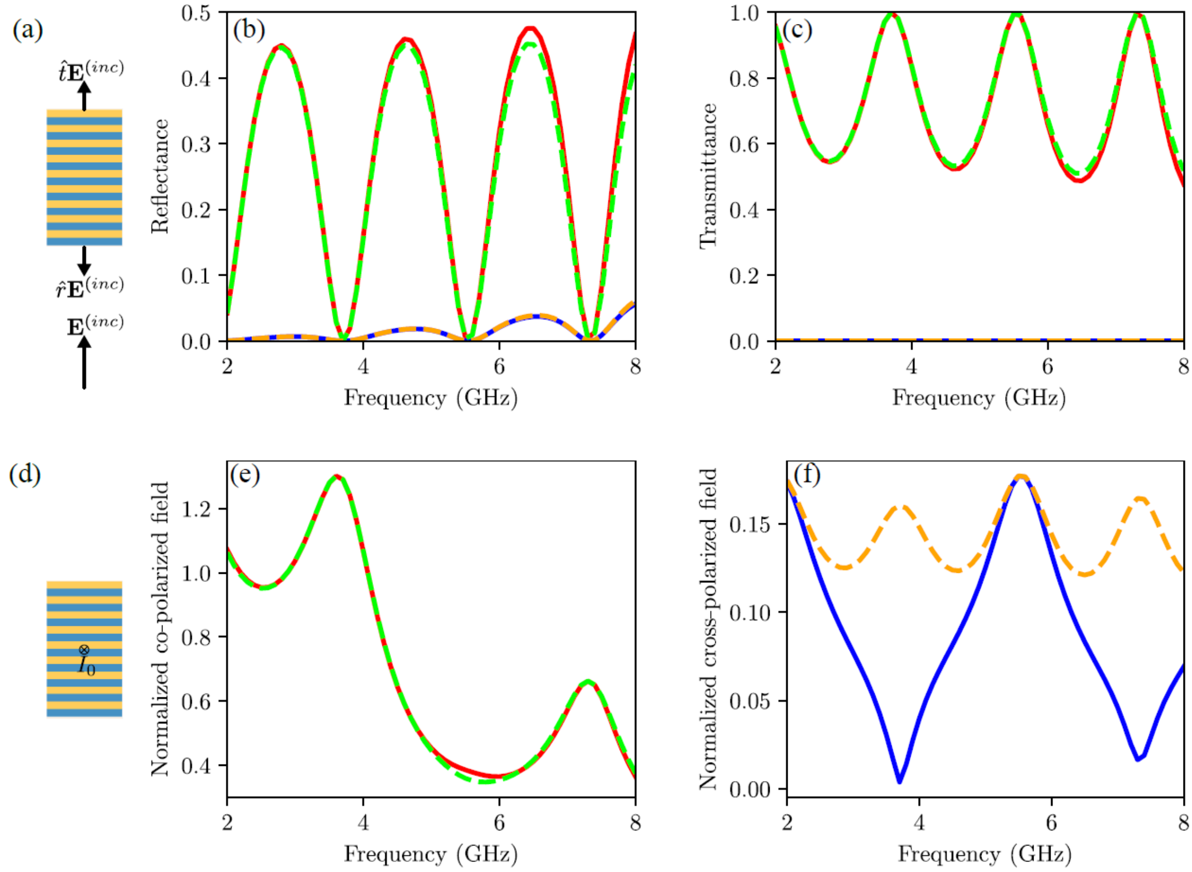

Figure R2. Exploring the distinction between axion and dual axion response in a planar geometry composed of 18 magnetized layers. (a) Scheme of a plane-wave excitaton of a multilayered slab. The period of the lattice $a_0 = 4$ mm, the parameters of the axion metamaterial are $\varepsilon = 5$, $\mu = 1$, $g_e = 4$, so $\chi = \pi/2\, g_e\, a_0/\lambda$, where λ is a wavelength. The parameters of the dual axion metamaterial are recalculated from the mapping Eqs. (22)–(24) of the main text, and magnetic gyrotropy is $g_m = 2/\pi\, \tilde{\chi}\, \lambda/a_0$. (b,c) The spectra of reflection and transmission coefficients modulo squared. Solid and dashed lines correspond to the axion and dual axion metamaterials, respectively. Red and green lines correspond to the co-polarized scattering, blue and orange lines correspond to the cross-polarized scattering. (d) Scheme of an infinite out-of-plane line current excitation. The parameters of the structures are the same as in (a). (e,f) Far fields above the multilayered structure normalized to the characteristic electric field of an isolated wire, $\mu_0 I_0 \omega/4$.

