nature portfolio

Peer Review File

Observing the dynamics of quantum states generated inside nonlinear optical cavities

Corresponding Author: Mr Seou Choi

This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
In this work, the authors propose and experimentally demonstrate a scheme to tomographically measure the quantum state inside a nonlinear optical cavity. The scheme leverages the single-photon-level sensitivity of an optical parametric oscillator (OPO) to the displacement of the initial state as the OPO collapses to one of the bistable states. By measuring the sensitivity of the branching ratio of the bistable states to an external bias field, a tomographic measurement of the Husimi-Q function of the intra-cavity state is achieved. The authors applied the scheme to unravel the dynamical formation of squeezed vacuum states inside an OPO.

In quantum optics, it is surprisingly difficult to experimentally study intra-cavity quantum states because of the lack of an easy means of "opening" the cavity. As a result, there have been only a very limited number of experimental observations of intra-cavity quantum-optical dynamics, despite the sheer volume of theoretical predictions and simulations. I find the authors' work quite significant, as it demonstrates the possibility of observing various dynamical phenomena that are predicted by theory but have never been observed in experiments. The experimental results show good agreement with the model, underscoring the reliability of their scheme. Overall, I would like to recommend the publication of this work in Nature Communications. Below are some comments for the authors to consider as they revise the manuscript:

1. The authors’ scheme differs from conventional quantum tomography methods in that it yields the Husimi-Q function, which is essentially a “blurred” version of the Wigner function. While it, in principle, carries equivalent information to the Wigner function, I expect that reconstructing a Wigner function or wavefunction from a Q-function requires a high signal-to-noise ratio, since this process essentially involves unblurring the phase-space profiles. For instance, will it be possible to use the scheme to measure highly non-Gaussian quantum states?
2. Related to the comment above: An optical parametric oscillator below threshold can perform some squeezing operations. By applying a squeezing operation before the measurement sequence, it seems that one could perform a projective measurement in a squeezed coherent state basis with an arbitrary squeezing parameters. Does this provide an advantage in performing quantum state tomography of exotic states with finer structures in phase space?
3. The idea of combining parametric amplification and bias displacement to realize quantum-state tomography was also theoretically analyzed in Quantum Sci. Technol. 9, 045054 (2024). Could the authors comment on the differences and connections between their scheme and this approach?
4. In Phys. Rev. Lett. 124, 160501 (2020), an approach for directly “opening” the cavity using nonlinear-optical gating was proposed, which would also enable one to take a snapshot of the intra-cavity state. What would be an advantage of the proposed scheme, compared to such a more direct approach?
5. A more conventional approach to studying intra-cavity quantum dynamics is quantum filtering through monitoring of the out-coupling optical fields, which may be worth a sentence of review.
6. What are the practical limitations of the scheme's applicability? As presented in the manuscript, it appears that the nonlinear cavity must possess quadratic optical nonlinearity for the scheme to be applicable. Also, I guess the OPO bifurcation dynamics need to occur fast compared to the other intra-cavity dynamics. What determines the time-resolution of the measurement?
7. Related to the comment above: Is it possible to use other bistabilities in quantum optics—e.g., in Kerr resonators (Phys. Rev. X 10, 021022 (2020)) or in Jaynes-Cummings-like models (IEEE Journal of Quantum Electronics 24, 1495 (1988))—to implement intra-cavity quantum-state tomography?

(Remarks on code availability)

Reviewer #2

(Remarks to the Author)
The authors present a method to explore the dynamics of the field in a nonlinear optical parametric oscillator by injecting a coherent field—referred to as the bias—that alters the macroscopic steady-state output of the cavity.
This approach provides information about the marginal values of the quasi-probability Q function of the state in the cavity.
The mapping is achieved by measuring the system's sensitivity to the injected coherent field.

While the method may be promising for probing the dynamics of quantum states in the cavity, I have some major concerns that I would like to address:

1. Calibration of the injected coherent field:
The calibration of the injected coherent field appears to be a critical issue, as improper calibration could introduce artefacts into the reconstructed Q-function. In my experience, accurately measuring losses across optical elements is either unreliable or poses a metrological challenge in itself. In homodyne or similar setups, calibration is typically performed by measuring detector values in the absence of a signal (i.e., the vacuum state), which establishes an auto-calibration process that does not rely on directly measuring the amplitude and phase of the local oscillator.
Is it possible to implement a similar calibration method here? If not, how can we certify the accuracy of the bias field measurement?

2. Measurement scheme and modeling:
The presentation of the method seems agnostic to the specific type of measurement performed at the cavity output. Only in the methods section is it revealed that the statistics of the OPO steady states are measured using an interferometric phase measurement. I do not understand why this aspect of the measurement is not incorporated into the theoretical modeling and sensitivity analysis.

Addressing these two points could significantly alter the interpretation and conclusions of the results presented.

Additional points:
• The Radon transform is known to introduce artefacts when reconstructing other quasi-probability distributions (such as the Wigner function), which is why maximum likelihood methods are often preferred. How can we be confident that similar artefacts are not present in the current experiment?
• The sentence:
"Using a standard homodyne detection method, the Husimi Q function can be measured by combining the unknown quantum state $|\psi\rangle$ with a reference coherent state $|\alpha\rangle$ using a beamsplitter."
is not entirely accurate.
Homodyne detection measures quadrature probability distributions, which are the marginals of the Wigner function. From a collection of such marginals, the Wigner function can be reconstructed tomographically, and subsequently the Q-function may be obtained. However, to sample the Q-function directly, the appropriate detection method is a double homodyne setup (often referred to as heterodyne detection in the quantum optics community).

