

Supplementary Information: Pseudo Point Nodal Superconducting Gap in Spin-Triplet UTe_2

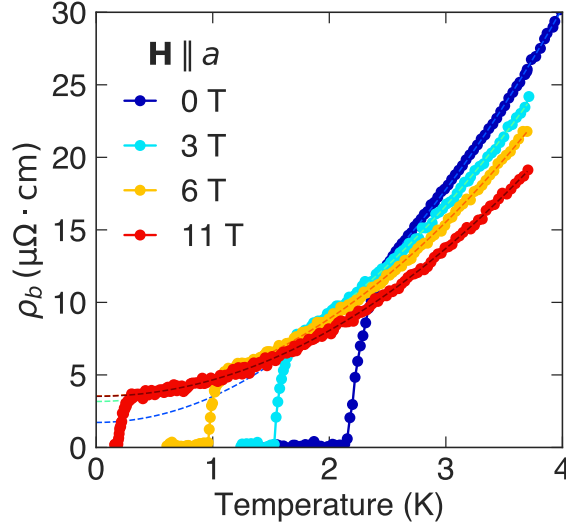
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SUPPLEMENTARY NOTE 1: ELECTRICAL RESISTIVITY MEASUREMENTS

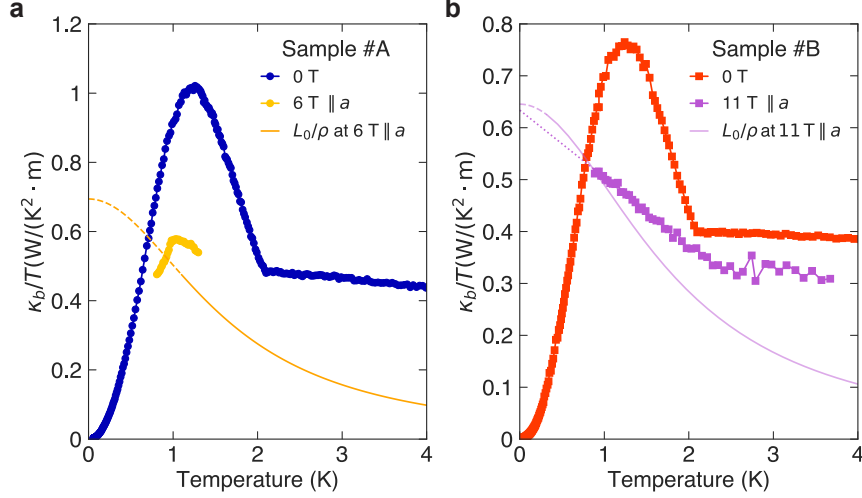


Supplementary Figure 1 | Temperature dependence of b -axis resistivity under magnetic fields parallel to the a axis. The dashed line represents a fit to Fermi-liquid behavior, $\rho_b(T) = \rho_b^0 + AT^2$.

Supplementary Figure 1 presents the temperature dependence of the b -axis resistivity ρ_b under several magnetic fields applied along $\mathbf{H} \parallel a$. At high temperatures, ρ_b exhibits negative magnetoresistance, a feature also observed for the other two current directions¹. Across the entire field range studied, the temperature dependence of ρ_b is well described by standard Fermi-liquid behavior, consistent with $\rho_b(T) = \rho_b^0 + AT^2$. We note that the upper critical field along the a axis, $H_{c2}^a \sim 11.7$ T for the samples used here, was determined from ρ_b measurements.

