

Reconfigurable non-Abelian integrated photonics

Corresponding Author: Professor Xu-Lin Zhang

This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

The focus of this work is non-Abelian integrated photonics, which harnesses non-Abelian physical mechanisms to design integrated photonic devices. Previously, the authors' group employed this concept to demonstrate the non-Abelian manipulation of photons in laser-written waveguide circuits. Subsequent studies by other groups have since extended this approach to alternative integrated photonic platforms, including silicon photonics. The reconfigurability demonstrated in this work is an important step towards practical application of devices of such a class. As such, it is likely to attract broad interest from researchers in both non-Abelian physics and integrated optics. I, in principle, recommend its publication in Nature Communications. But there are some issues need to be addressed.

(1) In the abstract, it is briefly mentioned that "Multi-mode geometric phase matrix... topologically protected." This is clearly incorrect. The braiding is path-dependent. The effect is geometrical. One may even say that the underlying principle has some topological consideration. But the evolution is certainly not topologically protected. But I see that "topological protection" is not mentioned again throughout the text (other than in referencing other works), so I guess it is perhaps careless writing. The wording should be removed.

(2) The non-Abelian braiding physically corresponds to the adiabatic evolution of degenerate eigenstates along closed trajectories on a Poincaré sphere. What analogous physical interpretation describes the case where waveguides are thermally modulated by electrodes?

(3) The present work has demonstrated 24 distinct group elements of the braid group B_4 within a single photonic circuit. Given that the braid group is an infinite group, a discussion on potential methodologies for implementing additional group elements would strengthen the analysis.

(4) I understand that the reconfigurability of the braiding matrix primarily stems from modifications to the diagonal terms of the effective Hamiltonian in Eq. (1) under heating operation. While Fig. 2(d) presents the calculated diagonal terms used in subsequent simulations, the derivation methodology for these critical changes remains unspecified. A detailed explanation should be provided.

(5) What is the design consideration underlying the implementation of incorporating curved waveguide segments in uncoupled regions of the photonic circuit?

(6) The heating electrodes primarily affect the waveguide directly beneath each heater. However, have the authors considered potential thermal crosstalk effects on adjacent waveguides in their analysis. Also, will heating up the waveguide cause any additional noise? How fast is the switching? (I imagine it is not very fast for one has to wait for the device to heat up / cool down.) How will the authors continue to improve the switching mechanism?

(Remarks on code availability)

Reviewer #2

(Remarks to the Author)

The manuscript "Reconfigurable non-Abelian integrated photonics" by Sun et al describes an approach to tune the Hamiltonian of a waveguide architecture for realizing reconfigurable on-chip non-Abelian photonic devices.

As a matrix valued generalization of the Berry phase, non-Abelian geometric phases offer great promise for robust broadband optical computation for both classical and quantum light. The possibility to achieve scalable and reconfigurable

non-Abelian photonic devices constitutes then a significant technological advance.

Even if, among various mechanisms for realizing reconfigurability, the thermo-optic effect proposed is not new for silicon, the demonstrated design framework makes it extremely promising for realizing holonomic photonic transformations and obtain efficient silicon photonic integrated circuits.

The manuscript, clearly written, describes a possible approach in a clear and convincing way. Moreover it gives experimental results that demonstrate the capabilities of the proposed architecture.

For this reason, I'd like to recommend the paper for publication.

I have only one suggestion for the authors that I hope they may wish to address.

My suggestion is to add some more views about potential applications in quantum computation, specifically holonomic quantum computation and implementation of quantum machine learning protocols. These are cases where reconfigurability may play an essential role.

(Remarks on code availability)

Reviewer #3

(Remarks to the Author)

The authors report on a photonics platform based on polymer waveguides that includes thermal elements to re-route light in a Mach-Zehnder-type interferometric setup. They show that this can be used for switching light in a braiding-type configuration by showing a set of switching operations for three and four modes of light.

I do not agree with the authors' assertion that they have shown a way to perform non-Abelian photonics. The shown transformations are switching operations for classical light that, although non-commuting, are not very useful unless either coherently superposed operations or operations on coherent superpositions of light (in particular nonclassical light, say single photons) can be shown. For the latter, I doubt that polymer waveguides would be a suitable platform due to its intrinsic propagation losses. In fact, the manuscript does not quantify or even mention losses at all which casts doubt on the validity of a Hamiltonian description (instead of a master equation description for density matrices).

I do not see that the manuscript provides the novelty and breakthrough that would warrant publication in Nature Communications. I see the manuscript as a solid technical report that would be more suited for a specialist optical journal.

(Remarks on code availability)

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

The revision is satisfactory and I recommend its publication.

(Remarks on code availability)

Reviewer #3

(Remarks to the Author)

The authors have made some amendments to their manuscript that answered my question regarding losses in their polymer structure, thereby confirming my point regarding the Hamiltonian description. A coupling loss of 1.4dB plus a 1.9dB/cm propagation loss is a loss budget that casts doubt on the Hamiltonian description instead of a master equation.

I maintain my viewpoint that the manuscript provides a technical improvement on a platform in which classical light can be manipulated. But the authors have still not convinced me that, given the quoted loss budget, anything other than classical light could be coherently manipulated.

I therefore have not changed my opinion that the manuscript provides an interesting platform that implements operations on classical light, but which is more suited to a more technical journal.

(Remarks on code availability)

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Referees' comments and Authors' Response

Dear Referees,

We are grateful for the constructive comments as well as the referees' time and efforts in accessing our work. In the following, please find (1) a table of list changes, (2) point-by-point response to the referees' comments, (3) a revised version of the main text with revision marks, and (4) a revised version of the Supplementary Information with revision marks. We thank the referees for their more efforts in accessing our revised manuscript.

Sincerely Yours,

Xu-Lin Zhang on behalf of all authors

List of changes.**Main text**

Revision	Addressing	Location
Revise the abstract by removing the wording “topologically protected”.	Referee 1, Comment 1	Abstract
Add a proposal on the realization of all the group elements associated with the braid group and the application for topological quantum computing.	Referee 1, Comment 3 Referee 2, main comment	Pages 7 and 8
Add details on how the effective Hamiltonian in Eq. (1) under heating operation is obtained.	Referee 1, Comment 4	Methods
Add details on the switching speed in experiments.	Referee 1, Comment 6	Page 6
Add an outlook on potential applications for holonomic quantum computing and deep learning.	Referee 2, main comment	Page 8
Add details on the loss of the fabricated chip.	Referee 3, Comment 3	Methods
Add three new references.	Referee 1, Comment 6 Referee 2, main comment Referee 3, Comment 3	Refs. 46,47,48

Supplementary Information

Revision	Addressing	Location
Add a new Supplementary Note 2 to theoretically study how the eigenvalues vary by exerting the thermal modulations.	Referee 1, Comment 2	Supplementary Note 2
Add a proposal on the realization of all the group elements associated with the braid group and the application for topological quantum computing.	Referee 1, Comment 3 Referee 2, main comment	Supplementary Fig. 15
Add an explanation on why curved waveguide segments are needed in the design.	Referee 1, Comment 5	Supplementary Fig. 16

Reply to Referee 1

The focus of this work is non-Abelian integrated photonics, which harnesses non-Abelian physical mechanisms to design integrated photonic devices. Previously, the authors' group employed this concept to demonstrate the non-Abelian manipulation of photons in laser-written waveguide circuits. Subsequent studies by other groups have since extended this approach to alternative integrated photonic platforms, including silicon photonics. The reconfigurability demonstrated in this work is an important step towards practical application of devices of such a class. As such, it is likely to attract broad interest from researchers in both non-Abelian physics and integrated optics. I, in principle, recommend its publication in Nature Communications. But there are some issues need to be addressed.

Reply: We thank the referee very much for reviewing our work and giving us positive comments and valuable suggestions. We have modified the manuscript according to these suggestions. In the following, we will answer the referee's questions in detail.

(1) In the abstract, it is briefly mentioned that "Multi-mode geometric phase matrix... topologically protected." This is clearly incorrect. The braiding is path-dependent. The effect is geometrical. One may even say that the underlying principle has some topological consideration. But the evolution is certainly not topologically protected. But I see that "topological protection" is not mentioned again throughout the text (other than in referencing other works), so I guess it is perhaps careless writing. The wording should be removed.