(Remarks on code availability)

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
The authors have sufficiently addressed the comments. Thus, I recommend the publication of the manuscript.

(Remarks on code availability)

Reviewer #2

(Remarks to the Author)
The author satisfactorily responded to the criticisms, so I recommend the article for publication.

(Remarks on code availability)

Response Letter to the reviewers

We are grateful for the constructive comments from all the reviewers on our manuscript.

The reviewer's comments are in *italic*, with our response below each specific comment. Moreover, in the revised main text, text that has been added to the manuscript is colored blue.

Reviewer: 1

Comment #1: *In this work, the authors propose and experimentally demonstrate a scheme to tomographically measure the quantum state inside a nonlinear optical cavity. The scheme leverages the single-photon-level sensitivity of an optical parametric oscillator (OPO) to the displacement of the initial state as the OPO collapses to one of the bistable states. By measuring the sensitivity of the branching ratio of the bistable states to an external bias field, a tomographic measurement of the Husimi-Q function of the intra-cavity state is achieved. The authors applied the scheme to unravel the dynamical formation of squeezed vacuum states inside an OPO.*

In quantum optics, it is surprisingly difficult to experimentally study intra-cavity quantum states because of the lack of an easy means of "opening" the cavity. As a result, there have been only a very limited number of experimental observations of intra-cavity quantum-optical dynamics, despite the sheer volume of theoretical predictions and simulations. I find the authors' work quite significant, as it demonstrates the possibility of observing various dynamical phenomena that are predicted by theory but have never been observed in experiments. The experimental results show good agreement with the model, underscoring the reliability of their scheme. Overall, I would like to recommend the publication of this work in Nature Communications. Below are some comments for the authors to consider as they revise the manuscript:

Response from the authors: We would like to thank Reviewer #1 for the very positive assessment of our work and for highlighting the importance of the demonstrated framework in enabling new experimental studies.

Comment #2: *The authors' scheme differs from conventional quantum tomography methods in that it yields the Husimi-Q function, which is essentially a "blurred" version of the Wigner function. While it, in principle, carries equivalent information to the Wigner function, I expect that reconstructing a Wigner function or wavefunction from a Q-function requires a high signal-to-noise ratio, since this process essentially involves unblurring the phase-space profiles. For instance, will it be possible to use the scheme to measure highly non-Gaussian quantum states?*

Response from the authors: We thank the reviewer for bringing up this important point regarding the reconstruction of different phase-space representations in our framework.

While conventional quantum tomography methods use the Wigner function, our experimental framework reconstructs the Husimi *Q* function, as it is naturally suited to our method. This is because our framework is based on evaluating the projection of cavity quantum states onto coherent states, which directly yields the Husimi *Q* function. While the Husimi *Q* function contains complete information about the quantum state, it is indeed a “blurred” version of the Wigner function due to intrinsic convolution with a Gaussian function. Recovering the Wigner function from the Husimi *Q* function involves a deconvolution problem, and as the reviewer rightly notes, is thus sensitive to the signal-to-noise ratio (SNR).

To address this, different numerical methods have been developed to reconstruct the Wigner function from the Husimi *Q* function, including convex optimization [1], phase-retrieval method [2], and series-expansion method [3]. These methods enable the deblurring of the Husimi *Q* function, but due to the ill-conditioned nature of deconvolution, they indeed typically require a high SNR to achieve high-fidelity reconstruction.

Our reconstruction framework inherently benefits from high sensitivity through the use of **phase-sensitive amplification**. This differs from standard quantum tomography methods that typically gain sensitivity by utilizing complex measurement setups such as homodyne detection or photon number resolving detectors. In contrast, our scheme infers cavity state field distribution by exploiting the sensitivity of the symmetry-breaking transition to small perturbations. Notably, the information extracted from the optical parametric oscillator (OPO) steady state is binary (0 or π phase), making the measurement process intrinsically robust to noise. Therefore, we can reconstruct the Husimi *Q* function of the quantum vacuum and squeezed vacuum with a high SNR. This opens the possibility for extending our framework to characterize more complex quantum states, including highly non-Gaussian ones, contingent on maintaining high measurement fidelity.

To address this point, we have added the following sentences in the Discussion section of our revised manuscript:

While the Husimi *Q* function contains complete information about the quantum state, it is indeed a “blurred” version of the Wigner function due to intrinsic convolution with a Gaussian function. Several mathematical algorithms have been developed to reconstruct the Wigner function from the Husimi *Q* function [1-3], and their reconstruction performance depends on the signal-to-noise ratio of the Husimi *Q* function. Our reconstruction protocol acquires high sensitivity during the phase-sensitive amplification. In addition, the measurement is less sensitive to noise as the information we extract from the OPO steady state is the binary phase information with only two possible outcomes (0 or π phase). Therefore, we expect the high signal-to-noise of our framework would allow high-fidelity reconstruction of the Wigner function from the experimentally measured Husimi *Q* function.

