

Kibble-Zurek Mechanism and Beyond in a Holographic Superfluid Disk

Corresponding Author: Dr András Grabarits

This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Attachments originally included by the reviewers as part of their assessment can be found at the end of this file.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

The paper titled "Kibble Zurek Mechanism and Beyond: Lessons from a Holographic Superfluid Disk" shows that the universal nature of defects generated along with its statistics is not only present in the slow quench regime, as per the original proposal, but is also present in the fast quench regime. Hence, this work extends the universal nature of the defects and its distribution to the non-Kibble Zurek region, i.e., to the fast quench regime. While the slow quench regime shows power law dependence of defect density with the quench rate where the exponents of the power law are related to equilibrium critical exponents (also known as Kibble Zurek scaling), the defect density in the fast quenches show universal features as a function of the final temperature. The work also proposes that a Poisson Binomial Distribution (PBD) can accurately capture the statistics of defects generated in the slow quench regime (already shown in PRL 2020), but also in the fast quench regime. The predictions are verified in the superfluid phase transition with the crossing of critical temperature in a disk geometry. The application of these results in 2 dimensional systems is also interesting. I find the work novel and interesting which deserves publication in Nature Communications. I have however, following suggestions/queries:

1. Below Eq. (5). If not freezing, then what else. It would be good expand this line. Also, is it equivalent to saying that through out the process, the relaxation time is always large so that adiabaticity is never possible, very essential for KZM physics.
2. What guarantees that t_f will match the minimum relaxation time, as discussed in the para below Eq. 5. Is it a possibility that it may not match at all!!!
3. Is Eq. 9 one of the main finding/observations of this work, or has it been pointed out before as well? Please cite, if needed.
4. It would be good to mention that $d=2$ in page 4, left column where the KZM exponents are predicted for this system. How is this validating mean field?
5. How is $\hat{t}$ calculated in Fig. 3
6. Please cite reference for binomial distribution , Eq. 14
7. In Eq. 16,17, will there be dimension d dependence?
8. Caption of Fig. 4. Please complete the sentence: $5 \times 10^5 \dots$ (is it trajectories??) What is the dashed line in Fig. 4 (inset)? More explanation on the inset is required. What is it representing etc etc..relative square distance of what from the numerical results?? Same in Fig. 5.
9. Page 6, right column where it is mentioned that pure Binomial distribution fitting give values of p outside the range. It it

possible to give a plot supporting this?

10. Page 7, left column. How is $r_m > 1$ and $\sum_m r_m = 1$ possible together?

11. Citations are done poorly. Please complete it properly. Example: The recent observation that the mean defect density is universal for fast quenches constitute the starting point of our study, see Discussion. But this recent reference is not cited.

~

(Remarks on code availability)

Reviewer #2

(Remarks to the Author)

(Remarks on code availability)

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

The authors have successfully addressed all the concerns. I now recommend its publication in the present form.

(Remarks on code availability)

Reviewer #2

(Remarks to the Author)

The authors have addressed most of my earlier concerns. However, one major issue remains regarding the motivation for using a holographic framework in this particular work.

Could the same questions not be explored using simpler testbeds, such as 1D models? What specific advantage does the present holographic model offer over these alternatives? I would like the authors to clarify this point.

In particular, just below Eq. 9, they state that "These results establish the universality of critical dynamics in rapid quenches in a holographic framework." It would be helpful if the authors could clarify whether they intend to highlight a specific class of systems for which their scaling results are expected to be applicable.

In addition, references for the equilibrium critical exponents mentioned on page 4 are missing and should be provided. For a 2D superfluid, one typically expects BKT criticality rather than $\nu = 1/2$, so it would help to understand the justification for the mean-field exponents used in the KZM scaling.

(Remarks on code availability)

Open Access This Peer Review File is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made.

In cases where reviewers are anonymous, credit should be given to 'Anonymous Referee' and the source.

The images or other third party material in this Peer Review File are included in the article's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.

To view a copy of this license, visit <https://creativecommons.org/licenses/by/4.0/>

We are pleased by the Reviewers' positive assessment, and we have thoroughly addressed each comment in an itemized way below. We are confident that with the resubmission, the quality of the presentation has improved.

We also attach a copy highlighting the changes in [blue](#). Throughout our response, we have reproduced the Reviewers' comments in *italics* while our reply is contained in the colored boxes.

,

Reviewer #1 (Remarks to the Author)

The paper titled "Kibble Zurek Mechanism and Beyond: Lessons from a Holographic Superfluid Disk" shows that the universal nature of defects generated along with its statistics is not only present in the slow quench regime, as per the original proposal, but is also present in the fast quench regime. Hence, this work extends the universal nature of the defects and its distribution to the non-Kibble Zurek region, i.e., to the fast quench regime. While the slow quench regime shows power law dependence of defect density with the quench rate where the exponents of the power law are related to equilibrium critical exponents (also known as Kibble Zurek scaling), the defect density in the fast quenches show universal features as a function of the final temperature. The work also proposes that a Poisson Binomial Distribution (PBD) can accurately capture the statistics of defects generated in the slow quench regime (already shown in PRL 2020), but also in the fast quench regime. The predictions are verified in the superfluid phase transition with the crossing of critical temperature in a disk geometry. The application of these results in 2 dimensional systems is also interesting. I find the work novel and interesting which deserves publication in Nature Communications. I have however, following suggestions/queries:

We thank the Reviewer for the careful reading of our manuscript and for the positive assessment of its novelty and significance. We are grateful for the constructive feedback and insightful comments, which helped us to improve the clarity and scope of the work. We carefully address all of the Reviewer's suggestions and queries in a point-by-point manner in our revision, ensuring that each aspect is explicitly clarified and appropriately incorporated into the manuscript.