Reply: We sincerely thank the referee for pointing out this incorrect writing. Our original intention is to state that non-Abelian holonomy is a non-resonant and global effect, and therefore the associated photonic device is robust to weak perturbations such as those introduced to the device length and coupling coefficients. Inspired by the referee, we realize that such robustness relies on a necessary condition that the path of the holonomy is kept the same. Therefore, the non-Abelian device is not topologically protected, since the braiding induced geometric phase matrix is path dependent.

In the revised abstract, we have removed the wording "topologically protected" and revised the sentence as: "Multi-mode geometric phase matrix, arising from the non-Abelian holonomy which is a non-resonant and global effect, has been recently introduced to integrated photonics for the design of broadband and robust on-chip photonic devices".

(2) The non-Abelian braiding physically corresponds to the adiabatic evolution of degenerate eigenstates along closed trajectories on a Poincaré sphere. What analogous physical interpretation describes the case where waveguides are thermally modulated by electrodes?

Reply: Thanks for this key question related to the mechanism of the proposed reconfigurable device. As stated by the referee, the two-mode braiding in a building block (i.e., a four-waveguide subsystem) corresponds to the case that two degenerate eigenmodes are smoothly rotating while they are evolving along a closed loop on a parameter sphere. The four-by-four Hamiltonian reads

$$H(z) = \begin{bmatrix} \beta_X & \kappa_{AX}(z) & \kappa_{BX}(z) & \kappa_{SX}(z) \\ \kappa_{AX}(z) & \beta_A & 0 & 0 \\ \kappa_{BX}(z) & 0 & \beta_B & 0 \\ \kappa_{SX}(z) & 0 & 0 & \beta_S \end{bmatrix}, \quad (\text{R1})$$

where $\beta_X = \beta_A = \beta_B = \beta_S = \beta_0$, with β_0 being the propagation constant of the fundamental mode of a single waveguide. The eigenvalues are $\beta_1 = \beta_2 = \beta_0$, $\beta_3 = \beta_0 - \sqrt{\kappa_{AX}^2(z) + \kappa_{BX}^2(z) + \kappa_{SX}^2(z)}$, and $\beta_4 = \beta_0 + \sqrt{\kappa_{AX}^2(z) + \kappa_{BX}^2(z) + \kappa_{SX}^2(z)}$. Here the two-fold degeneracy is governed by the chiral

symmetry of the Hamiltonian: $\Gamma^{-1}H(z)\Gamma = -H(z)$, where $\Gamma = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. The non-Abelian

braiding occurs in this degenerate subspace, i.e., the two working modes follow the eigenstates associated with β_1 and β_2 .

In the case that the waveguides are thermally modulated by electrodes, the refractive index of the nearby waveguides (i.e., A or B) is significantly lowered down. As a result, a negative detuning term Δ is introduced to β_A or β_B , which then breaks the chiral symmetry of the Hamiltonian. To show this point, we take step II for example, where both waveguides A and B are affected by the electrode heater. The Hamiltonian takes the form

$$H(z) = \begin{bmatrix} \beta_0 & \kappa_{AX}(z) & \kappa_{BX}(z) & 0 \\ \kappa_{AX}(z) & \beta_0 + \Delta & 0 & 0 \\ \kappa_{BX}(z) & 0 & \beta_0 + \Delta & 0 \\ 0 & 0 & 0 & \beta_0 \end{bmatrix}. \quad (\text{R2})$$

The corresponding eigenvalues are $\beta_1 = \beta_0$, $\beta_2 = \beta_0 + \frac{1}{2}(\Delta + \sqrt{\Delta^2 + 4(\kappa_{AX}^2(z) + \kappa_{BX}^2(z))})$, $\beta_3 = \beta_0 + \frac{1}{2}(\Delta - \sqrt{\Delta^2 + 4(\kappa_{AX}^2(z) + \kappa_{BX}^2(z))})$, and $\beta_4 = \beta_0$. Here the eigenstates associated with β_1 and β_2 are still the working modes located in waveguides A and B, but the degeneracy is lifted off since $\beta_1 \neq \beta_2$. Numerical results are shown in the lower panel of Fig. 2d of the main text. As a result, the

two working modes are evolving separately, accumulating dynamical phases only. This is more like a scenario where they are evolving on a trivial plane independently, with no braiding effect and geometric phase accumulation, leading to $U_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ which relates the input and output of the building block.

In the revised manuscript, we have added the above analysis as **Supplementary Note 2**.

(3) The present work has demonstrated 24 distinct group elements of the braid group B_4 within a single photonic circuit. Given that the braid group is an infinite group, a discussion on potential methodologies for implementing additional group elements would strengthen the analysis.

Reply: The braid group B_4 has three generators: σ_1 , σ_2 , and σ_3 . Their corresponding schemes using waveguide arrays are shown in Fig. R1a, where the realization schemes of σ_1^{-1} , σ_2^{-1} , and σ_3^{-1} are also presented. These six elements can be regarded as the basic braids associated with the braid group B_4 .

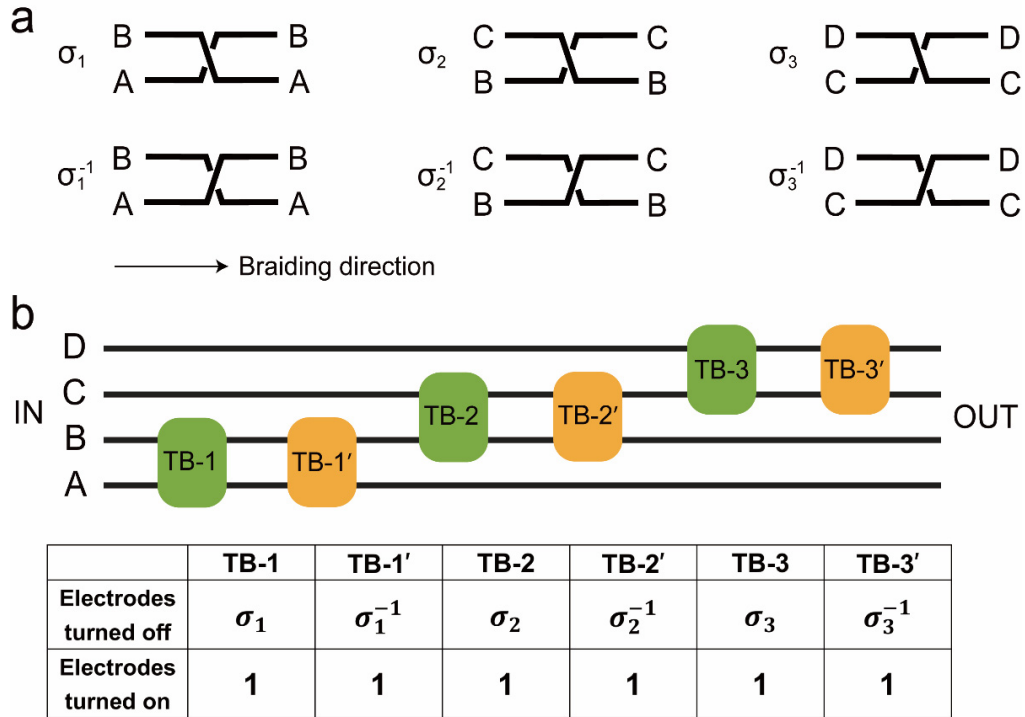


Fig. R1 | **a**, The six basic braids of braid group B_4 and their realization schemes using waveguide arrays. **b**, A potential solution to the realization of a general braiding operation $(\sigma_1)^{n_1}(\sigma_2)^{n_2}(\sigma_3)^{n_3}(\sigma_1)^{n_4}(\sigma_2)^{n_5}(\sigma_3)^{n_6} \dots$, where $n_i \in \mathbb{Z}$, by using a reconfigurable device with six building blocks.