SUPPLEMENTARY NOTE 2: THERMAL CONDUCTIVITY MEASUREMENTS

As a consistency check of the thermal conductivity measurements, we examined the Wiedemann-Franz (WF) law in the zero-temperature limit. As shown in Supplementary Figure 2, the measured κ_b/T at $\mu_0\mathbf{H} = 6$ T ($\mathbf{H} \parallel a$) for sample #A exhibits a pronounced kink at 1 K, corresponding to T_c , then decreases rapidly upon further cooling. The orange solid and dashed lines represent the electronic contribution estimated from the WF relation



Supplementary Figure 2 | Verification of Wiedemann-Franz law for samples #A and

#B. **a** Blue and orange circles represent the κ_b/T measured at zero field (blue) and at 6 T magnetic field applied parallel to the a -axis (orange) for sample #A. The orange solid and dashed lines represent the electronic contribution calculated from the WF relation L_0/ρ_b and its extrapolation based on the assumption $\rho_b(T) = \rho_b^0 + AT^2$ below T_c , respectively. **b** Similar analysis for sample #B. Red and purple circles represent κ_b/T measured at zero field (red) and at 11 T with the magnetic field applied parallel to the a -axis (purple). The dotted line indicates a simple extrapolation to $T \rightarrow 0$, which approaches the electronic contribution expected from the WF relation (purple solid line).

L_0/ρ_b , where $L_0 = 2.44 \times 10^{-8} \text{ W}\Omega\text{K}^{-2}$ is the Lorenz number. At $T_c \approx 1 \text{ K}$, κ_b/T approaches L_0/ρ_b . The observed minor deviation is attributed to other contributions such as phonons. Upon further cooling, suppression of these contributions is expected to drive κ_b/T toward the WF expectation L_0/ρ_b . This behavior supports the internal consistency of our thermal conductivity measurements.

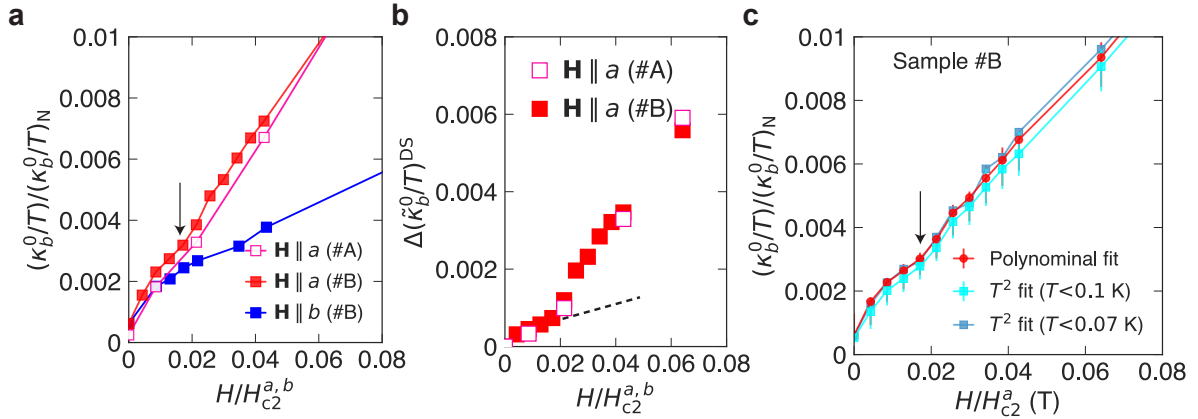
We also examined the WF law for sample #B at a higher magnetic field of 11 T ($\mathbf{H} \parallel a$), as shown in Supplementary Figure 2b. Due to the limited sample dimensions and the intrinsically large thermal conductivity of high-quality samples at high magnetic fields, the resulting temperature gradients became extremely small at low temperatures. This technical limitation restricted the accessible temperature range in the high-field WF-law measurements. Nevertheless, the available low-temperature data approach the WF expectation

within experimental resolution, and a simple extrapolation to $T \rightarrow 0$ yields a value close to L_0/ρ_b . The consistency between samples #A and #B further supports the reliability of the thermal conductivity measurements. We also examined the WF law for sample #B at a higher magnetic field of 11 T ($\mathbf{H} \parallel a$), as shown in Supplementary Figure 2b. Due to the limited sample dimensions and the intrinsically large thermal conductivity of high-quality samples at high magnetic fields, the resulting temperature gradients became extremely small at low temperatures. This technical limitation restricted the accessible temperature range in the high-field WF-law measurements. Nevertheless, the available low-temperature data approach the WF expectation within experimental resolution, and a simple extrapolation to $T \rightarrow 0$ yields a value close to L_0/ρ_b . The consistency between samples #A and #B further supports the reliability of the thermal conductivity measurements.