Below Eq. (5). If not freezing, then what else. It would be good expand this line. Also, is it equivalent to saying that through out the process, the relaxation time is always large so that adiabaticity is never possible, very essential for KZM physics.

We thank the Reviewer for requesting clarification and for pointing out the terminology. Eq. (5) expresses the condition for the breakdown of the KZM, and in this regime, no Kibble–Zurek scaling is expected. In particular, following the universal scaling argument

of H.-B. Zeng, C.-Y. Xia, and A. del Campo, Phys. Rev. Lett., 130, 060402 (2023), the finite quench depth scaling for rapid processes can be obtained by identifying the freeze-out time and correlation length with their equilibrium values at the end of the quench, for fast quenches.

Consistent with this picture, the dynamics remains in the freeze-out regime throughout the process. In perfect agreement with the Reviewer's observation, adiabaticity is largely absent, as the relaxation time remains large compared to the quench rate. In such a scenario, the final equilibrium correlation length serves as the effective freeze-out (nonequilibrium) length scale, and, together with the corresponding relaxation time, it sets the characteristic temporal and spatial scales that determine the typical vortex density. This gives rise to the universal power-law behavior described by Eqs. (6) and (7).

The discussion around Eq. (5) has been emphasized for improved clarity.

What guarantees that t_f will match the minimum relaxation time, as discussed in the para below Eq. 5. Is it a possibility that it may not match at all!!!

In the conventional KZM theory, the freeze-out time $\hat{t}$ is defined by equating the time elapsed after crossing the critical point to the instantaneous equilibrium relaxation time, i.e., as the solution to $\hat{t} \equiv \tau(\hat{t})$. Using the power-law expression for the equilibrium relaxation time $\tau = \tau_0/|\varepsilon|^{z\nu}$ and assuming a linear quench $\varepsilon(t) = t/\tau_Q$, the solution to $\hat{t} \equiv \tau_0/|\hat{t}/\tau_Q|^{z\nu}$ is the KZ expression $\hat{t} = (\tau_0\tau_Q^{z\nu})^{\frac{1}{1+z\nu}}$. Note that this corresponds to $\varepsilon(\hat{t}) = (\tau_0/\tau_Q)^{\frac{1}{1+z\nu}}$. As $\varepsilon(t) = |T(t) - T_c|/T_c$, all these KZ scaling formulae assume that the quench continues with no end, but experiments generally involve a finite value of T_f and t_f . It is thus essential to extend the definition of the quench to a piecewise schedule in which the control parameter ($T(t)$, and similarly, $\varepsilon(t)$) remains constant after a time t_f . This carries over the resulting expressions for $\hat{t}$, $\varepsilon(\hat{t})$, $\xi(\hat{t})$.

This provides a consistent framework for analyzing defect generation within a KZM-inspired framework. As the dynamics remains effectively frozen throughout the whole process, the final equilibrium correlation length determines the typical separation between vortices and sets the characteristic length scale of adiabatically driven domains. In turn, this length scale determines the average spacing and number of defects, capturing the universal scaling behavior predicted by the Kibble–Zurek framework.

When the variation of the control parameter occurs over a limited range of values, as is the case in experiments, below the characteristic quench time τ_Q^{cl} in Eq. (9), one finds $\hat{t} = \tau(\hat{t}) = \tau(t_f)$. For $\tau_Q < \tau_Q^{cl}$, one finds $\hat{t} = \tau(t_f)$ while for $\tau_Q > \tau_Q^{cl}$ one recovers the conventional KZM scaling $\hat{t} \sim \tau_Q^{\frac{\nu z}{1+\nu z}}$.

This discussion has been included as a new section under Methods.

Is Eq. 9 one of the main finding/observations of this work, or has it been pointed out before as well? Please cite, if needed.

Eq. (9) represents a scaling argument that identifies the breakdown timescale of the KZM for varying final quench depths. This relation was proposed previously (H.-B. Zeng, C.-Y. Xia, and A. del Campo, Phys. Rev. Lett., 130, 060402 (2023)) and serves as an important ingredient leading to the independent main findings presented in the main text in the Results section. In particular, it illustrates that the earlier the process stops (i.e.,

the smaller ϵ_f), the faster the system must be driven for KZM scaling to emerge. Below this threshold, the universal fast-quench dynamics dominates: the driving time becomes irrelevant, and vortex formation is determined exclusively by the final quench depth, ϵ_f .

The main contributions of our work lie in the full statistical characterization of vortex formation under rapid quenches, and in the first clear observation of deviations from the accepted binomial paradigm in both slow and fast quench regimes. Specifically, we show that cumulants of the vortex number distribution scale with the final quench depth in rapid processes, while their ratios follow a non-trivial Poisson-binomial distribution that converges to a Gaussian for large system sizes. Importantly, the same Poisson-binomial structure governs vortex statistics in both fast and slow driving regimes within the numerical precision available. Notably, for the case of slow driving, our results clearly rule out the expected binomial distribution.