Here the device structure associated with σ_1 , σ_2 , and σ_3 is shown in Fig. 1b of the main text, while that associated with σ_1^{-1} , σ_2^{-1} , and σ_3^{-1} can be achieved by simply reversing the device structure in Fig. 1b along the x axis. With these definitions, the reconfigurable device in Fig. 1a is associated with a group element $(\sigma_1)^{n_1}(\sigma_3)^{n_2}(\sigma_2)^{n_3}(\sigma_1)^{n_4}(\sigma_3)^{n_5}(\sigma_2)^{n_6}$, where n_i can be 0 or 1, corresponding to the case that the associated electrode heater TB- i is turned on or off, respectively. In this way, our reconfigurable device can realize a total of $2^6 = 64$ sets of braiding operations. In our experiment, we have demonstrated 24 sets out of them, which exactly form a permutation group S_4 . The braid group B_4 is an infinite group, and its general element can be represented by $(\sigma_1)^{n_1}(\sigma_2)^{n_2}(\sigma_3)^{n_3}(\sigma_1)^{n_4}(\sigma_2)^{n_5}(\sigma_3)^{n_6} \dots$, where $n_i \in \mathbb{Z}$. This general element can potentially be realized in a reconfigurable device with six tunable building blocks (see Fig. R1b), which are associated with the six basic braids of the group, respectively (see the table). By controlling the array of electrode heaters according to the values of n_i and repeatedly feeding back the output of the device to the input in situ using auxiliary circuits, the reconfigurable non-Abelian braiding device can in principle be employed for the task of topological quantum computing. By introducing necessary working waveguides, our reconfigurable scheme can also be used to design any element belonging to other braid groups B_i .

In the revised manuscript, we have added a detailed discussion in **Page 7 of the main text**: “The braid group B_4 has three generators: σ_1 , σ_2 , and σ_3 , which can be realized using a two-mode braiding building block with waveguides A & B (see Fig. 1b), B & C, and C & D, respectively. With these definitions, the group element representing the reconfigurable device in Fig. 1a is given by $(\sigma_1)^{n_1}(\sigma_3)^{n_2}(\sigma_2)^{n_3}(\sigma_1)^{n_4}(\sigma_3)^{n_5}(\sigma_2)^{n_6}$, where each $n_i \in \{0, 1\}$ depends on whether the corresponding electrode heater TB- i is turned on ($n_i = 0$) or off ($n_i = 1$). In this way, our device can be reconfigured to realize a total of $2^6 = 64$ settings of braiding operations”. We have also added another discussion in **Page 8 of the main text**: “The braid group B_4 is an infinite group, and a general element can be written as $(\sigma_1)^{n_1}(\sigma_2)^{n_2}(\sigma_3)^{n_3}(\sigma_1)^{n_4}(\sigma_2)^{n_5}(\sigma_3)^{n_6} \dots$, where $n_i \in \mathbb{Z}$. A universal solution could be a reconfigurable device in which six tunable building blocks associated with the six basic braids of the group are cascaded in sequence, i.e., σ_1 , σ_1^{-1} , σ_2 , σ_2^{-1} , σ_3 , and σ_3^{-1} , where the inverse braids (i.e., σ_1^{-1} , σ_2^{-1} , σ_3^{-1}) can be realized by simply reversing the device structure of Fig. 1b along the x axis. By controlling the array of electrode heaters according to the values of n_i and repeatedly feeding back the output of the device to the input in situ using auxiliary circuits (see Supplementary Fig. 15 for a schematic), the reconfigurable non-Abelian braiding device can in

principle be employed for the task of topological quantum computing. In addition, by adding or removing necessary working waveguides (i.e., grey ones in Fig. 1a), a reconfigurable braiding belonging to a general braid group B_l can also be achieved". We have also added the above Fig. R1 as **Supplementary Fig. 15** in the revised manuscript.

(4) I understand that the reconfigurability of the braiding matrix primarily stems from modifications to the diagonal terms of the effective Hamiltonian in Eq. (1) under heating operation. While Fig. 2(d) presents the calculated diagonal terms used in subsequent simulations, the derivation methodology for these critical changes remains unspecified. A detailed explanation should be provided.

Reply: We thank the referee for reminding us of the missing of a detailed explanation on the modified Hamiltonian in our original manuscript. This modified Hamiltonian under heating operation is obtained using the following procedure. We first calculate the temperature distribution with the electrode heater by using a finite element method. A typical result is shown in Supplementary Fig. 2. The modified distribution of the refractive index in the device can then be obtained by considering the thermo-optic coefficient of the polymer material ($\sim -1 \times 10^{-4}/\text{K}$). It is intuitive that the refractive index of the waveguide is significantly more affected when it is closer to the electrode heater, and vice versa. Then we consider the proposed two-mode building block in which the distance between the electrode heater and the waveguide (A, B, or X) varies along the waveguiding direction. We study the three waveguides respectively. For each waveguide, we choose a variety of cross sections along the waveguiding direction and calculate the effective mode index of the fundamental mode in the corresponding waveguide (A, B, or X) by using the heating induced refractive index distribution. In this regard, we obtain modified $\beta_A(z)$, $\beta_B(z)$, and $\beta_X(z)$, which are shown in the upper panel of Fig. 2d of the main text. Since the waveguide A (or B) is parallel with the electrode heater, $\beta_A(z)$ (or $\beta_B(z)$) is constant in steps I and II (or II and III). Meanwhile, $\beta_X(z)$ continuously varies with z since the waveguide X (i.e., X1, X2 or X3) is not parallel with the electrode heater. In addition, $\beta_S(z)$ is the same as the case without heating since the waveguide S is sufficiently far away from the electrode heater. The coupling strengths are assumed the same as the case without the electrode heater, since the heating only affects the effective mode index, while the eigenfield profile is nearly not changed. In this way, we have obtained the modified Hamiltonian with electrode heaters turned on. The lower

panel of Fig. 2d of the main text is calculated directly using the finite element method, i.e., the propagation constant in a variety of cross sections of the building block is calculated with the heating induced refractive index distribution. In fact, we can also calculate the eigenvalues of the modified Hamiltonian, which are almost the same as the lower panel of Fig. 2d of the main text, indicating that the modified Hamiltonian works well for the system with electrode heaters turned on.

To make this point clear, we have rewritten the **Methods**: “*Model Hamiltonian*. The fitted coupling strengths (i.e., the off-diagonal terms of the Hamiltonian) in the upper panel of Fig. 2a are obtained by calculating the propagation constants of a corresponding two-waveguide system (see Supplementary Note 1 and Supplementary Fig. 1 for details). The on-site energies of the system (i.e., the diagonal terms of the Hamiltonian) with electrodes turned on, shown in the upper panel of Fig. 2d, are obtained using the following procedure. We first calculate the temperature distribution with the electrode heater by using a finite element method. A typical result is shown in Supplementary Fig. 2. The modified distribution of the refractive index in the device can then be obtained by considering the thermo-optic coefficient of the polymer material ($\sim -1 \times 10^{-4}/\text{K}$). Then we consider the proposed two-mode building block in which the distance between the electrode heater and the waveguide (A, B, or X) varies along the waveguiding direction. We study the three waveguides respectively. For each waveguide, we choose a variety of cross sections along the waveguiding direction and calculate the effective mode index of the fundamental mode in the corresponding waveguide (A, B, or X) by using the heating induced refractive index distribution. In this regard, we obtain modified $\beta_A(z)$, $\beta_B(z)$, and $\beta_X(z)$, as shown in the upper panel of Fig. 2d. In addition, $\beta_S(z)$ is the same as the case without heating since the waveguide S is sufficiently far away from the electrode heater. The coupling strengths are assumed the same as the case without the electrode heater, since the heating only affects the effective mode index, while the eigenfield profile is nearly not changed. In this way, we have obtained the modified Hamiltonian with electrode heaters turned on.

Eigenvalues of the system. The lower panels of Fig. 2a,d are obtained by using the finite element method, i.e., we calculate the propagation constants in a variety of cross sections along the waveguiding direction. In fact, we can also calculate the eigenvalues by directly solving the model Hamiltonian (i.e., upper panels of Fig. 2a,d). The corresponding results are found to be almost the same as the results in the lower panels of Fig. 2a,d, indicating that the fitted Hamiltonian works well for the proposed system”.

(5) What is the design consideration underlying the implementation of incorporating curved waveguide segments in uncoupled regions of the photonic circuit?