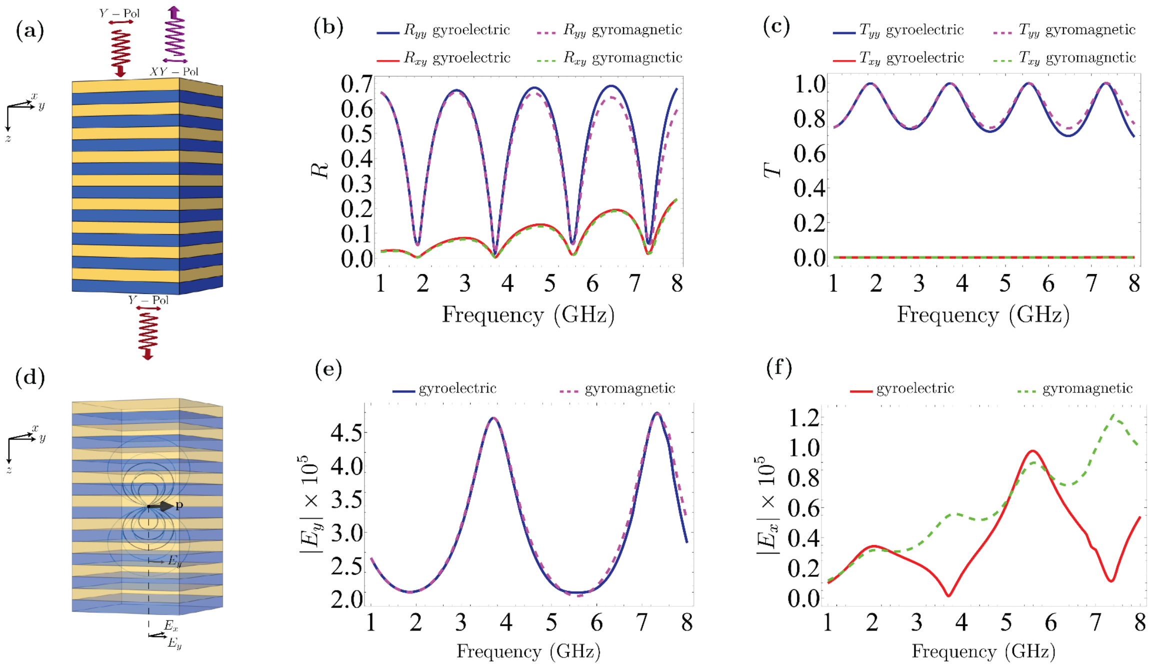

Figure R3. Distinguishing axion and dual axion responses in a multilayered metamaterial in planar geometry. We consider a multilayered stack composed of 18 alternating gyrotropic layers (9 unit cells) applying periodic boundary conditions in the transverse (x and y) directions. The effective permittivity and permeability of the layers are chosen as $\varepsilon = 5, \mu = 1$, while the gyrotropic coefficients are $g_e = 4$, $g_m =- 0.73397$ and are related via the mapping, Eq. (25) of the main text. (a) A scheme, illustrating the typical configuration used to probe the material response in reflection and transmission experiments. (b,c) Amplitude reflection and transmission coefficients computed for χ (gyroelectric) and $\tilde{\chi}$ (gyromagnetic) media show near-perfect agreement, in accordance with the mapping that renders these responses indistinguishable in the absence of sources. (d) Fields from the metamaterial structures excited by the point electric dipole placed in the middle of the multilayered stack and oriented along the y axis. (e,f) The electric field measured along the z-axis reveals a distinction in the E_x component between axion and dual axion media, particularly between 3–4 GHz, while the E_y components remain nearly identical.

Response to Reviewer #2

Q1. How feasible is such an experimental set up? Has a similar experiment been performed with a meta-material before? If not, is it in principle possible?

A1. We thank the Referee for raising this important question. Very recently, there has been substantial progress in realizing metasurfaces with effective Tellegen response [Asadchy, *et al*. arXiv:2503.22184 (2025); Q. Yang, *et al,* Nature Communications **16**, 151 (2025)]. These structures provide the first instances of axion electrodynamics in metamaterials enhancing the effect by two orders of magnitude compared to such celebrated condensed matter structures as Cr_2O_3.

So far, the reported realizations were restricted only to a single layer of non-reciprocal meta-atoms. However, fabricating more layers to create a bulk metamaterial structure is certainly feasible, especially in the microwave range.