It would be good to mention that $d=2$ in page 4, left column where the KZM exponents are predicted for this system. How is this validating mean field?

We thank the Reviewer for this important comment, we added a comment on the dimensionality. We used a minimal two-dimensional holographic model for which the numerical solutions are exact without any mean field approximations.

How is $\hat{t}$ calculated in Fig. 3

We thank the Reviewer for helping us to clarify this aspect of the exposition. Numerically, the freeze-out time $\hat{t}$ is defined at the moment when the mean value of the order parameter reaches 10% of its maximum value, as we have clarified in the caption of Fig. 3. We note that the results reported are robust with respect to variations in the percentage used in this definition. Likewise, similar results hold when one estimates the freeze-out time by the transition from exponential to adiabatic growth of the order parameter, sometimes termed equilibration or lag time in related literature, e.g., in Refs. [20,21].

Please cite reference for binomial distribution , Eq. 14

We added the citation to Phys. Rev. Lett. 124, 240602 (2020), also cited in the abstract, which appears as Ref. [62]. We thank the Reviewer for this comment, helping us to enhance the readability of the text.

In Eq. 16,17, will there be dimension d dependence?

As these equations are already derived in two-dimensional polar coordinates, they naturally hold only in two dimensions ($d = 2$) for vortices, (which are zero-dimensional topological defects (The generalization to point-like defects in d -spatial dimensions is direct by replacing the factors of 2 by d). We have modified the preceding lines around Eqs. (14) and (15) to make this explicit. We have also changed the notation slightly, replacing $\mathcal{N}_d$ (where the choice of the subscript for defect was somewhat unfortunate, as it could be confused with the spatial dimension) by $\mathcal{N}$.

Caption of Fig. 4. Please complete the sentence: $5 \times 10^5 \dots$ (is it trajectories??) What is the dashed line in Fig. 4 (inset)? More explanation on the inset is required. What is it representing etc etc...relative square distance of what from the numerical results?? Same in Fig. 5.

We thank the Reviewer for pointing out these missing details, which we have provided in the revision. Indeed, we meant 5×10^5 trajectories. The blue dots connected by the solid line denote the relative square errors of the sum of the first three cumulants from the theoretical values of the Poisson-binomial distribution with the best-fitting parameters. The dashed line stands for the τ_Q (Fig. 4) and ϵ_f (Fig. 5) averages of these values. We clarified this in the captions of Figs. 4 and 5.

Page 6, right column where it is mentioned that pure Binomial distribution fitting give values of p outside the range. It is possible to give a plot supporting this?

This statement can most conveniently be demonstrated by the cumulant ratios that for a binomial distribution with success probability p are given by $\kappa_1/\kappa_2 = 1/(1-p)$ and $\kappa_1/\kappa_3 = 1/((1-p)(1-2p))$. However, the numerically obtained cumulative ratios cannot match the binomial ones for any value of $p \in [0, 1]$. To fully address the Reviewer's comment, we added a new section in Methods "Numerical demonstration of the inadequacy of the binomial approximation" with the new Figure 8 showing how the deviations between the binomial and numerical cumulants change with varying $p \in [0, 1]$. The inability of the binomial approximation to simultaneously describe the observed cumulative values in the holographic model rules out this distribution for describing the defect statistics.

Page 7, left column. How is $r_m > 1$ and $\sum_m r_m = 1$ possible together?

We are grateful to the Reviewer for pointing out this error. Indeed, the probability r_m that m protodomains meet is at most 1, i.e., $r_m \leq 1$. We corrected this in the revised manuscript.

Citations are done poorly. Please complete it properly. Example: The recent observation that the mean defect density is universal for fast quenches constitutes the starting point of our study, see Discussion. But this recent reference is not cited.

We corrected the citations throughout the revised manuscript and added a code availability statement.

Reviewer #2 (Remarks to the Author)

The authors study dynamical aspects of spontaneous vortex formation in a two-dimensional holographic superfluid undergoing a phase transition driven by the ramp of an external parameter. The Kibble–Zurek mechanism (KZM) provides a well-known framework for predicting universal features of such transitions in terms of the ramp time, within the adiabatic–impulse–adiabatic (AIA) approximation. Over the years, a large body of work has extended KZM by modifying defect-scaling predictions, incorporating complex coarsening dynamics, and analyzing vortex correlations that often reveal behavior beyond the original framework. This manuscript targets the fast-quench regime and demonstrates universality in terms of the final time/parameter by analyzing the defect distribution in detail. The authors show that a Poisson–Binomial distribution continues to capture scaling laws, consistent with results obtained by some of the same authors in the slow-quench regime. The manuscript reads with clarity, however, prior to recommending a publication, I would like to mention some concerns, outlined below.

We thank the Reviewer for their positive assessment and the comments/critiques that helped us improve the quality of the revised manuscript.

1. ***Quench protocol.*** The protocol is not entirely clear. Do the authors start exactly at the critical point, where AIA already predicts freezing? They state that fast quenches correspond to cases where the final time is shorter than the relaxation time at that point. Why does this prevent freezing? Are their predictions restricted purely to critical dynamics?

