

Supplementary information for “Local Clustering Decoder as a fast and adaptive hardware decoder for the surface code”

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Appendix A: Leakage

This section reviews some key aspects of leakage. [1–3] influence our exposition.

1. Leakage overview

In many quantum computing platforms qubits are a two-dimensional computational subspace $\{|0\rangle, |1\rangle\}$ of the larger (often infinite-dimensional) physical Hilbert space of a device. If the device’s state is contained wholly within the computational subspace we say it is “sealed”. However, when the state has a non-negligible overlap outside of the computational subspace, represented by $|2\rangle$ in this work, we say the device has “leaked”. Throughout the work, when we refer to “qubit”, we mean a single device which ideally behaves as a qubit but has the potential to leak. While this general definition of leakage encompasses many physical non-computational error mechanisms, such as atom loss [4, 5], “erasure qubits” [6, 7], and leakage in ions/atoms to non-computational states, in this work we focus on leakage in the superconducting context where the device resides in a higher energy level outside the computational space.

Leakage is often long-lived and can therefore be a source of highly correlated noise [8]. This is particularly damaging in the context of QEC, where protocols work under the assumption that noise is statistically independent. These correlations can negate the benefits of error correction [1]. Total avoidance of leakage is, however, very difficult. Furthermore, some platforms directly exploit the higher-dimensional Hilbert space to implement gates [9–12]. Leakage is pernicious in error correction because two-qubit gates are designed for pairs of sealed qubits. When one qubit is leaked, the gate is out of spec and can damage the other. This continual introduction of noise in subsequent gates makes leakage a correlated noise channel.

Let us stress some fundamental differences between the leakage and erasure noise channels [3]. An erasure is said

to have occurred when a qubit becomes maximally mixed at a known circuit location [13]. By contrast, leakage can only be detected upon measurement. The leakage event can, in principle, occur at any time after qubit reset, and it will go undetected until it is heralded upon measurement. Also note that a leaked qubit introduces subsequent new noise at every two-qubit gate, whereas the erased qubit does not introduce more noise besides it being fully depolarised. More generally, erasure is largely seen as a benign channel, so much so that it is desirable to trade other noise sources for erasures—boosting the performance of error-correction protocols [14, 15]. Instead, leakage is generally seen as highly detrimental, and specific devices such as LRUs are needed to not completely forego the benefits of error correction.

2. Patch wiggling

To restore error suppression in the presence of leakage, specialised leakage reduction units (LRUs) are required [8]. Several types of LRU have been proposed. Direct-type LRUs use dedicated gate-type devices engineered to directly address leaked states and do nothing otherwise [12, 16–19]. Swap-type LRUs are specific quantum circuit gadgets that exchange the roles of auxiliary and data qubits, ensuring that all physical qubits are periodically reset [2, 8]. Here we will focus on a specific swap-type LRU for the surface code: patch wiggling [20]. Unlike other swap-type LRUs, patch wiggling achieves a swap between the roles of data and auxiliary qubits during a round of syndrome extraction without increasing the depth of the circuit. This swap ensures that each qubit is reset every two rounds of syndrome extraction.

Patch wiggling ensures that a qubit will not stay leaked for more than two rounds of error correction, but a leaked qubit can still introduce many errors during these two rounds. Making the decoder aware of where this error may have occurred can greatly improve accuracy. Leakage can be heralded when we measure qubits—if a measurement returns $|2\rangle$ instead of $|0\rangle$ or $|1\rangle$ [12, 21]. Upon such heralding, we know that the measured qubit leaked at some time since the last reset, and we also know that from the leakage event onwards, errors in nearby qubits will be much higher. Without knowledge of the precise location of the leakage event, we should assume that the

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probability of the leakage event having occurred increases from zero to one between reset and measurement linearly with ‘time’ [2]. A simpler approximation is to assume leakage occurred on the previous reset, and maximally increase our prior for the errors connected to the leaked qubit between the last reset and its leaked measurement, see Fig. 1 of the main text. As we have seen, this simple approximation recovers accuracy.

3. Distance restoration via leakage-aware decoding

It is well-known that, in the presence of leakage, the effective distance of a code can halve [1, 3]. This is because an error in an auxiliary qubit can spread to any two nearby data qubits. We can, therefore, generate undecodable error strings on $d/2$ data qubits with just $d/4$ auxiliary-qubit leakage errors, half the errors we would usually require. In other words, the correlated errors that leakage introduces are like “bad hooks” in syndrome-extraction schedules.

We now explain the effect we have seen in the main text—that being leakage aware squares Λ , which doubles, and hence restores, the effective distance, from $d/2$ to d . We discuss this in a simplified setting in which we ignore wiggling. Consider the $d = 7$ surface code patch of Fig. Supplementary1. At this distance, an unrecoverable error contains a chain of at least 4 Pauli errors. In the figure, two auxiliary qubits have leaked, subsequently depolarising neighbouring data qubits. One instance of these depolarisations is a chain of Z errors in some of the data qubits (bolts), with a signature of a single detector lighting up in the left side of the X decoding graph. Without any leakage information, the decoder will pair this detection with the boundary vertex through the left of the patch, producing a logical error. We see that the smallest number of independent errors producing a logical failure has reduced from 4 Pauli errors to 2 leakage events. However, when the decoder is aware that leakage occurred to the right of the patch, it clusters the defect with the boundary through the right boundary of the patch, avoiding logical failure.

The mechanism by which leakage-awareness restores

the distance of the code is reminiscent of the doubling of the code distance under erasures [15], but there are some differences. Most importantly, a heralded erasure yields an exact space-time location, while leakage can be heralded only upon measurement. When we herald leakage on a qubit, we know that, assuming perfect resets, that qubit leaked at some point since the last reset, but we do not know exactly when. In the main text we have seen evidence that assuming leakage happened just after that reset is a good enough for restoring the distance.

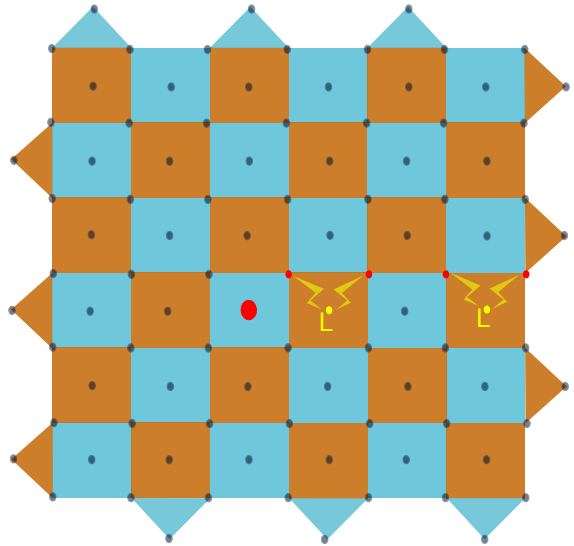


FIG. Supplementary1. $d = 7$ surface code patch. Two leaked auxiliary qubits (yellow) can introduce 4 subsequent data qubit errors (red, bolts) aligned with a logical string, producing a single detection event (big red dot in the blue plaquette). Without any leakage information, the decoder will connect this detection event with the left boundary of the patch, completing the logical string and introducing a logical failure. By heralding the leakage of the auxiliary qubits (yellow), we become aware that errors are more likely to the right than left of the patch, biasing the decoder to the right boundary, and recovering from these two leakage errors.

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