In light of these advances, we are convinced that the shortest path to observe dual axion response experimentally is to utilize Tellegen meta-atoms already fabricated by the other groups and perform two sorts of measurements:

1. Measurement of cross-polarized reflection (r_{xy}) and transmission (t_{xy}) for the incident plane wave. Observation of the nonzero r_{xy} along with $t_{xy} = 0$ excludes the conventional magneto-optical response providing a smoking gun of the axion or dual axion field. These measurements have already been performed in the experimental papers cited above.
2. In the second experiment, we suggest exciting the metamaterial structure by the dipole antenna introduced inside. Measuring cross- and co-polarized fields outside of the structure and comparing the results with the effective medium prediction or numerical simulations, one can infer whether the structure features axion or dual axion response.

To support this proposal, we have performed two independent sets of full-wave numerical simulations that implement the same underlying physics in slightly different geometries. Both simulations model planar multilayered metamaterials featuring either χ or $\tilde{\chi}$ response with parameters chosen to satisfy the mapping. In Figure R2, we show the results of a two-dimensional simulation with periodic boundary conditions applied along the transverse direction. In Figure R3, we present a three-dimensional simulation using periodic boundary conditions in both transverse directions.

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Next, we consider the excitation of the structure by the oscillating sources in both setups. In the 2D case, we simulate an infinite periodic linear array of thin wires, each carrying an oscillating current [Fig. R1(d)]. The lattice of such wires produces a plane wave polarized along the wires and a set of rapidly decaying higher-order Floquet harmonics. Due to the influence of the axion medium, the field outside acquires not only co-polarized, but also the cross-polarized component. The frequency dependence of those components is depicted in Fig. R1(e,f). Quite remarkably, we observe that the cross-polarized fields in χ and $\tilde{\chi}$ cases are profoundly different. We therefore conclude that such experiments allow to clearly distinguish the two versions of the axion-type response. Even though this particular measurement has not been done before, it is certainly feasible for microwave metamaterials.

In the 3D case, we simulate the excitation of the structure by an infinite periodic planar array of electric dipoles localized at the center of the slab [Fig. R2(d)]. Periodic boundary conditions are applied in both transverse directions to exclude the influence of the side surfaces and to emulate an infinite array of identical dipoles, analogous to the wire array in the 2D setup. As in the previous case, the axion-type response induces a cross-polarized electric field component in the far field. The retrieved E_x field along the z-axis shows a pronounced and frequency-dependent difference between the χ and $\tilde{\chi}$ metamaterials [Fig. R2(f)], in close agreement with the 2D results. This further validates the robustness and feasibility of the proposed experimental strategy, independent of the specific excitation scheme. Notably, the suggested scheme utilizes a planar geometry which could be readily implemented in experiments.

These complementary simulations prove that such an experiment is not only theoretically well founded but also practically realizable using existing microwave metamaterials. In light of the recent advances in the design and fabrication of metasurfaces and metamaterials featuring effective axion response [V. Asadchy, *et al*. "Realization of the Tellegen Effect in Resonant Optical Metasurfaces", arXiv:2503.22184 (2025); Q. Yang, *et al*, "Gigantic Tellegen responses in metamaterials", Nature Communications **16**, 151 (2025)], we are convinced that the predicted dual axion response can be observed experimentally in the near future.