The quench was initiated from the critical point by starting the chemical potential at μ_c and then gradually increasing it. Consistent with the universal scaling argument of H.-B. Zeng, C.-Y. Xia, and A. del Campo, Phys. Rev. Lett., 130, 060402 (2023), even fast quenches can be treated in the spirit of the Kibble–Zurek framework by identifying the freeze-out regime final parameters. This sets a lower bound to the relaxation time and correlation length, naturally capturing the freeze-out time and distance as $\hat{t} \equiv t_f$ and $\hat{\epsilon} \equiv \epsilon_f$. The latter, in turn, determines the average spacing and number of the emergent vortices. Since the universal scaling argument assumes that the correlation length diverges and that vortices start to form only at the critical point, the corresponding protocol must also be initiated from the critical point. This ensures consistency with the analytical predictions and makes the comparison more transparent.

Importantly, this construction does not prevent the system from freezing. On the contrary, the dynamics remain in the freeze-out regime throughout the process, with the correlation length set by its equilibrium value at the end of the quench.

In perfect agreement with this picture, the Kibble–Zurek mechanism also remains valid for slow quenches initiated at the critical point. In this case, domains begin forming immediately after the quench, but as the system escapes the freeze-out regime, their growth halts, with their final size determined by the freeze-out time. The essential point is that the freeze-out time and freeze-out length are governed by the same principle as in the conventional KZM, leaving the overall scaling unchanged. Possible deviations appear only as constant prefactors at leading order due to the missing freeze-out regime before the critical point. However, this latter does not alter the overall timescale of the freeze-out regime.

2. ***Post-quench evolution.*** If one waits long enough after the quench ends, should one not observe nontrivial time dependence in the cumulants, even if non-universal? Some clarification is needed.

The Reviewer is right. The subsequent dynamics upon completion of the quench, and in certain cases even before, generally exhibits coarsening, which is expected to dominate the long-time behavior as the system approaches thermal equilibrium. Defect annihilation mechanisms reduce the defect density and are expected to lead to deviations from the universal scaling laws in both fast and slow quenches, for the mean, as well as for higher-order cumulants. We do not claim or expect the Poisson-Binomial distribution to account for the long-time equilibrium behavior of the vortex number (even if it may still be accurate). We have clarified this immediately after quoting the scaling laws for the cumulants of the defect number distribution.

3. ***Figure 1.*** Why does $(-O-)$ saturate around 1.5? An explanation of this feature would be helpful. How are the boundary contributions to the Poincar’e-Hopf theorem taken care of?

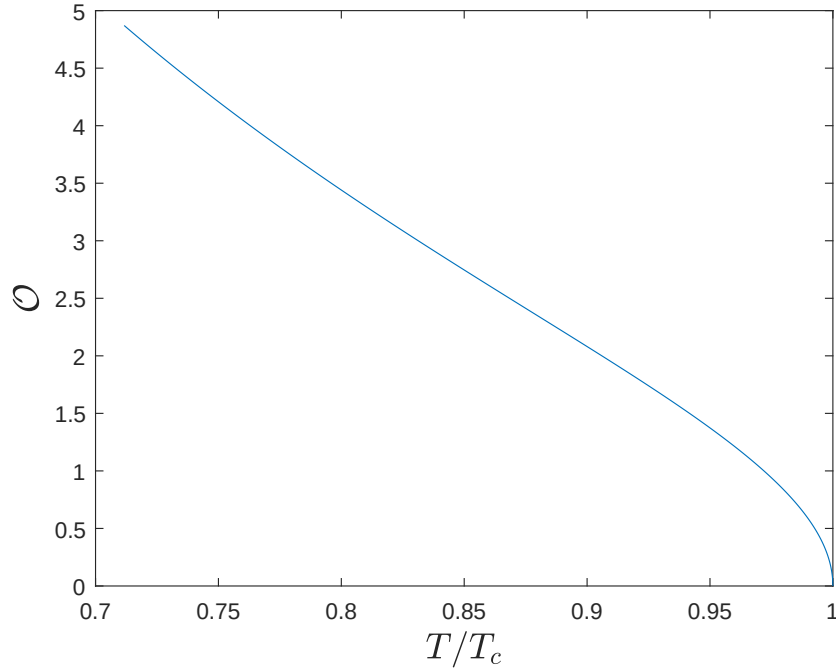


Figure 1: Order parameter versus temperature.

We thank the reviewer for this question. The equilibrium value of the order parameter is determined by the final temperature. The dependence of the equilibrium value of the order parameter on temperature is shown in Fig. 1. Specifically, the equilibrium value of the order parameter corresponding to T_c is 1.5, i.e., when the final temperature is $0.94T_c$. This has been clarified in the caption of Fig. 1 by adding the sentence “The equilibrium value of the order parameter corresponding to the final temperature is close to 1.5.”

In the numerical study of the Kibble-Zurek mechanism (KZM) in two dimensions, periodic boundary conditions are commonly adopted. The corresponding geometry of such a system is a torus, which has an Euler characteristic of zero and satisfies the Poincaré-Hopf theorem: the total topological charge must vanish identically and the defect number is conditioned to even outcomes (when defects have equal absolute value of the topological charge, such as the case of vortices with winding number ± 1).

