

Reprogrammable metamaterial robot with embodied versatile computation and mechanical intelligence

Corresponding Author: Professor Yi-Ze Wang

This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

Mechanical computing, which leverages structural architectures to perform calculations and respond to mechanical stimuli, offers a promising pathway toward realizing embodied computational autonomy and mechanical intelligence in smart, adaptive, and robotic materials. In this work, the authors propose a mechanical metamaterials platform capable of integrating a broad range of computational functions—including analog operations such as differentiation and integration, digital logic operations such as AND, OR, and XOR, as well as qubit-like logics—into robotic locomotion. The study is technically sound and interesting, and I recommend it for publication in Nature Communications. The following comments should be considered for further improving the manuscript:

- (1) In this paper, the term “universal” seems to be intended to highlight that mechanical analog, logic, and quantum-inspired computing can be integrated within a unified framework. However, in computational theory, a universal computing machine usually refers to a system that can, in principle, simulate any real-world computer given unlimited time and memory. Since the proposed metamaterial does not possess information storage capabilities, its computational power may not fully align with this definition. Thus, describing its computational capabilities as “universal” is potentially misleading to readers.
- (2) Could the authors provide a more detailed explanation of the claimed parallel computing in the frequency domain? In Fig. 4(f) and 4(g), the input frequencies are 70 Hz and 140 Hz, respectively, yet the corresponding computational outputs are both observed to be 70 Hz (Fig. 4o). If two inputs at different frequencies (70 Hz and 140 Hz) are applied simultaneously, it seems that the outputs would overlap at 70 Hz, making it difficult to distinguish the contributions of different inputs. Does this suggest that the system cannot simultaneously perform calculations at both 70 Hz and 140 Hz? If so, how to understand the parallel computing capability in the frequency domain emphasized in the paper?
- (3) On page 11 (second paragraph), the authors state: “Based on the eigenstate diagram in Fig. 4(e), three incident frequencies with $f_e = 70, 140$ and 300 Hz are selected in Figs. 4(f)–4(h).” However, the third paragraph on the same page lists a different incident-frequency value, which appears to be a clerical inconsistency. Please verify the intended frequencies and correct the manuscript so that the values reported in the text, figure captions, and Fig. 4 itself are consistent.
- (4) I agree that this paper provides an interesting perspective on the modulation of acoustic waves by metamaterials within the framework of quantum computing. Nevertheless, a more rigorous discussion of acoustic quantum computing could further enrich the work. For example, Ref. 32 reports the realization of acoustic quantum analogue computing based on acoustic phi-bits, which were argued to satisfy DiVincenzo’s five criteria for the physical implementation of a quantum computer (Runge, K., Deymier, P. A., Hasan, M. A., Lata, T. D. & Levine, J. A. Acoustic metamaterials for realizing a scalable multiple phi-bit unitary transformation, AIP Advances 14, 025010 (2024)). It would be helpful if the authors could clarify the distinction between the acoustic phi-bits and the mechanical qubits presented here, and discuss to what extent the proposed mechanical qubits may be compatible with or fall short of DiVincenzo’s criteria.

Reviewer #2

(Remarks to the Author)

Comments for Author

- What are the noteworthy results?

A mixed signal architecture that integrates analog and logic operations, allowing for nested and hierarchical operations. The architecture is also shown to be able to switch between multiple different modes providing multi-task ability. The implementations is based on electric wave metamaterials. The extensive experimentation demonstrates the various computing methodologies and implements a simple crawler robot.

• Will the work be of significance to the field and related fields? How does it compare to the established literature? If the work is not original, please provide relevant references.

The work is similar to a broad swathe of literature in mechanical computing, which has seen a notable amount of popularity over the past few years. The core claim of mixed signal and multi-task has previously been shown to some extent in, e.g., <https://www.pnas.org/doi/10.1073/pnas.2407431121>. There is also a string of uncited works covering e.g., 'ghost circuits', etc. There's a lot of work on the embodied side e.g. <https://arxiv.org/abs/2110.15245>, especially Helmut Hauser's work e.g. <https://link.springer.com/article/10.1007/s00422-012-0471-0>. The work is therefore partially original. It would likely be of interest to specialists in the field. It is too dense and specialised to be understandable by many non-specialists.

One weakness with this work is that the robotic implementation is simple – this is fine as a proof of concept but really shows the inability to turn this technology into something useful. Despite claiming to present a universal architecture, it is unclear why this is an important problem and the justification for using their implementation is weak/underdeveloped.

• Does the work support the conclusions and claims, or is additional evidence needed?

The work would benefit from a degree of 'hand holding' – e.g. for each experiment, what are you showing, why are you showing it, etc? The paper needs to be navigable.

• Are there any flaws in the data analysis, interpretation and conclusions? Do these prohibit publication or require revision? Results seem sound.

• Is the methodology sound? Does the work meet the expected standards in your field?

There are large numbers of equations without much guidance between them. The figures are average quality. In general this paper was very hard to read and understand. There is a lot of quantum theory interspersed into large walls of text. It is therefore a huge effort, requiring a background in quantum, to see if the methodology is sound, and if the work is reproducible.

• Concerns

Unclear how this is novel compared to some of the recent literature that appears to do the same thing.

Although I've been told not to comment overly on the language, use of language in this manuscript is of a poor standard in the introduction, and can lead to confusion. I frequently found myself becoming lost, or having to re-read a passage multiple times to try and decode what the authors are trying to say. I highly recommend the authors use a professional editing service. Many of the sentences are not legal sentences, or incredibly unclear, e.g.

1. "The flexible designs from bioinspired materials to molecular crystalline structures endow metamaterials with extraordinary mechanical properties, such as acoustic cloaking, negative refractive and topological insulators [5–8]."

2. "In comparison, novel progress in metamaterials has developed wave-based analog computing [26–30], which enables continuous processing of complex information in spatial."

Fig 1 the caption does not explain the figure, which is very dense and needs much clearer explanation.

Reviewer #3

(Remarks to the Author)

Referee report for

Nature Communications

Manuscript NCOMMS-25-641411

Reprogrammable metamaterial robot with embodied universal mechanical computations

by W. Zhou and Y.-Z. Wang

The manuscript presents a metamaterial robotic system that has three modes for computations. The first mode is conformal change in the metamaterial structure, a second mode is in the vibrational spectrum supported in the robotic system. Finally, the system can use locomotion as a means to accomplish computation. The authors claim that they have demonstrated binary and quantum logic operation using the robotic system. Further, they claim that the system is capable of both serial and parallel operation. Finally, the authors predict that the system will perform robustly in harsh environments. The current manuscript has a number of drawbacks including the lack of quantitative measures for the fidelity of the computations. Also, the concept of quantum computing is not well represented by this system as it lacks both scalability and flexibility with respect to input states. The length of supplementary material, both written and video, is an impediment to the assessment of this submission. I recommend resubmission of this manuscript after major revision.

The concept of using multiple aspects of the metamaterial robotic system for computational means is of interest and would be advantageous for exploration in harsh environments. It is, however, unclear how the authors would propose to accomplish inputs from the environment to adapt an instruction set for flexible operation. Further, attention must be paid to the accuracy of the computed result, the authors fail to report any measure of the uncertainty related to any of the computations performed. It is clear from the figures, e.g., Figure 2, that the accuracy of the computation is limited by the nature of the robotic system. A measure of uncertainty is required for each of the computations performed.

While acoustic systems analogous to quantum computers have been reported in the literature and cited by the authors, some requirements for quantum computation are not met by the current system. In particular, following Divincenzo's criteria, the system fails to be scalable. Recall that quantum advantage is usually considered to require 50 qubits. This robotic system does not scale to such a large number, or at least such scaling is not clear to this reviewer. Secondly, for general quantum computing the initial state must be able to accommodate a wide range of states available on the Bloch sphere. Again, it is not clear that the current system will support such diversity in specifying the initial state.

With regard to the supplementary material, it does not appear to this reviewer that the video material adds to the content of

the manuscript. While the written supplementary material does support the manuscript, its length could easily be reduced without losing the vital supporting information the manuscript requires.

I strongly recommend that the authors include quantitative measure of the uncertainty related to each of the computations performed in the manuscript. Further, references to quantum computation should be removed from the manuscript. Operations that have performed on so-called 'few qubit states' can be reported as matrix manipulations rather than quantum logic operations.

I, therefore, recommend that the article be resubmitted after these major revisions.

Reviewer #4

(Remarks to the Author)

I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

Reviewer #5

(Remarks to the Author)

I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)

All my concerns have been addressed in the revised paper. It is now recommended for publications in NC.

Reviewer #2

(Remarks to the Author)

I would like to thank the authors for their thoughtful response to the reviewers comments. From reading the revised manuscript, I can see that the authors have taken steps to make the article easier to follow, and to more acceptably place their work within the wider related literature.

I believe the revised manuscript to be suitable for publication.

Reviewer #3

(Remarks to the Author)

We appreciate the revisions made by the authors and now find the manuscript appropriate for publication.

Reviewer #4

(Remarks to the Author)

I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

Reviewer #5

(Remarks to the Author)

I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

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Responses to Reviewers' Comments

Journal: Nature Communications

Manuscript number: NCOMMS-25-62680-T

Article title: Reprogrammable metamaterial robot with embodied versatile computation and mechanical intelligence

Point-by-point responses to Reviewer 1's comments

Reviewer 1

General comment: Mechanical computing, which leverages structural architectures to perform calculations and respond to mechanical stimuli, offers a promising pathway toward realizing embodied computational autonomy and mechanical intelligence in smart, adaptive, and robotic materials. In this work, the authors propose a mechanical metamaterials platform capable of integrating a broad range of computational functions—including analog operations such as differentiation and integration, digital logic operations such as AND, OR, and XOR, as well as qubit-like logics—into robotic locomotion. The study is technically sound and interesting, and I recommend it for publication in Nature Communications. The following comments should be considered for further improving the manuscript:

Responses:

The authors greatly appreciate Reviewer 1 for your positive comments and high recognition of our work. We wish that our work could provide some effective strategies for the applications of elastic wave metamaterials in mechanical computing and robotics in the future!

We also sincerely appreciate Reviewer 1 for your careful reading and helpful suggestions on our work. We have carefully checked our manuscript and substantially improved the expression in the revised version. Our responses to your valuable comments and suggestions are listed below with point-by-point responses. We wish that these revised statements and improvements could better convey the main features of our present work.

Reviewer 1

Comment 1: In this paper, the term “universal” seems to be intended to highlight that mechanical analog, logic, and quantum-inspired computing can be integrated within a unified framework. However, in computational theory, a universal computing machine usually refers to a system that can, in principle, simulate any real-world computer given unlimited time and memory. Since the proposed metamaterial does not possess information storage capabilities, its computational power may not fully align with this definition. Thus, describing its computational capabilities as “universal” is potentially misleading to readers.

Responses:

The authors greatly appreciate your careful reading and insightful comment about the use of “universal”. We quite agree with Reviewer 1 about that in computational theory, the “universal computing” is usually related to the Turing-completeness and ability to simulate any real-world computer in principle, which typically requires additional information storage and retrieval functions.

Our main attention is focused on the ability to perform multifunctional and versatile computing, rather than implying its universality and completeness. Such a robotic system aims to integrate multiple different computational functions within a unified framework, from mechanical analog computing to binary and quantum-inspired logic computing. It can crawl while performing mechanical computing and its locomotion states can be fully represented by computational results. We quite agree with your valuable comments and sincerely appreciate the opportunity to further improve this statement.

Revisions:

In order to present more precise description, we use a more proper word “**versatile**” to replace the term “**universal**” and revised the manuscript throughout. We wish that the statement “versatile mechanical computing” could better express our main idea of integrating multiple computational modes with robotic locomotion. The title, abstract and relevant sections have been modified accordingly and please refer to blue sentences in the revised manuscript. We wish that the revised statements could address most of your concerns.

Reviewer 1

Comment 2: Could the authors provide a more detailed explanation of the claimed parallel computing in the frequency domain? In Fig. 4(f) and 4(g), the input frequencies are 70 Hz and 140 Hz, respectively, yet the corresponding computational outputs are both observed to be 70 Hz (Fig. 4o). If two inputs at different frequencies (70 Hz and 140 Hz) are applied simultaneously, it seems that the outputs would overlap at 70 Hz, making it difficult to distinguish the contributions of different inputs. Does this suggest that the system cannot simultaneously perform calculations at both 70 Hz and 140 Hz? If so, how to understand the parallel computing capability in the frequency domain emphasized in the paper?

Responses:

Thank you very much for your valuable comments on this work with your careful reading of our manuscript. We apologize for not clearly conveying the principle of parallel computing in the frequency domain. We have improved the statements and further explained the relation between parallel/cooperative computing and transformable computing in the following parts:

As mentioned in Fig. 3, the authors have proposed two kinds of computational modes with the $\mathcal{F}_1$ - H - $\mathcal{F}_2^*$ - PLS layout, including the **cooperative computing** in Fig. 3(j) as well as **transformable computing** in Figs. 3(k)–3(m).

(1) Cooperative computing

Firstly, when both fundamental and harmonic waves locate at pass bands, the robot allows propagations of different harmonic components within the same structure, which corresponds to the “**cooperative computing**” or “**parallel computing**”. For this computational mode, all harmonic components are independent and their computational results behave as functions of their harmonic orders r . As a result, the total output fields $g(y, t)$ can be expressed as the superposition of different harmonic results as illustrated in Eq. (6):

$$g(y, t) \propto C_{r1} g_0(y) \cos(\omega_0 t + \varphi_{r1}) + \sum_{r2} C_{r2} d^{nr-n} g_0(y) / dy^{nr-n} \cos(r\omega_0 t + \varphi_{r2}) \\ + \sum_{r3} C_{r3} \int \dots \int g_0(y) \underbrace{dy \dots dy}_{(nr-n)/r} \cos(\omega_0 t / r + \varphi_{r3}). \quad (R.1)$$

We can observe from Eq. (R.1) (i.e., Eq. (6)) that in the frequency domain, each harmonic component has its own computational result and all components operate at the same time. It indicates that for this computational mode, the system allows parallel computing of different harmonic components, which corresponds to the “parallel computing in the frequency domain” in the previous version.