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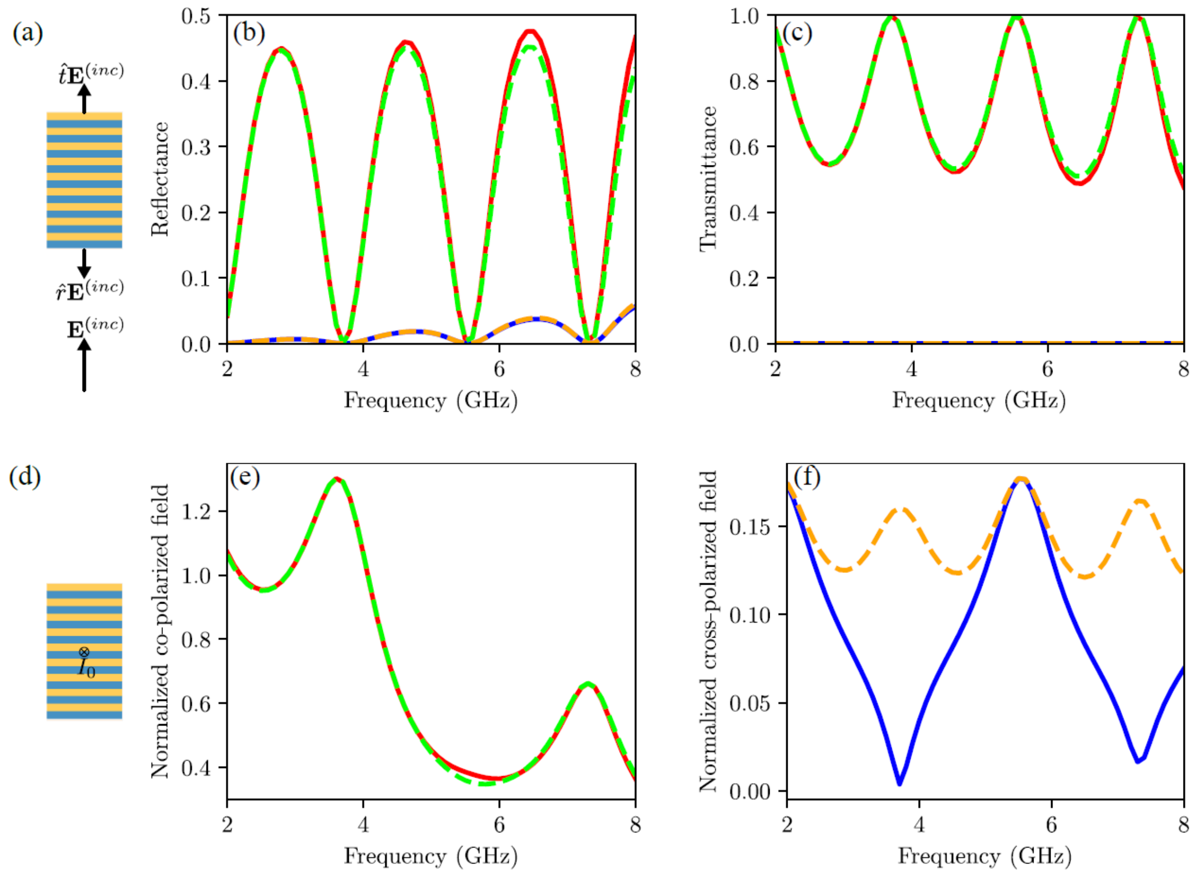

Figure R2. Exploring the distinction between axion and dual axion response in a planar geometry composed of 18 magnetized layers. (a) Scheme of a plane-wave excitaton of a multilayered slab. The period of the lattice $a_0 = 4$ mm, the parameters of the axion metamaterial are $\varepsilon = 5$, $\mu = 1$, $g_e = 4$, so $\chi = \pi/2\, g_e\, a_0/\lambda$, where λ is a wavelength. The parameters of the dual axion metamaterial are recalculated from the mapping Eqs. (22)–(24) of the main text, and magnetic gyrotropy is $g_m = 2/\pi\, \tilde{\chi}\, \lambda/a_0$. (b,c) The spectra of reflection and transmission coefficients modulo squared. Solid and dashed lines correspond to the axion and dual axion metamaterials, respectively. Red and green lines correspond to the co-polarized scattering, blue and orange lines correspond to the cross-polarized scattering. (d) Scheme of an infinite out-of-plane line current excitation. The parameters of the structures are the same as in (a). (e,f) Far fields above the multilayered structure normalized to the field of an isolated wire, $\mu_0 I_0 \omega/4$.