However, in this study, the superfluid disk can be fully embedded in a plane, meaning that the disk also inherits the Euler characteristic of 2 from the plane. As a result, the topological charge in the superfluid disk is no longer constrained by this theorem, allowing the number of positive and negative vortices to be unequal and leading to vortices following a Poisson binomial distribution. Numerically, we applied Neumann boundary conditions at the maximum radius of the superfluid disk to achieve the effect of an open boundary. Since the superfluid near the boundary occupies a very small fraction of the total area of the superfluid disk, it is reasonable to infer that the presence of the boundary will not cause significant deviations in the KZM. Our numerical simulation results confirm this. In practice, real experiments typically involve material samples with boundaries. Open boundary conditions can be simulated experimentally by considering a subregion of the

sample [e.g., as done in Refs. [95-96]]. To our knowledge, open boundaries do not cause deviations in the KZM (provided that $\hat{\xi}$ is much smaller than the system size, as reflected by the vortex number), while periodic or twisted boundary conditions can, in particular, affect cumulants of the defect number distribution [see e.g., Phys. Rev. B 106, 134302 (2022)].

4. ***Role of the final parameter.** The authors emphasize that after the quench, the final parameter value alone dictates the growth of the defect density. But given that fast quenches are defined through Eq. (5), and no other relevant scale appears in the problem (the relaxation time remains much larger), why should this be surprising?*

We agree with the Reviewer that, as defined in Eq. (5), for fast quenches the final parameter value indeed dictates the average defect density, since no other relevant scale enters and the relaxation time remains much larger. This scaling argument has already been established in H.-B. Zeng, C.-Y. Xia, and A. del Campo, Phys. Rev. Lett., 130, 060402 (2023). However, it only constrains the scaling of the average number of defects under specific assumptions. A first indication of the novelty of our results is to confirm the validity of this scaling in driven holographic models, capturing the non-equilibrium dynamics of strongly correlated many-body systems.

More importantly, our analysis goes beyond averages and uncovers the full statistical structure of defect formation. We show that the cumulants remain proportional for fast quenches as well, with a Gaussian limiting distribution, which cannot be inferred from the scaling arguments of the mean vortex number alone. In addition, we show that the statistics follow a Poisson–binomial distribution, with individual success probabilities independent of the final parameter ϵ_f . While the average defect density follows the scales as expected with the final parameter ϵ_f , our results reveal that the underlying statistics are richer than the mean alone, uncovering a universality previously unnoticed. That the same distribution describes fast and slow quenches is highly nontrivial, given the different phenomenology involved near adiabatic dynamics and near sudden quenches.

5. ***Motivation for holography.** The overall rationale for employing a holographic system is not sufficiently clear. A more detailed introduction and explanation of its relevance would strengthen the manuscript.*

Holography provides a powerful, nonperturbative framework to study strongly correlated quantum many-body systems in and out of equilibrium, a regime often beyond the reach of perturbative or conventional numerical techniques.

In particular, the AdS/CFT correspondence maps such problems in quantum field theory to (classical) gravity in higher dimensions, where the dynamics can be computed reliably. This approach has been successfully applied to investigate quantum critical dynamics, transport, and defect formation in strongly interacting systems; see, for example, S. R. Das, T. Nishioka, and T. Takayanagi, JHEP 07, 071 (2010); M. J. Bhaseen, J. P. Gauntlett, B. D. Simons, J. Sonner, and T. Wiseman, Phys. Rev. Lett. 110, 015301 (2013); A. Adams, P. M. Chesler, and H. Liu, Science 341, 368 (2013). By now, the use of AdS/CFT correspondence to probe critical dynamics has been recognized in some advanced monographs [e.g., Sean A. Hartnoll, Andrew Lucas and Subir Sachdev, Holographic Quantum Matter (MIT Press, 2018)].

In the present work, we employ a holographic superfluid on a disk geometry, modeled via an Einstein–Abelian-Higgs theory in AdS_4 , whose boundary corresponds to a strongly coupled 2+1D superfluid. This setup allows us to simulate quench dynamics self-consistently and track vortex formation in real time, including in regimes of fast quenches and high defect density where conventional analytic or lattice approaches are limited.

More crucially, holography enables us to test universality beyond the scaling of average defect densities: we are able to establish cumulant scaling for both varying final quench depth in rapid quenches and for varying driving time in slow processes, and reveal a Poisson–binomial structure underlying defect distributions with a limiting Gaussian statistics. These results highlight an additional layer of universality in defect formation that, to our knowledge, has not been accessible in previous approaches.

In summary, holography provides (i) nonperturbative control in a strongly correlated regime, (ii) real-time access to nonequilibrium vortex dynamics, and (iii) the ability to uncover universal statistical features beyond the mean, making it a natural and powerful framework for the questions addressed in this work. In this revision, we emphasize this in the opening paragraph in the introduction and while introducing the holographic setting in Methods.

6. ***Origin of (r_m) .** The new parameter (r_m) (introduced in Eq. (19)) appears central to the analysis. Is there a physical mechanism or intuition supporting its definition and significance?*

In line with the Reviewer’s suggestion, we emphasize that the parameter r_m is indeed central to our analysis. Its origin can be traced back to the geodesic rule, first discussed in Kibble’s seminal work, to the best of our knowledge.