Comment #3: *Related to the comment above: An optical parametric oscillator below threshold can perform some squeezing operations. By applying a squeezing operation before the measurement sequence, it seems that one could perform a projective measurement in a squeezed coherent state basis with an arbitrary squeezing parameters. Does this provide an advantage in performing quantum state tomography of exotic states with finer structures in phase space?*

Response from the authors: We thank Reviewer #1 for the insightful question regarding the potential role of squeezed coherent bias fields on the reconstruction performance.

Introducing a squeezed bias field modifies not only the mean displacement but also the quantum noise properties of the cavity, enabling the intriguing possibility of tailoring the measurement basis in phase space, potentially improving resolution of fine and exotic features of a quantum state.

In contrast to a coherent bias, which introduces a displacement without changing the noise properties of the cavity state, a squeezed bias can affect both the mean and the variance of the quantum state field distribution. To analyze how the bias - probability curve changes when using a squeezed bias, we consider the master equation of a degenerate optical parametric oscillator (DOPO) coupled to a squeezed reservoir as below [4]:

$$\begin{aligned}\dot{\rho} = \quad & \frac{\gamma\lambda}{2}[a^{\dagger 2} - a^2, \rho] + \gamma b[a^\dagger - a, \rho] + \gamma(N+1)(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) + \gamma N(2a^\dagger \rho a - aa^\dagger \rho - \rho aa^\dagger) \\ & + \gamma M^*(2a\rho a - a^2\rho - \rho a^2) + \gamma M(2a^\dagger \rho a^\dagger - a^{\dagger 2}\rho - \rho a^{\dagger 2}) + \frac{g^2\gamma}{2}(2a^2\rho a^{\dagger 2} - a^{\dagger 2}a^2\rho - \rho a^{\dagger 2}a^2)\end{aligned}$$

Equation 1. Master equation of the DOPO in a squeezed reservoir.

$N = sinh^2(r)$ and $M = -cosh(r)sinh(r)e^{i\theta}$ are determined by the squeeze parameter r and the squeezing angle θ. γ denotes the cavity decay rate and *g* the quantum noise level [5]. We convert this master equation to a stochastic differential equation (SDE) for *P* representation as below:

$$\begin{bmatrix} \frac{dX}{d\tau} \\ \frac{dY}{d\tau} \end{bmatrix} = \begin{bmatrix} -1+\lambda & 0 \\ 0 & -1-\lambda \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} \sqrt{2}b \\ 0 \end{bmatrix} + \begin{bmatrix} 2N+\lambda+M+M^* & -i(M-M^*) \\ -i(M-M^*) & 2N+\lambda-M-M^* \end{bmatrix}^{\frac{1}{2}} \begin{bmatrix} \eta_X \\ \eta_Y \end{bmatrix}$$

Equation 2. SDE of the DOPO in a squeezed reservoir.

By numerically solving the SDE, we obtain bias - probability relationships under three different levels of squeezing, shown in Figure 1. These numerical results confirm that a squeezed bias reduces the total noise in the cavity state, resulting in a sharper bias - probability curve, enhancing the system's sensitivity to variations in the input field, and thereby improving the resolution of the phase-space reconstruction.

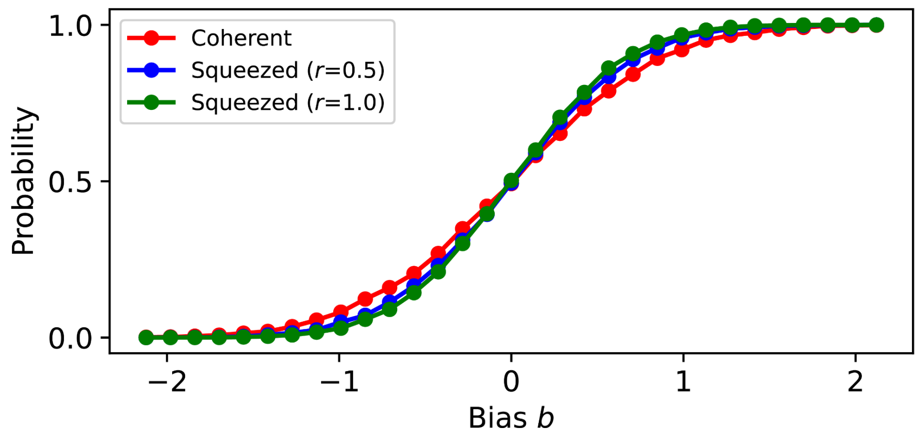

Figure 1. Bias - probability curves at different squeezing levels. The initial cavity state is the quantum vacuum. As the bias field is more squeezed, the total noise of the cavity quantum state decreases. Therefore, squeezing the bias field makes the bias - probability curve shaper.

However, we should emphasize that the interpretation of the measurement outcomes differs when using a squeezed bias. The system no longer performs projections onto the coherent states, and the derivative of the bias - probability curve does not directly give the 1D marginal distribution of the Husimi *Q* function. This is because the squeezed bias field not only displaces the cavity quantum state but also affects the variance of the quantum state. As a result, an additional reconstruction step is needed to relate the bias - probability curve measured with a squeezed bias to a coherent bias.