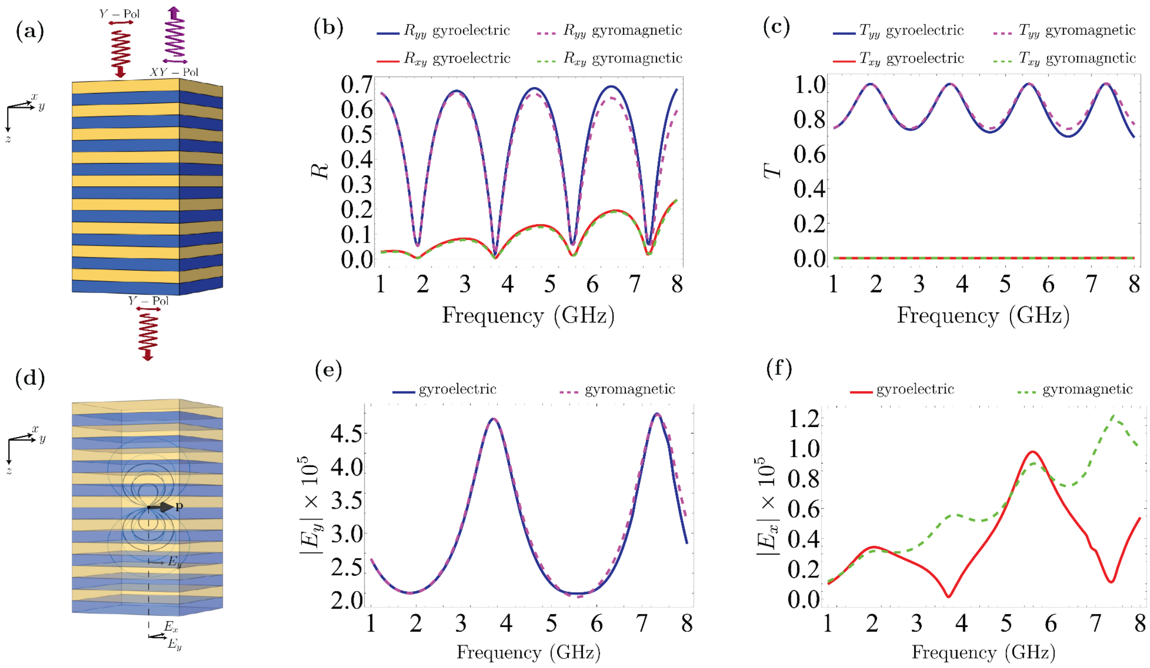

Figure R3. Distinguishing axion and dual axion responses in a multilayered metamaterial in planar geometry. We consider a multilayered stack composed of 18 alternating gyrotropic layers (9 unit cells) applying periodic boundary conditions in the transverse (x and y) directions. The effective permittivity and permeability of the layers are chosen as $\varepsilon = 5$, $\mu = 1$, while the gyrotropic coefficients are $g_e = 4$, $g_m = -0.73397$ and are related via the mapping, Eq. (25) of the main text. (a) A scheme, illustrating the typical configuration used to probe the material response in reflection and transmission experiments. (b,c) Amplitude reflection and transmission coefficients computed for χ (gyroelectric) and $\tilde{\chi}$ (gyromagnetic) media show near-perfect agreement, in accordance with the mapping that renders these responses indistinguishable in the absence of sources. (d) Fields from the metamaterial structures excited by the point electric dipole placed in the middle of the multilayered stack and oriented along the y axis. (e,f) The electric field measured along the z-axis reveals a distinction in the E_x component between axion and dual axion media, particularly between 3–4 GHz, while the E_y components remain nearly identical.

Q2. The authors state that any voids must be smaller than the lattice spacing, such that material can be treated as homogeneous. (In contrast to the case of a larger void, in which there is no observable difference between axion and dual axion materials.) Presumably the antenna generating the dipole field must also be smaller than the lattice spacing in order to be considered “inside” the material. Is this feasible?

A2. We thank the Referee for this question. In short, the size of the antenna is not crucial for observing the predicted physics. As we have shown previously [Fig. 3(d-f) of the main text], exciting the metamaterial structure by the relatively large antenna, one could observe a clear distinction between axion and dual axion responses [Fig. 3(f) of the main text]. On the other hand, in our new simulations we have analyzed the excitation of the planar metamaterial structure by the point dipole antenna, see Fig. R3 above. Still, the difference between χ and $\tilde{\chi}$ responses is clearly visible. A similar distinction is also observed when exciting the structure by the linear current, i.e. a very long antenna [Fig. R2 above]. Based on these simulations, we conclude that the difference between axion and dual axion responses is robust and can be probed experimentally in the different settings. For clarity, we mention that in the revised manuscript.

Yet another important question touched by the Referee is when one can assume that the sources are “inside” the material and when they are “outside”. To elaborate more on this, we have simulated a spherical metamaterial geometry with a void inside [Figure R4] for the two representative cases: (a,c) the size of the antenna is smaller than the void; (b,d) the size of the antenna is larger than the diameter of the void.

In the former case, the fields outside of the structure computed for χ and $\tilde{\chi}$ cases are identical [see Fig. R4(c)]. This result agrees with the mapping, which states that the response of the two media to the arbitrary field incident from outside (including the cavity) is the same.

In the latter case, the antenna is larger and slightly penetrates into the metamaterial [Fig. R4(b)]. Quite remarkably, the difference between χ and $\tilde{\chi}$ media already shows up resulting in the different values of the magnetic dipole moment produced by the entire structure [Fig. R4(d)].

To clarify this interesting physics to the readers, we have added a brief discussion in the manuscript text and included the results of our additional simulations to the Supplementary Materials, Sec. XI.