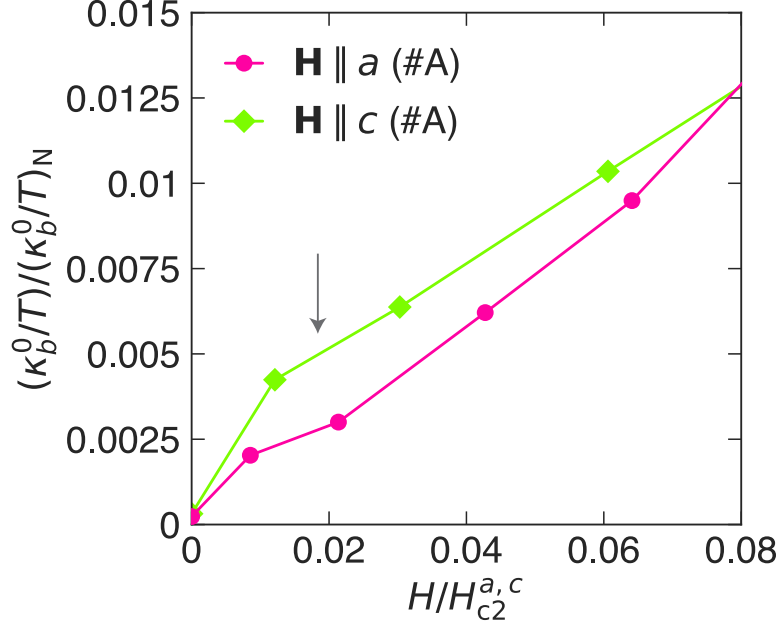
Beyond the scope of the present study, we briefly comment on the normal-state behavior of κ_b ($T > 1$ K). The electrical resistivity exhibits a clear T^2 dependence, consistent with Fermi-liquid-like inelastic quasiparticle scattering, likely influenced by magnetic fluctuations. The reduction of ρ under magnetic fields (Supplementary Figure 1) further suggests that these inelastic scattering processes are suppressed. However, despite the reduction in ρ , the total thermal conductivity at 11 T is lower than that at zero field. This indicates that changes in the electronic contribution alone cannot account for the observed behavior. As discussed above, the phonon contribution remains substantial in this temperature region. A natural interpretation is that the phonon contribution is reduced under magnetic field through field-dependent modifications of phonon scattering processes. We cannot exclude that an additional magnetic contribution to the thermal transport may also play a role. A detailed understanding of these mechanisms is beyond the scope of the present work and requires further investigation.

SUPPLEMENTARY NOTE 3: ANALYSIS OF RESIDUAL THERMAL CONDUCTIVITY

Our main conclusion supporting the pseudo-point-node scenario is based on the anomaly observed in the field dependence of the residual thermal conductivity. Since the extrapolation to $T \rightarrow 0$ inevitably introduces some uncertainty, and different fitting forms may slightly modify the apparent curvature, we additionally performed an alternative analysis using a T^2 -linear fit motivated by the clear T^2 dependence of κ_b/T at zero field (see Fig. 2b of the main text), alongside the polynomial fit used in the main text. This procedure allows us to evaluate the stability of the extrapolated residual term with respect to the choice of fitting function. As shown in Supplementary Figure 3, overall field dependence and the kink anomaly in κ_b^0/T remain robust against the choice of fitting procedure, reinforcing the intrinsic nature of the threshold behavior. In addition, we examined the sensitivity of the extrapolated residual term to the choice of fitting temperature range. Although the absolute value shows a systematic variation with fitting window, the threshold-like behavior remains clearly visible within this uncertainty (Supplementary Figure 3c).