The geodesic rule was tested in a series of beautiful experiments by Sadler et al (PRL 2006) merging multiple independent BECs. It was observed that the probability for defect formation when m BECs merge depends on m . We denote this probability in the manuscript as p_m and should be understood as the conditional probability for vortex formation provided that m -protodomains merge. This motivated early theoretical work for slow quenches [F. J. Gómez-Ruiz, J. J. Mayo, and A. del Campo, Phys. Rev. Lett. 124, 240602 (2020)].

There is an element of heuristic modeling in transitioning from the experimental scenario discussed by Sadler et al. to our setting: As $U(1)$ symmetry breaking takes place, the order parameter transiently exhibits regions with nearly constant, slowly varying phase, which are termed protodomains. The merging of such protodomains at a given location makes it possible for a vortex to be formed. For that, it is not sufficient that different domains merge at a point (an event to which we assign a probability r_m), but that their respective phases have a well-defined circulation (to which we assign a probability p_m). This latter aspect is the goal of the geodesic rule. The heuristics come from the fact that such protodomains are not precisely defined and there is an element of approximation and abstraction in mapping $U(1)$ symmetry breaking to the above scenario.

Note that our prediction does not rely on the estimate of r_m and p_m ; rather, we use such classical modeling of $U(1)$ symmetry breaking to predict the form of the pdf, which is then verified numerically.

That such modeling extends from slow to fast quenches is truly remarkable, given the very different dynamics involved, from nearly-adiabatic to sudden quenches.

Falsifying the prediction for the pdf is numerically demanding, since vortex counts alone provide only limited precision. Instead, the cumulant ratios offer a more sensitive probe. In practice, the first three cumulants can unambiguously reveal a Poisson–binomial distribution with two distinct success probabilities, which is why we focused on this case in our exposition. This restricted case is not only numerically tractable but also conceptually illustrative, as it demonstrates in the clearest possible terms that a single binomial law is insufficient.

Building on this, we stress that our results provide the first clear evidence for the breakdown of a purely binomial description in higher-dimensional systems. Although extending the analysis to more refined Poisson–binomial forms requires extracting higher-order cumulants (a numerically challenging task), our present results mark a crucial step forward, providing both physical intuition and a concrete statistical framework, giving a huge push for related future research.

7. ***Critical model description.*** The discussion of the model in Eq. (28) is not clearly presented. What is the physical meaning of the critical parameters introduced here?

We appreciate the Reviewer’s comment and are glad to provide clarification. The holographic model is a gravitational theory defined in $3 + 1$ dimensional space time (the black hole background) and is dual to a $2 + 1$ dimensional $U(1)$ field theory on the boundary of the black hole. The Hawking temperature of the black hole determines the thermal state of the boundary theory, and as it is lowered to a critical value, the $U(1)$ field develops a nonzero solution corresponding to a symmetry-broken phase—realizing a continuous phase transition within the AdS/CFT correspondence. In our numerical simulations, we solve the dynamical equations of motion with a time-dependent temperature, enabling a complete characterization of the phase-transition dynamics and the associated defect formation.

The model in Eq. (28) captures the statistical nature of defect formation during a quench. It is based on the idea that vortices form locally and independently, so that the total number of defects, N , can be expressed as the sum of independent random variables associated with each potential vortex location. This description provides a natural framework to connect microscopic formation events with the emergent statistical properties of the system.

For slow quenches, it is well established within the Kibble–Zurek framework that the driving rate primarily sets the number of possible vortex formation locations, while the individual formation probabilities are independent of the driving rate and the events themselves occur independently. This separation between the density of potential sites and the stochastic formation process is a key ingredient behind the emergence of Gaussian statistics for the total number of defects in the large-system limit.

Our analysis shows that the same assumption applies to fast quenches, provided that the final quench depth, ϵ_f , is not too large. In this regime, the potential formation sites remain sufficiently dilute so that vortices continue to form as independent events. The critical parameters in Eq. (28) therefore characterize both the effective number of independent vortex sites, N_{max} , and their individual formation probabilities, p . This framework most generally leads to an overall Poisson–binomial description for vortex statistics, for which our analysis provides the first concrete evidence. In the large-system limit, this Poisson–binomial distribution converges to a Gaussian form, connecting the microscopic picture of independent

local defect formation to the emergent macroscopic statistics discussed in Sec. VI, and showing that universal features of defect formation extend beyond mean quantities to the full distribution.

8. ***Beyond KZM through correlations** Correlations among defects often display unusual features that fall beyond the KZM predictions. I would like to draw the authors' attention to some work like Phys. Rev. B. 104.014406, 2021.*

We thank the Reviewer for this reference, providing a complementary characterization of the correlations beyond KZM, which we have cited in the revised manuscript, in the added sentence in p. 2 column 2 under “Beyond KZM”: “A complementary characterization based on spatial correlations between defects has also been reported” [Phys. Rev. B. 104.014406, 2021].

Addressing these points would significantly enhance the clarity and impact of the work.

We thank the Reviewer for his careful reading and helpful comments, helping to improve the quality of the manuscript. We hope the revised version meets the Reviewer's expectations for publication in Nature Communications.

Dear Reviewers,

We are pleased by the positive assessment and thankful for the recommendation for publication. In the revised manuscript, we have thoroughly addressed the remaining comment of Reviewer 2.

We also attach a copy highlighting the changes in [blue](#). Throughout our response, we have reproduced the Reviewers' comments in *italics* while our reply is contained in the colored boxes.