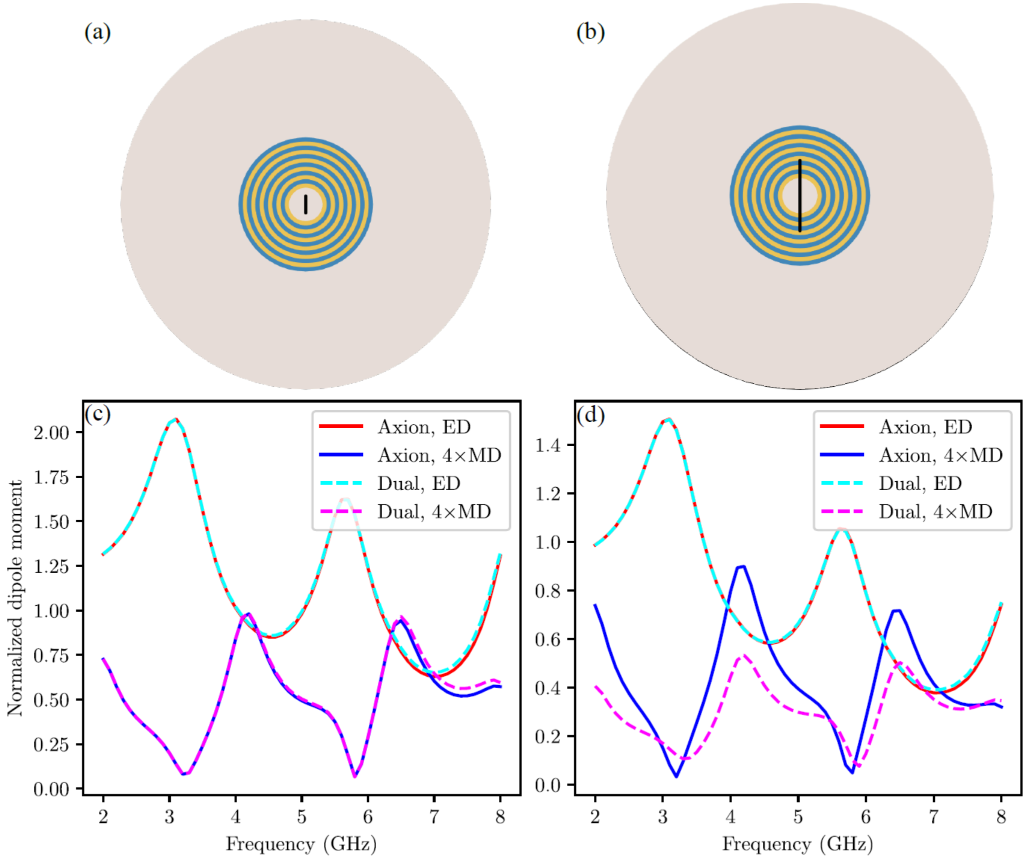

Figure R4. Probing the distinction between axion and dual axion responses using antennas of the different sizes. If antenna is small enough and fits inside the cavity, the distinction is not observed. However, if the antenna touches the material crossing at least few magnetized layers, the difference between χ and $\tilde{\chi}$ starts to show up. Solid and dashed lines correspond to the axion and the dual axion metamaterials, respectively. Red and cyan lines depict the electric dipole moment. Blue and magenta lines correspond to the magnetic dipole moment multiplied by the factor of 4 for better visibility. Parameters of the simulations correspond to those in Fig. 3 of the main text.

Q3. Even in the case that the antenna is smaller than the lattice spacing, it introduces an impurity into the material, which is not taken into account in the authors' calculations. Can the authors justify that this may be neglected?

A3. We would like to highlight an important distinction between (i) an impurity in the material and an (ii) external source.

An impurity is a local modification of the material properties, e.g. permittivity, permeability or magnetization, which alters the propagation and scattering of light. An example of such a kind is a spherical cavity in our metamaterial.

In contrast, the external source arises as nonzero term (ρ, j) on the right-hand side of Maxwell's equations. In particular, the oscillation of external sources defines the frequency of the fields in the structure, while the polarization of the source determines the polarization of the excited fields.

In this sense, the antennas which we introduce in our numerical or analytical calculations are sources, not impurities, and this is fully taken into account in all our calculations.

Of course, to introduce an antenna into the realistic metamaterial, one has to reserve some space for it. However, based on our experience with microwave metamaterials, this can be readily done without creating the impurities but rather by preparing some empty spaces in advance. An example of such kind for 3D microwave metamaterials is provided e.g. in [Y. Yang, et al. Nature 565, 622 (2019)].

Q4. The authors assume that the fields outside the sphere take the standard forms for the fields generated by an electric or magnetic dipole moment (equations S158 and S159). It is not obvious to me that this is correct. The presence of the metamaterial may change the form of these fields in a way not simply described by an induced dipole moment. Can the authors show why these ansatz fields are sufficient?

A4. We thank the Referee for this comment. In the general case, an arbitrary distribution of the fields inside and outside of the sphere can be presented in the form of multipole expansion. Each multipole has a unique angular distribution of the field captured by the vector spherical harmonic characterized by the two indices, l and m. The multipole expansions for the fields inside and outside must be matched at the boundary using the boundary conditions.