Supplementary Figure 3 | Field-dependent residual term using a T^2 -linear fit a,b Same as Figs. 4b,c of the main text, but the residual term is extracted using linear fits of κ_b/T as a function of T^2 . The overall field dependence and the threshold-like kink remain clearly visible. c Comparison of the extrapolated residual term for sample #B with $H \parallel a$ obtained using different fitting temperature ranges. Error bars represent the systematic variation associated with the choice of fitting temperature range, rather than random statistical uncertainty.



Supplementary Figure 4 | Low-field behavior of the residual thermal conductivity for $\mathbf{H} \parallel c$. The gray arrow marks the relative threshold field determined for sample #B with $\mathbf{H} \parallel a$, shown here for comparison.

Based on the relation $H^*/H_{c2} = a^{-2}(\Delta_{\min}/\Delta_0)^2$, the threshold field for $\mathbf{H} \parallel c$ is also expected to occur at a comparable value of H/H_{c2} as for $H \parallel a$. However, the field resolution for $\mathbf{H} \parallel c$ was insufficient to resolve a possible low-field threshold feature. The measurements on Sample #A were performed using a single-axis rotator, and the field steps along the c direction were not optimized for detecting such a small onset field.

Despite this limitation, the available data do not contradict the expected onset scale and remain broadly consistent with it within experimental uncertainty (Supplementary Figure 4). The slightly larger increase of the residual term for $\mathbf{H} \parallel c$ compared with $\mathbf{H} \parallel a$ may reflect the quasi-two-dimensional character of the Fermi surface. Higher-resolution measurements for $\mathbf{H} \parallel c$ would provide a useful future test of the pseudo-point-node scenario.

Regarding the justification of the subtraction procedure, the background field dependence as a function of $H/H_{c2}^{a,b}$ need not be strictly identical for the two field orientations, since effects such as Fermi-surface anisotropy, field and temperature dependence of the vortex-core size, and details of the vortex-state structure may all contribute. The exact weak field dependence of the residual thermal conductivity far below H_{c2} in a fully gapped supercon-

ductor is therefore itself a difficult problem, and a fully consistent theoretical treatment that accounts for all of these effects on equal footing is challenging. Nevertheless, below the threshold field, where the Doppler energy remains smaller than the gap minimum and nodal quasiparticles are not yet significantly activated, the dominant field response is expected to arise mainly from generic mixed-state effects rather than orientation-selective nodal excitations. In this regime, it is therefore reasonable as a first approximation to expect a similar smooth low-field background evolution for the two field orientations. Under this assumption, subtracting the $\mathbf{H} \parallel b$ data provides a useful phenomenological way to suppress the common background contribution and highlight the anisotropic onset feature associated with nodal quasiparticles more clearly.

SUPPLEMENTARY NOTE 4: CALCULATION OF THERMAL CONDUCTIVITY

Guided by the calculations showing that the Fermi surface of UTe_2 is nearly square in the a - b plane, we assume a “box” Fermi surface with a quasipoint node described by $\Delta^2(k) = \Delta_{\min}^2 + v_{\Delta}^2 k^2$ with k the distance to the node on the face of the box. The projection of this Fermi surface to the a - b plane is shown in Supplementary Figure 5. Given that we are interested only in low-energy behavior, we focus on the segments of the Fermi surface normal to k_b , and adopt the low energy description similar to that of Ref.², where we introduce a high momentum cutoff $v_{\Delta}^2 k_0^2 + \Delta_{\min}^2 = \Delta_0^2$, where Δ_0 is the gap maximum. The isotropic impurity scattering is determined in the self-consistent T -matrix approximation parameterized by the scattering rate Γ and the phase shift of scattering $c = \cot \delta$. Within this approximation the self-consistency equation for the impurity-renormalized frequency is³

$$\tilde{\omega} = \omega + \Gamma \frac{g_0}{c^2 - g_0^2}, \quad (\text{S1})$$

with the momentum-integrated (local) Green’s function

$$g_0(\tilde{\omega}) = -i \left\langle \frac{\tilde{\omega}}{\sqrt{\tilde{\omega}^2 - \Delta^2(k)}} \right\rangle, \quad (\text{S2})$$

