

Supplementary Information for
“Partial coherence control delivers skyrmionic topological
resilience and transitions”

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S1. ANALYTICAL CALCULATION FOR THE PROPAGATION OF A PARTIALLY COHERENT SKYRMION BEAM

In this section, we derive analytical expressions for the propagation of a partially coherent skyrmion beam through a paraxial linear ABCD optical system. The input cross-spectral density (CSD) matrix is given by Eq. (3) in the main text, $\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2) = \mathbf{E}^*(\mathbf{r}_1)\mathbf{E}^T(\mathbf{r}_2)\mu(\mathbf{r}_1, \mathbf{r}_2)$. Here we consider a Gaussian degree of spatial coherence $\mu(\mathbf{r}_1, \mathbf{r}_2) = \exp[-(\mathbf{r}_1 - \mathbf{r}_2)^2/(2\delta_0^2)]$, where δ_0 controls the transverse coherence width. The deterministic vector field $\mathbf{E}(\mathbf{r})$ is synthesized from a left-circularly polarized LG_0^0 (Gaussian) mode and a right-circularly polarized LG_0^1 mode,

$$\mathbf{E}(\mathbf{r}) = a_1 \text{LG}_0^0(\mathbf{r}) \hat{\mathbf{e}}_l + a_2 e^{i\gamma} \text{LG}_0^1(\mathbf{r}) \hat{\mathbf{e}}_r, \quad (\text{S1})$$

where $a_{1,2} \geq 0$ set the relative modal weights (amplitude ratio $\eta = a_2/a_1$), γ is the relative phase, and $\hat{\mathbf{e}}_{r,l}$ are unit vectors for right- and left-circular polarization.

In the circular polarization basis, the CSD matrix can be written as

$$\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2) = \begin{bmatrix} W_{rr}(\mathbf{r}_1, \mathbf{r}_2) & W_{rl}(\mathbf{r}_1, \mathbf{r}_2) \\ W_{lr}(\mathbf{r}_1, \mathbf{r}_2) & W_{ll}(\mathbf{r}_1, \mathbf{r}_2) \end{bmatrix}, \quad (\text{S2})$$

with elements

$$W_{rr}(\mathbf{r}_1, \mathbf{r}_2) = \frac{4a_2^2}{\pi\omega_0^4} r_1 r_2 e^{i(\theta_1 - \theta_2)} \exp\left(-\frac{r_1^2 + r_2^2}{\omega_0^2} - \frac{|\mathbf{r}_2 - \mathbf{r}_1|^2}{2\delta_0^2}\right), \quad (\text{S3})$$

$$W_{ll}(\mathbf{r}_1, \mathbf{r}_2) = \frac{2a_1^2}{\pi\omega_0^2} \exp\left(-\frac{r_1^2 + r_2^2}{\omega_0^2} - \frac{|\mathbf{r}_2 - \mathbf{r}_1|^2}{2\delta_0^2}\right), \quad (\text{S4})$$

$$W_{rl}(\mathbf{r}_1, \mathbf{r}_2) = \frac{2\sqrt{2} a_1 a_2}{\pi\omega_0^3} e^{-i\gamma} r_1 e^{i\theta_1} \exp\left(-\frac{r_1^2 + r_2^2}{\omega_0^2} - \frac{|\mathbf{r}_2 - \mathbf{r}_1|^2}{2\delta_0^2}\right), \quad (\text{S5})$$

$$W_{lr}(\mathbf{r}_1, \mathbf{r}_2) = W_{rl}^*(\mathbf{r}_2, \mathbf{r}_1), \quad (\text{S6})$$

where (r, θ) denote the polar coordinates in the transverse plane and ω_0 is the waist parameter of the LG modes.

Within the paraxial approximation, the CSD matrix elements at the output plane of an ABCD system are related to those at the input plane via the Collins integral,

$$W_{ab}(\mathbf{r}_1, \mathbf{r}_2) = \iint W_{ab}(\mathbf{r}'_1, \mathbf{r}'_2) H^*(\mathbf{r}'_1, \mathbf{r}_1) H(\mathbf{r}'_2, \mathbf{r}_2) d^2\mathbf{r}'_1 d^2\mathbf{r}'_2, \quad (\text{S7})$$

where $a, b \in \{r, l\}$ and $\mathbf{r}'$ and $\mathbf{r}$ are the transverse position vectors in the input and output planes, respectively. The impulse response function is

$$H(\mathbf{r}', \mathbf{r}) = \frac{-i}{\lambda B} \exp\left(\frac{ikA}{2B} \mathbf{r}'^2 + \frac{ikD}{2B} \mathbf{r}^2 - \frac{ik}{B} \mathbf{r}' \cdot \mathbf{r}\right), \quad (\text{S8})$$

where A , B , and D are the ABCD matrix elements, k is the wavenumber, and λ is the wavelength.