Reply: We have indeed introduced curved segments such as those in the crossing step. These segments are necessary to ensure that light injected via any waveguide could accumulate the same dynamical phase so that the dynamical phase effect could be compensated. To give a detailed analysis, we show a top-view of the building block in Fig. R2. At the input part, waveguides A and B share the same length L_{in} so that light propagating in them would accumulate the same dynamical phase. The length of a straight waveguide in step I, II, or III is L_0 . In the crossing step, waveguides A and B are both curved and the corresponding length is designed the same and denoted by L_c . The waveguide S has curved segments in both step II and the crossing step, where the length is represented by L_{s2} and L_{sc} , respectively.

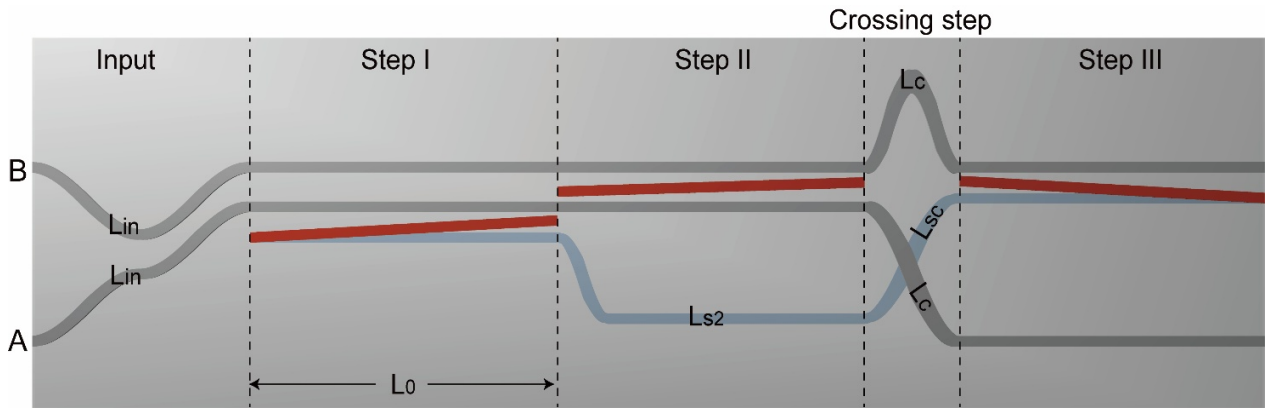


Fig. R2 | Top view of the designed two-mode braiding building block, where the electrodes are not drawn.

We can now calculate the total length of the waveguides when they guide light from the input to the output in this building block. We consider the two cases with electrode heaters turned on and off, respectively. For each case, the input has two choices, i.e., via the waveguide A or B. There are thus a total of four cases, and the corresponding results are given in the following table.

Case No.	Input waveguide	Electrode heater	Total length of waveguides that guide light
1	A	Turned off	$L_{in} + 2 L_0 + L_{s2} + L_{sc}$
2	B	Turned off	$L_{in} + 3 L_0 + L_c$
3	A	Turned on	$L_{in} + 3 L_0 + L_c$
4	B	Turned on	$L_{in} + 3 L_0 + L_c$

In our design, we set $L_0 + L_C = L_{S2} + L_{SC}$. In this way, all the four cases share the same total waveguide length. The dynamical phase effect is therefore compensated by using these specially designed curved segments, leaving the output solely attributed to the geometric phase effect, which is the key to the non-Abelian braiding. We should emphasize that when the electrode heaters are turned on, the refractive index of waveguide A or B is lowered down, resulting in that the accumulated dynamical phase is different from the case without heating. Such difference can further be compensated by introducing additional electrode heaters, as discussed in Supplementary Fig. 17.

To make this point clear, we have added the above Fig. R2 and discussion as **Supplementary Fig. 16** in the revised manuscript.

(6) The heating electrodes primarily affect the waveguide directly beneath each heater. However, have the authors considered potential thermal crosstalk effects on adjacent waveguides in their analysis. Also, will heating up the waveguide cause any additional noise? How fast is the switching? (I imagine it is not very fast for one has to wait for the device to heat up / cool down.) How will the authors continue to improve the switching mechanism?

Reply: We thank the referee for the questions.

For questions regarding the crosstalk, we have considered potential thermal crosstalk effects on adjacent waveguides in our theoretical analysis and numerical simulations. For example, you may find in the upper panel of Fig. 2d that $\beta_X \neq \beta_0$, indicating that the adjacent waveguides X1, X2 and X3 are also affected by the electrode heaters. In fact, the cladding material is also affected. More details on how the thermal crosstalk effects are treated in theoretical analysis have been added in **Methods** (see the reply to comment 4). We think that the heating will only introduce a shift of the refractive index of the nearby polymer materials through the thermal-optic effect. The good coincidence between numerical simulations using RSoft (see Fig. 2e,f) and analytical calculations using fitted Hamiltonian (see Supplementary Fig. 3c,d) indicate that heating up the waveguide would not cause additional evident noises.

For questions regarding the switching speed, the demonstrated device exhibits a response time of ~ 1 ms, primarily attributed to the low thermal conductivity inherent to polymer materials. The response time of the device can be further improved by employing the polymer/silica hybrid waveguide structure [e.g., see *Photonix* 4,13 (2023)], which leverages a silica under-cladding layer

to facilitate efficient heat dissipation from the waveguide core via the underlying silicon substrate (the presence of the thin silica layer is common to all such devices). While our experimental demonstration utilizes a polymer waveguide platform, the proposed architecture is scalable to other integrated photonic platforms, including silicon photonics and lithium niobate photonics. This scalability supports the development of fast-responding on-chip non-Abelian photonic devices.

In the revised manuscript, we have added a discussion on the switching speed in **Page 6 of the main text**: “The switching exhibits a response time of ~ 1 ms, primarily attributed to the low thermal conductivity inherent to polymer materials. The response time of the device can be further improved by employing the polymer/silica hybrid waveguide structure⁴⁶”.

Reply to Referee 2

The manuscript “Reconfigurable non-Abelian integrated photonics” by Sun et al describes an approach to tune the Hamiltonian of a waveguide architecture for realizing reconfigurable on-chip non-Abelian photonic devices.

As a matrix valued generalization of the Berry phase, non-Abelian geometric phases offer great promise for robust broadband optical computation for both classical and quantum light. The possibility to achieve scalable and reconfigurable non-Abelian photonic devices constitutes then a significant technological advance.

Even if, among various mechanisms for realizing reconfigurability, the thermo-optic effect proposed is not new for silicon, the demonstrated design framework makes it extremely promising for realizing holonomic photonic transformations and obtain efficient silicon photonic integrated circuits.

The manuscript, clearly written, describes a possible approach in a clear and convincing way. Moreover it gives experimental results that demonstrate the capabilities of the proposed architecture.

For this reason, I’d like to recommend the paper for publication.

Reply: We sincerely thank the referee for reviewing our work and giving us positive remarks.

I have only one suggestion for the authors that I hope they may wish to address. My suggestion is to add some more views about potential applications in quantum computation, specifically holonomic quantum computation and implementation of quantum machine learning protocols. These are cases where reconfigurability may play an essential role.

Reply: We appreciate this valuable comment from the referee. In what follows, we will first show how our reconfigurable non-Abelian braiding device can be employed for topological quantum computing, and then discuss on the application of the reconfigurable scheme for other applications including holonomic quantum computing and machine learning.

(1) Proposal and outlook for application of topological quantum computing

The non-Abelian holonomy effect can be employed for various applications, while the non-Abelian braiding proposed in this work is one of them. The non-Abelian braiding is an essential

process for topological quantum computing. We give a potential proposal for realizing universal non-Abelian braiding, i.e., all the possible and infinite braiding operations within a given braid group such as B_4 can be achieved in a single reconfigurable device. Before showing the proposal, we first add an analysis on the proposed four-mode braiding device in Fig. 1a from the viewpoint of braid group, which will shed light on the design of a universal proposal for topological quantum computing.