(2) Transformable computing

Due to the band gap property of periodic layers PLs , the mechanical metamaterial allows the filtering of harmonic components and can transform them into different frequency regions. By tuning incident frequencies within different harmonic stop and pass bands, the mixed waves can be directly output as multi-task results in Eq. (R.1) or separated as single-task results in Figs. 3(k)–3(m). The latter is considered as the “**transformable computing**”.

For example, as your valuable comments on Fig. 4, The two inputs with frequencies $f_{e1} = 70$ Hz and $f_{e2} = 140$ Hz are injected into the metamaterial but their corresponding outputs are both 70 Hz. It is because that elastic waves with $f_{e2} = 140$ Hz locate at the stop band of fundamental waves but fall into the pass band of subharmonic waves. Due to the nonlinearity, the inputs with $f_{e2} = 140$ Hz can yield subharmonic components with frequency $f_{sub} = 70$ Hz. Then, the fundamental signal with $f_{e2} = 140$ Hz can be eliminated by periodic layers PLs but the subharmonic one with $f_{sub} = 70$ Hz remains.

It is worth noting that there are some advantages for the “transformable computing”. Different from conventional single-tasking devices with sole response or multi-tasking devices with mixed responses, our system can present multiple independent results according to the incident frequency. On the other hand, it can transform different harmonic inputs into the same frequency while maintaining their computational results. This function provides possibilities of subsequent operations requiring identical frequency condition, e.g., the spatial addition/subtraction operations in Fig. 7. In this case, two input signals f_1 and f_2 with different frequencies (e.g., 70 and 140 Hz) can yields the results $g = d^2f_1/dy^2 \pm df_1/dy$ (70 Hz), which is not possible for the frequency-domain parallel computing. As a result, Fig. 4 is used to illustrates the “transformable computing” instead of presenting the “parallel computing in the frequency domain”.

We quite agree with your comments for not clearly explaining the “transformable computing” and frequency-domain “parallel computing”. In the revised manuscript, we have added additional figure and discussion to further illustrate the frequency-domain “parallel computing” function.

Revisions:

In the previous version, we verified the “transformable computing” in Fig. 4 but discussion about “parallel computing” in frequency domain remains several sentences. In this revision, we add figure and discussion to further illustrate the parallel and cooperative computing functions achieved in the frequency domain. As discussed above, the “cooperative computing” requires that all harmonic components locate at their corresponding pass bands. As a result, we consider the $\mathcal{F}_1$ -H- $\mathcal{F}_2$ layout without periodic layers PLs . This layout can be seen as a degeneracy of the $\mathcal{F}_1$ -H- $\mathcal{F}_2^*$ - PLs layout and is equivalent to the $\mathcal{F}_1$ -H- $\mathcal{F}_2^*$ - PLs layout without stop bands. The

transfer layer H is designed as the 2nd derivative $g_0(y) = d^2f(y)/dy^2$ for 70 Hz waves and can be theoretically transformed to $g_r(y) = \int \dots \int g_0(y) dy \dots dy^{(2r-2)/r}$ for the r -th harmonics.

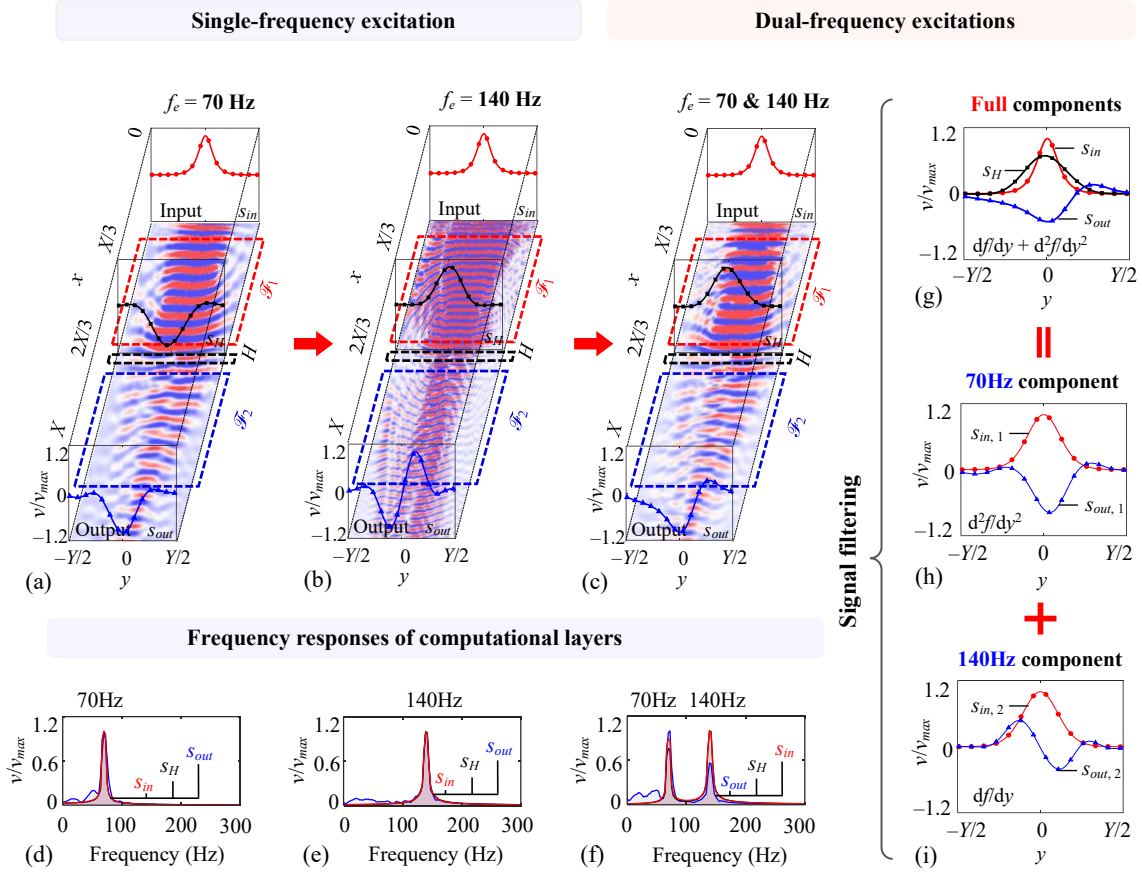


Fig. R1. Parallel and cooperative computing in the frequency domain. (a) and (b) Velocity responses of unit cells under single-frequency excitation with (a) $f_e = 70$ Hz and (b) $f_e = 140$ Hz. (c) Velocity responses of unit cells under dual-frequency excitation with $f_{e1} = 70$ Hz and $f_{e2} = 140$ Hz. (d)–(f) frequency responses of unit cells corresponding to (a)–(c). The red s_{in} , black s_H and blue s_{out} curves represent signals generated by input, transfer function and output layers. (g) Full-frequency input/output responses of (c) under dual-frequency excitation. (h) and (i) Filtered input/output responses of (c) corresponding to (h) 70 Hz and (i) 140 Hz components.

As shown in Fig. R1, the elastic wave metamaterials with single- and dual- frequency excitations are considered to illustrate the parallel and cooperative computing. Firstly, the single-frequency excitation is performed in Figs. R1(a) and R1(b) with different frequencies 70 Hz and 140 Hz. It can be observed from Figs. R1(a) and R1(b) that with the same input profile s_{in} and transfer layer, the outputs s_{out} for 70 Hz and 140 Hz behave as odd and even functions, respectively. This phenomenon agrees with theoretical results of 2nd and 1st derivatives. In addition, the frequency responses in Figs. R1(d) and R1(e) show that for both 70 Hz and 140 Hz excitations, the input s_{in} and output s_{out} almost keep their original frequencies and do not change during wave propagation.

It shows the ability of shifting computing modes while maintaining their frequencies, which are different from the transformable computing in Figs. 4(f) and 4(g).

In order to support the function of simultaneously processing different tasks, Figs. R1(c) and R1(f) presents the spatial and frequency responses of the metamaterial under dual-frequency excitation. We can see from Fig. R1(c) that the output profile s_{out} behaves as the superposition of two harmonic outputs and becomes asymmetric to the midpoint $y = 0$. The frequency responses in Fig. R1(f) show that both 70Hz and 140Hz components exist in the output signals, which agree with the input ones. In order to further verify the parallel property, the combined input s_{in} and output s_{out} profiles in Fig. R1(c) are filtered into two harmonic components $s_{in,1-2}$ and $s_{out,1-2}$ in Figs. R1(g)–R1(i). It is obvious from Figs. R1(h) and R1(i) that the 70 Hz and 140 Hz components can present almost the same computing results as those with single-frequency excitations. These phenomena indicate that the system allow the independent calculation of different frequency components, which show the ability of parallel and cooperative computing in the frequency domain.

We have added the figure and explanations above to Supplementary Information and modified statements about the parallel computing in the revised Manuscript. Please refer to the blue sentences we modified in the second paragraph on page 5, the second paragraph on page 10 in the revised **Manuscript**. And please also refer to Fig. S4 and the blue sentences we added on pages 10 and 11 in revised **Supplementary Information**. We wish that these revised statements and improvements could address most of your concerns.

Reviewer 1

Comment 3: On page 11 (second paragraph), the authors state: “Based on the eigenstate diagram in Fig. 4(e), three incident frequencies with $f_e = 70, 140$ and 300 Hz are selected in Figs. 4(f)–4(h).” However, the third paragraph on the same page lists a different incident-frequency value, which appears to be a clerical inconsistency. Please verify the intended frequencies and correct the manuscript so that the values reported in the text, figure captions, and Fig. 4 itself are consistent.

Responses:

The authors quite agree with Referee 1 and greatly appreciate your careful reading. We apologize for the incorrect labeling of the incident frequencies, which is indeed a clerical inconsistency. We have revised the frequencies in the discussion about Fig. 4 and checked other sentences carefully in the whole work. The modifications are provided as follows.

Revisions:

The incident frequencies in the previous version are revised as 70, 140 and 300 Hz, respectively. Please refer to the blue sentences in the first paragraph on page 12 in the revised manuscript.

Reviewer 1

Comment 4: I agree that this paper provides an interesting perspective on the modulation of acoustic waves by metamaterials within the framework of quantum computing. Nevertheless, a more rigorous discussion of acoustic quantum computing could further enrich the work. For example, Ref. 32 reports the realization of acoustic quantum analogue computing based on acoustic phi-bits, which were argued to satisfy DiVincenzo's five criteria for the physical implementation of a quantum computer (Runge, K., Deymier, P.A., Hasan, M.A., Lata, T.D. & Levine, J. Acoustic metamaterials for realizing a scalable multiple phi-bit unitary transformation, AIP Advances 14, 025010 (2024)). It would be helpful if the authors could clarify the distinction between the acoustic phi-bits and the mechanical qubits presented here, and discuss to what extent the proposed mechanical qubits may be compatible with or fall short of DiVincenzo's criteria.

Responses:

Thank you very much for your positive comments and constructive suggestions on our present work. We quite agree with Reviewer 1 and perform more comprehensive discussion about the mechanical qubit-like computing.

1. Comparisons between acoustic phi-bits and mechanical qubits

Firstly, we sincerely appreciate Reviewer 1 for highlighting the pioneering contributions of Runge et al. [R1] and their group's related investigations on acoustic phi-bit computing [R2–R5]. These studies have extended the applications of acoustic/elastic wave metamaterials to mechanical computing. The authors have followed your kind suggestions and summarized some similarities and differences between the **acoustic phi-bit computing** and **mechanical qubit-like computing** presented here.

(1) Physical forms

According to the studies of Runge, Deymier and Hasan et al. [R1–R5], their group members have developed many effective strategies which can perform acoustic phi-bit unitary transformations analogous to the qubit computing. As Runge et al. previously discussed [R1], the logical phi-bit is “a two-state DOF in the spectral domain of nonlinear acoustic modes”. It is capable of operating in a coherent superposition state with complex amplitude coefficients. Based on the spectral responses in Refs. [R1, R2], the logical phi-bits are individual frequency points in the spectral domain and their logic states are fully characterized by their relative phase relation φ_{21} and φ_{31} . However, in our present work, the mechanical qubit behaves as a pair of wave signals in the spatial domain and their logic states are represented by both wave amplitudes $\{A_1, A_2\}^T$ and phase differences $\Delta\varphi = \varphi_1 - \varphi_2$. Adjacent mechanical qubits are divided through spatially discrete arrays rather than spectral frequency intervals employed in the phi-bit system.

(2) Excitations and measurements

As reported by Runge et al. [R1], the acoustic phi-bits which are represented by individual harmonic signals in the spectral domain are generated by a couple of excitation frequencies f_1 and f_2 in different nonlinear waveguides. Then, the detected outputs are the Fourier sum of acoustic modes with f_1 and f_2 and their harmonic components. Their logic states are measured by detecting the phase differences of two nonlinear waveguides relative to the fundamental one. Different from phi-bits, our mechanical qubit originates from a couple of parallel traveling waves with identical excitation frequency. By measuring the amplitudes and relative phases of two traveling waves, the logic states can be determined.

(3) Computational principles

As mentioned in Ref. [R1], the acoustic phi-bits are correlated via the nonlinear elasticity of their physical system. Accordingly, the coherent superposition of logical phi-bit states can be realized by modulating physical parameters such as the frequency, relative phase or intensity of driving forces. For instance, in Ref. [R1], phase exchange and the associated several unitary transformations can be achieved by changing the driving parameters e.g., frequency and phase relation. However, in our present work, the computations are achieved by the wave interaction of two incident signals within the architected structure, such as superposition, interference, diffraction, as well as wave focusing, reflection and division. More complex operations can be achieved by the serial and parallel designs in the spatial domain.

It is worth mentioning that both computational methods have their own advantages. **The acoustic phi-bits** are distributed in several small intervals in the spectral domain, which have special advantages of processing large information and multitasking. For multi-bit computations, it does not require decomposition into smaller-scale bit gates. **Our present designs** allow versatile serial and parallel modes in the spatial domain and are compatible with previously mentioned analog and digital logic operations. In particular, the computational results can be directedly output as waveform signals or robot locomotion states without additional Fourier transformations.