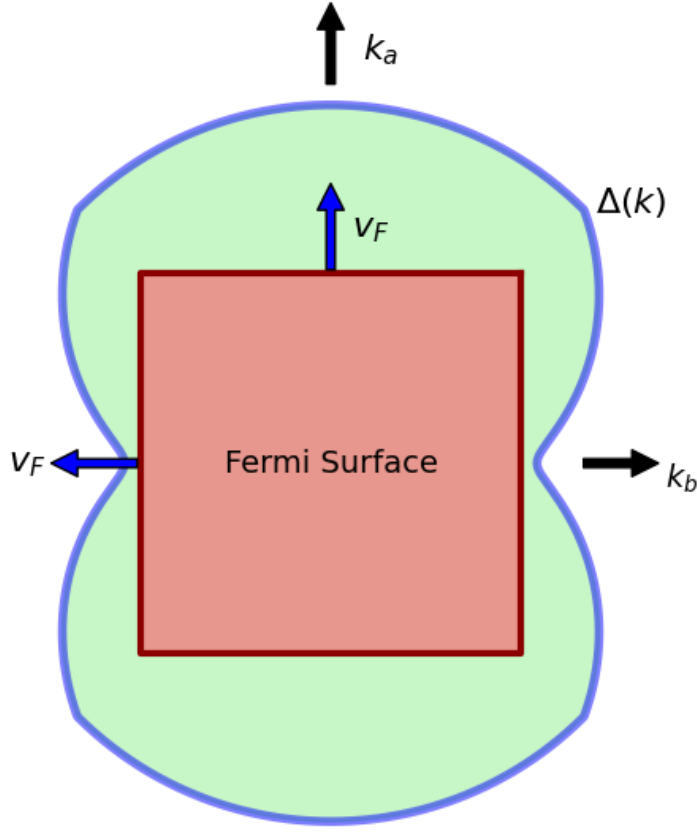
and $\langle \cdots \rangle$ denotes the normalized Fermi surface average. Once the renormalized frequency is determined self-consistently, we define the thermal conductivity kernel

$$\kappa_c(k, \tilde{\omega}) = \frac{1}{2} \frac{1}{\text{Re} \sqrt{\Delta^2(k) - \tilde{\omega}^2}} \left[1 + \frac{\tilde{\omega}^2 - \Delta^2(k)}{|\Delta^2(k) - \tilde{\omega}^2|} \right] \quad (\text{S3})$$

Now the low temperature thermal conductivity for each of the two nodal patches as a function of temperature is given by (we set $\hbar = k_B = 1$)

$$\frac{\kappa}{N_0 v_F^2 T} = \frac{N_0 v_F^2}{2} \int_0^\infty \frac{d\omega}{T} \left(\frac{\omega}{T} \right)^2 \langle \kappa_c(k, \tilde{\omega}) \rangle, \quad (\text{S4})$$

where the flatness of the Fermi surface implies that the direction of the Fermi velocity is independent of the location on the patch.



Supplementary Figure 5 | Sketch of the Fermi surface and the gap function for the theoretical model used in the calculations.