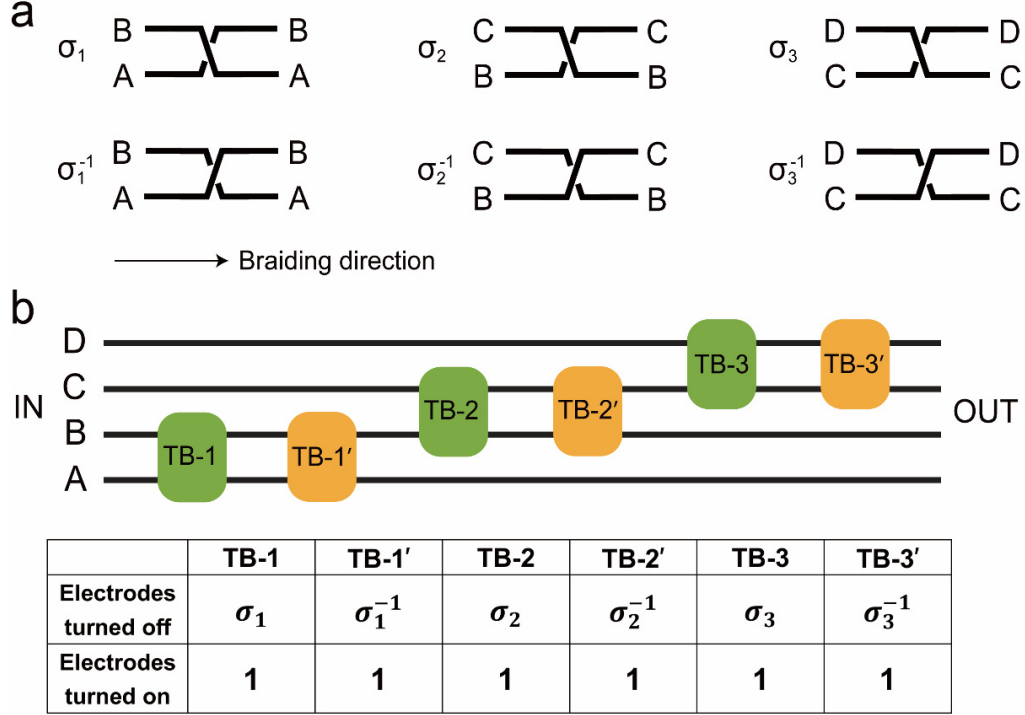


Fig. R3 | **a**, The six basic braids of braid group B_4 and their realization schemes using waveguide arrays. **b**, A universal proposal for topological quantum computing by using a reconfigurable device with six building blocks, in which a general braiding operation $(\sigma_1)^{n_1}(\sigma_2)^{n_2}(\sigma_3)^{n_3}(\sigma_1)^{n_4}(\sigma_2)^{n_5}(\sigma_3)^{n_6} \dots$, where $n_i \in \mathbb{Z}$, can be realized.

We take the braid group B_4 for example. This non-Abelian group has three generators: σ_1 , σ_2 , and σ_3 . Their corresponding schemes using waveguide arrays are shown in Fig. R3a, where the realization schemes of σ_1^{-1} , σ_2^{-1} , and σ_3^{-1} are also presented. These six elements can be regarded as the basic braids associated with the braid group B_4 . Here the device structure associated with σ_1 , σ_2 , and σ_3 is shown in Fig. 1b of the main text, while that associated with σ_1^{-1} , σ_2^{-1} , and σ_3^{-1} can be achieved by simply reversing the device structure in Fig. 1b along the x axis. With these definitions, the reconfigurable device in Fig. 1a is associated with a group element $(\sigma_1)^{n_1}(\sigma_3)^{n_2}(\sigma_2)^{n_3}(\sigma_1)^{n_4}(\sigma_3)^{n_5}(\sigma_2)^{n_6}$, where n_i can be 0 or 1, corresponding to the case that the

associated electrode heater TB- i is turned on or off, respectively. In this way, our reconfigurable device can realize a total of $2^6 = 64$ sets of braiding operations.

We now give a universal proposal for realizing all the possible braiding operations within the braid group B_4 , which is an infinite group. A general group element can be represented by $(\sigma_1)^{n_1}(\sigma_2)^{n_2}(\sigma_3)^{n_3}(\sigma_1)^{n_4}(\sigma_2)^{n_5}(\sigma_3)^{n_6} \dots$, where $n_i \in \mathbb{Z}$. Figure R3b gives a proposal for realizing this general braiding operation, where a reconfigurable device with six tunable building blocks, which are respectively associated with the six basic braids of the group (see the table), is designed. By controlling the array of electrode heaters according to the values of n_i and repeatedly feeding back the output of the device to the input in situ using auxiliary circuits, this reconfigurable non-Abelian braiding device can in principle achieve all the braiding operations belonging to the braid group B_4 . In addition, braiding operations associated with a general braid group B_i can be achieved by adding or removing the working waveguides (i.e., grey ones in Fig. 1a of the main text).

The above proposal and outlook for topological quantum computing would require future experimental works, where some technical challenges need to be addressed including the in-situ feedback mechanism.

(2) Outlook for application of holonomic quantum computing

The non-Abelian holonomy can also be employed for the application of holonomic quantum computing. Different from the application of topological quantum computing where the geometric phase matrices are associated with the braid group, the application of holonomic quantum computing requires geometric phase matrices associated with continuous non-Abelian groups such as $SO(N)$ and $U(N)$. Two recent works [*Nat. Phys.* 19, 30 (2023) and *Nat. Commun.* 16, 3650 (2025)] focused on this issue and successfully demonstrated that arrays of tripod configuration of photonic waveguides can be employed to achieve geometric phase matrices associated with the $SO(N)$ group. These geometric phase matrices can be used to construct holonomic quantum gates for application of holonomic quantum computing. However, the devices proposed in these two works are not reconfigurable.

Our reconfigurable scheme has the potential to be employed for achieving tunable geometric phase matrices associated with the $SO(N)$ group. A potential scheme would be by employing an electrode heater to tune the refractive index of one of the waveguides of the tripod structure [see Fig. 2a in *Nat. Phys.* 19, 30 (2023)]. As the induced refractive index change is continuous, the $SO(N)$ matrix could be tuned continuously as well. In this way, our reconfigurable scheme would be

especially useful for realizing tunable geometric phase matrices belonging to continuous non-Abelian groups, which are essential entities for application of holonomic quantum computing. This would require future theoretical and experimental works, where the key challenge lies in the design of full tunability of the geometric phase matrix.

(3) Outlook for implementation of machine learning protocols

The pioneering work for deep learning using photonic chips was conducted by Dr Shen and published in [*Nat. Photon.* 11, 441 (2017)]. The photonic chip is composed of 56 programmable Mach–Zehnder interferometers (MZIs), each of which comprised a thermo-optic phase shifter. The MZIs, based on the mechanism of dynamical phase accumulation, are used for generating tunable unitary matrices. In principle, these MZIs can be replaced by our non-Abelian building blocks which can also generate the required reconfigurable unitary matrices via the non-Abelian geometric phase effect instead. Since the non-Abelian mechanism is non-resonant and global, the resulting device exhibits broadband and robust features—key advantages over the MZIs based device. This is a potential application of the proposed reconfigurable non-Abelian photonic device for deep learning applications.

(4) Revision in the manuscript

In the revised manuscript, we have added the discussion on the application of topological quantum computing in **Page 8 of the main text**: “The braid group B_4 is an infinite group, and a general element can be written as $(\sigma_1)^{n_1}(\sigma_2)^{n_2}(\sigma_3)^{n_3}(\sigma_1)^{n_4}(\sigma_2)^{n_5}(\sigma_3)^{n_6} \dots$, where $n_i \in \mathbb{Z}$. A universal solution could be a reconfigurable device in which six tunable building blocks associated with the six basic braids of the group are cascaded in sequence, i.e., $\sigma_1, \sigma_1^{-1}, \sigma_2, \sigma_2^{-1}, \sigma_3$, and σ_3^{-1} , where the inverse braids (i.e., $\sigma_1^{-1}, \sigma_2^{-1}, \sigma_3^{-1}$) can be realized by simply reversing the device structure of Fig. 1b along the x axis. By controlling the array of electrode heaters according to the values of n_i and repeatedly feeding back the output of the device to the input in situ using auxiliary circuits (see Supplementary Fig. 15 for a schematic), the reconfigurable non-Abelian braiding device can in principle be employed for the task of topological quantum computing. In addition, by adding or removing necessary working waveguides (i.e., grey ones in Fig. 1a), a reconfigurable braiding belonging to a general braid group B_i can also be achieved”. We have also added the above Fig. R3 as **Supplementary Fig. 15**.