Substituting Eqs. (S3)–(S6) and (S8) into Eq. (S7), and performing the Gaussian integrations, we obtain the polarization matrix at the output plane, $\mathbf{\Phi}(\mathbf{r}) = \mathbf{W}(\mathbf{r}, \mathbf{r})$:

$$\Phi_{rr}(\mathbf{r}) = \frac{4a_2^2}{\pi\omega_0^3\Delta^2} \exp\left(-\frac{2r^2}{\omega_0^2\Delta}\right) \left[r^2 + \frac{2B^2}{k^2\delta_0^2} \left(1 - \frac{2r^2}{\omega_0^2\Delta}\right)\right], \quad (\text{S9})$$

$$\Phi_{ll}(\mathbf{r}) = \frac{2a_1^2}{\pi\omega_0^2\Delta} \exp\left(-\frac{2r^2}{\omega_0^2\Delta}\right), \quad (\text{S10})$$

$$\Phi_{rl}(\mathbf{r}) = \frac{2\sqrt{2}a_1a_2e^{-i\gamma}}{\pi\omega_0^3\Delta^2} \left(A + \frac{iB}{z_R}\right) r e^{-i\theta} \exp\left(-\frac{2r^2}{\omega_0^2\Delta}\right), \quad (\text{S11})$$

$$\Phi_{lr}(\mathbf{r}) = \Phi_{rl}^*(\mathbf{r}), \quad (\text{S12})$$

where

$$\Delta = A^2 + \frac{B^2}{z_R^2} + \frac{\omega_0^2 B^2}{z_R^2 \delta_0^2}, \quad (\text{S13})$$

and $z_R = k\omega_0^2/2$ is the Rayleigh range.

The components of the renormalized Stokes vector $\bar{\mathbf{S}}$ are then obtained as

$$\bar{S}_1(\mathbf{r}) = \frac{2|\Phi_{lr}|\cos(\theta - \phi_G + \gamma)}{\sqrt{(\Phi_{rr} - \Phi_{ll})^2 + 4|\Phi_{rl}|^2}}, \quad (\text{S14})$$

$$\bar{S}_2(\mathbf{r}) = \frac{-2|\Phi_{lr}|\sin(\theta - \phi_G + \gamma)}{\sqrt{(\Phi_{rr} - \Phi_{ll})^2 + 4|\Phi_{rl}|^2}}, \quad (\text{S15})$$

$$\bar{S}_3(\mathbf{r}) = \frac{\Phi_{rr} - \Phi_{ll}}{\sqrt{(\Phi_{rr} - \Phi_{ll})^2 + 4|\Phi_{rl}|^2}}, \quad (\text{S16})$$

where θ is the azimuthal coordinate in the output plane and $\phi_G = \tan^{-1}(B/z_R)$ denotes the B -dependent Gouy-phase difference between the LG_0^0 and LG_0^1 components (for free-space propagation, $B = z$). From Eqs. (S14) and (S15) we directly obtain

$$\frac{\bar{S}_2}{\bar{S}_1} = -\tan(\theta - \phi_G + \gamma), \quad (\text{S17})$$

which shows that the vorticity is identical to that of the fully coherent counterpart and is independent of the spatial coherence.

Substituting Eqs. (S9)–(S12) into Eq. (S16) yields the explicit analytical expression for $\bar{S}_3$:

$$\bar{S}_3(\mathbf{r}) = \frac{\frac{2\eta^2}{\omega_0^2\Delta} \left[r^2 + \frac{4B^2}{k^2\delta_0^2} \left(1 - \frac{2r^2}{\omega_0^2\Delta} \right) \right] - 1}{\sqrt{\left(\frac{2\eta^2}{\omega_0^2\Delta} \left[r^2 + \frac{4B^2}{k^2\delta_0^2} \left(1 - \frac{2r^2}{\omega_0^2\Delta} \right) \right] - 1 \right)^2 + \frac{8\eta^2 r^2}{\omega_0^2\Delta^2} \left(A^2 + \frac{B^2}{z_R^2} \right)}}. \quad (\text{S18})$$

In the coherent limit ($\delta_0 \rightarrow \infty$), Eq. (S18) reduces to

$$\bar{S}_3(\mathbf{r}) = 1 - \frac{2\omega_0^2\Delta}{\omega_0^2\Delta + 2\eta^2 r^2}, \quad (\text{S19})$$

which decreases monotonically from -1 at $r = 0$ to $+1$ as $r \rightarrow \infty$ for a fixed propagation distance. In this limit, the skyrmion number is preserved by any paraxial ABCD system, while the skyrmionic texture rotates due to the Gouy-phase term ϕ_G that enters through $\bar{S}_2/\bar{S}_1$.