We account for the effect of the magnetic field in the semiclassical Doppler shift picture, where the bare energy of a quasiparticle with momentum $\mathbf{k}_F$ at the Fermi surface located at position $\mathbf{r}$ in real space is shifted by $m\mathbf{v}_F(\mathbf{k}_F) \cdot \mathbf{v}_s(\mathbf{r})$, where $\mathbf{v}_s(\mathbf{r})$ is the local supervelocity due to vortices. For the field along the nodal direction in our geometry there is no Doppler shift since the quasiparticle velocity is normal to the supercurrent field due to vortices. In the

commonly taken approximation of a circular unit cell of the vortex lattice^{4,5} the quasiparticle energy on the two segments of interest is locally shifted, $\omega \rightarrow \omega \pm \epsilon(r, \phi)$ with

$$\epsilon(r, \phi) = \frac{E_H}{\rho} \cos \phi, \quad (\text{S5})$$

where $\rho = r/R$ is the distance from the center of the vortex normalized to the average intervortex distance, $R = a\xi_0\sqrt{H_{c2}/H}$, with $a \simeq 1$. The real space angle ϕ describes winding of the supercurrent around the vortex core. The energy scale $E_H = \eta\Delta_0\sqrt{H/H_{c2}}$ with $\eta \sim 1$ is the minimal Doppler shift in the vortex unit cell (at $r = R$). The Doppler shift approximation is valid only outside of the vortex core, i.e. for $\rho \geq E_H/\Delta_0$.

We now define the local Green's function with the Doppler shift, and recalculate the impurity scattering rate and the local value of the thermal conductivity kernel averaged over the Fermi surface κ_c . At $T = 0$ we set $\omega' = \pm\epsilon(r, \phi)$ on the two patches of the Fermi surfaces, see Supplementary Figure 5, and perform the self-consistency loop. The sum of the contributions of the two patches defines the local thermal conductivity, $\kappa(\mathbf{r})$. In the spirit of Ref. 5, we define the conductivity normal to the vortices via the analogy with resistors in series,

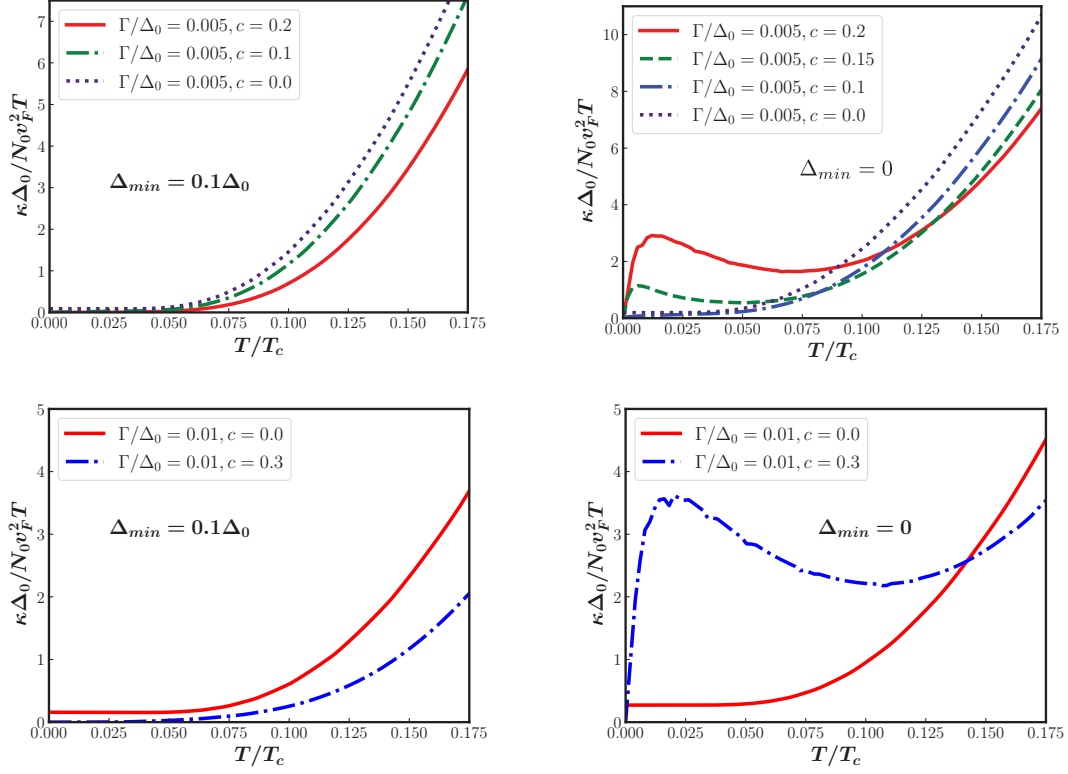
$$\kappa(H) = \left[\frac{1}{\pi R^2} \int_C d^2\mathbf{r} \frac{1}{\kappa(\mathbf{r})} \right]^{-1}, \quad (\text{S6})$$