To address this point, we have added a subsection in the Supplementary Information Section S2 of our revised manuscript to discuss the impact of non-coherent bias field in measuring the cavity quantum state:

In this section, we briefly discuss how non-coherent bias fields can potentially improve our reconstruction performance. In standard homodyne detection, injecting a squeezed vacuum state reduces the noise [6]. We can consider a similar situation when we inject a squeezed bias to the cavity. From the master equation of the DOPO with a squeezed bias (squeezing ratio *r*, angle θ), we can write the SDEs as:

$$\begin{bmatrix} \frac{dX}{d\tau} \\ \frac{dY}{d\tau} \end{bmatrix} = \begin{bmatrix} -1+\lambda & 0 \\ 0 & -1-\lambda \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} \sqrt{2}b \\ 0 \end{bmatrix} + \begin{bmatrix} 2N+\lambda+M+M^* & -i(M-M^*) \\ -i(M-M^*) & 2N+\lambda-M-M^* \end{bmatrix}^{\frac{1}{2}} \begin{bmatrix} \eta_X \\ \eta_Y \end{bmatrix}$$

where $N = sinh^2(r)$ and $M = -cosh(r)sinh(r)e^{i\theta}$. The squeezed bias changes the noise of the cavity quantum state, which results in having a more complex diffusion term compared to Equation S7. By solving Equation S12 numerically, we get bias - probability curves for different types of bias fields as in Figure S2. As the squeezed bias field not only displaces the quantum state but also reduces the noise of the cavity quantum state, the variance of bias - probability curve is reduced as we introduce the squeezed bias field to the cavity. Therefore, the system reacts with higher sensitivity to the same mean bias field, promoting quantum state reconstruction with higher resolution. However, the squeezed bias field also affects the overall noise of the cavity quantum state, which requires an additional step to extract the 1D marginal Husimi *Q* function from the bias - probability curve. Therefore, while using squeezed bias can improve sensitivity and recover finer features of the Husimi *Q* function into the bias - probability curve, it adds complexity in the reconstruction procedure, as it alters both the displacement and variance of the cavity quantum state.

We also added a sentence in the Results section of our revised manuscript to guide readers who want to know more about the impact of the non-coherent bias field.

We also discuss the potential applications of using non-coherent bias fields in the measurement protocol in the Supplementary Information Section S2.

Comment #4: *The idea of combining parametric amplification and bias displacement to realize quantum-state tomography was also theoretically analyzed in Quantum Sci. Technol. 9, 045054 (2024). Could the authors comment on the differences and connections between their scheme and this approach?*

Response from the authors: We thank the reviewer for pointing out the related work. We appreciate the opportunity to clarify the distinctions and connections between that approach and ours.

In the paper *Quantum Sci. Technol. 9, 045054 (2024)*, the authors apply both parametric amplification and displacement to reconstruct the Wigner function. Amplification with an optical parametric amplifier (OPA) allows more precise measurement of the intensity distribution, since direct measurement of weak quantum states can be challenging. In their approach, displacement is used to shift the amplified quantum state to one side of the quadrature (i.e., positive or negative), promoting one-to-one correspondence between the intensity distribution of the amplified quantum state and the actual field distribution. This allows for the reconstruction of the Wigner function of complex and asymmetric quantum states. While both their scheme and our approach apply parametric amplification and displacement to reconstruct the quantum state, their functionalities differ significantly.

1) Role of parametric amplification: In their scheme, parametric amplification is essential for mapping the measurable intensity distribution onto the Wigner function of the quantum state. It directly amplifies the field for detection with either an intensity detector or a homodyne detector. On the other hand, in our system, parametric amplification maps the cavity field distribution to the probability of measuring certain steady states. In other words, our scheme leverages the sensitivity of the symmetry-breaking during parametric amplification.
2) Strength of the displacement: In their scheme, post-amplification displacement is introduced to push the amplified quantum state to either the positive or negative side of the quadrature axis, mapping the intensity distribution to the field quadrature values. In

contrast, our framework applies only the pre-amplification displacement via a weak bias field. The strength of this bias is carefully chosen to be on the same order as the cavity quantum state's field amplitude. Rather than amplifying the signal for intensity measurement, the purpose of our displacement is to probe the cavity field in a phase sensitive manner. It enables a direct mapping between the cavity field strength and the probability of observing a particular steady-state (0 or π phase), leveraging the high sensitivity of symmetry-breaking to initial conditions. This distinction highlights how our method emphasizes noise-resilient, probabilistic readout rather than intensity-based reconstruction.

3) Ability to measure cavity quantum states: In their scheme, the cavity quantum state should be extracted and injected into the OPA. Our framework can do both generation and measurement in a single cavity, enabling fundamental studies of cavity quantum state dynamics.

To address this point, we have added the following sentence in the Discussion section of our revised manuscript:

While both concepts – parametric amplification and displacement of quantum states – have been utilized to reconstruct quadrature probability distributions from the measured intensity distributions without photon number resolving detectors [7], our framework differs in that it measures the steady-state probability distribution of the cavity phase, which encodes the field distribution of the displaced cavity quantum state.

Comment #5: *In Phys. Rev. Lett. 124, 160501 (2020), an approach for directly "opening" the cavity using nonlinear-optical gating was proposed, which would also enable one to take a snapshot of the intra-cavity state. What would be an advantage of the proposed scheme, compared to such a more direct approach?*

Response from the authors: We thank the reviewer for highlighting this important point.