In our analysis, we examine the excitation of χ or $\tilde{\chi}$ sphere by the point electric or magnetic dipole parallel to the Oz axis. Since the problem has full rotational symmetry with respect to Oz axis, all harmonics with $m \neq 0$ vanish. Moreover, the spherical shape of the boundary ensures that the boundary conditions only couple electric and magnetic multipoles of the same order l. As a result, the multipole expansion of the fields inside and outside of the sphere includes only harmonics with $l = 1$ (dipole field) and $m = 0$ (no dependence of the field on the azimuthal angle).

These considerations bring us to the ansatz provided in Secs. VIII and IX of the Supplementary Materials. To clarify the situation to the reader, we have added these clarifications to Sec. IX of the Supplementary Materials.

On a side note, the technical details of the multipole decomposition technique for such kind of problems were summarized in our paper [T.Z. Seidov, M.A. Gorlach. “Hybridization of electric and magnetic responses in the effective axion background”, Physical Review A **108**, 053515 (2023)] which examines the field produced by the oscillating dipole in a static axion medium and which is cited in the revised manuscript.

Response to Reviewer #3

Q1,2. Unfortunately, I do not believe this work represents a significant scientific advance that warrants publication in Nat Comm. The duality of E&M is ancient and well-understood. Indeed it is richer in a medium, but fundamentally there is nothing new. As the authors mentioned themselves, the "dual axion electrodynamics" only leads to quantitative differences, not qualitatively new results. The key results in the main text, such as effective magnetic monopoles, already exists in the "original" axion electrodynamics, known as the Witten effect.

A1,2. We thank the Referee for careful reading of our manuscript and their comments. However, we respectfully disagree with their assessment of the impact and scientific advance of the present work. We believe that **our study opens an entirely new research avenue** which can have profound implications for condensed matter, photonics and, potentially, other disciplines. Below, we provide our arguments in support of that view.

1. We demonstrate that **even if the two materials respond identically to the arbitrary incident field, their electromagnetic properties could still differ**.

We have shown that for the suitable choice of parameters axion χ and dual axion $\tilde{\chi}$ media scatter the incident electromagnetic waves identically [see e.g. Fig. 4(b,c) of the revised manuscript]. However, their interaction with the external sources introduced inside the medium is quite different. From the first glance, this result might seem technical. However, it has important implications for material science and beyond.

It is a common approach to retrieve the information on the properties of the medium from the scattering of light. Notably, this approach is utilized not only in condensed matter and photonics, but also in astrophysics while analyzing the scattered light from the distant sources.

Now, we demonstrate that the scattering data alone is insufficient. This provides a paradigm shift for the materials science and calls for a more thorough analysis while retrieving the electromagnetic responses. Potentially, this also suggests that some electromagnetic responses could have been misidentified previously.

2. **The existence of a dual axion response results not only in the quantitative corrections to axion electrodynamics, but also in the qualitatively new physics**

To illustrate that, we consider a spherical particle (meta-atom) featuring such combination of axion and dual axion response that $\chi = \frac{\varepsilon}{\mu}\tilde{\chi}$. According to our analysis, an arbitrary excitation of such a meta-atom by the incident wave will not reveal any kind of non-reciprocity. The system will scatter just as a usual dielectric particle with some permittivity and permeability [see Supplementary Materials, Sec. IV, Eq. (S84)].

However, the situation changes when the particle is excited by the point electric dipole placed inside it. If d_0 is a dipole moment of the source electric dipole, the far field detected outside of the meta-atom will feature both electric and magnetic dipole contributions captured by the formulas

$$d = \frac{3(\mu + 2)(\chi\tilde{\chi} + 1)}{\varepsilon(\mu\chi\tilde{\chi} + \mu + 2\chi\tilde{\chi}^2 + 2) + 2(\mu\chi^2 + \mu + 2\chi\tilde{\chi} + 2)} d_0,$$
$$m = \frac{3(\chi\tilde{\chi} + 1)(\mu\chi + 2\tilde{\chi})}{\varepsilon(\mu\chi\tilde{\chi} + \mu + 2\chi\tilde{\chi}^2 + 2) + 2(\mu\chi^2 + \mu + 2\chi\tilde{\chi} + 2)} d_0.$$

Note that the magnetic dipole contribution is non-zero and describes the non-reciprocity of electromagnetic response which does not show up in the scattering experiments.

We thus arrive to the concept of ***nonreciprocal optical decoy***. By any scattering experiment, an outside observer cannot retrieve the nonreciprocal response of the meta-atom. Therefore, when observing electric d and magnetic m dipole moments of the structure, the observer concludes that d and m are the actual dipole moments of the source. However, this is wrong, as the structure is only excited by the electric dipole d_0.