For finite coherence width, Eq. (S18) shows that $\bar{S}_3 \rightarrow 1$ as $r \rightarrow \infty$, whereas at the beam center

$$\bar{S}_3(r = 0) = \text{sgn} \left[\frac{8\eta^2 B^2}{k^2 \delta_0^2 \omega_0^2 \Delta} - 1 \right], \quad (\text{S20})$$

where $\text{sgn}(\cdot)$ is the sign function. Therefore, the central value of $\bar{S}_3$ depends on the amplitude ratio η , the beam waist ω_0 , the coherence width δ_0 , and the system parameter B . The transition of $\bar{S}_3(0)$ provides a convenient criterion for the onset of topological corruption in propagation. Following this criterion, the preservation condition can be recast as

$$A^2 z_R^2 - \frac{2\omega_0^2}{\delta_0^2} \left(\eta^2 - \frac{\delta_0^2}{2\omega_0^2} - 1 \right) B^2 > 0, \quad (\text{S21})$$

which implies that if $\eta^2 \leq 1 + \delta_0^2/(2\omega_0^2)$ the inequality is satisfied for all B , whereas for larger η the skyrmionic texture becomes corrupted once B exceeds a critical value B_c ,

$$B_c = \frac{A\delta_0 z_R}{\omega_0 \sqrt{2(\eta^2 - \delta_0^2/(2\omega_0^2) - 1)}}. \quad (\text{S22})$$

S2. CONVOLUTION APPROACH FOR BEAM PROPAGATION

Obtaining closed-form analytical expressions for the polarization matrix and Stokes vector of partially coherent skyrmion beams becomes increasingly difficult for general (non-Gaussian) coherence functions. For beams obeying Schell-model correlations, however, i.e.,

when the degree of coherence depends only on the coordinate difference $\mathbf{r}_2 - \mathbf{r}_1$, one can efficiently compute propagation numerically by recasting the problem into a convolution form.

Substituting the Collins kernel in Eq. (S8) into Eq. (S7), the propagated CSD element after an ABCD paraxial system can be written as

$$W_{ab}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{(\lambda B)^2} \iint W_{ab}(\mathbf{r}'_1, \mathbf{r}'_2) \exp \left[-\frac{ikA}{2B} \mathbf{r}'_1{}^2 + \frac{ik}{B} \mathbf{r}'_1 \cdot \mathbf{r}_1 - \frac{ikD}{2B} \mathbf{r}'_1{}^2 \right] \times \exp \left[+\frac{ikA}{2B} \mathbf{r}'_2{}^2 - \frac{ik}{B} \mathbf{r}'_2 \cdot \mathbf{r}_2 + \frac{ikD}{2B} \mathbf{r}'_2{}^2 \right] d^2 \mathbf{r}'_1 d^2 \mathbf{r}'_2, \quad (\text{S23})$$

where $a, b \in \{r, l\}$. For a Schell-model field at the source plane,

$$W_{ab}(\mathbf{r}_1, \mathbf{r}_2) = E_a^*(\mathbf{r}_1) E_b(\mathbf{r}_2) \mu(\mathbf{r}_2 - \mathbf{r}_1), \quad (\text{S24})$$

Eq. (S23) becomes

$$W_{ab}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{(\lambda B)^2} \exp \left[-\frac{ikD}{2B} (\mathbf{r}_1^2 - \mathbf{r}_2^2) \right] \iint A_a^*(\mathbf{r}'_1) A_b(\mathbf{r}'_2) \mu(\mathbf{r}'_2 - \mathbf{r}'_1) \times \exp \left[\frac{ik}{B} (\mathbf{r}'_1 \cdot \mathbf{r}_1 - \mathbf{r}'_2 \cdot \mathbf{r}_2) \right] d^2 \mathbf{r}'_1 d^2 \mathbf{r}'_2, \quad (\text{S25})$$

where we have defined the quadratic-phase-modified fields

$$A_a(\mathbf{r}) = E_a(\mathbf{r}) \exp \left(\frac{ikA}{2B} \mathbf{r}^2 \right), \quad A_b(\mathbf{r}) = E_b(\mathbf{r}) \exp \left(\frac{ikA}{2B} \mathbf{r}^2 \right). \quad (\text{S26})$$