In the paper *Phys. Rev. Lett. 124, 160501 (2020)*, the authors propose methods to dynamically control the opening of the cavity with optical nonlinearity. By controlling the amplitude and phase of a control field, they modulate the coupling between the cavity mode and the waveguide, enabling functions such as Controlled-Phase Gate and, in principle, extraction of intracavity states.

Dynamical switching of an optical cavity for quantum tomography is a conceptually appealing method as high-fidelity reconstruction of non-classical states with stationary output coupling requires extraction efficiency to be very close to unity [8]. Although experimental demonstrations of quantum tomography with dynamical switching of the cavity have been proposed [9,10], their schemes require additional setups to precisely control the opening of the cavity. By contrast, a key advantage of our proposed scheme is that it avoids the need to extract the cavity quantum state altogether. This allows us to probe the quantum state *in situ*, without requiring external coupling or suffering the loss and noise associated with extraction. In addition, direct opening of the cavity still requires a sensitive measurement setup (e.g., homodyne detection) to reconstruct the quantum state. On the contrary, our method offers noise robust and sensitive quantum state tomography by leveraging the sensitivity of symmetry-breaking.

We acknowledge that describing approaches for the direct extraction of the cavity quantum states and their current challenges would highlight the importance of our work. Therefore, we added the following sentence in the Introduction section of our revised manuscript.

Dynamical switching of the cavity can provide direct access to the cavity quantum state [9,10,11], which requires precise control of the switching behavior and still suffers from the loss during the extraction.

Comment #6: *A more conventional approach to studying intra-cavity quantum dynamics is quantum filtering through monitoring of the out-coupling optical fields, which may be worth a sentence of review.*

Response from the authors: We thank the reviewer for introducing a different cavity quantum state reconstruction method.

Quantum filtering is based on Bayesian correction, which allows real-time estimation of cavity quantum states with previously monitored out-coupled optical fields. Our method also allows us to reconstruct the dynamics of the cavity quantum states, but with a different approach.

To address this point, we have added the following sentence in the Introduction section of our revised manuscript:

Although quantum filtering can estimate cavity quantum states, it is fundamentally limited by the detector efficiency [12].

Comment #7: *What are the practical limitations of the scheme's applicability? As presented in the manuscript, it appears that the nonlinear cavity must possess quadratic optical nonlinearity for the scheme to be applicable. Also, I guess the OPO bifurcation dynamics need to occur fast compared to the other intra-cavity dynamics. What determines the time-resolution of the measurement?*

Response from the authors: We thank the reviewer for bringing up the possible future improvement of our experiment.

The practical limitations of our current reconstruction scheme can be summarized by two factors: time-resolution of the measurement and the extinction ratio of the bias field.

The time-resolution can be improved when we increase the lifetime of the OPO cavity (i.e., decreasing the Q factor of the cavity). When the cavity has a longer lifetime, the effective gain of the system is weak, which makes the overall intracavity dynamics slowly evolving before it reaches a steady state. Therefore, we can capture more snapshots of the intracavity dynamics. Note that the rise time mentioned in the discussion part of the manuscript is different from the lifetime of the cavity. The rise time in our experiment is mainly limited by the bandwidth of the electronics (e.g., voltage amplifier), while the lifetime is determined by the loss of the cavity.

Higher extinction ratio of the bias field allows the reconstruction of the later stage of the cavity dynamics, including the OPO bifurcation dynamics. When reconstructing the dynamics of cavity quantum states, a bias field that is not completely attenuated can already displace the cavity state before we actually inject the bias to measure the bias - probability curve. While we discussed the impact of the finite extinction ratio in measuring the bias - probability relationship in the Supplementary Information Section S4 of our original manuscript, we expect integrated platforms can improve the extinction ratio [13].

For the applicability of our reconstruction protocol to different types of nonlinearities, we expect we can apply similar methods to the other systems if they undergo phase transitions due to symmetry breaking. As the reviewer brought up a similar question in the next comment, we will address how reconstruction protocols can be applied to different nonlinear systems later.

To address this point, we have added the following sentences in the Discussion and Results sections of our revised manuscript:

Reducing the rise time of the optical signals and increasing the lifetime of the cavity would open the avenue for capturing the DOPO dynamics with a better time resolution. While short rise time of the optical signal would also allow fast switching between different dynamical phases, longer cavity lifetime allows each dynamical phase to remain longer, thus enabling detailed reconstruction of intracavity dynamics.

Because a stronger bias field is required to displace and probe a cavity state at a later stage, the finite extinction ratio of our system can unintentionally displace the cavity state even before the intended injection of the bias.

Additional sentences to discuss whether we can extend our reconstruction protocol to other types of nonlinearities are answered in the next comment.

Comment #8: *Related to the comment above: Is it possible to use other bistabilities in quantum optics—e.g., in Kerr resonators (Phys. Rev. X 10, 021022 (2020)) or in Jaynes-Cummings-like models (IEEE Journal of Quantum Electronics 24, 1495 (1988))—to implement intra-cavity quantum-state tomography?*

Response from the authors: We thank the reviewer for raising this insightful question regarding the broader applicability of our reconstruction protocol to other bistable systems in quantum optics.