2. DiVincenzo's five criteria

The authors are also grateful for your kind suggestions of discussing what extent the proposed mechanical qubits may be compatible with the DiVincenzo's criteria. It would help readers to have a more thorough understanding of the mechanical qubit-like computing presented here. We have followed your valuable suggestions and summarized as follows:

(1) A scalable physical system with well-characterized qubit

As discussed above in the “**Physical forms**” section on page 8, our proposed mechanical qubits are characterized as a pair of traveling wave signals in the spatial domain and their logic states can be represented by both the wave amplitudes $\{A_1, A_2\}^T$ and phase differences

$\Delta\varphi = \varphi_1 - \varphi_2$. As the exciting waveforms and phases change, the system can present different qubit-like states as the superposition of two basis states $|0\rangle = \{1, 0\}^T$ and $|1\rangle = \{0, 1\}^T$, i.e., $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$. The logic ‘0’ and ‘1’ states in $|0\rangle$ and $|1\rangle$ correspond to the existence and nonexistence of traveling signals. And α and β are complex parameters related to $\{A_1, A_2\}^T$ and $\varphi_0 - \varphi_1$. Since the metamaterial can be designed with multiple waveguides for traveling wave propagating, the physical system can be also scaled by the serial and parallel strategies of fundamental qubit-like gates in the spatial domain.

(2) The ability to initialize the state of the qubits to a simple fiducial state

The ability to initialize the state of the qubits to a simple fiducial state requires all qubits to be a known, pure initial state before computations. As discussed above, the fiducial states $|0\rangle = \{1, 0\}^T$ and $|1\rangle = \{0, 1\}^T$ in our system are characterized by two single-mode excitations, i.e., $(A_1 = 1, A_2 = 0)$ and $(A_1 = 0, A_2 = 1)$, respectively. For example, for single-qubit gates like Hadamard and Pauli X gates, the fiducial state $|0\rangle$ can be initialized by setting the wave amplitude $\{A_1, A_2\}^T$ as $\{1, 0\}^T$, which corresponds to wave propagations only along the upper half-space.

(3) Long relevant decoherence times

Similar to the acoustic phi-bits discussed in Ref. [R1], the coherent superpositions of phi-bit states are acoustic complex amplitudes. These states are not subjected to decoherence, which are different from the probability amplitude of quantum systems composed of qubits. In our present work, the physical forms for achieving mechanical qubit-like computing are elastic waves with deterministic amplitude and phase information. Before computation completed, the coherent superpositions of mechanical qubits can be sustained during wave propagation, which exhibits long relevant decoherence times.

(4) A ‘universal’ set of quantum gates

Qubit-like gates: Our proposed qubit-like computing is achieved by the interactions of traveling wave pairs, such as interference, diffraction, superposition, as well as wave focusing, reflection and division. As previously verified in Figs. 5 and S5, multiple fundamental and combinational qubit-like operations are achieved, including the single-qubit Pauli X, Pauli iY and Hadamard gates, two-qubit CNOT gate, as well as more complex serial and parallel operations. Among these designs, the system behaves as a mechanical bit processor that can perform different qubit-like unitary operations.

Universal requirements: On the other hand, the ‘universal’ conditions require that the system can theoretically perform arbitrary unitary operations. As reported by Runge et al. [R1], largescale multiple-qubit matrix operations can be achieved by their decompositions of multiple single- and two- qubit ones. It requires that our system accommodate a wide range of states available on the Bloch sphere.

In quantum computing theory, an arbitrary qubit state can be expressed as

$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi} \sin(\theta/2)|1\rangle, \quad (\text{R.2})$$

where θ and ϕ are Bloch sphere coordinates covering the entire sphere. As mentioned above, our presented qubit-like states can be characterized by the normalized wave amplitudes $\{A_1^*, A_2^*\}^T$ and phases $\{\varphi_1, \varphi_2\}^T$ of the traveling wave pair. Considering the complex amplitudes $A_1^* \exp(i\varphi_1)$ and $A_2^* \exp(i\varphi_2)$ of the wave pair, we can obtain the equivalent qubit-like state as

$$|\psi\rangle = A_1^* e^{i\varphi_1} |0\rangle + A_2^* e^{i\varphi_2} |1\rangle, \quad (\text{R.3})$$

where A_1^* and A_2^* satisfy $A_1^{*2} + A_2^{*2} = 1$. Setting φ_1 as the fiducial phase, the phase relation is actually determined by the relative phase $\Delta\varphi = \varphi_2 - \varphi_1$. We neglect the overall phase factor $\exp(i\varphi_1)$ and the qubit-like state can be written as $|\psi\rangle = A_1^* |0\rangle + A_2^* \exp(i\Delta\varphi) |1\rangle$. We assume that $\alpha = A_1^*$ and $\beta = A_2^* \exp(i\Delta\varphi)$, which results in the qubit-like state as

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle, \quad (\text{R.4})$$

where α and β satisfy $|\alpha|^2 + |\beta|^2 = 1$ and are equivalent to the Bloch sphere representation as $\alpha = \cos(\theta/2)$ and $\beta = \exp(i\phi) \sin(\theta/2)$. By tuning the wave amplitudes to satisfy $A_1^{*2} + A_2^{*2} = 1$ and adjusting relative phase $\Delta\varphi = \varphi_2 - \varphi_1$, the system can accommodate a wide range of states on the Bloch sphere.

During the computations, the wave splitter in Fig. 5(i) can adjust the wave amplitude ratio A_1/A_2 , which corresponds to α and β varying in the real number range. The relative phase $\Delta\varphi = \varphi_2 - \varphi_1$ can be adjusting by tuning the mass distribution m_H of transfer function layers to yield phase differences, e.g., π for Hadamard gate in Fig. 5(e), which corresponds to β varying in the imaginary range. Their combinations as well as spatial serial/parallel strategies provide possibilities of more complex unitary operations.

(5) A qubit-specific measurement capability

As mentioned in Ref. [R1], measurements in conventional quantum computing approaches entail the collapse of the wavefunction. In contrast, measurements in our present mechanical qubit-like systems do not alter the wave function, which persists as long as the drivers are applied. In particular, the computational results can be directedly output as waveform signals and robot motion states without additional Fourier transformations and filtering devices.

Revisions:

Following the Reviewer 1's valuable suggestions, we have added discussion about the mechanical qubit-like computing and compared with the well-established quantum computing and acoustic phi-bit computing. The response letter also discusses what extent this approach can satisfy the DiVincenzo's five criteria for quantum computing. Limited by writing space, explicit discussion is

listed in the Supplementary Information. Please refer to the blue sentences we added on pages 13 and 14 in Supplementary Information and Refs. [32–34] on page 26 in the revised Manuscript. We wish that these revised statements and improvements could address most of your concerns.

Once again, we sincerely appreciate your thorough review and valuable feedback that helps us to comprehensively improve our manuscript!

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Point-by-point responses to Reviewer 2's comments

Reviewer 2

Comment 1: *What are the noteworthy results?*

A mixed signal architecture that integrates analog and logic operations, allowing for nested and hierarchical operations. The architecture is also shown to be able to switch between multiple different modes providing multi-task ability. The implementation is based on electric wave metamaterials. The extensive experimentation demonstrates the various computing methodologies and implements a simple crawler robot.

Responses:

Thank you very much for your thorough reading and comprehensive summary of our present work. We are sincerely grateful for your recognition of the potential significance of this study.

As noted in your comments, the main idea of our present work is to develop a versatile mechanical architecture of metamaterial robots which can integrate multiple analog and logic operations with robot motion. This hybrid approach enables the nested and hierarchical operations within a single robotic platform, allowing compatible and highly flexible designs. Moreover, the ability to reconfigure the system to different computational modes endows the metamaterial robot with parallel and multitasking functions, which is particularly promising for adaptive robot applications. In addition, a proof-of-concept robot is designed to further verifies the feasibility of applying such computational designs to embodied robotic systems.

We wish that these clarifications could better convey the main idea of our work and align with your understanding. Thank you again for your valuable feedback!

Reviewer 2

Comment 2: *Will the work be of significance to the field and related fields? How does it compare to the established literature? If the work is not original, please provide relevant references.*

2.1. The work is similar to a broad swathe of literature in mechanical computing, which has seen a notable amount of popularity over the past few years. The core claim of mixed signal and multi-task has previously been shown to some extent in, e.g., <https://www.pnas.org/doi/10.1073/pnas.2407431121>. There is also a string of uncited works covering e.g., ‘ghost circuits’, etc. There’s a lot of work on the embodied side e.g. <https://arxiv.org/abs/2110.15245>, especially Helmut Hauser’s work e.g. <https://link.springer.com/article/10.1007/s00422-012-0471-0>. The work is therefore partially original. It would likely be of interest to specialists in the field. It is too dense and specialized to be understandable by many non-specialists.

Responses:

Thank you very much for your valuable comments on the mechanical computing and robotic implementation presented in our work. We appreciate your careful reading and would like to address your concerns in detail.

2.1. Relationship to recent advances in mechanical computing

Firstly, we sincerely appreciate Reviewer 2 for highlighting the pioneering contributions of Mousa et al. (*PNAS* 2024) [R1], Nanayakkara et al. (*Springer Nature* 2024) [R2], Roy et al. (*arXiv* 2021) [R3] and Hauser et al. (*Biological Cybernetics* 2021) [R4], in multi-tasking processing, ghost circuits, as well as embodied computing fields, respectively. These papers have significantly extended the applications of mechanical computing in embodied intelligence and robotics. The authors have summarized some similarities and differences between their and our present works.

A. Parallel and multi-tasking processing (Mousa et al., *PNAS* 2024)

As reported by Mousa et al. [R1], they have developed a time-modulated acoustic metasurface that allows parallel and multitasking computing in the spectral domain. Due to the nonlinear characteristics, the incident signals generate several individual harmonic components that calculate independently. Then, **the received outputs are expressed by the Fourier sum of acoustic modes and their computational results are measured by filtering specific output frequencies.** In their work, the relation between computational functions and frequencies are established by designing the transmission responses of acoustic Helmholtz cavity at specific frequencies.

We agree with Review 2’s valuable comments about that the work mentioned in *PNAS* is an interesting study on mixed and multi-tasking computing. However, there are some obvious differences between our and their works, which include the **working principles, functions**

and architectures, application scenarios, as well as our core focus.

(1) Differences between PNAS parallel computing and our transformable computing

The main focus of Mousa et al. [R1] is to validate the capability of acoustic metasurfaces to perform tasks parallelly in the frequency domain. However, in Figs. 3(k)–3(m) and 4, our system introduces the concept of “transformable computing” instead of previously mentioned “parallel computing”. With the nonlinear band gaps of periodic layers PLs , the system can present filtering, tuning and transformable functions of different harmonic signals. Such an architecture not only enables “parallel computing” (pass band properties) in Fig. S4, but also allows wave filtering and transformation of different harmonic inputs into a same frequency (stop band properties) in Fig. 4. This function allows algebraic superposition of different computational results.

For the conventional “parallel computing”, computational results are encoded across distinct harmonic components. Since these harmonics are physically decoupled, the signals cannot be directly output through elementary arithmetic operations. Conventional parallel strategies in frequency domain usually require additional Fourier transforms and digital filtering devices. **However, the “transformable computing” can integrate the multi-frequency information into a single-frequency mode while maintaining computational results. Such a property provides platforms for subsequent operations requiring identical frequency condition, e.g., spatial addition/subtraction operations.**

In addition, as mentioned in Comment 2 of Reviewer 1, conventional “parallel computing” can be considered as a special case of “transformable computing” in our system. Such a computing mode can be directly realized through the $\mathcal{F}_1$ - H - $\mathcal{F}_2$ layout without periodic layers PLs , which is a degeneracy of the $\mathcal{F}_1$ - H - $\mathcal{F}_2^*$ - PLs layout for “transformable computing”. As a result, this work extends Mousa et al.’s work [R1] from frequency-domain parallel computing to a hybrid cooperative and transformable computing mode.

(2) Differences in core focus and system architecture

The paper of Mousa et al. [R1] and our work are not only different in functionality but also in system architecture. The acoustic metasurface of Mousa et al. exhibit as a “**specialized analog processor**”, which only focuses on the parallel function in the frequency domain. In contrast, our system aims to develop a “**versatile computing architecture**” that can integrate multiple computational modes within a single platform, including the following key aspects:

(a) Versatility in computing modes. Our work can support spatial analog operations such as integration and differentiation, binary logic operations including AND, OR and XOR gates, as well as qubit-like operations like Hadamard, Pauli-X and CNOT gates. **(b) Cross-domain compatibility.** The proposed platform allows serial, parallel and nested operations in different computational domains such as addition, subtraction and multiplication operations in **spatial**

domain, parallel and transformable computing in **frequency domain**, as well as multilevel and combinational logics in **bitwise domain**. **(c) Transformability**. As discussed above, the "transformable computing" serves as a bridge between spatial and frequency domains, which can transform multi-frequency information into a single frequency. Such function provides platforms for follow-up spatial serial/parallel operations.

As a result, the compatibility and flexibility suggest that our framework provides a hybrid and versatile computational platform instead a task-specific tool.

(3) Differences in mechanism and engineering application

The physical mechanisms are different: The acoustic metasurface of Mousa et al. [R1] is realized by acoustic waves in fluid media and its computational function is achieved by transmission responses of **acoustic Helmholtz cavity**. Our present work is based on the propagation of elastic waves in solid media with **periodic spring-mass structures**, in which signals are manipulated by tuning the mass, stiffness and damping coefficients. Different from acoustic systems, elastic wave metamaterials are more robust to large deformation and allow large-amplitude vibrations, which align with the working conditions of robotic systems. As a result, this work shows the advantages of elastic wave metamaterials in robot designs and introduces the mechanical computing to robot crawling.

(4) Realizations of robot crawling and embodied computing in our work

A key attempt of our work is to achieve the metamaterial computing from static and table-top demonstrations to dynamic crawling robots. Previous studies about metamaterial mathematical computing have reported many theoretical approaches from analog to logic operations (e.g., Science 343, 160–163, 2014 [R5]; Science 363, 1333–1338, 2019 [R6]). However, most of them are mainly based on fixed and stationary platforms which process signals without interactions of physical motion. The integration of robotic locomotion with mathematical computing and especially with mechanical metamaterials, provides a new strategy for embedding mechanic intelligence into robotics.