We have added the outlook for applications of holonomic quantum computing and deep learning as a new section “**Prospective applications in related fields**” in **Page 9 of the main text**: “Beyond non-Abelian braiding, our reconfigurable approach for the non-Abelian geometric phase matrix is also expected to be employed for more applications such as holonomic quantum computing. Unlike topological quantum computing where the geometric phase matrices are associated with the braid group, holonomic quantum computing relies on geometric phase matrices associated with continuous non-Abelian groups such as $SO(N)$. A scalable approach utilizing arrays of tripod-configured photonic waveguides has been experimentally demonstrated in femtosecond-laser-written photonic chips²⁷, with subsequent implementation extended to the silicon nitride platform²⁰. By implementing our reconfigurable approach to these devices, the geometric phase matrix associated with the $SO(N)$ group has the potential to be tuned continuously, since the thermal effect induced refractive index change is continuous. These tunable geometric phase matrices facilitate the realization of reconfigurable holonomic quantum gates, serving as the fundamental operational units for holonomic quantum computing. By replacing conventional tunable Mach-Zehnder interferometers with these reconfigurable units based on continuous non-Abelian groups, the resulting broadband and robust non-Abelian device may also enable novel deep learning applications⁴⁷”.

Reply to Referee 3

The authors report on a photonics platform based on polymer waveguides that includes thermal elements to re-route light in a Mach-Zehnder-type interferometric setup. They show that this can be used for switching light in a braiding-type configuration by showing a set of switching operations for three and four modes of light.

I do not agree with the authors' assertion that they have shown a way to perform non-Abelian photonics. The shown transformations are switching operations for classical light that, although non-commuting, are not very useful unless either coherently superposed operations or operations on coherent superpositions of light (in particular nonclassical light, say single photons) can be shown. For the latter, I doubt that polymer waveguides would be a suitable platform due to its intrinsic propagation losses. In fact, the manuscript does not quantify or even mention losses at all which casts doubt on the validity of a Hamiltonian description (instead of a master equation description for density matrices).

I do not see that the manuscript provides the novelty and breakthrough that would warrant publication in *Nature Communications*. I see the manuscript as a solid technical report that would be more suited for a specialist optical journal.

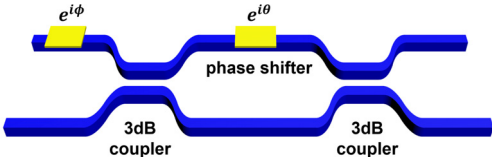
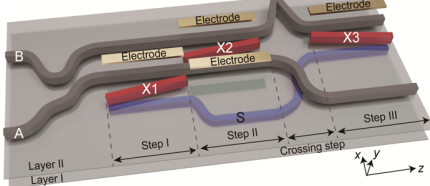
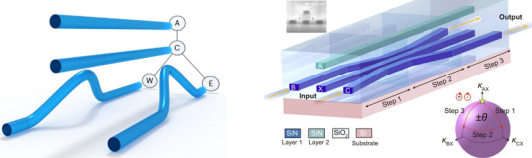
Reply: We thank the referee very much for reviewing our work. We feel sorry that the referee may misunderstand the content and novelty of our work. We first give a brief reply to the comment and then give details. First of all, our device is not based on the Mach-Zehnder-type interferometric (MZI) setup which is related to a local effect that utilizes the accumulation of dynamical phase. Instead, our device is based on the non-Abelian holonomy effect, i.e., a global effect with the emergence of geometric phase matrix that can be employed to realize broadband and robust photonic devices. The architecture of the proposed device is completely different from the traditional Mach-Zehnder-type device. Our system is indeed a non-Abelian photonic system, which can be supported by previous publications: *Nat. Photon.* 16, 390 (2022), *Nat. Phys.* 18, 1080 (2022), *Nat. Phys.* 19, 30 (2023), *Nat. Commun.* 15, 9311 (2024), *Nat. Commun.* 16, 3650 (2025), *Sci. Adv.* 11, eadt7224 (2025) (only some representative experimental works are listed). However, all these works have only demonstrated the scalability of non-Abelian integrated photonics, while the functionality of the non-Abelian photonic device cannot be reconfigured. For practical applications, the non-Abelian photonic devices must meet two key requirements: scalability and reconfigurability. Towards this goal, the novelty of

our work is that we have proposed the first approach and experimental realization of reconfigurable non-Abelian photonic devices. Although we have performed experiments using classical light in this work, the proposed device can also manipulate nonclassical light such as single photons. Actually, there are already some experimental works using single photons in non-reconfigurable non-Abelian photonic devices [see *Nat. Photon.* 16, 390 (2022), *Nat. Phys.* 19, 30 (2023), *Nat. Commun.* 16, 3650 (2025)], indicating that the non-Abelian architecture is compatible with quantum optics. Regarding the issue of loss, selecting an appropriate polymer material can enable the polymer waveguide to achieve a propagation loss as low as 0.32 dB/cm. While our experimental demonstration uses a polymer waveguide platform, the proposed scheme can be extended to other integrated photonic platforms including silicon photonics and lithium niobate photonics.

In what follows, we will give detailed proof of the above reply, and also summarize the novelty and importance of our work.

(1) Detailed reply to “The authors report on a photonics platform based on polymer waveguides that includes thermal elements to re-route light in a Mach-Zehnder-type interferometric setup. I do not agree with the authors' assertion that they have shown a way to perform non-Abelian photonics”

Table R1. A comparison of integrated photonic devices based on MZI and non-Abelian holonomy principles.

	Architecture	Effect	Phase	Matrix
MZI		Local effect	Dynamic phase	$\begin{bmatrix} e^{i\phi} \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \\ e^{i\phi} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} \end{bmatrix}$
Non-Abelian braiding		Global effect	Geometric phase	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \text{heating off}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \text{heating on}$
Non-Abelian holonomy with SO(N)	 Nat. Phys. 19, 30 (2023) Nat. Commun. 16, 3650 (2025)			$\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$

The proposed integrated photonic device is not based on the MZI architecture. We give a detailed comparison between our non-Abelian device and the MZI device using the above Table R1. A MZI structure typically consists of two directional couplers with two thermal phase shifters. By manipulating the accumulation of the two dynamical phases, i.e., $\phi = 2\pi\Delta n_1 L_1/\lambda_0$ and $\theta = 2\pi\Delta n_2 L_2/\lambda_0$, the output and input of the MZI structure can be related via a 2×2 matrix, which acts as a building block for various applications such as optical computing. Since the dynamical phases ϕ and θ depend on the length of the phase shifter (i.e., L_1 and L_2) as well as the wavelength λ_0 , the MZI structure is a resonant component based on a local effect. As a result, the integrated photonic device based on a series of MZI building blocks would exhibit a narrow working bandwidth, and the performance is sensitive to local perturbations that affect the accumulation of the dynamical phases.

The proposed reconfigurable non-Abelian braiding device is based on the non-Abelian holonomy effect, which is a global and non-resonant effect. To be specific, the non-Abelian braiding is realized by constructing a variety of degenerate eigenmodes that adiabatically evolve in a parameter space, where a geometric phase matrix appears with the matrix element $U_{mn} = \mathcal{P} \exp(i \int A_{mn}(\lambda) d\lambda)$, where $A_{mn}(\lambda) = i\langle \psi_m(\lambda) | \partial_\lambda | \psi_n(\lambda) \rangle$ is the Berry connection, ψ_m is the m th eigenvector, and $\mathcal{P}$ is the path-ordering operator. In this way, the non-Abelian geometric phase matrix is accumulated in the entire process, indicating that it comes from a global effect and can therefore be immune from local perturbations. In photonic waveguides, the multiple degeneracy required by the non-Abelian holonomy can be achieved by enforcing adiabatic variations of the coupling strengths within the waveguide array satisfying the chiral symmetry. By mapping the required position-dependent coupling strengths to gap distances, we have designed a building block for two-mode braiding, as shown in the second row of Table R1. In fact, the first experiment for non-Abelian braiding of photonic modes was conducted by our group using femtosecond laser written waveguides [see *Nat. Photon.* 16, 390 (2022)], where the scalability of the photonic chip was demonstrated in both classical laser and single photon experiments. The idea has recently been extended to the silicon nitride platform [see *Nat. Commun.* 16, 3650 (2025)]. However, the functionality of the device cannot be reconfigured, which is essential for practical applications. This problem has been addressed in this work.