As we mentioned in the previous question, we can implement a similar intra-cavity quantum state reconstruction protocol in Kerr resonators. The Heisenberg-Langevin equation of motion for the Kerr-nonlinearity based OPO system can be described below [14]:

$$\dot{a} = -a + \lambda a^{\dagger} + i g^{2} (a^{\dagger} a) a + F(t) + b$$

Equation 3. Heisenberg-Langevin equation of motion for the Kerr-nonlinearity based OPO system.

$F(t)$ is an operator-valued Langevin noise. The difference is that we have an intensity-dependent nonlinearity instead of a phase-dependent nonlinearity. Despite this distinction, Kerr resonators also exhibit bistability and enable a form of parametric amplification due to four-wave mixing. Therefore, our reconstruction approach can, in principle, be extended to such platforms. Note that the phase for the Kerr OPO steady state is rotated relative to the 0 or π phase due to the Kerr nonlinearity [14].

In Jaynes-Cummings-like models (JC models), the switching behavior between bistable steady states induced by quantum vacuum fluctuations depends on the number of photons inside the cavity [15]. While we can reconstruct the cavity quantum state by observing the change in the switching behavior at different levels of the bias field, the same reconstruction protocol cannot be applied to the JC model without significant modifications. This is because the JC model does not involve a parametric amplification process, which allows mapping between weak cavity quantum states to the macroscopic observables such as the phase of the OPO steady state.

To address this point, we have added the following sentences in the Discussion section of our revised manuscript:

Beyond the DOPO platform proposed here, which exhibits bistability through phase-sensitive parametric amplification, our framework can be generalized to other bistable systems with parametric amplification, such as Kerr-nonlinearity based OPOs [14].

Reviewer: 2

Comment #1: *The authors present a method to explore the dynamics of the field in a nonlinear optical parametric oscillator by injecting a coherent field—referred to as the bias—that alters the macroscopic steady-state output of the cavity.*

This approach provides information about the marginal values of the quasi-probability Q function of the state in the cavity. The mapping is achieved by measuring the system's sensitivity to the injected coherent field.

While the method may be promising for probing the dynamics of quantum states in the cavity, I have some major concerns that I would like to address:

Response from the authors: We appreciate the reviewer for the positive assessment of our work and the helpful comment on the manuscript.

Comment #2: *Calibration of the injected coherent field: The calibration of the injected coherent field appears to be a critical issue, as improper calibration could introduce artefacts into the reconstructed Q-function. In my experience, accurately measuring losses across optical elements is either unreliable or poses a metrological challenge in itself. In homodyne or similar setups, calibration is typically performed by measuring detector values in the absence of a signal (i.e., the vacuum state), which establishes an auto-calibration process that does not rely on directly measuring the amplitude and phase of the local oscillator. Is it possible to implement a similar calibration method here? If not, how can we certify the accuracy of the bias field measurement?*

Response from the authors: We thank the reviewer for this important comment regarding the calibration of the injected coherent (bias) field.

As the reviewer correctly mentioned, accurate calibration of the probe signal in quantum metrology is the key for an accurate quantum state reconstruction. As pointed out, in homodyne detection, calibration is often performed indirectly by measuring the detector response to the vacuum state. Inspired by this, we proposed a conceptually similar calibration method in our system, based on using the quantum vacuum state as a reference.

Although our initial approach involved estimating the bias field strength by measuring the loss at each optical element, we can implement a data-driven calibration method that extracts the amplitude of the bias field by measuring the bias - probability curve of the ground state of the cavity, which is the quantum vacuum state. As demonstrated in both our previous work [5] and our current work, the field distribution of the quantum vacuum state can be accurately reconstructed from the bias - probability curve and compared with theoretical predictions derived from Eq. (13) in the main text. This provides a robust calibration standard because the quantum vacuum is the ground state of the cavity and is always experimentally accessible.

From Eq. (13) in the main text, the bias - probability curve of the quantum vacuum state follows the equation below:

$$p(b) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{\sqrt{2}b}{\sqrt{\lambda}\sqrt{\lambda - 1}}\right)\right]$$

Equation 4. Bias - probability curve of the quantum vacuum state.

The sensitivity of our bias field measurement is mainly determined by the statistical error in probability measurement due to finite sampling of OPO steady states [5]. The maximum statistical error occurs near the linear region of the bias - probability curve (i.e., when *b* ~ 0), and scales as $\sqrt{\frac{\pi}{4N}}$ for a given sampling number *N*. Consequently, the uncertainty in the bias field amplitude σ_{b} follows the relationship below:

$$\sigma_b \sim \sqrt{\frac{\lambda(\lambda-1)}{N}}$$

Equation 5. Uncertainty in the bias field amplitude σ_b .

By increasing the *N* or decreasing the pump field λ, we can reduce σ_b. When we measured the amplification of the quantum vacuum in Fig. 3 of the main text of our manuscript, we set λ = 2 and *N* = 10,000, resulting in an estimated uncertainty of $\sigma_b \sim 0.01$. Because $|b|^2$ is the mean photon number entering the cavity, *b* is kept close to a unity (i.e. single photon) when reconstructing the cavity quantum state. Therefore, we can accurately measure the bias field from the bias - probability curve of the quantum vacuum state.