To proceed, we introduce the sum and difference coordinates $\mathbf{r}_s = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and $\mathbf{r}_d = \mathbf{r}_2 - \mathbf{r}_1$, and express $A_{a(b)}$ via its Fourier transform:

$$A_{a(b)}(\mathbf{r}) = \frac{1}{(\lambda B)^2} \iint \tilde{A}_{a(b)} \left(\frac{\mathbf{u}}{\lambda B} \right) \exp \left(-\frac{ik}{B} \mathbf{r} \cdot \mathbf{u} \right) d^2 \mathbf{u}, \quad (\text{S27})$$

where $A_{a(b)}(\mathbf{r})$ and $\tilde{A}_{a(b)}(\mathbf{u}/\lambda B)$ form a Fourier-transform pair.

Since the polarization properties at the output plane are fully determined by the polarization matrix $\Phi_{ab}(\mathbf{r}) = W_{ab}(\mathbf{r}, \mathbf{r})$, we set $\mathbf{r}_1 = \mathbf{r}_2 = \mathbf{r}$ (i.e., $\mathbf{r}_d = 0$) in Eq. (S25) and obtain a reduced expression. Using the identity $\int \exp(i \mathbf{q} \cdot \mathbf{x}) d^2 \mathbf{x} = (2\pi)^2 \delta(\mathbf{q})$, integration over the intermediate variables yields

$$\Phi_{ab}(\mathbf{r}) = \frac{1}{(\lambda B)^4} \iint \tilde{A}_a^* \left(\frac{\mathbf{u}}{\lambda B} \right) \tilde{A}_b \left(\frac{\mathbf{u}}{\lambda B} \right) \mu(\mathbf{r}_d) \exp \left[\frac{ik}{B} (\mathbf{u} + \mathbf{r}) \cdot \mathbf{r}_d \right] d^2 \mathbf{r}_d d^2 \mathbf{u}. \quad (\text{S28})$$

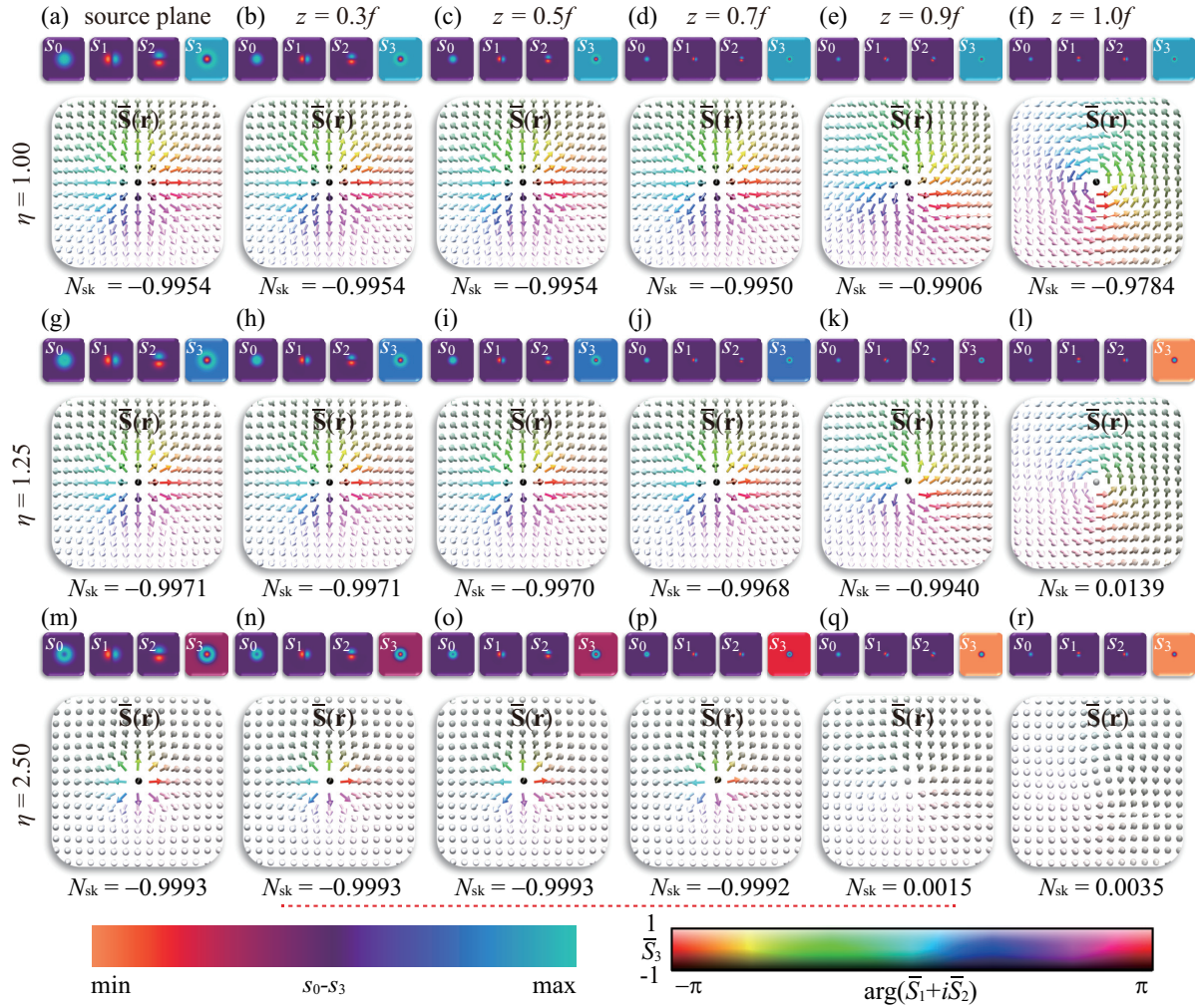
We then introduce the power spectrum $p(\cdot)$ as the Fourier transform of the degree of coherence:

$$p \left(-\frac{\mathbf{u} + \mathbf{r}}{\lambda B} \right) = \frac{1}{(\lambda B)^2} \int \mu(\mathbf{r}_d) \exp \left[\frac{ik}{B} (\mathbf{u} + \mathbf{r}) \cdot \mathbf{r}_d \right] d^2 \mathbf{r}_d, \quad (\text{S29})$$

which leads to the desired convolution form for the polarization matrix elements,

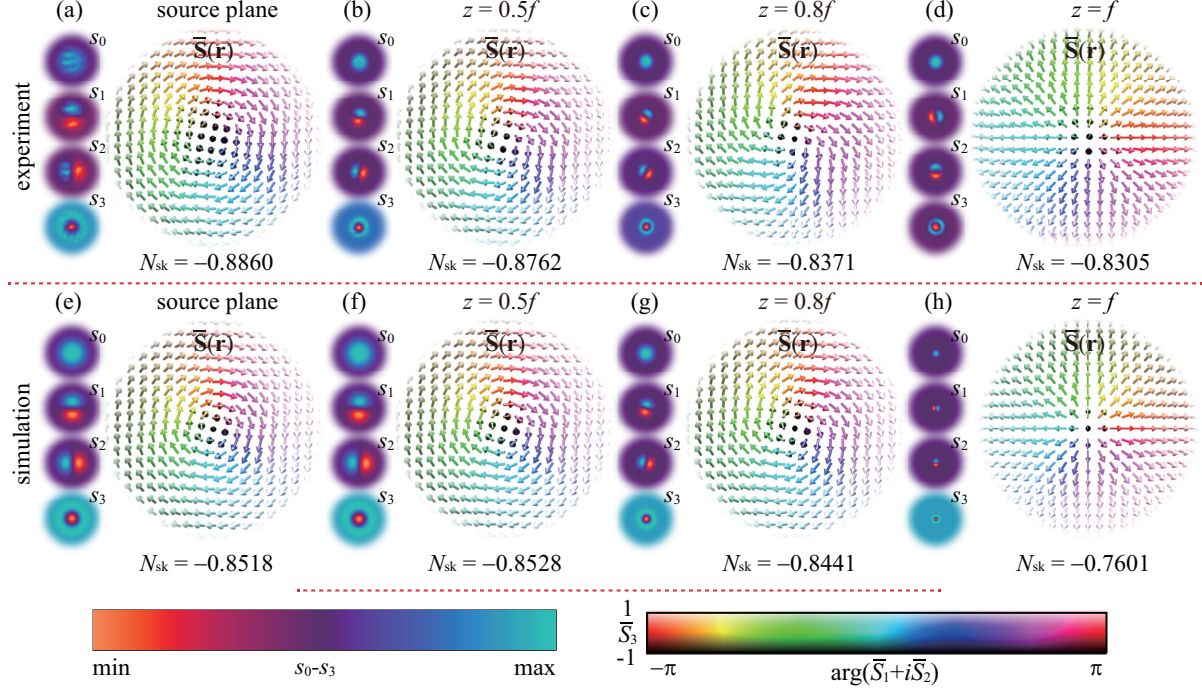
$$\Phi_{ab}(\mathbf{r}) = \frac{1}{(\lambda B)^2} \left[\tilde{A}_a^*(\mathbf{r}/\lambda B) \tilde{A}_b(\mathbf{r}/\lambda B) \right] \otimes p(\mathbf{r}/\lambda B), \quad (\text{S30})$$