Sincerely,

Chuan-Yin Xia, Hua-Bi Zeng, András Grabarits, and Adolfo del Campo

Reviewer #1 (Remarks to the Author)

The authors have successfully addressed all the concerns. I now recommend its publication in the present form.

We thank the Reviewer for their positive assessment and recommendation for publication.

Reviewer #2 (Remarks to the Author)

The authors have addressed most of my earlier concerns. However, one major issue remains regarding the motivation for using a holographic framework in this particular work. Could the same questions not be explored using simpler testbeds, such as 1D models? What specific advantage does the present holographic model offer over these alternatives? I would like the authors to clarify this point.

In particular, just below Eq. 9, they state that “These results establish the universality of critical dynamics in rapid quenches in a holographic framework.” It would be helpful if the authors could clarify whether they intend to highlight a specific class of systems for which their scaling results are expected to be applicable. In addition, references for the equilibrium critical exponents mentioned on page 4 are missing and should be provided.

We thank the Reviewer for this important question. We have revised the manuscript to clarify the motivation for employing a holographic framework in this work.

There are several motivations for our setting.

Regarding dimensionality, 1D systems, like the 1D time-dependent Ginzburg-Landau theory (a common testbed for KZM studies since the seminal works by Laguna and Zurek in '98, such as Ref. P. Laguna and W. H. Zurek, Phys. Rev. D, 58, 085021, (1998)), support kinks the simplest version of topological defects and describe the breaking of a discrete symmetry such as parity. Moreover, 1D systems, along with 1D quantum Ising models, artificially enhance defect-defect interactions in ways that are unrepresentative of higher-dimensional systems. In short, standard 1D testbeds are often weakly coupled, integrable, or effectively described by free or near-free theories, and thus fail to capture the

strongly correlated, finite-temperature critical dynamics of interest. They are also better suited for probing discrete rather than continuous symmetry breaking.

We focus on a 2D system supporting vortices as true topological defects. A similar study could have been conducted using a complex Ginzburg-Landau theory that breaks $U(1)$ symmetry, giving rise to point-like vortices. However, the holographic setting allows us to demonstrate the validity of our results in a broader context, namely, at strong coupling.

Holography provides a first-principles description of strongly interacting systems undergoing thermal phase transitions. It thus offers a conceptually broader and more generic testbed, in which vortex formation, dissipation, and rapid-quench dynamics emerge naturally from the strongly coupled dynamics. Within this wider and more correlated setting, it is natural to regard the scaling behavior we uncover as reflecting universal features that hold even in strongly interacting systems, which cannot be captured by simpler weakly coupled or free models. From this perspective, scalings of universal critical dynamics established in the strongly correlated regime generally hold at weak coupling, while the converse is not true. [Ref. Chesler, P. M., Liu, L., Phys. Rev. X 5, 021015 (2015)] offers a counterexample to this expectation, which has not been refuted in the literature, though logarithmic corrections claimed to be exclusive of strongly coupled systems can also be found in a much broader context, e.g., via BKT physics.

Outside holography, finite-temperature dynamics are typically modeled by extending the stochastic Gross–Pitaevskii equation, with temperature dependence and dissipation introduced by hand. In contrast, the holographic dual realizes both intrinsically: the black hole serves as a thermal reservoir, driving the superfluid transition and absorbing energy during a quench. This provides a fully first-principles mechanism for dissipation and thermalization.

This intrinsic treatment makes holography a natural and self-contained framework for exploring universal quench dynamics in strongly coupled 2D superfluids, beyond what standard 1D or weakly coupled models can capture.

In addition, references for the equilibrium critical exponents mentioned on page 4 are missing and should be provided.

We thank the Reviewer for this important comment. We have carefully double-checked the list of references for the original works reporting the mean-field critical exponents of the holographic superfluid phase transition, and we cite Refs [11,20,86]. In contrast to holographic superconductors, there is relatively little literature on holographic superfluid quantum phase transitions, which further motivates providing explicit references for these mean-field exponents.

For a 2D superfluid, one typically expects BKT criticality rather than $\nu = 1/2$, so it would help to understand the justification for the mean-field exponents used in the KZM scaling.

We agree with the Reviewer’s concerns about the validity of the mean field approximation. In the holographic duality of the gauge/gravity correspondence, the boundary quantum field theory describing the 2D superfluid on a disk is defined at large N , denoting the rank of the gauge group, and at strong coupling. The dual gravitational description is given by the Einstein–Abelian–Higgs system in an AdS_4 black hole background [Herzog, C. P., Kovtun, P., Son, D. T., Phys. Rev. D 79, 066002 (2009); Maeda, K., Natsuume, M., Okamura, T., Phys. Rev. D 79, 126004 (2009)]. In this large- N classical limit, fluctuations of the order

parameter are suppressed by $1/N$, and the dominant saddle-point dynamics reproduces the mean-field equilibrium critical exponents, e.g., $\nu = 1/2$, for the thermal phase transition in the holographic superfluid [Zeng, H.-B., Gao, X., Jiang, Y., Zong, H.-S., Phys. Rev. D 82, 126014 (2010); Lai, X.-Y., Pan, Q.-Y., Jing, J., Wang, Y., Phys. Lett. B 774, 298–303 (2017)]. This suppression of fluctuations is precisely the reason why holographic superfluids exhibit mean-field universality at leading order, even in two spatial dimensions.