Importantly, we also highlight that for subsequent experiments, the exact calibration of the bias field is not necessary. We can reconstruct the Husimi *Q* function of the unknown cavity quantum state relative to the Husimi *Q* function of the quantum vacuum state. The exact value of the bias field is not necessary because it is a relative measurement, directly providing the amount of squeezing. It is worth noting that this procedure is conceptually similar to the mentioned auto-calibration in homodyne techniques. In the manuscript, we showcase the scenario without any prior reference, where one can use the measured bias field strength to reconstruct an arbitrary intracavity field distribution (i.e., vacuum or squeezed state).

We added the following sentences in the Methods section and the Supplementary Information Section S4 of our revised manuscript, respectively, to guide readers who may be curious about how we can precisely calibrate the bias field.

Although we initially estimated the bias field by measuring the power loss at each optical element, more precise calibration of the bias field can be achieved by measuring the bias - probability relationship of the quantum vacuum state. Details of the calibration method can be found in the Supplementary Information Section S4.

In our current work and previous work [5], we have shown that we can successfully reconstruct the quantum vacuum state by measuring the bias - probability curve. Because the quantum vacuum state is the ground state of the cavity, the bias - probability distribution for the quantum vacuum state can also be theoretically calculated using Equation 13 in the main text. From Equation 13, we can estimate the strength of the bias field by knowing the probability *p* and the pump field above the threshold λ, which can be measured easily by comparing the power at the operation and at the threshold. The main uncertainty in the probability *p* originates from the statistical error due to finite sampling of OPO steady states during experiment [5]. The maximum statistical error occurs at the linear region of the bias - probability curve, which is the order of $\sqrt{\frac{\pi}{4N}}$ (*N* is the number of samples). The error of the bias field σ_b due to the error in the probability *p* follows the equation below:

$$\sigma_b \sim \sqrt{\frac{\lambda(\lambda-1)}{N}}$$

When we reconstructed the quantum vacuum state in Fig. 3 of the main text, we set λ = 2 and N = 10,000. In this scenario, $\sigma_b \sim 0.01$. Because $|b|^2$ is the mean photon number incident to the cavity, b is kept close to unity when reconstructing the cavity quantum state. Therefore, we can achieve accurate measurement of the bias field by measuring the bias - probability curve for the quantum vacuum state. For better calibration, we can decrease λ and increase the number of samplings N.

Although we have shown that we can precisely calibrate the bias field from the quantum vacuum state, we also propose another approach to reconstruct the cavity state without knowing the exact strength of the bias field. We first reconstruct the Husimi Q function of the quantum vacuum state with an arbitrary unit (i.e., without knowing the exact strength of the bias field), and then reconstruct the Husimi Q function of the unknown cavity quantum state. By comparing the relative size of two Husimi Q functions, we can reconstruct the unknown quantum state with actual phase-space coordinate values, as we know the theoretical expression of the quantum vacuum state. This is similar to how we can avoid direct calibration of the LO in homodyne detection [16]. While we could use this approach in this manuscript, the key message of this paper is to reconstruct the quantum state without any prior information, so we measure the bias field and use this value for the reconstruction.

Comment #3: *Measurement scheme and modeling: The presentation of the method seems agnostic to the specific type of measurement performed at the cavity output. Only in the methods section is it revealed that the statistics of the OPO steady states are measured using an interferometric phase measurement. I do not understand why this aspect of the measurement is not incorporated into the theoretical modeling and sensitivity analysis. Addressing these two points could significantly alter the interpretation and conclusions of the results presented.*

Response from the authors: We thank the reviewer for pointing out the aspect of the measurement scheme we used to collect the statistics of the OPO steady states.

In our experiment, we used a simple interferometer setup to extract the phase of the OPO steady-state and map it to binary information using a photodiode. The local oscillator (LO) is derived from the same laser source used to generate the bias. The LO and OPO signals are temporally aligned so they can properly interfere. Although our measurement scheme shares similar features to the homodyne detector (i.e., mixing signal with the LO, using photodetectors to measure the mixed signal), it differs fundamentally in both the purpose and implementation. In standard homodyne detection, both the amplitude and the phase of the LO should be precisely calibrated, as the signal we measure is continuous. In contrast, our scheme is tailored to detect discrete phases of the OPO steady-states, which can only take one of two values, 0 or π. The role of the interferometer is simply to convert this binary phase information into “low” or “high” intensity levels at the photodiode, which we can count to determine the probability distribution of the steady-state. The phase of the LO is only adjusted to ensure sufficient contrast between the two levels. Hence, we do not need to incorporate the theoretical modeling or the sensitivity analysis of the detection module. This is because the information we extract from the cavity – the probability of measuring certain steady states – is not dependent on the exact LO strength and the detector efficiency.

It is important to note, however, that our protocol requires careful selection of the OPO mode that is degenerate. To ensure that the system only has two possible steady states, we first operate the OPO without any bias and measure the steady-state statistics with a relatively large number of samples, up to 200 Mbits [5], to confirm the statistical independence of consecutive samples and show that the OPO mode is a degenerate bistable system.

In the revised manuscript, we added additional sentences in the Results section of our revised manuscript to specify how we measure the OPO steady states with a simple interferometer setup, and how it differs from standard homodyne detection.