Different from Mousa et al.'s paper [R1], this work not only focuses on the computing functions but also consider how to integrate them at the system level: **(a) Firstly, this work establishes a closer relation between computation and locomotion**. Mousa et al.'s work [R1] is more focused on isolated analog computing without actuation or behavioral output. However, our robot can crawl while performing mechanical computing and its locomotion states can be fully represented by the computational results. **(b) Secondly, our approach is supported by analytical derivations, numerical simulations, as well as experiments**, which provides both theoretical and engineering feasibilities. **However, Mousa et al.'s [R1] paper is mainly verified by numerical simulations. As a result, we have further extended the theoretical work to experimental validation and practical application.**

B. Ghost circuits (Nanayakkara et al., *Springer Nature* 2024)

The discussion above has illustrated the differences between our and previous works about parallel and multi-tasking processing. Next, we will focus on the following two concepts: “ghost circuits” and “embodied computing” [R2–R4].

As reported by Nanayakkara et al. [R2], the “ghost circuit” describes a form of distributed dynamic network which originates from dynamic coupling between a physical body and its environment. Its function is neither pre-programmed nor governed by a central controller. It is generated from the real-time interactions among the physical body, control system and neighboring environment. As a result, the “ghost circuit” is a system-environment interactive system in specific conditions, and it will emerge and dissipate when the interaction changes. As mentioned by Nanayakkara et al. [R2], **its function is analogous to a dead fish that can sway with water vortex, which forms an interactive system**. When the vortex disappears, the fish will no longer move [R2].

Table R1. Comparisons with ghost circuits

Feature	Ghost Circuits [R2]	Our Metamaterial Robot
System architecture	Systematic and distributed dynamic network	Structural and integrated physical body
Formation	Coupled with specific environmental conditions	Active pre-design with programmability and multifunctionality
Core idea	System-environment coupling	Computational-structural integration
System characteristics	Temporariness, low predictability, high environmental perception	Sustainability, high predictability, high computational compatibility
Computational mode	Single mode: Analog operation in coupled physical fields	Hybrid mode: Analog, digital logic and mechanical matrix operations
Relationship with environments	Dependent on specific environments (e.g., turbulence, spider webs)	Self-contained and adaptive to different environments
Analogy function	A dead fish swaying with flows (leveraged by water vortex)	A programmable computer (processing tasks by structured hardware)
System level	Interactive systems: Focusing on how subsystems couple and interact with each other	Integrated system: Integrating multiple subsystems into a unified and cooperative system

In our work, the robot is designed as a computational-structural integrated system that can integrate multiple subfunctions into a unified physical body. Such a system employs a reprogrammable architecture, in which different sub cells cooperate to generate **deterministic mathematical results**. Before computations are completed, these results can be sustained and

do not change by environments, exhibiting long relevant decoherence times. It does not mean that our system is fully isolated, and we can also find the interaction between the robot and environment. One example is the ability of using differential operations to achieve target tracking in Fig. 6. The “ghost circuits” focuses on the dynamical analogue of coupled physical fields. But our system aims to provide a hybrid computational mode of analog, digital logic and matrix operations, as well as their transformations. **Different from the environmental interaction of “ghost circuits”, the concept of “embodied computing” [R3] in this work focuses on the system itself serving as a signal processor to regulate its own locomotion.**

In order to better compare the “ghost circuits” and our metamaterial robot, we have listed their main features in Table R1 to further show the differences.

C. **Embodied morphological computing** (*arXiv, Biological Cybernetics* 2011)

Similar to the “ghost circuits” mentioned above, the “morphological computing” [R4], as a kind of “embodied computing” [R3], has also served as a novel strategy for integrating embodied intelligence into mechanical systems. In Ref. [R4], Hauser et al. developed two mechanical filters which use “morphological computing” to simulate arbitrary nonlinear and time-invariant dynamic systems. These systems are composed of two parts: **(a) Physical body:** A mechanical component includes mass-spring array/network, which is used to provide time integration and memory, as well as nonlinear mapping functions. **(b) Digital algorithm:** A subsequent linear regressor or digital neural network is used to process and iterate outputs, which requires additional digital training algorithm.

We quite agree with Reviewer 2 that the “morphological computing” is an interesting work which combines the mechanical system with digital computing. However, after carefully reading of Hauser et al.’s studies, we can find many differences between our work and their paper from **operating mechanism, computational mode and analogy function to their engineering applications.**

(1) Differences in operating mechanisms

In Hauser et al.’s work, computations are achieved through the cooperation of mechanical mass-spring network and digital regressor/neural network. The mechanical network act as a **dynamic feature extractor/filter**, and the digital regressor processes and outputs signals after training. The function of the mechanical mass-spring network is similar to the convolutional layers of neural networks which can extract information from complex inputs. In contrast, our robot is a **computational-structural integrated system**, in which computations can be performed by the robot itself without additional training and iteration. Its function is similar to a movable mathematical calculator which can process tasks by the inner structured hardware.

(2) Differences in computational modes

In Hauser et al.’s work, the inputs to the mechanical mass-spring network are **time-domain**

vibration signals. The mechanical mass-spring network functions as a time-domain analog operator, in which dynamic responses are described by the **ordinary differential equations** (i.e. vibration equations). Then, signals after the mass-spring network behave as implicit results which require additional training and iteration to generate target results. In our work, the inputs to the metamaterial robot are **spatial-domain waveform signals** $f(y)$, which is obviously different from the **time-domain vibration signals in vibration system**. When waves propagate through different processing components in our work, dynamic behaviors can be described by the **partial differential equations** (i.e. wave equations). Our system can support three computational modes, including the spatial-domain analog computing, binary logic operations, as well as qubit-like logic operations. After calculations, it can generate explicit waveform output that directly drive and regulate the robot motion.

(3) Differences in engineering applications

As described by Hauser et al., they have demonstrated how mechanical mass-spring systems can be employed to emulate arbitrary, time-invariant and nonlinear filters with fading memory. Such system has theoretically verified the ability of morphological computing to simplify calculating tasks. Their design is more suitable to applications requiring a fixed-parameter processor for dynamic control, in which morphological computing can effectively reduce the computational complexity. As a result, their paper provides an effective method to design **specialized and dynamic feature extractors**.

Table R2. Comparisons with morphological computing

Feature	Morphological Computing [R4]	Our Metamaterial Robot
System architecture	Dynamic feature extractor /filter	Computational-structural integrated robot
Computational mode	Time-domain analog operations	Hybrid mode: Spatial-domain analog, binary logic and qubit-like operations
Computational result	Implicit results that require training and iterations	Explicit results that can directly drive and modulate locomotion
Mechanical model	Solving ordinary differential equations (vibration equation, ODE)	Solving partial differential equations (wave equation, PDE)
System parameters	Random and unchangeable: mass/stiffness is unchangeable	Deterministic and reprogrammable: mass/stiffness varies as function changes
System completeness	Partially complete: Requiring subsequent digital regressors	Complete: the computation and robotic locomotion are integrated
Analogy function	Neural network (feature extracting, temporal relevance and nonlinearity)	Mathematical calculator (processing tasks by structured hardware)

However, our robot is a computational-structural integrated robot that can perform multiple mathematical functions: including spatial analog operations (differential/integral), binary logics (AND, OR, XOR), as well as mechanical qubit-like operations (Hadamard, Pauli X, CNOT gates). Such a system can support hybrid computational modes with reprogrammable functions, which is more suitable to create a **versatile and multitasking robotic device**.

We have listed their main features in Table R2 to further show these differences. We wish that these statements and comparisons could address most of your concerns.

2.2. One weakness with this work is that the robotic implementation is simple – this is fine as a proof of concept but really shows the inability to turn this technology into something useful. Despite claiming to present a universal architecture, it is unclear why this is an important problem and the justification for using their implementation is weak/underdeveloped.

2.2 Robotic implementation and experimental verification

The authors also appreciate Reviewer 2 for your concerns about the validity and justification for the robotic implementation. Following your valuable comments, we have improved the statements about robotic implementation and added more explanations about the applications of our computational architecture. We wish that these statements and improvements could better clarify the significance of our work.

(1) The “simple” proof of robotic implementation

The authors acknowledge that our demonstrated robotic model is simple in experimental verification. However, as explained in **Comment 1**, the main idea of this work is to design a simple and effective framework that can perform diverse computing functions with robotic locomotion. The prototype is used to verify whether the elastic wave metamaterials can be effectively integrated with robotic motion, instead of testing its functional complexity. **During the experiments, such a simple framework can show its advantage in visualizing outputs. It can present a clear and explicit relation between the computational result and robotic locomotion.** As a result, the simple and direct experimental demonstration is a more robust method to verify the basic computing principles.

In addition, it is worth mentioning that our robot is a highly scalable platform that is not limited to our present function. (a) For example, our robot can be applied to emulate the parallel computing which is achieved by acoustic Helmholtz cavity (Mousa et al. *PNAS* 2024, [R1]). Because their function is only verified by simulations without experiment, our robot can extend their work to engineering applications with robotic crawling. (b) **Similarly, both the “ghost circuits” [R2] and morphological computing [R4] have verified that such simple mass-spring frameworks can be used for complex computational behaviors.**

(2) Significance and justification of universal architecture

Recently, a lot of attention has been focused on the multifunctional computing which can integrate multiple computational modes into a single platform [R7–R10]. These designs allow hardware to dynamically adapt the target function, which changes from specialized to general computing. Similar conclusion is highlighted by Mousa et al. [R1] as “**while there is added complexity to enable the multiplexing, the extra cost, space and error rate of multiple single-task components has exponentially exceeded the complexity brought upon by multitasking**”.

For instance, a recent work by Zhao et al. (*Nature* 640, 931–940) [R7] employs modular chiral metamaterials to design multiple deformable devices with multimodal, multi-stable and reconfigurable functions. In their work, some simple prototypes of metamaterials are used to demonstrate a variety of complex robotic and computing functions. Their applications range from robotic morphogens, thermal regulation, mechanical memory to information encryption and energy absorption. Similar strategies have also been reported by El Helou et al. [R9] (*Nature* 608, 699–703), Mei et al. [R8] (*Nature Communications* 12, 7234) and Hu et al. [R10] (*Nature Communications* 14, 6709), etc. **In their papers, the universal architectures of computing metamaterials are introduced, which are extended to various programmable mechanical logics and verified by simple prototypes.**

In our work, the mechanical spatial analog, binary logic and qubit-like operations are not isolated functions. They can cooperate and transform through serial, parallel, nested, as well as transformable computing. Such advantages have not been reported by conventional parallel computing and references about analog computing [R1, R5, R6]. In particular, **the capability to integrate a wide range of computing functions into a single robotic system is also a key indicator for assessing its versatility and intelligence level.** For example, based on the parallel addition/subtraction operations, the universal architecture can emulate digital logic signals to achieve the binary logic AND, OR and XOR operations. Furthermore, our system can also emulate the acoustic “parallel computing” (Mousa et al. *PNAS* 2024, [R1]) by the universal architecture. Based on the serial modular design, the “parallel computing” can be further extended to “transformable computing”, which is also a justification of the universal architecture.

Finally, we would like to further convey our main ideas. This work aims to provide a versatile computation framework which not limited to our present designs. Our robot implementation is used to help readers to better understand the integrated multiple functions embedded in a single system. In our future work, we wish to design more intelligent robotic devices with integrated, adaptive and multitasking capabilities.

References:

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- [R10] Hu, X.Y., Tan, T., Wang, B.L., Yan, Z.M., 2023. A reprogrammable mechanical metamaterial with origami functional-group transformation and ring reconfiguration. *Nature Communications* 14, 6709.

Reviewer 2

Comment 3: *Does the work support the conclusions and claims, or is additional evidence needed?*

The work would benefit from a degree of ‘hand holding’ – e.g. for each experiment, what are you showing, why are you showing it, etc.? The paper needs to be navigable.

Responses:

The authors greatly appreciate your careful reading and valuable comments, which help us to improve our work. We quite agree with Reviewer 2 that more explicit guidance of experiments would help readers to better understand this work. Following your suggestion, we have added more introduction and explanation to each experiment and carefully revised the statements in the whole manuscript. We wish that these revised statements and improvements could better address most of your concerns.

Revisions:

The authors have improved descriptions about all experiments in our work, which includes the spatial analog computing in **Fig. 2**, transformable computing in **Fig. 4**, robotic crawling with embodied computing in **Fig. 6**, spatial serial and parallel operations in **Fig. 7**, multifunctional performance during locomotion in **Fig. 8**, combinational analog computing in **Fig. S2**, binary logic operations in **Fig. S3**, mechanical matrix (qubit-like) computing in **Fig. S5**, robotic crawling with band gap properties in **Fig. S6**, crawling velocity for different exciting frequencies in **Fig. S8**, multi-gait crawling and turning behaviors in **Figs. S9**, as well as effects of wave amplitudes and mass ratios on the robot crawling velocity in **Fig. S10**.

Please refers to the captions and sentences we marked in the **Manuscript**: the caption of Fig. 2 on page 7, the blue sentences for Fig. 4 in the first paragraph on page 12, the blue sentences for Fig. 6 in the last paragraph on page 16, the blue sentences for Fig. 6 in the second paragraph on page 18, the blue sentences for Fig. 7 in the first paragraph on page 20, the blue sentences for Fig. 8 in the last paragraph on page 20.

Please also refers to the captions and sentences we modified in the **Supplementary Information**: the caption of Fig. S2 on page 7, the caption of Fig. S3 on page 9, the caption of Fig. S5 on page 12, the blue sentences for Fig. S8 in the last paragraph on page 19, as well as the blue sentences for Figs. S9 and S10 on page 26.

Reviewer 2

Comment 4: *Are there any flaws in the data analysis, interpretation and conclusions? Do these prohibit publication or require revision?*

Results seem sound.

Responses:

The authors greatly appreciate Reviewer 2 for your thorough reading and positive feedback on our present work.

Following the revisions and improvements to previous comments, we have further clarified the correlations between the theoretical results and experimental validations in the revised manuscript. The data analysis, interpretation and conclusions are rechecked to ensure that they are consistent and well supported by the evidence. We wish that the revised manuscript could better convey the main features of our work!

Reviewer 2

Comment 5: *Is the methodology sound? Does the work meet the expected standards in your field?*

There are large numbers of equations without much guidance between them. The figures are average quality. In general this paper was very hard to read and understand. There is a lot of quantum theory interspersed into large walls of text. It is therefore a huge effort, requiring a background in quantum, to see if the methodology is sound, and if the work is reproducible.