In addition to the non-Abelian braiding, which is the main point of this work, the non-Abelian holonomy effect has also been employed to realize the geometric phase matrix belonging to the $SO(N)$ group, as shown in the third row of Table R1. Representative experimental works include [*Nat. Phys.*

19, 30 (2023)] and [*Nat. Commun.* 16, 3650 (2025)]. The proposed reconfigurable scheme in this work can also be employed to reconfigure the geometric phase matrix belonging to the $SO(N)$ group. Recent works on non-Abelian photonic Thouless pumps are also based on the non-Abelian holonomy effect [see *Nat. Phys.* 18, 1080 (2022); *Nat. Commun.* 15, 9311 (2024)].

With the above explanation, we can conclude safely that our device is a non-Abelian photonic device based on the non-Abelian holonomy effect, which is intrinsically different from the traditional MZI device. The non-Abelian device as well as the proposed reconfigurable scheme are essential for global effect required applications such as topological quantum computing and holonomic quantum computing.

(2) Detailed reply to “The shown transformations are switching operations for classical light that, although non-commuting, are not very useful unless either coherently superposed operations or operations on coherent superpositions of light (in particular nonclassical light, say single photons) can be shown”

The main reason that we only perform classical-light experiments in this work is that non-classical light experiments in non-Abelian integrated photonic devices have been reported in a variety of previous works. To be specific, the first experiment of non-Abelian braiding on photonic chips was conducted by our group in [*Nat. Photon.* 16, 390 (2022)], where we have used both classical light and single photons to demonstrate that the device can work in both classical and quantum regimes. A recent work in [*Nat. Commun.* 16, 3650 (2025)] has extended our original idea to the silicon nitride platform, where experiments using single photons are also provided. Another recent work in [*Nat. Phys.* 19, 30 (2023)] has conducted a two-photon experiment in femtosecond laser written non-Abelian photonic chips. These three works have already convincingly shown that the non-Abelian photonic device based on the non-Abelian holonomy effect is compatible with quantum experimental setup, regardless of the experimental platform. Therefore, performing non-classical light experiments is not novel in this research field. Instead, achieving the reconfigurability of the non-Abelian photonic device is the novelty of this work, which has not been realized before. We will discuss the novelty in detail later.

(3) Detailed reply to “In fact, the manuscript does not quantify or even mention losses at all which casts doubt on the validity of a Hamiltonian description”

In this work, we employ an established polymer platform with the waveguide featuring a propagation loss of ~ 1.9 dB/cm [*Opt. Lett.* 40, 3125 (2015)] and the fiber-waveguide coupling loss of ~ 1.4 dB to provide the experimental demonstration, as its fabrication flexibility and high thermo-optic coefficient are crucial for realizing the non-Abelian operations. In the proposed design, the propagation loss of the waveguide does not affect the validity of a Hamiltonian description. To address this concern, we re-simulate the performance of the device with the propagation loss of the waveguides of 1.9 dB/cm, as shown in Fig. R4. It is observed that the output power of the light is decreased, but the reconfigurable non-Abelian braiding is still realized.

For future practical implementations, we agree that it is necessary for the device to have low loss. Over the recent years, numerous studies on photonic integrated circuits have explored leveraging low-loss polymers [*Adv. Funct. Mater.* 13, 453 (2003); *Photonics Res.* 9, 181 (2021); *Nat. Commun.* 14, 4578 (2023); *Opt. Lett.* 49, 582 (2024)]. These materials could directly replace our current platform to significantly reduce the propagation loss. Figure R5 shows the calculated light intensity distributions in the two-mode braiding building block by using the perfluorinated polymer (LFR series) produced by ChemOptics Co. in [*Photonics Res.* 9, 181 (2021)]. The refractive index of the core and cladding materials are 1.396 and 1.371, respectively, and the propagation loss is ~ 0.32 dB/cm at 1550 nm. Due to the change in the refractive index of the waveguide, we redesign the structural parameters of the waveguides in step I, step II and step III (see Supplementary Fig. 10 for the definition of the parameters). The interlayer distance is $4.4\ \mu\text{m}$ and all the waveguides share the same rectangular cross section of $3.5\ \mu\text{m} \times 3\ \mu\text{m}$. The values of g_1 and g_2 at step I (or step III) are designed to be $1.4\ \mu\text{m}$ and $6.8\ \mu\text{m}$, while their values at step II are $0.8\ \mu\text{m}$ and $3.4\ \mu\text{m}$, respectively. We find from Fig. R5 that the two-mode non-Abelian braiding also works well.

In the revised manuscript, we have added details on the loss in **Methods**: “The propagation loss of light in the fabricated waveguide is ~ 1.9 dB/cm, which can be further reduced by leveraging lower-loss polymers⁴⁸. The fiber-waveguide coupling loss is ~ 1.4 dB in the measurement”.

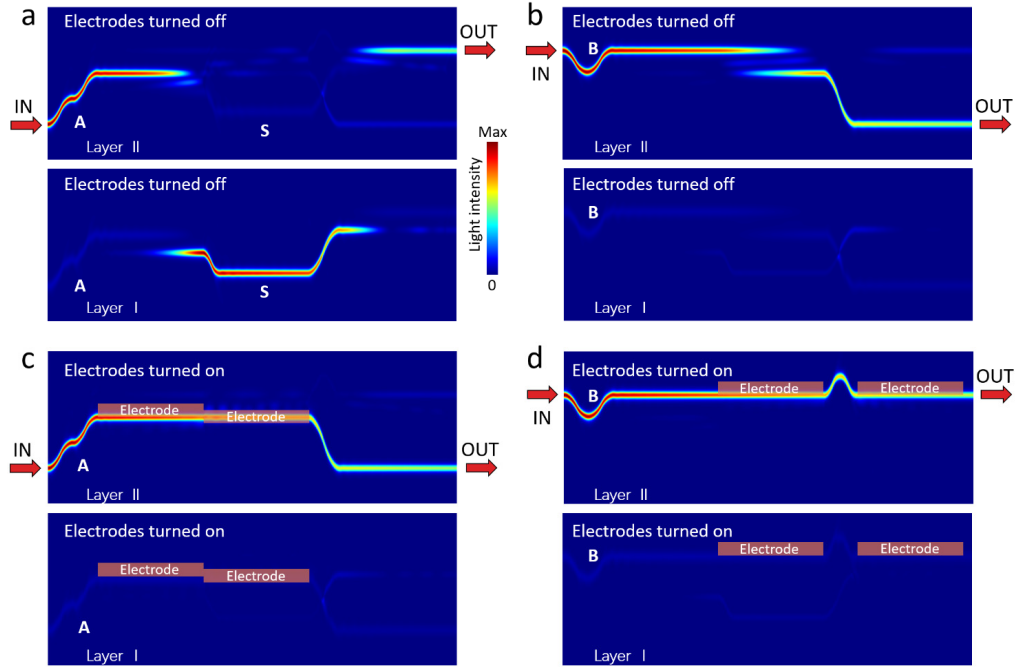


Fig. R4 | **The simulated light intensity distributions in the two-mode braiding building block after adding the loss of the waveguides.** a,b Light input via the waveguide A (a) and B (b) with electrodes turned off. c,d Light input via the waveguide A (c) and B (d) with electrodes turned on.