By measuring the intensity of the interference between the out-coupled OPO signal and the local oscillator (LO), we extract the phase of the OPO signal. Unlike standard homodyne detection, which requires precise calibration of the LO amplitude and phase to resolve continuous quadrature values, our protocol only distinguishes between two phase states (i.e., 0 or π phase). We then count the number of high-intensity outcomes to determine the probability distribution. Because the outcome is binary and insensitive to the exact intensity level, we do not require exact calibration of the amplitude of the LO. Moreover, we set the phase of the LO to maximize the contrast between low and high level signals by moving a piezo-driven delay stage. Details about the experimental setup can be found in the Methods section.

Comment #4: *The Radon transform is known to introduce artefacts when reconstructing other quasi-probability distributions (such as the Wigner function), which is why maximum likelihood methods are often preferred. How can we be confident that similar artefacts are not present in the current experiment?*

Response from the authors: We thank the reviewer for the comment.

While maximum likelihood methods can potentially outperform inverse Radon transforms, we expect artifacts would be minimally introduced while using inverse Radon transforms for three reasons.

Direct access to 1D marginal distribution of the Husimi Q function. The key message of our paper is that we can obtain 1D marginal distributions of the Husimi *Q* function at different angles by simply taking a derivative of bias - probability curves. After marginal distributions are obtained at different pump phases, reconstructing the Husimi *Q* function becomes straightforward with inverse Radon transform. Several studies that measured the 1D marginal distributions at different angles have chosen inverse Radon transform as their reconstruction method [17,18].

Husimi Q function is smoothly defined over the entire phase space. Husimi Q function is a non-negative quasi-probability distribution. Also, the Husimi Q function is the convolution between the Wigner function and the Gaussian filter, which provides smoothness in its probability distribution. Therefore, we expect fewer chances of introducing artifacts when using inverse Radon transform.

Our reconstructed Husimi Q function is robust to measurement noise. Compared to inverse Radon transform, the maximum likelihood method shows better performance when the input data is noisy. This is because the regularization term reduces the noise during the optimization. However, our reconstruction protocol is less sensitive to measurement noise, as we answered in comment #2 from Reviewer 1 and comment #3 from Reviewer 2. Therefore, we can minimize the artifacts that can possibly occur when performing inverse Radon transform with noisy input data.

While our experimental results show that the inverse Radon transform can accurately reconstruct the Husimi Q function of the cavity quantum states, we acknowledge that both the inverse Radon transform and maximum likelihood method can be used under different circumstances. If we want to reconstruct the complex dynamics of cavity quantum states that are not trivial (i.e., generation and collapse of the squeezed vacuum state), then the maximum likelihood method would be a better option. As the maximum likelihood method gives better performance when we have prior information about the image we want to reconstruct, we apply an inverse Radon transform first to get a rough estimate of the quantum state that we want to reconstruct. Then we use the reconstruction result as prior information for the maximum likelihood methods, promoting better reconstruction with appropriate selection of the optimal regularization term.

We included the discussions above in the Supplementary Information Section S2 in the revised manuscript, describing how we can utilize different reconstruction algorithms to reconstruct more complex dynamics of cavity quantum states.

While maximum likelihood methods can be utilized to reconstruct the 2D Husimi Q function, the direct access to the 1D marginal distribution at different angles allows more straightforward reconstruction when using the inverse Radon transform during our experiment. Moreover, our reconstruction protocol is less sensitive to the detector noise, adding minimal artifacts during the inverse Radon transform. However, maximum likelihood methods can outperform the inverse Radon transform when the cavity quantum state we want to reconstruct is complex. For example, reconstructing the dynamics of Wigner functions of Fock states can be limited when using the inverse Radon transform. In this scenario, we can first use the inverse Radon transform to obtain the rough sketch of the cavity quantum state, and then we leverage this prior information while using the maximum likelihood method. With this hybrid approach, the prior information obtained during the inverse Radon transform will benefit us while deciding the initial conditions and the regularization terms in maximum likelihood methods.

Comment #5: *The sentence: "Using a standard homodyne detection method, the Husimi Q function can be measured by combining the unknown quantum state $|\psi\rangle$ with a reference coherent state $|\alpha\rangle$ using a beamsplitter." is not entirely accurate. Homodyne detection measures quadrature probability distributions, which are the marginals of the Wigner function. From a collection of such marginals, the Wigner function can be reconstructed tomographically, and subsequently the Q-function may be obtained. However, to sample the Q-function directly, the appropriate detection method is a double homodyne setup (often referred to as heterodyne detection in the quantum optics community).*

Response from the authors: We thank the reviewer for clarifying the sentence that may mislead the readers. We rewrite the paragraph so we can provide accurate information about the standard homodyne detection method.

The Husimi *Q* function can be either reconstructed with a standard homodyne detection setup that combines the unknown quantum state $|\psi\rangle$ with a reference coherent state $|\alpha\rangle$ [19] or directly measured by using a heterodyne setup [20], which is also known as a double homodyne setup.

In addition to the revision made to answer the reviewers' comments: while submitting the code in the online repository (https://codeocean.com/capsule/5289451/tree) after the initial submission, we realized that the earlier version of Figure 3 was uploaded during the initial submission. While the latest version of Figure 3 is already available in the online repository, the same version is included in the revised manuscript. The changes made in Figure 3(d,e) do not affect the interpretation or conclusions of the manuscript.

We again thank the reviewers for their insightful comments and hope that our revised manuscript is now in good shape to be published in Nature Communications.

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