Responses:

We sincerely appreciate Reviewer 2 for your insightful comments about the formula derivations and concept introductions. We quite agree with your valuable suggestions and carefully revise each of these points. Your feedback has helped us substantially improve our work and further clarify the logical structure of the manuscript. We wish that the revised manuscript could better convey the main features of our work.

Revisions:

The following illustrates the key changes we have modified in response to your feedback:

(1) **Firstly, we have deleted a large number of repeated and complex derivations which may make readers confused. Some key steps and explanatory notes are retained.**

(a) Some derivations about spatial analog computing in previous Supplementary Note 2 have been deleted. Then, only computational principles of fundamental analog operations, serial and parallel operations are retained and integrated into the revised Supplementary Note 1.

(b) The nonlinear dynamic equations and their related vectors and matrices in previous Supplementary Note 4 have been simplified. The remaining equations have been integrated into Supplementary Notes 3 and 5.

(c) Derivations of nonlinear harmonic band structures in Supplementary Note 3 and robot crawling velocity in Supplementary Note 5 have been simplified and integrated with the nonlinear dynamic equations mentioned above.

After the simplification, the equation number has been reduced from **55** to **30** and the chapter number of Supplementary Information has been reduced from **8** to **6**. The numbers of figures and pages have been reduced from **12** to **10** and **34** to **29**, respectively. The revised equations are integrated into the following three parts: (i) **computational principles**, (ii) **harmonic band structures** and (iii) **robot crawling velocity**. These equations have been collected into Supplementary Notes **1**, **3** and **5** with their corresponding theoretical and experimental results in Supplementary Notes **2**, **4** and **6**. The revised equation number of each part is about **10** within **3** pages to improve readability.

- (2) **We have also improved the expressions of theoretical derivations and discussions. More explanations are introduced to adjacent equations and sections, as well as equations are provided with clearer guidance and supplementary references.**

(a) **Manuscript:** Please refer to Eqs. (1) and (2) with the corresponding blue sentences on pages 5 and 6; Eqs. (4) and (5) with the corresponding blue sentences on page 8; the blue sentences to explain Eq. (6) on page 10; as well as Eqs. (7)–(10) we added to explain the Hadamard and CNOT operations on pages 14–16 in the revised Manuscript.

(b) **Supplementary Information:** Please also refer to the blue sentences before Eqs. (S1) and (S3) on page 4; the blue sentences before Eq. (S5) on page 5 and before Eq. (S8) on page 6; the blue sentences before Eqs. (S11), (S17) and (S21) on pages 15–17; the blue sentences before Eqs. (S23), (S24), (S26), (S27) and (S29) on pages 22 and 23 in the revised Supplementary Information.

- (3) **Furthermore, we have also thoroughly modified the discussion about the mechanical qubit-like computing and added more explanations about the principles for achieving multi-bit matrix operations.**

(a) We have removed the abstract concepts related to quantum theory and introduced our ideas from the classical matrix operations. The principle for encoding spatial analog signals into the mechanical multi-bit states has been further discussed in the revised manuscript.

(b) In order to avoid unfamiliar concepts of qubit-like states $|0\rangle$ and $|1\rangle$, we have provided their states $\{\alpha, \beta\} = \alpha|0\rangle + \beta|1\rangle$ with classical wave representations $\alpha = A_1 \exp(i\varphi_1)$ and $\beta = A_2 \exp(i\varphi_2)$. The proposed qubit-like matrix operations are also added with equations to explain how the logical mappings perform.

(c) Furthermore, we have also presented the comparisons between the mechanical qubit-like matrix operations and acoustic phi-bit computing in Supplementary Note 2 - Part E in the revised Supplementary Information.

Please refer to the blue sentences about mechanical matrix operations from pages 12 to 16 and Eqs. (7)–(10) in the revised **Manuscript**. And please also refer to the blue sentences from pages 13 and 14 in the **Supplementary Information** to see the comparisons between our present work and conventional acoustic computing.

Reviewer 2

Comment 6: Concerns

1. Unclear how this is novel compared to some of the recent literature that appears to do the same thing.
2. Although I've been told not to comment overly on the language, use of language in this manuscript is of a poor standard in the introduction, and can lead to confusion. I frequently found myself becoming lost, or having to re-read a passage multiple times to try and decode what the authors are trying to say. I highly recommend the authors use a professional editing service. Many of the sentences are not legal sentences, or incredibly unclear, e.g.
 - (1) "The flexible designs from bioinspired materials to molecular crystalline structures endow metamaterials with extraordinary mechanical properties, such as acoustic cloaking, negative refractive and topological insulators [5–8]."
 - (2) "In comparison, novel progress in metamaterials has developed wave-based analog computing [26–30], which enables continuous processing of complex information in spatial."
 - (3) Fig 1 the caption does not explain the figure, which is very dense and needs much clearer explanation.

Responses:

The authors great appreciate Reviewer 2 for your comprehensive reading and valuable comments about the novelty and English expression. We quite agree with you about that the language used in the introduction is proper. We have carefully checked our manuscript and substantially improved the English expression in the revised version. Furthermore, we have added comparisons between our work and recent studies about mechanical computing. The caption of Fig. 1 is modified with clear explanation. We wish the revised manuscript could better convey the main features of our work.

1. Comparisons between recent literatures

In the previous responses to **Comment 2**, we have illustrated detailed comparisons between recent literatures about mechanical computing and our present work. The similarities and differences between our and their works are listed with point-to-point responses.

In response to Mousa et al.'s work (*PNAS* 2024) [R1] about parallel computing, we have compared our differences in working principles, engineering applications and core computing functions. The parallel computing function reported by Mousa et al. can be considered as a special case of our proposed "transformable computing". The ability to integrate waves of different frequencies into a single frequency and maintain their computational results has not been reported in their work. Different from conventional single-tasking devices with sole

response or multi-tasking devices with mixed responses, our system can present multiple independent results according to the incident frequency without additional filtering devices. On the other hand, this work not only focuses on the basic analog functions, but also considers binary logic and matrix operations with robotic motion. Mousa et al.'s paper is more focused on the isolated analog computing without actuation and behavioral output. However, our robot can crawl while performing mechanical computing, and its locomotion states can be represented by the computational results. In addition, our experiments extend their purely theoretical verification of acoustic metasurfaces to the practical applications in elastic wave robotics.

Furthermore, the authors also compare our work with studies of Nanayakkara et al. [R2], Roy et al. [R3] and Hauser et al. [R4] about ghost circuits, embodied and morphing computing. We have explained our differences in engineering applications, computational functions and the core principles. The “ghost circuits” emphasize on the interaction between the individual and the environment as a whole system. However, our work mainly focuses on the individual itself which can present different computational functions with integrated subsystems. As for the “morphing computing”, our core working principles are different. Their work is similar to a mechanical neural network that requires additional digital filter to iterate outputs. In particular, the inputs to “morphing computing” are **time-domain analog signals**. However, our work is similar to a movable mathematical calculator that can directly generate waveform signal and control robotic locomotion. Our system can process the **spatial-domain analog signals** as well as the **encoded binary and qubit-like signals**.

Details about the comparisons and related tables are illustrated in the previous responses to **Comment 2**. We wish that these statements and explanations could address most of your concerns.

2. Grammar revision

The authors also appreciate your valuable comments about the English expression. We are pleased to have the opportunity to comprehensively improve our work. Following your suggestions, we have carefully checked our grammar in the introduction and improved the English expressions. The previous statements about **mechanical metamaterials** and **spatial analog computing** have been replaced by more precise descriptions. Furthermore, we have also checked the whole manuscripts and revised the discussion. The caption of Fig. 1 is added with more explanations.

Please refer to the introduction with the corresponding blue sentences on pages 1–4 in the revised manuscript. Please also refer to the caption of Fig. 1 on pages 3 and 4 in the revised manuscript for further illustrations.

Once again, we sincerely appreciate your thorough review and valuable feedback, which has helped us to comprehensively improve our manuscript!

References:

- [R1] Mousa, M., Nouh, M., 2024. Parallel mechanical computing: metamaterials that can multitask. *Proceedings of the National Academy of Sciences of the United States of America* 2024, e2407431121.
- [R2] Nanayakkara, T., Laschi, C., Mazzolai, B., Godage, I., Walker, I., Srivastava, M., Hirai, S., Duriez, C., Rojas, N., Sung, C., Paik, J., 2024. *Handbook on soft robotics*. Springer Nature, Switzerland.
- [R3] Roy, N., Posner, I., Barfoot, T., Beaudoin, P., Bengio, Y., Bohg, J., Brock, O., Depatie, I., Fox, D., Koditschek, D., Lozano-Perez, T., Mansinghka, V., Pal, C., Richards, B., Sadigh, D., Schaal, S., Sukhatme, G., Therien, D., Toussaint, M., Van de Panne, M, 2021. From machine learning to robotics: challenges and opportunities for embodied intelligence. *ArXiv*, 2110.15245.
- [R4] Hauser, H., Ijspeert, A.J., Fuchslin, R.M., Pfeifer, R., Maass, W., 2021. Towards a theoretical foundation for morphological computation with compliant bodies. *Biological Cybernetics* 105, 355–370.

Point-by-point responses to Reviewer 3's comments

Reviewer 3

General comment: The manuscript presents a metamaterial robotic system that has three modes for computations. The first mode is conformal change in the metamaterial structure, a second mode is in the vibrational spectrum supported in the robotic system. Finally, the system can use locomotion as a means to accomplish computation. The authors claim that they have demonstrated binary and quantum logic operation using the robotic system. Further, they claim that the system is capable of both serial and parallel operation. Finally, the authors predict that the system will perform robustly in harsh environments. The current manuscript has a number of drawbacks including the lack of quantitative measures for the fidelity of the computations. Also, the concept of quantum computing is not well represented by this system as it lacks both scalability and flexibility with respect to input states. The length of supplementary material, both written and video, is an impediment to the assessment of this submission. I recommend resubmission of this manuscript after major revision.

Responses:

We sincerely appreciate Reviewer 3 for your thorough review and thoughtful comments on our present work. We have carefully considered your comments and suggestions and subsequently revised our manuscript. For instance:

1. **Quantitative measures:** we have improved the quantitative indicators for both theoretical calculations and experimental measurements. Each of the computational results is marked with computational uncertainty which is represented by the Pearson correlation coefficient r . Details are provided in the following responses to **Comments 1** and **4**. Please refer to Figs. 2, 4, 5 and 7 as well as Figs. S2, S3 and S5 in the revised version.
2. **Mechanical matrix operations:** we have removed the concepts related to quantum theory and introduced our ideas from the classical matrix operations. The principle for encoding spatial analog signals into mechanical multi-bit states has been further discussed in the revised manuscript. Details are provided in the following responses to **Comments 2** and **4**. Please refer to the “mechanical matrix operations” section modified in the revised manuscript.
3. **Simplified contents:** we have reduced complex derivations and movies which would make readers confused. More explanations are introduced to adjacent sections and equations with clear guidance and references. Details are provided in the following responses to **Comment 3**. Please refer to Movies 1–9 and Supplementary Notes 1, 3 and 5 modified in the revised Supplementary Information.

In the following discussions, we provide the response to each of your suggestions in detail. Some additional changes are presented in the revised **Manuscript** and **Supplementary Information** and please refer to the blue sentences. We wish that these revised statements and improvements could address most of your concerns. Once again, we truly appreciate your valuable feedback and the time you have dedicated to our work.

Reviewer 3

Comment 1: The concept of using multiple aspects of the metamaterial robotic system for computational means is of interest and would be advantageous for exploration in harsh environments. It is, however, unclear how the authors would propose to accomplish inputs from the environment to adapt an instruction set for flexible operation. Further, attention must be paid to the accuracy of the computed result, the authors fail to report any measure of the uncertainty related to any of the computations performed. It is clear from the figures, e.g., Figure 2, that the accuracy of the computation is limited by the nature of the robotic system. A measure of uncertainty is required for each of the computations performed.

Responses:

Thank you very much for your positive feedback and recognition on our metamaterial robot. We sincerely appreciate Reviewer 3 for your valuable comments on the environmental input and computational accuracy.

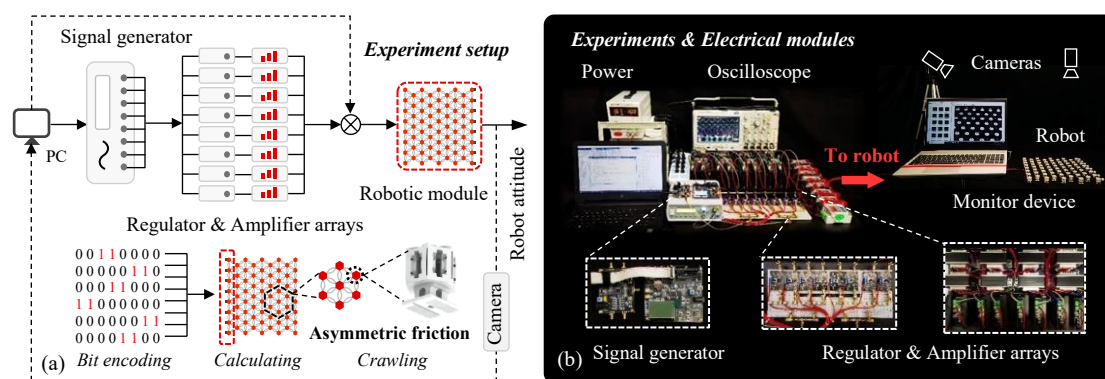


Fig. R2. (a) Experimental setups and working principle for computational crawling and (b) electrical modules and monitor devices during experiments.

Firstly, we would like to explain the principle for flexible operations according to the environment. As shown in Fig. R2, our robotic system is composed of the following four parts: (a) a computer with signal generator to produce harmonic inputs; (b) the regulator and amplifier arrays to adjust and amplify signals; (c) the robot with an excitation array on its head; (d) the monitor device to record the robot attitudes and transmit them to the computer. These devices form a closed-loop system, in which the environment inputs are represented by the recorded robot attitudes. The monitor device in our present work is a set of cameras on the top and side of the experiment platform, which is used to capture both the real-time position and crawling direction. It is worth mentioning that in addition to our present design, the environmental inputs can be detected by more integrated strategies.