where the integral is taken over a unit cell of the vortex lattice (assumed circular), with the core excluded.

SUPPLEMENTARY NOTE 5: IMPURITY SCATTERING AND THE THERMAL AND TRANSPORT PROPERTIES.

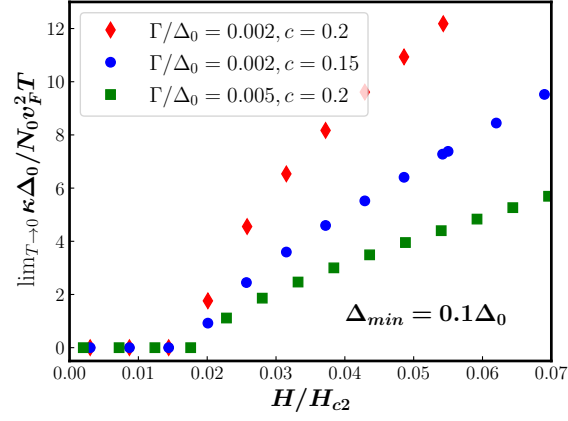
We performed calculations of the temperature dependence of the thermal conductivity and the heat capacity as a function of temperature for different values of the order parameters, with the representative results shown in Supplementary Figure 6. For true point nodes moving away from the unitary scattering yields a low-temperature peak in κ/T . For the unitary scattering generically a finite residual linear term in the thermal conductivity is observed.

Supplementary Figure 7 shows the field dependence of the residual term in the thermal conductivity, with the kink field essentially independent of the details of scattering in the

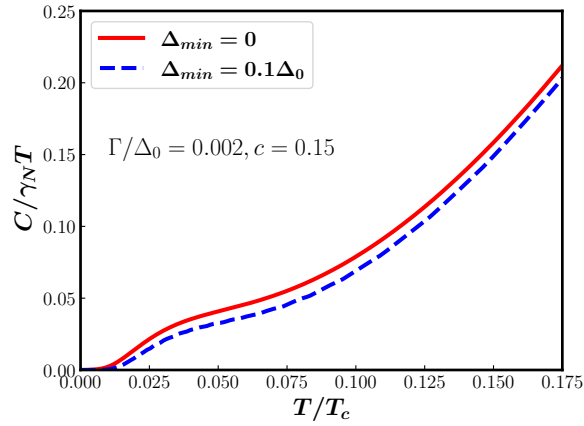


Supplementary Figure 6 | Thermal conductivity as a function of temperature in the presence of a minimal gap (left column) and for true point nodes (right column) for different values of the scattering rate and phase shift.

clean limit. Supplementary Figure 8 demonstrates that the temperature dependence of the heat capacity is insensitive to the opening of the small gap.



Supplementary Figure 7 | Thermal conductivity as a function of field for $\Delta_{min} = 0.1\Delta_0$ and different scattering rate in the clean limit.



Supplementary Figure 8 | Heat capacity, normalized to the Sommerfeld constant in the normal state, $\gamma_N = \pi^2 N_0/3$, as a function of temperature for point nodes and for $\Delta_{min} = 0.1\Delta_0$.

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