Because the classical gravity approximation corresponds to the leading order large- N limit of the boundary theory, it does not capture strong phase fluctuations or vortex–antivortex unbinding responsible for Berezinskii–Kosterlitz–Thouless (BKT) criticality in weakly interacting 2D systems. Genuine BKT behavior arises from infrared fluctuations, which are subleading in $1/N$ and absent in the classical bulk treatment; such fluctuation effects would appear only through quantum-bulk loop corrections or additional degrees of freedom beyond the classical gravity limit. This large- N mean-field character of holographic transitions has been further confirmed in studies of holographic KZM scaling [Natsuume, M., Okamura, T., Phys. Rev. D 95, 106009 (2017)] and in classical phase-diagram analyses [Basü, P., Mukherjee, A., Krishnan, C., J. High Energ. Phys. 2010, 6; Chesler, P. M., Liu, L., Phys. Rev. X 5, 021015 (2015)], which considers classical bulk solutions undergoing second-order transitions consistent with mean-field exponents.

Interestingly, to the best of our knowledge, in a holographic setting, no thermal BKT transition has been found to date. Studies of BKT physics in holography are currently limited to a zero-temperature quantum BKT transition that disappears at finite temperature [see Ref. K. Jensen, A. Karch, D. T. Son, and E. G. Thompson Phys. Rev. Lett. 105, 041601 (2010) and ensuing citing works].

Thus, within the classical holographic framework employed in our manuscript, the use of mean-field critical exponents is fully justified: the dual large- N , strongly coupled boundary theory exhibits a genuine second-order thermal transition, fluctuations are suppressed, and the corresponding KZM scaling naturally follows from these mean-field exponents. We have added this discussion and explicit references in the revised manuscript.

However, we note that studies such as Jelić, and Cugliandolo, Quench dynamics of the 2d XY model, J. Stat. Mech. (2011) P02032 and J. Dziarmaga and W. H. Zurek, Quench in the 1D Bose-Hubbard model: Topological defects and excitations from the Kosterlitz-Thouless phase transition dynamics, Sci. Rep. 4, 5950 (2014), or A. Polkovnikov, Phys. Rev. B 72, 161201 (2005) attempts to extend the KZM scaling to true BKT systems. In the first case, the post-quench defect dynamics are described using a coarsening-based argument, while in the latter examples the BKT correlation length divergence is approximated by an effective $\nu \rightarrow \infty$ power-law with logarithmic corrections. These approaches are heuristic and only partially controlled, highlighting the intrinsic caveats of applying KZM to BKT transitions: the dynamics is governed by infrared vortex–antivortex unbinding, not simple power-law divergences, so the resulting “effective” KZM scaling is at best an approximate description rather than a rigorous prediction. This illustrates why the mean-field large- N limit of holographic superfluids offers a more controlled basis for the KZM.

The authors study dynamical aspects of spontaneous vortex formation in a two-dimensional holographic superfluid undergoing a phase transition driven by the ramp of an external parameter. The Kibble–Zurek mechanism (KZM) provides a well-known framework for predicting universal features of such transitions in terms of the ramp time, within the adiabatic–impulse–adiabatic (AIA) approximation. Over the years, a large body of work has extended KZM by modifying defect-scaling predictions, incorporating complex coarsening dynamics, and analyzing vortex correlations that often reveal behavior beyond the original framework.

This manuscript targets the fast-quench regime and demonstrates universality in terms of the final time/parameter by analyzing the defect distribution in detail. The authors show that a Poisson–Binomial distribution continues to capture scaling laws, consistent with results obtained by some of the same authors in the slow-quench regime. The manuscript reads with clarity, however, prior to recommending a publication, I would like to mention some concerns, outlined below.

1. **Quench protocol.** The protocol is not entirely clear. Do the authors start exactly at the critical point, where AIA already predicts freezing? They state that fast quenches correspond to cases where the final time is shorter than the relaxation time at that point. Why does this prevent freezing? Are their predictions restricted purely to critical dynamics?
2. **Post-quench evolution.** If one waits long enough after the quench ends, should one not observe nontrivial time dependence in the cumulants, even if non-universal? Some clarification is needed.
3. **Figure 1.** Why does $\langle |O| \rangle$ saturate around 1.5? An explanation of this feature would be helpful. How are the boundary contributions to the Poincaré–Hopf theorem taken care of?
4. **Role of the final parameter.** The authors emphasize that after the quench, the final parameter value alone dictates the growth of the defect density. But given that fast quenches are defined through Eq. (5), and no other relevant scale appears in the problem (the relaxation time remains much larger), why should this be surprising?
5. **Motivation for holography.** The overall rationale for employing a holographic system is not sufficiently clear. A more detailed introduction and explanation of its relevance would strengthen the manuscript.
6. **Origin of (r_m) .** The new parameter (r_m) (introduced in Eq. (19)) appears central to the analysis. Is there a physical mechanism or intuition supporting its definition and significance?
7. **Critical model description.** The discussion of the model in Eq. (28) is not clearly presented. What is the physical meaning of the critical parameters introduced here?
8. **Beyond KZM through correlations** Correlations among defects often display unusual features that fall beyond the KZM predictions. I would like to draw the authors’ attention to some work like Phys. Rev. B. 104.014406, 2021.

Addressing these points would significantly enhance the clarity and impact of the work.