For example, in Fig. 8, we have verified that the robot can carry loads which are several times of its own weight. A micro camera or sonar device can be installed on the robot, which can provide position information with less interference. The micro camera and sonar devices are both well-established systems which can be applied to robot vision and path planning fields. Using integrated and onboard monitor devices, the robot can generate instructions itself for subsequent computing based on environments. Since our attention is mainly focused on the theoretical design and robot validation, more integrated designs for monitor systems are not discussed in detail. We will focus on more integrated and adaptive robotic devices in our future work.

Revisions:

Secondly, following your scientific suggestion, we have added quantitative indicators for both theoretical calculations and experimental measurements to show their computational accuracy. Each result is associated with computational uncertainty which is represented by Pearson correlation coefficient r . We know that Pearson correlation coefficient r is a statistical measure used to quantify the correlation between two data sets $\{x_1, x_2, \dots, x_n\}$ and $\{y_1, y_2, \dots, y_n\}$ as

$$r = \frac{n \sum_{j=1}^n x_j y_j - \sum_{j=1}^n x_j \sum_{j=1}^n y_j}{\sqrt{n \sum_{j=1}^n x_j^2 - (\sum_{j=1}^n x_j)^2} \cdot \sqrt{n \sum_{j=1}^n y_j^2 - (\sum_{j=1}^n y_j)^2}}, \quad (\text{R.5})$$

where $r \in [-1, 1]$; $n = 2, 3, \dots$ is the sample size.

In this work, the correlation coefficient r is employed to indicate the degree of linear agreement, which serves as a representation for the calculating uncertainty. An r closer to 1 denotes a stronger agreement. The computational accuracy with $r \in [0.8, 0.9]$ indicates that two curves exhibit similar trends and those with $r > 0.9$ exhibit almost the same trend. The following discussion illustrates the specific data in terms of the computational accuracy:

1. Manuscript:

- (1) In Fig. 2, the correlation coefficients corresponding to the theoretical simulations of **focusing**, **reflection**, **integral**, **differential**, **convolution** and **solving equation** are calculated as $r = 0.8182, 0.9154, 0.9445, 0.9566, 0.9614$ and 0.9665 , respectively. The correlation coefficients for experimental measures are derived as $r = 0.8045, 0.9605, 0.9601, 0.9259, 0.9083$ and 0.9365 , respectively. Most of the theoretical and experimental results agree well with the theory.
- (2) In Fig. 4, the correlation coefficients corresponding to the **2nd differential of fundamental waves** are calculated as $r = 0.9045$ for simulations and $r = 0.8957$ for experiments. The correlation coefficients corresponding to the **1st differential of subharmonic waves** are

derived as $r = 0.9656$ for simulations and $r = 0.9028$ for experiments. Both simulations and experiments show reasonable accuracy.

- (3) In Fig. 5, the correlation coefficients corresponding to **Hadamard operation** are derived as $r = 0.9187, 0.9717$ and 0.9281 for $|0\rangle, |+\rangle$ and $|1\rangle$ inputs, respectively. The correlation coefficient corresponding to the **arbitrary operation** in Fig. 5(h) is calculated as $r = 0.9607$. The correlation coefficients corresponding to **CNOT operations** in Fig. 5(i) are derived as $r = 0.9409$ and 0.9192 for $|00\rangle$ and $|01\rangle$ inputs, respectively. These results show good agreements with theoretical ones.
- (4) In Fig. 7, the correlation coefficients for the **integral, differential, solving equation I, solving equation II, addition** and **subtraction** operations are calculated as $r = 0.9880, 0.9363, 0.9297, 0.9714, 0.9028$ and 0.9452 , respectively. Furthermore, the robot crawling behaviors during experiments also agree with the theoretical results.

2. Supplementary Information

- (5) In Fig. S2, the correlation coefficients corresponding to theoretical simulations of **addition I, addition II, subtraction I, subtraction II, serial I** and **serial II** operations are calculated as $r = 0.9486, 0.9044, 0.9429, 0.9250, 0.9293$ and 0.8996 , respectively. In addition, the correlation coefficients for experiment measurements are calculated as $r = 0.9497, 0.8809, 0.8921, 0.9033, 0.9127$ and 0.8869 , respectively. Both simulations and experiments show reasonable accuracy.
- (6) In Fig. S3, the correlation coefficients corresponding to **binary AND operations** are calculated as $r = 0.8575, 0.8712$ and 0.9940 for $(1, 0), (0, 1)$ and $(1, 1)$ inputs. The correlation coefficients corresponding to **binary OR operations** are calculated as $r = 0.9437, 0.9362$ and 0.8974 for $(1, 0), (0, 1)$ and $(1, 1)$ inputs. The correlation coefficients corresponding to **binary XOR operations** are calculated as $r = 0.9102, 0.9348$ and 0.9063 for $(1, 0), (0, 1)$ and $(1, 1)$ inputs, respectively.
- (7) In Fig. S5, the correlation coefficients corresponding to **Pauli X operations** are calculated as $r = 0.9663, 0.9698$ and 0.9393 for $|0\rangle, |1\rangle$ and $|+\rangle$ inputs. The correlation coefficients corresponding to **Pauli iY operations** are calculated as $r = 0.9296, 0.9258$ and 0.9305 for $(1, 0), (0, 1)$ and $(1, 1)$ inputs. The correlation coefficients corresponding to **Hadamard operations** are calculated as $r = 0.9242, 0.9761$ and 0.9765 for $(1, 0), (0, 1)$ and $(1, 1)$ inputs, respectively.

Please refer to Figs. 2, 4, 5 and 7 in the revised **Manuscript**, as well as Figs. S2, S3 and S5 in revised **Supplementary Information** for further illustrations.

Reviewer 3

Comment 2: While acoustic systems analogous to quantum computers have been reported in the literature and cited by the authors, some requirements for quantum computation are not met by the current system. In particular, following DiVincenzo's criteria, the system fails to be scalable. Recall that quantum advantage is usually considered to require 50 qubits. This robotic system does not scale to such a large number, or at least such scaling is not clear to this reviewer. Secondly, for general quantum computing the initial state must be able to accommodate a wide range of states available on the Bloch sphere. Again, it is not clear that the current system will support such diversity in specifying the initial state.

Responses:

Thank you very much for your careful reading and insightful comments about the mechanical qubit-like computing. The following responses aim to discuss the scalability of our work and the ability to cover the Bloch sphere.

1. Scalability in mechanical computing

In our previous version, we have reported the single-qubit operations and their combinations to more complex multi-qubit logics. For example, based on the serial and parallel strategies, the multi-qubit operations are illustrated in Figs. R3(d) and R3(e) to verify its theoretical scalability. However, during the experiments, our metamaterial robot is designed with 9 and 8 unit cells along the x and y directions, respectively. Such a system is mainly used to illustrate the feasibility of qubit-like computing instead of showing its scalability. Although it can be scaled with the spatial serial and parallel strategies, the large size for a 50-qubit system can lead to giant spatial occupation and energy consumption.

Following your valuable suggestion, we have removed the concepts related to quantum computing and introduced our ideas from classical matrix operations. **According to DiVincenzo's criteria, quantum computing systems require at least 50 qubits to achieve the computing advantage. The scalable dimensions for classical matrix operations are relatively flexible without rigorous requirements.** Previous single-qubit operations, such as Hadamard, Pauli X and iY gates, can be considered as 2×2 mechanical matrix operations. The two-qubit CNOT gate achieved by several 1:2 wave splitters in Fig. R3(e) can be considered as 4×4 matrix operations with 4 computational channels. In addition, the combinational serial/parallel operations of several Hadamard gates in Fig. R3(d) can be considered as the 4×6 matrix operations. Furthermore, the principle of encoding spatial analog signals to mechanical matrix operations is further discussed in the revised manuscript.

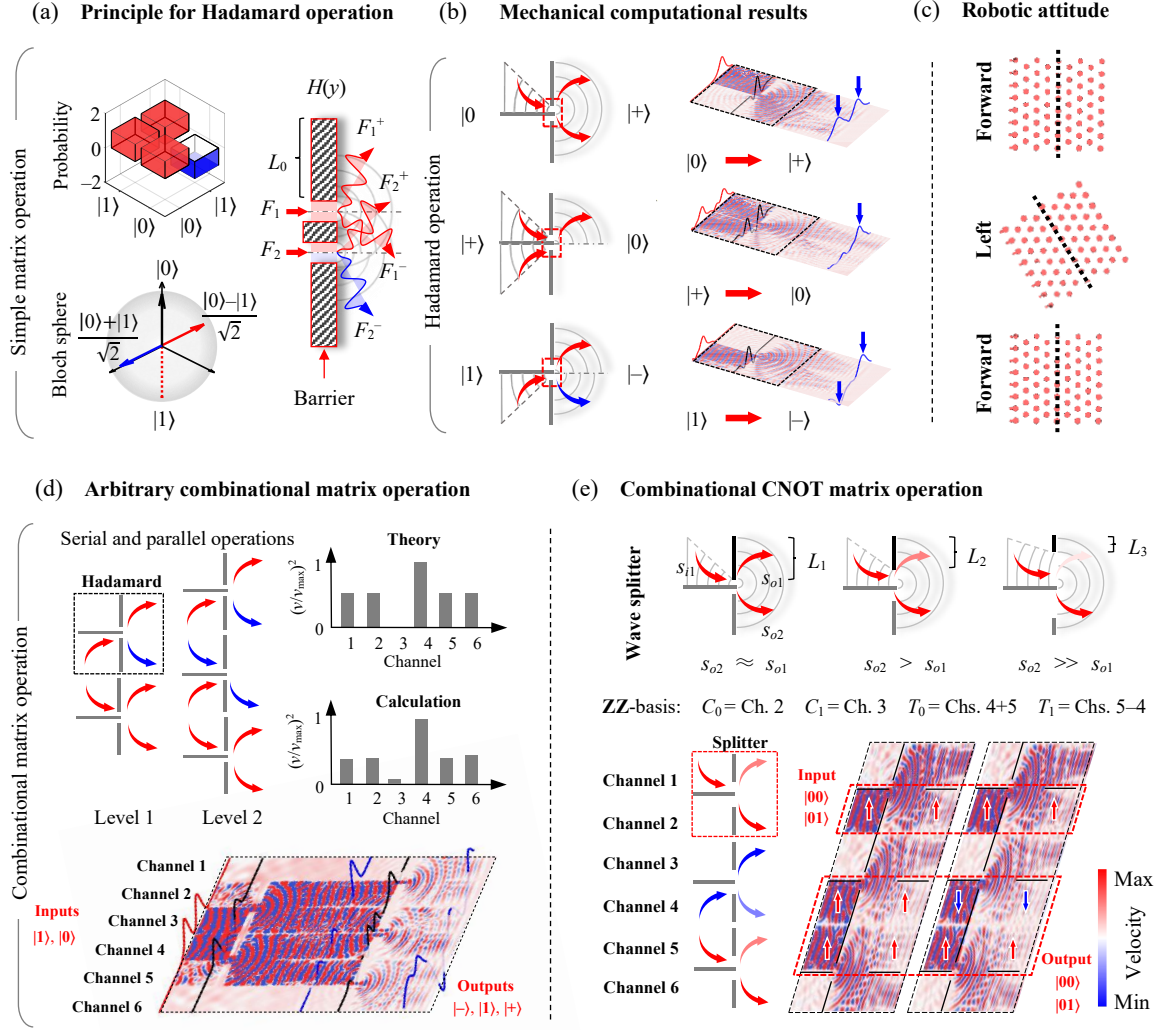


Fig. R3. Mechanical realizations of reprogrammable matrix operations. (a)–(c) Principle for mechanical Hadamard operation and the computational results of $|0\rangle$, $|+\rangle$ and $|1\rangle$ inputs. (d) Combinational matrix operation with several Hadamard operators and (e) the combinational four-channel CNOT operation. The correlation coefficients are calculated as $r = 0.9187, 0.9717$ and 0.9281 for $|0\rangle, |+\rangle$ and $|1\rangle$ inputs in (b), $r = 0.9607$ in (d), as well as $r = 0.9409$ and 0.9192 for $|00\rangle$ and $|01\rangle$ inputs in (e).

For more complex matrix operations, we can employ a small-scale experimental model by decreasing the unit size. During our experiments, the side length of hexagonal blocks a and the initial length of springs b are 1.5 and 3.2 cm, which will be obviously large as the unit number increases. As reported by Zhang et al. [R1] and He et al. [R2], they have designed some super-compact quantum logic gates in which the footprints can be minimized to 20~200 μm^2 . Although our purpose is not to design such a small-scale robot, we can apply similar ideas to minimize the robot size. In our future applications, the robot framework will be directly fabricated by small-scale 3D printing. It can reduce the unit size to a millimeter level and significantly

enhance the scalability of matrix operations. It is worth mentioning that this work is focused on a general strategy from elastic wave metamaterials to crawling robots with embodied computing functions. Such strategy is not limited to our present design and can be extended to more versatile and integrated designs in future applications.

[R1] Zhang, M., Feng, L.T., Li, M., Chen, Y., Zhang, L., He, D.Y., Guo, G.P., Guo, G.C., Ren, X.F., Dai, D.X., 2021. Supercompact photonic quantum logic gate on a silicon chip. *Physical Review Letters* 126, 130501.

[R2] He, L., Liu, D.N., Gao, J.X., Zhang, W.X., Zhang, H.Z., Feng, X., Huang, Y.D., Cui, K.Y., Liu, F., Zhang, W., Zhang, X.D., 2023. Super-compact universal quantum logic gates with inverse-designed elements. *Science Advances* 9, adg6685.