We believe that this result alone could initiate a new research avenue in applied electromagnetism, metamaterials and beyond having presumably not only fundamental interest, but also practical applications.

Besides that, this result suggests that the coexisting axion and dual axion responses can suppress each other in the scattering experiments, while showing strong nonreciprocity when probed by the localized sources.

3. **The physics of our structure is governed by electrodynamics with magnetic charge featuring $\nabla \cdot B \neq 0$ - in contrast to the axion electrodynamics**

Magnetic monopoles have so far proven elusive as elementary particles. However, in condensed matter physics, magnetic charges are sometimes introduced as emergent collective excitations. In particular, they arise as image magnetic monopoles for the point electric charge placed above the surface of a strong 3D topological insulator [X.-L. Qi, et al. Science 323, 1184-1187 (2009); A. Uri, et al. Nature Physics 16, 164-170 (2020)]. The notion of magnetic charges is also utilized when describing transport phenomena in spin glasses [S.T. Bramwell, et al. Nature 461, 956–959 (2009); D.J.P. Morris, et al. Science 326, 411–414 (2009)].

Our present work provides a novel twist for this research area deriving an effective medium description of a nonreciprocal metamaterial with $\nabla \cdot B \neq 0$. To the best of our knowledge, our work is the first to introduce such a description of composite structures. This novel description may stimulate further advances in condensed matter and photonics and potentially shine light on the behavior of complex magnetic systems such as altermagnets, spin spirals and spin ice.

4. Finally, **the physics we predicted could be straightforwardly implemented and tested experimentally**.

While this work is theoretical, our prediction can be readily verified experimentally in the near future. Below we list the key prerequisites for that:

-we have shown that the axion and dual axion responses can be distinguished in a planar layered geometry when inserting dipole antennas inside the structure and measuring the field outside. Such a setup can be readily fabricated. To illustrate the potential experiments, we have added the results of new simulations to the main text ("Suggested experiments" section) and Supplementary Materials (Sec. XI).

-recently, there was an experimental demonstration of the Tellegen metamaterial [Q. Yang, et al, “Gigantic Tellegen responses in metamaterials”, Nature Communications 16, 151 (2025)], where the effective Tellegen response was retrieved from the scattering data. Notably, the effective axion response of that structure was 2 orders of magnitude larger than in condensed matter systems, which greatly simplifies the observation and the measurement of the associated physics.

-there is a broad interest to the axion physics in condensed matter community, as evidenced by the recent work [J.-X. Qiu, et al. (Su-Yang Xu group), “Observation of the axion quasiparticle in 2D MnBi2Te4”, Nature, DOI: 434 10.1038/s41586-025-08862-x (2025)].

Therefore, we are optimistic that the materials with the dual axion response could be readily implemented experimentally, opening novel opportunities and applications.

Given conceptual novelty, multidisciplinary scope and experimental feasibility of our results, we kindly ask the Referee to reassess the suitability of our manuscript for Nature Communications.

Q3. The main text of the paper is written in a rather technical fashion, while the essence could have been transparently conveyed by Eqs. S87 and S93 in the SM (of the arXiv version).

A3. We thank the Reviewer for their comment. While we provide the Lagrangian formulation of our results in the Supplementary Materials, we choose the language of the modified Maxwell’s equations in the main text. This choice is motivated by the following considerations:

1. As we elaborated in our reply above, the predicted physics can be verified experimentally in the near future. In our opinion, the metamaterial platform is the most promising for such experiments. We therefore did our best to present our findings in the form understandable to the photonics and condensed matter community. In this area, different responses are typically cast in a form of the modified Maxwell’s equations with various material relations. Hence, the chosen form makes our main results easily interpretable by the community. Notably, this form of presentation has been appreciated by the Reviewers #1 and #2.

2. The Lagrangian description has several important limitations. In particular, it is challenging to incorporate losses. Introducing frequency-dependent χ or $\tilde{\chi}$ (as is the case for our system) is feasible but makes the expressions bulky. The Lagrangian description of the medium with coexisting χ and $\tilde{\chi}$ becomes too complicated, as one needs to introduce two kinds of potentials and then incorporate additional constraints to keep the number of degrees of freedom unchanged.

To sum up, adding extra physical factors to the Lagrangian description quickly complicates the situation. Therefore, it is not a coincidence that the early works exploring axion electrodynamics indeed utilized the Lagrangian language [Essin, et al. Phys. Rev. Lett. 146805 (2009)], while later more detailed studies turned to a different description [J. Ahn, et al. Nature Commun. 13, 7615 (2022)].