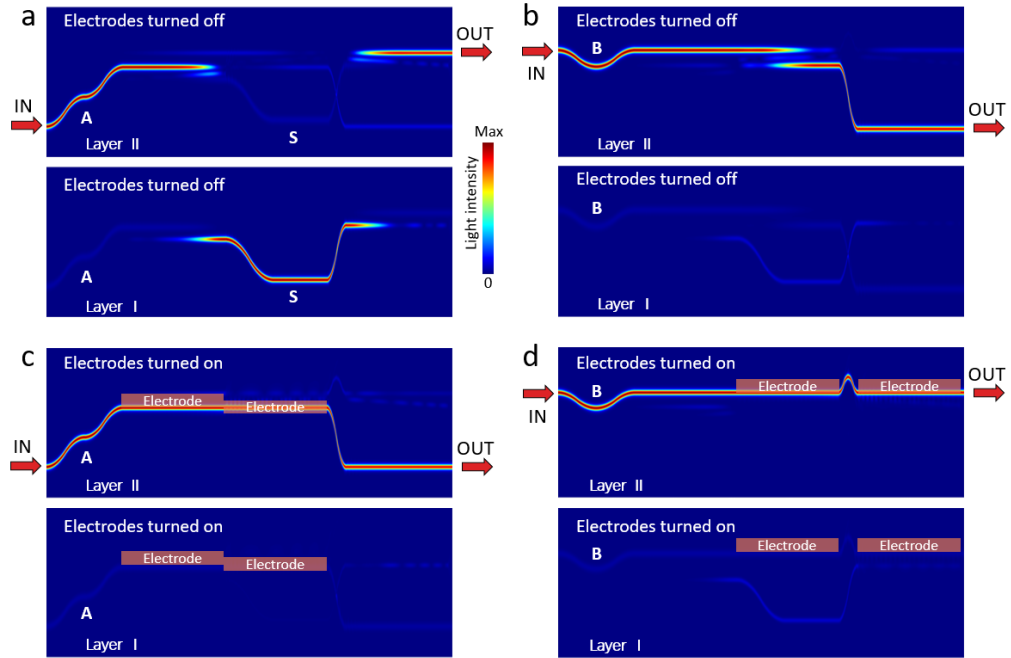


Fig. R5 | **The simulated light intensity distributions in the two-mode braiding building block by using the low loss polymer waveguide material.** a,b Light input via the waveguide A (a) and B (b) with electrodes turned off. c,d Light input via the waveguide A (c) and B (d) with electrodes turned on.

(4) The novelty of our work

We now state the novelty of our work. Non-Abelian physics has important implications in modern science. The recent combination of non-Abelian physics and integrated photonics has given rise to non-Abelian integrated photonics that exploits the non-Abelian holonomy principle. It has innovated the traditional integrated photonics which typically relies on dynamical phase associated resonant components (e.g., Mach–Zehnder interferometers), which exhibit a narrow working bandwidth and perturbation-sensitive performance. In contrast, the non-Abelian holonomy is a global and non-resonant effect, and therefore the associated non-Abelian geometric phase matrix can be exploited to design broadband and robust on-chip photonic devices. In addition, the non-Abelian holonomy can generate arbitrary unitary matrices that are essential to various integrated application scenarios, thus non-Abelian integrated photonics shows promising prospects for optical communications, optical computing, and topological quantum computing. The following Fig. R6 gives a roadmap of non-Abelian integrated photonics.

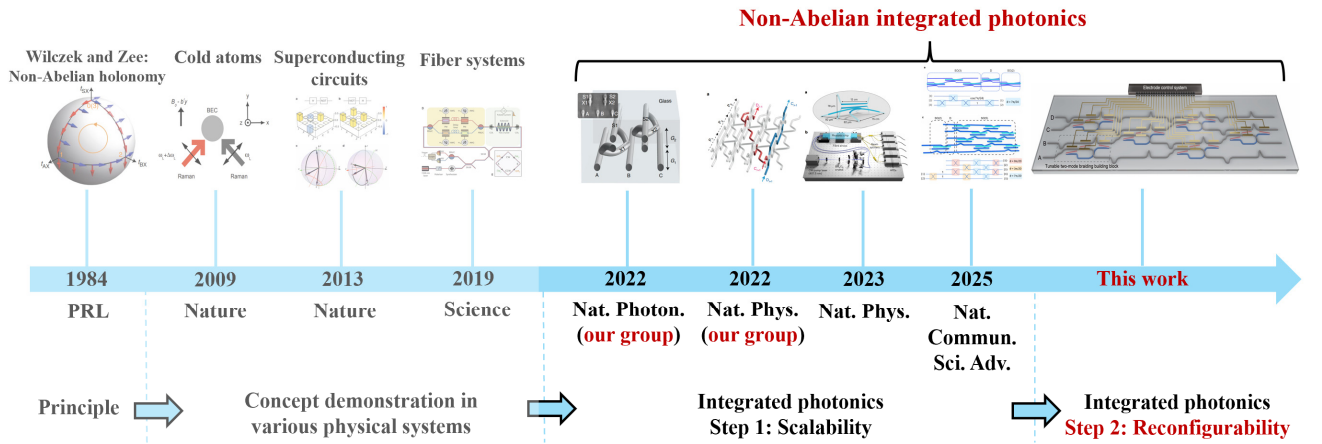


Fig. R6 | Development of non-Abelian integrated photonics

To put forward non-Abelian integrated photonics towards practical applications, the developed on-chip non-Abelian photonic devices must meet two key requirements: **scalability** and **reconfigurability**. The former comes from the demand of increasing information capacity and computing scale, and scalable non-Abelian photonic chips have been successfully demonstrated in various recent works (see Fig. R6). **The latter one, i.e., making non-Abelian devices reconfigurable such that one photonic circuit can be programmed to implement a full plethora of classical or quantum algorithms, however, has not been realized yet.**

In this work, we propose and experimentally realize a general approach for reconfigurable non-Abelian integrated photonics. We design a tunable on-chip non-Abelian building block in which the non-Abelian geometric phase matrix can be reconfigured by employing the thermo-optic effect. To verify the concept, we fabricate a non-Abelian photonic device with six sets of tunable building blocks that performs the task of non-Abelian braiding of four photonic modes. The device can be reconfigured to realize at most 64 sets of unitary matrices, and in our experiment, we demonstrate 24 settings out of them which exactly form a permutation group S_4 .

Our approach is scalable so that the reconfigurable non-Abelian braiding of an arbitrary number of photonic modes can be realized. Besides non-Abelian braiding, other types of non-Abelian photonic devices such as non-Abelian Thouless pumps can also be constructed using our reconfigurable approach. More importantly, the proposed approach is general so that the idea is easy to be transplanted to other integrated platforms such as silicon photonics and lithium niobate photonics towards the design of reconfigurable, broadband, and robust on-chip non-Abelian photonic devices.

We appreciate the referee's comment and hope the referee could reconsider our manuscript.

Referees' comments and Authors' response

Dear Referees,

We are grateful for the new comments. In the following, please find a point-by-point response to the referees' comments and a revised version of the main text with revision marks. We thank the referees for their more efforts in accessing our revised manuscript.

Sincerely Yours,

Xu-Lin Zhang on behalf of all authors

Reply to Referee 1

The revision is satisfactory and I recommend its publication.

Reply: We sincerely thank the referee for his/her effort in reviewing our work.

Reply to Referee 3

The authors have made some amendments to their manuscript that answered my question regarding losses in their polymer structure, thereby confirming my point regarding the Hamiltonian description. A coupling loss of 1.4dB plus a 1.9dB/cm propagation loss is a loss budget that casts doubt on the Hamiltonian description instead of a master equation. I maintain my viewpoint that the manuscript provides a technical improvement on a platform in which classical light can be manipulated. But the authors have still not convinced me that, given the quoted loss budget, anything other than classical light could be coherently manipulated. I therefore have not changed my opinion that the manuscript provides an interesting platform that implements operations on classical light, but which is more suited to a more technical journal.

Reply: We thank the referee very much for reviewing our work. We agree with the referee that there is a loss budget in our polymer platform, and as a result, our platform is currently useful for classical light applications. However, the proposed reconfigurable approach is general, which is actually the novelty of this work. By replacing our polymer material with low-loss polymer such as the perfluorinated polymer (LFR series) with a propagation loss of ~ 0.32 dB/cm at 1550 nm [*Photonics Res.* 9, 181 (2021)], the system has the potential for quantum applications including topological quantum computing and holonomic quantum computing. We will work along this direction in the future.

In the revised manuscript, we have added a clarification on the above point in the **Discussion part (Page 9)**: “Since the polymer waveguide used in our experiments exhibits a non-ignorable propagation loss for light, we have only demonstrated the application of the reconfigurable photonic device for manipulating classical light in this work. By replacing our material with lower-loss polymers⁴⁸, the proposed platform may be employed for quantum applications in the future”.

In addition, we have deleted all the overclaimed statements related to quantum applications, including topological quantum computing and holonomic quantum computing, in both the **main text** and **Supplementary Information**. You may refer to the following main text with revision marks to check all the revisions.

We believe that after revision, our manuscript now focuses solely on applications for classical light, while proposing the use of low-loss polymers for quantum applications as a future prospect.