2. Completeness of initial states

The authors are also very grateful for your kind suggestions to discuss whether the initial state could accommodate a wide range of states on the Bloch sphere. It would help readers to have a more thorough understanding of the matrix computing. We have followed your suggestion and summarized as follows:

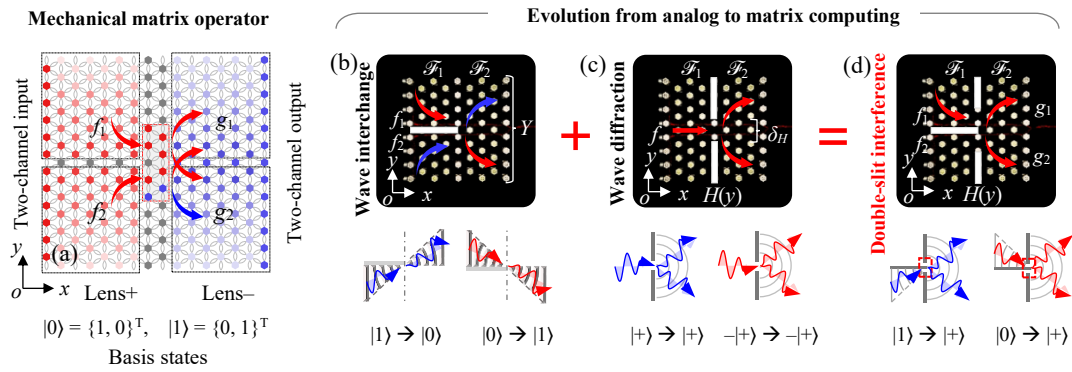


Fig. R4. Principle for achieving mechanical matrix operations. (a) Robot configuration for achieving mechanical matrix operations. (b)–(d) Evolution from analog to matrix computing: the matrix operation in (d) is achieved by the double-slit interference effect with combinations of (c) analog wave interchange and (d) wave diffraction.

(1) Definition of mechanical bits

As shown in Fig. R4(a), the metamaterial employs a similar multilayer $\mathcal{T}_1$ - H - $\mathcal{T}_2$ layout to the spatial analog computing. A mass barrier array $m_b \gg m_0$ is placed along the axial centerline $y = Y/2$ which divides the input f into a signal pair $\{f_1, f_2\}^T$. For harmonic signals with an identical frequency, the signal pair $\{f_1, f_2\}^T$ defines a 2D complex state vector $\{A_1^* \exp(i\varphi_1), A_2^* \exp(i\varphi_2)\}^T$ with $A_1^{*2} + A_2^{*2} = 1$. This vector is characterized by the normalized amplitudes and phases A_1^* , A_2^* , φ_1 and φ_2 of $\{f_1, f_2\}^T$. We consider the state vector as a linear combination $\{\alpha, \beta\}^T = \alpha \{1, 0\}^T +$

$\beta\{0, 1\}^T$, in which the basis states $\{1, 0\}^T$ and $\{0, 1\}^T$ are defined by the single-mode excitations ($A_1^* = 1, A_2^* = 0$) and ($A_1^* = 0, A_2^* = 1$). In order to better describe the two-channel signals, we adopt the symbolic notation from qubit computing, which labels the classical basis states $\{1, 0\}^T$ and $\{0, 1\}^T$ as the mechanical qubit-like states $|0\rangle$ and $|1\rangle$. Then, a general input state is written as $\{\alpha, \beta\}^T = \alpha|0\rangle + \beta|1\rangle$ with $\alpha = A_1^* \exp(i\varphi_1)$ and $\beta = A_2^* \exp(i\varphi_2)$. It is worth mentioning that this notation is only a purely mathematical convention without quantum implication. Similar discussion is added in the manuscript and the responses to Comment 4 of Review 1.

(2) Ability to cover the Bloch sphere

In the quantum computing theory, an arbitrary qubit state can be expressed as

$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi} \sin(\theta/2)|1\rangle, \quad (\text{R.6})$$

where θ and ϕ are Bloch sphere coordinates covering the entire sphere. As mentioned above, our mechanical states are characterized by the normalized wave amplitudes $\{A_1^*, A_2^*\}^T$ and phases $\{\varphi_1, \varphi_2\}^T$ of the traveling wave pair. Considering the complex amplitudes $A_1^* \exp(i\varphi_1)$ and $A_2^* \exp(i\varphi_2)$ of the wave pair, we can obtain the equivalent qubit-like state as

$$|\psi\rangle = A_1^* e^{i\varphi_1} |0\rangle + A_2^* e^{i\varphi_2} |1\rangle, \quad (\text{R.7})$$

where A_1^* and A_2^* satisfy the relation $A_1^{*2} + A_2^{*2} = 1$. Setting φ_1 as the fiducial phase, the phase relation is actually determined by the relative phase $\Delta\varphi = \varphi_2 - \varphi_1$. We neglect the overall phase factor $\exp(i\varphi_1)$ and the qubit-like state can be written as $|\psi\rangle = A_1^* |0\rangle + A_2^* \exp(i\Delta\varphi) |1\rangle$. Assume that $\alpha = A_1^*$ and $\beta = A_2^* \exp(i\Delta\varphi)$, the qubit-like state can be expressed as

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad (\text{R.8})$$

where α and β satisfy $|\alpha|^2 + |\beta|^2 = 1$ and are equivalent to the Bloch sphere representation $\alpha = \cos(\theta/2)$ and $\beta = \exp(i\phi)\sin(\theta/2)$. With the normalization $A_1^{*2} + A_2^{*2} = 1$, the initial state can be fully characterized by the amplitude ratio A_1^*/A_2^* and relative phase $\Delta\varphi = \varphi_2 - \varphi_1$. Then, it can be visualized on a Bloch sphere by the following mapping relation:

$$\theta = 2 \arccos(A_1^*) \text{ or } 2 \arcsin(A_2^*), \quad \theta \in [0, \pi], \quad (\text{R.9})$$

$$\phi = \varphi_2 - \varphi_1, \quad \phi \in [0, 2\pi]. \quad (\text{R.10})$$

During our experiments, both wave amplitudes and phases can be adjusted by the arbitrary waveform generator. By tuning the wave amplitudes to satisfy $A_1^{*2} + A_2^{*2} = 1$ and relative phase $\Delta\varphi = \varphi_2 - \varphi_1$, the initial state can accommodate a wide range of states on the Bloch sphere.

Reviewer 3

Comment 3: With regard to the supplementary material, it does not appear to this reviewer that the video material adds to the content of the manuscript. While the written supplementary material does support the manuscript, its length could easily be reduced without losing the vital supporting information the manuscript requires.

Responses:

The authors greatly appreciate Reviewer 3 for your careful reading and insightful comments on our work. We quite agree with your comments about that the length of supplementary materials affects the readability of our work. We have followed your suggestions and substantially revised our manuscript. The following discussion illustrates the detailed changes we modified in response to your suggestions. We wish that the revised version could better convey the main features of this present work.

Revisions:

1. Firstly, we have removed several movies in which the contents are repeated in the text materials. The remaining movies illustrate the key functions of our metamaterial robot and its experimental performances.

(a) The principle of robotic crawling in previous Movie S2 is deleted, which has been explained in Fig. 1 and Supplementary Note 1, Part A.

(b) The principle of spatial serial operations in previous Movie S4 is deleted, which has been explained in Fig. 7 and Supplementary Note 1, Part C.

(c) The principle of spatial parallel operations in previous Movie S6 is deleted, which has been explained in Fig. 7 and Supplementary Note 1, Part D.

(d) The principle of mechanical matrix operations in previous Movie S8 is deleted, which has been explained in Fig. 4 and Supplementary Note 2, Part D.

(e) The experiment of Pauli X operation in previous Movie S10 is deleted, which has been demonstrated in Fig. S5 and Supplementary Note 2, Part D.

(f) The experiments of amplitude dependency and parameter regulating effects in previous Movie S13 are deleted, which has been demonstrated in Fig. S9 and Supplementary Note 6.

After the simplification, the movie number has been reduced from **15** to **9**. The remaining movies mainly illustrate the following contents, i.e., (i) **Movie S1**: fundamental principle of reprogrammable mechanical computing; (ii) **Movies S2–S5**: experiments of reprogrammable analog computing, spatial serial computing, spatial parallel computing, as well as mechanical

matrix operations; (iii) **Movies S6 and S7**: band gap property and velocity response during robot crawling; as well as (iv) **Movies S8 and S9**: multi-gait locomotion and multifunctional application.

2. We have also deleted several complex derivations and repeated figures that would make readers confused. Some key steps and explanatory notes are retained in the revised Supplementary Information.

(a) Some derivations of spatial analog computing in Supplementary Note 2 have been deleted. Then, only fundamental principle of reprogrammable analog computing, serial and parallel operations are retained and integrated into the revised Supplementary Note 1.

(b) The nonlinear dynamic equations and their related complex vectors and matrices in the previous Supplementary Note 4 have been simplified. The remaining equations have been integrated into the revised Supplementary Notes 3 and 5.

(c) Derivations of nonlinear harmonic band structures in Supplementary Note 3 and robot crawling velocity in Supplementary Note 5 have been simplified and integrated with the nonlinear dynamic equations mentioned above.

(d) Repeated figures and discussion about eigenstate diagram and robot multifunctional illustrations in previous Supplementary Figures 6 and 11 have been removed. An additional figure has been added to show the parallel computing in the frequency domain in the revised Supplementary Figure 4.

After the simplification, the equation number has been reduced from **55** to **30** and the chapter number of Supplementary Information has been reduced from **8** to **6**. The numbers of figures and pages have been reduced from **12** to **10** and **34** to **29**, respectively. The revised equations are integrated into three parts, i.e., **computational principles**, **harmonic band structures** and **robot crawling velocity**. These equations are collected into Supplementary Notes **1**, **3** and **5** with their corresponding discussion in Supplementary Notes **2**, **4** and **6**. The revised equation number of each part is about **10** within **3** pages to improve readability. Similar revisions are added in the responses to Comment 5 of Reviewer 2.

3. Furthermore, we have improved the expressions of theoretical derivations and their discussion. More explanations are introduced to adjacent sections and equations with clearer guidance and references.

(a) **Manuscript**: Please refer to Eqs. (1) and (2) with the corresponding blue sentences on pages 5 and 6; Eqs. (4) and (5) with the corresponding blue sentences on page 8; the blue sentences to explain Eq. (6) on page 10; as well as Eqs. (7)–(10) we added to explain the Hadamard and CNOT operations on pages 14–16 in the revised Manuscript. Please also refer to other blue sentences we modified in the whole manuscript.

(b) **Supplementary Information:** Please also refer to the blue sentences before Eqs. (S1) and (S3) on page 4; the blue sentences after Eq. (S5) on page 5 and before Eq. (S8) on page 6; the blue sentences before Eqs. (S11), (S17) and (S21) on pages 14–16; the blue sentences before Eqs. (S23), (S24), (S26), (S27) and (S29) on pages 22–23 in the revised Supplementary Information.

In addition, in order to improve readability, Supplementary Figures S1–S9 and Table S1 are embedded in their corresponding text materials of Supplementary Notes S1–S6.

In summary, we have deleted a large number of complex derivations and figures which would affect the readability of this work. More explanations are introduced to adjacent sections and equations with clearer guidance and references. In addition, we have removed several movies in which the contents are repeated in the text materials. Please refer to the revised Supplementary Figures S1–S10, Notes S1–S6 and Movies S1–S9 for further illustrations. We wish that these improvements and modifications could address most of your concerns.

Reviewer 3

Comment 4: I strongly recommend that the authors include quantitative measure of uncertainty related to each of the computations performed in the manuscript. Further, references to quantum computation should be removed from the manuscript. Operations that have performed on so-called ‘few qubit states’ can be reported as matrix manipulations rather than quantum logic operations. I, therefore, recommend that the article be resubmitted after these major revisions.

Responses:

1. Quantitative measure of uncertainty

The authors sincerely appreciate your thorough review and constructive suggestions which help us better improve our present work. Following your valuable suggestions, we have carefully checked the manuscript and added quantitative indicators for both theoretical and experimental results. Each computational result is marked with computational uncertainty, which is represented by the Pearson correlation coefficient r . The Pearson correlation coefficient r is a statistical measure used to quantify the degree of linear correlation and the calculating uncertainty. Detailed revisions of computational accuracy have been illustrated in the responses to **Comment 1**. The relevant data for each simulation (Sim.) and experiment (Exp.) results are relisted in the following tables:

Table R3. Quantitative measure of uncertainty in Manuscript

Figure	Computing function	Correlation coefficient r
Fig. 2 (Sim. & Exp.)	(i) focusing	(i) 0.8182 (Sim.), 0.8045 (Exp.)
	(ii) reflection	(ii) 0.9154 (Sim.), 0.9605 (Exp.)
	(iii) integral	(iii) 0.9445 (Sim.), 0.9601 (Exp.)
	(iv) differential	(iv) 0.9566 (Sim.), 0.9259 (Exp.)
	(v) convolution	(v) 0.9614 (Sim.), 0.9083 (Exp.)
	(vi) solving equation	(vi) 0.9665 (Sim.), 0.9365 (Exp.)
Fig. 4 (Sim. & Exp.)	(i) 2nd differential of fundamentals	(i) 0.9045 (Sim.), 0.8957 (Exp.)
	(ii) 1st differential of subharmonics	(ii) 0.9656 (Sim.), 0.9028 (Exp.)
Fig. 5 (Sim.)	(i) Hadamard operations	(i) 0.9187, 0.9717 and 0.9281 (Sim.)
	(ii) arbitrary operations	(ii) 0.9607 (Sim.)
	(iii) CNOT operations	(iii) 0.9409 and 0.9192 (Sim.)
Fig. 7 (Sim.)	(i) integral, (ii) differential, (iii) solving equation I, (iv) solving equation II, (v) addition and (vi) subtraction	(i) 0.9880 (Sim.), (ii) 0.9363 (Sim.), (iii) 0.9297 (Sim.), (iv) 0.9714 (Sim.), (v) 0.9028 (Sim.), (vi) 0.9452 (Sim.)

Table R4. Quantitative measure of uncertainty in Supplementary Information

Figure	Computing function	Correlation coefficient r
Fig. S2 (Sim. & Exp.)	(i) addition I	(i) 0.9486 (Sim.), 0.9497 (Exp.)
	(ii) addition II	(ii) 0.9044 (Sim.), 0.8809 (Exp.)
	(iii) subtraction I	(iii) 0.9429 (Sim.), 0.8921 (Exp.)
	(iv) subtraction II	(iv) 0.9250 (Sim.), 0.9033 (Exp.)
	(v) serial I	(v) 0.9293 (Sim.), 0.9127 (Exp.)
	(vi) serial II	(vi) 0.8996 (Sim.), 0.8869 (Exp.)
Fig. S3 (Sim.)	(i) binary AND operations	(i) 0.8575, 0.8712 and 0.9940 (Sim.)
	(ii) binary OR operations	(ii) 0.9437, 0.9362 and 0.8974 (Sim.)
	(iii) binary XOR operations	(iii) 0.9102, 0.9348 and 0.9063 (Sim.)
Fig. S5 (Sim.)	(i) Pauli X operations	(i) 0.9663, 0.9698 and 0.9393 (Sim.)
	(ii) Pauli iY operations	(ii) 0.9296, 0.9258 and 0.9305 (Sim.)
	(iii) Hadamard operations	(iii) 0.9242, 0.9761 and 0.9765 (Sim.)

Please refer to Figs. 2, 4, 5 and 7 in the revised **Manuscript**, as well as Figs. S2, S3 and S5 in revised **Supplementary Information** for further illustrations.

2. Mechanical matrix operations

Furthermore, following your constructive suggestions, we have removed the concept related to quantum computing and introduced our ideas from classical matrix operations. More explanations about the principles of mechanical matrix operations have been added. Please refer to the modified “**mechanical matrix operations**” section in the revised manuscript.

- (1) The concept related to quantum computing has been removed and the previous qubit computing has been revised as classical matrix operation. The principle for encoding spatial analog signals into mechanical multi-bit states has been further discussed in the revised manuscript.
- (2) In order to avoid unfamiliar concepts of qubit-like states $|0\rangle$ and $|1\rangle$, we provide their states $\{\alpha, \beta\}^T = \alpha|0\rangle + \beta|1\rangle$ with classical wave representations $\alpha = A_1 \exp(i\varphi_1)$ and $\beta = A_2 \exp(i\varphi_2)$. The mechanical matrix operations are added with equations to explain their logical mapping relation.
- (3) Furthermore, we also present the comparisons between the mechanical matrix operations and conventional acoustic computing in Supplementary Note 2 in the revised Supplementary Information.

Revisions:

The following discussion illustrates detailed modifications and improvements related to the “mechanical matrix operations” in the revised Manuscript:

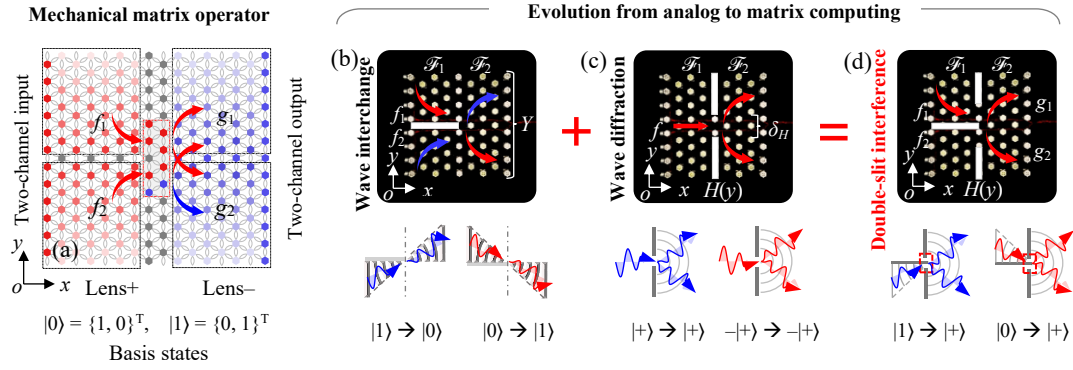


Fig. R4. Principle for achieving mechanical matrix operations. (a) Robot configuration for achieving mechanical matrix operations. (b)–(d) Evolution from analog to matrix computing: the matrix operation in (d) is achieved by the double-slit interference effect with combinations of (c) analog wave interchange and (d) wave diffraction.

(1) [Paragraph 2 on Page 12](#)

“**Mechanical matrix operations.** In addition to spatial analog computing, the elastic wave metamaterial can be also applied to matrix operations on encoded mechanical signals. In the following discussion, we employ the versatile computational framework to realize multiple matrix operations which are commonly used in information processing. Different from spatial analog computing with single input f , the matrix operations are achieved by several encoded signals $\{f_1, f_2, \dots, f_N\}^T$ with a N -dimensional transformation matrix $\mathbf{M}^{(N)}$. For example, we can encode the two-channel analog inputs $\{f_1, f_2\}^T$ and outputs $\{g_1, g_2\}^T$ as vectors in a classical 2D state space. Then, the metamaterial can achieve linear transformation $\{g_1, g_2\}^T = \mathbf{M}^{(2)}\{f_1, f_2\}^T$ by a 2D matrix $\mathbf{M}^{(2)}$. Such transformation can realize fundamental 2×2 operations such as Hadamard or Pauli-X type matrices in linear algebra. Accordingly, this framework can be also extended to higher-dimensional state spaces by increasing the number of wave channels, e.g., the 4×4 controlled-NOT (CNOT) matrix. It is worth noting that many high-dimensional linear and invertible transformations can be decomposed into sequences of 2×2 operations. As a result, a robust and reprogrammable $\mathbf{M}^{(2)}$ transformation device is essential to achieve more complex and scalable higher-dimensional operations.”

(2) [Paragraph 1 on Page 14](#)

“As shown in Fig. R4(a), the metamaterial robot employs a similar multilayer $\mathcal{T}_1$ - H - $\mathcal{T}_2$ layout to spatial analog computing. A mass barrier array $m_b \gg m_0$ is placed along the axial centerline

$y = Y/2$ which divides the input f into a signal pair $\{f_1, f_2\}^T$. For the harmonic signals with identical frequency, the signal pair $\{f_1, f_2\}^T$ defines a 2D complex state vector $\{A_1^* \exp(i\varphi_1), A_2^* \exp(i\varphi_2)\}^T$ with $A_1^{*2} + A_2^{*2} = 1$. This vector is characterized by the normalized amplitudes and phases A_1^*, A_2^*, φ_1 and φ_2 of $\{f_1, f_2\}^T$. We consider the state vector as a linear combination $\{\alpha, \beta\}^T = \alpha\{1, 0\}^T + \beta\{0, 1\}^T$, in which the basis states $\{1, 0\}^T$ and $\{0, 1\}^T$ are defined by single-mode excitations ($A_1^* = 1, A_2^* = 0$) and ($A_1^* = 0, A_2^* = 1$). In order to better describe the two-channel signals, we adopt the symbolic notation from acoustic phi-bit computing, which labels classical basis states $\{1, 0\}^T$ and $\{0, 1\}^T$ as mechanical qubit-like states $|0\rangle$ and $|1\rangle$. Then, a general input state is written as $\{\alpha, \beta\}^T = \alpha|0\rangle + \beta|1\rangle$ with $\alpha = A_1^* \exp(i\varphi_1)$ and $\beta = A_2^* \exp(i\varphi_2)$. It is worth mentioning that this notation is a purely mathematical convention without relevance to quantum computing. With the normalization $A_1^{*2} + A_2^{*2} = 1$, the mechanical state can be fully characterized by the amplitude ratio A_1/A_2 and relative phase $\Delta\varphi = \varphi_2 - \varphi_1$. Then, it can be visualized on a Bloch sphere by the mapping of two polar coordinates $\theta = 2\arccos(A_1^*)$ and $\phi = \Delta\varphi$. Such representation is a classical analogous mapping to provide visualization without quantum implication.”

(3) Paragraph 2 on Page 14

“In order to realize the $\mathbf{M}^{(2)}$ transformation function, Figs. R4(b)–R4(d) provide the evolution from spatial analog to mechanical matrix computing. The metamaterials in Figs. R4(b) and R4(c) are designed with mass barriers along the axial and vertical centerlines $y = Y/2$ and $x = X/2$, respectively. In Fig. R4(b), the input f is divided into a signal pair $\{f_1, f_2\}^T$ by the axial barrier $y = Y/2$ and the logic states can be swapped after two Fourier lenses. In Fig. R4(c), a tunable slit is designed at the center of the vertical barrier $x = X/2$. Such a design allows the conversion of incident waves into point wave sources and presents the diffraction effect. After the Fourier layer $\mathcal{F}_2$, the point source can be further transformed as a parallel waveform and equally divided into $\{g_1, g_2\}^T$ as the superposition state $|+\rangle = \{\sqrt{2}/2, \sqrt{2}/2\}^T$, i.e., $\alpha = \beta$. Then, combining Figs. R4(b) and R4(c), incident waves with either $|0\rangle = \{1, 0\}^T$ or $|1\rangle = \{0, 1\}^T$ input in Fig. R4(d) can be transformed into their superposition states $|+\rangle = \{\sqrt{2}/2, \sqrt{2}/2\}^T$.”

(4) Paragraph 1 on Page 15

“Based on the discussion above, the evolution $|0\rangle \rightarrow |+\rangle$ can be achieved by the robot configuration in Fig. R4(d). In order to obtain the $|1\rangle \rightarrow |-\rangle$ output, the transfer layer H in Fig. R5(a) is designed as a double-slit layout, in which the lower slit has two mass arrays with π phase difference. When the input $|1\rangle = \{0, 1\}^T$ is applied, the input f_2 can be focused into the lower slit as F_2 and divided into two separated wave fronts $F_2^\pm$ with opposite phases. Based on the Huygens-Fresnel principle, the separated wave fronts can be considered as two half-space wave sources which input into the Fourier layer $\mathcal{F}_2$. Then, they can generate two opposite output states, i.e., $|-\rangle = \{\sqrt{2}/2, -\sqrt{2}/2\}^T$. Similarly, for the $|+\rangle = \{\sqrt{2}/2, \sqrt{2}/2\}^T$ input, the

output corresponding to the upper half-space g_1 can be enhanced by two in-phase wave fronts $F_{1,2}^+$ but the lower output g_2 can be eliminated by two out-phase $F_{1,2}^-$. Then, they can yields the $|0\rangle = \{1, 0\}^T$ output, i.e., $|+\rangle \rightarrow |0\rangle$. Similar phenomena can be observed from the numerical results and robot attitudes during experiments in Figs. R5(f) and R5(g), respectively. For comparisons, Supplementary Figure S5 and Movies S5 further provide several mechanical realizations of 2×2 matrix operations with experimental supports, including Pauli X, Pauli iY and Hadamard transformations.”

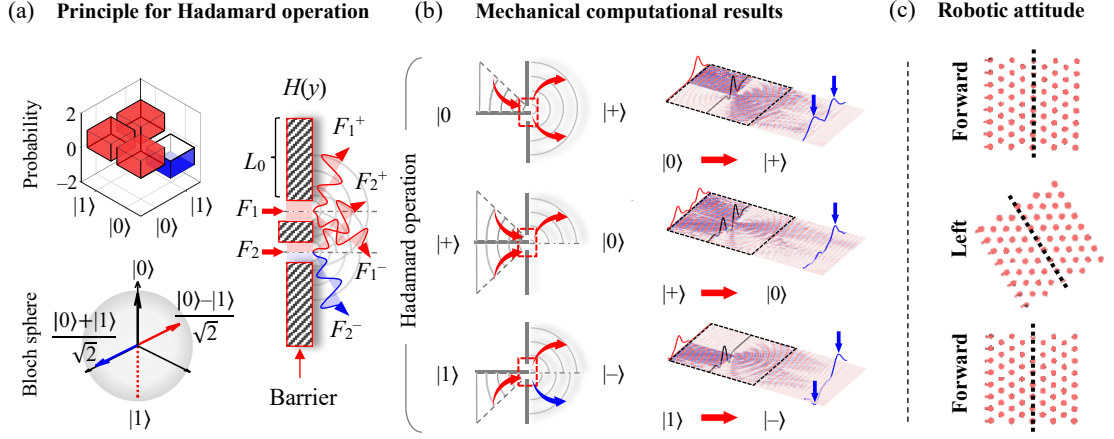


Fig. R5. Mechanical realizations of reprogrammable matrix operations. (a)–(c) Principle for mechanical Hadamard operation and the computational results of $|0\rangle$, $|+\rangle$ and $|1\rangle$ inputs.

Please refer to the blue sentences about mechanical matrix operations from pages 12 to 16 and Eqs. (7)–(10) in the revised **Manuscript**. In addition to the modifications above, the relationship between the mechanical matrix operation and acoustic phi-bit computing are also discussed in Supplementary Note 2, Part E. Limited by space, please refer to the blue sentences on pages 13 and 14 in the revised **Supplementary Information**. We wish that these modifications and improvements could better convey the main features of our present work.

Once again, we sincerely appreciate your thorough review and valuable feedback that has helped us to comprehensively improve our manuscript!

Responses to Reviewers 4 & 5' comments

Reviewers: 4 & 5

General comment: I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

Responses:

The authors greatly appreciate Reviewers 4 and 5 for your thorough review and constructive feedback on our manuscript. We are sincerely grateful for the time and effort you have dedicated to the insightful comments, which have helped us significantly improve our work.

Following your valuable suggestions, we have carefully checked our work and comprehensively improved the theoretical framework, result presentation, as well as our English expression. A point-by-point response has been provided to each of your comments and the places we modified have been marked in the revised manuscript. Furthermore, the writing logic from spatial analog computing to matrix and logic operations has been revised with more explanations. We wish that these improvements and revisions could align with your expectations and better convey the main features of our work.

Finally, we thank all the reviewers and editors for encouraging us to substantially improve our manuscript with your valuable comments and kind suggestion!

Responses to Reviewers' Comments

Reviewer 1

All my concerns have been addressed in the revised paper. It is now recommended for publications in NC.

The authors sincerely appreciate Reviewer 1 for your positive feedback and valuable suggestions during the review, which have enabled us to comprehensively improve our manuscript!

Reviewer 2

I would like to thank the authors for their thoughtful response to the reviewers comments. From reading the revised manuscript, I can see that the authors have taken steps to make the article easier to follow, and to more acceptably place their work within the wider related literature.

The authors sincerely appreciate Reviewer 2 for your valuable suggestions and thoughtful comments during the review. We wish that the revised manuscript could make it easier for readers to understand the main feature of this work!

Reviewer 3

We appreciate the revisions made by the authors and now find the manuscript appropriate for publication.

The authors sincerely appreciate Reviewer 3 for your thorough review and constructive comments on our work, which have helped us to better combine the computational functions with our robot!

Reviewers: 4 & 5

I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

We are sincerely grateful for the time and effort you have dedicated to the insightful comments, which have helped us significantly improve our work!

Finally, we thank all the reviewers and editors for encouraging us to substantially improve our manuscript with your valuable comments and kind suggestion!