where $\otimes$ denotes a two-dimensional convolution. Equation (S30) provides an efficient numerical route for propagating partially coherent skyrmion beams with arbitrary Schell-model coherence functions.



S3. PARTIALLY COHERENT SKYRMION BEAM IN FREE SPACE

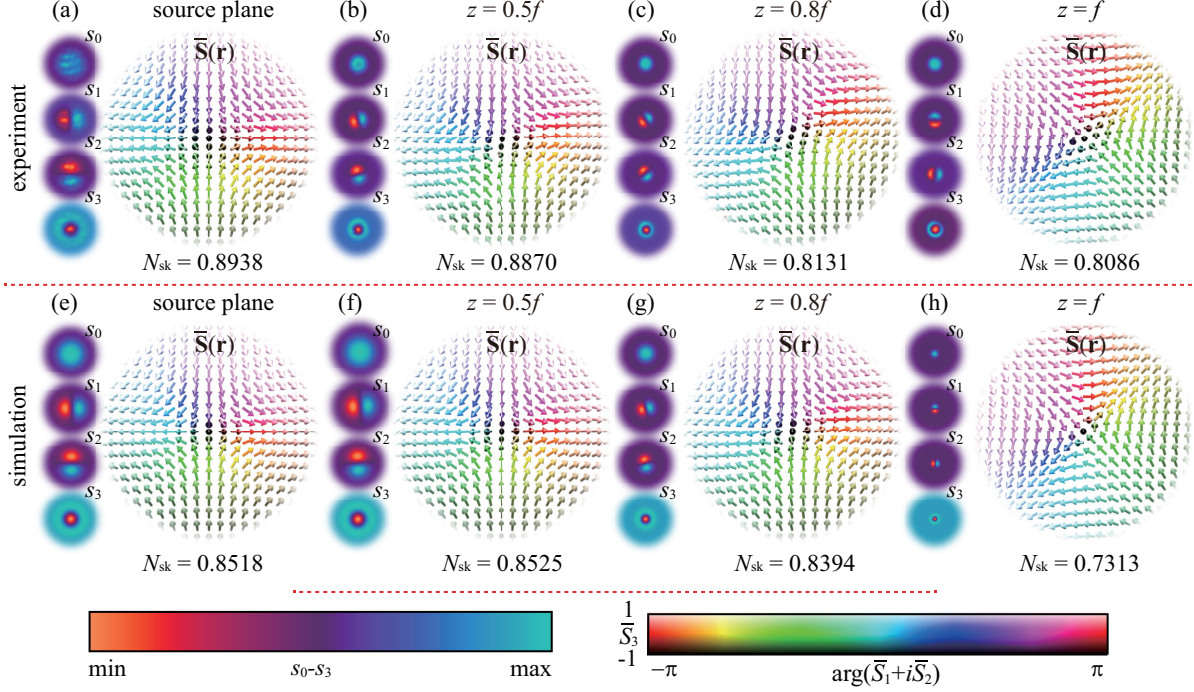
In this section, we present numerical results obtained from the analytical expressions and the convolution approach described in Secs. S1 and S2. The corresponding experimental results are also present in the supplemental material or in the main text.



Supplementary Figure 2: Propagation dynamics of a partially coherent skyrmion beam whose left- and right-circular components are partially coherent LG_0^0 and LG_0^1 modes (Gaussian degree of coherence). (a)–(d) Experimental results and (e)–(h) simulation results. Parameters: $\gamma = \pi/2$, $\lambda = 532$ nm, $\omega_0 = 1.0$ mm, $\delta_0 = 1.0$ mm, and $\eta = 1$.

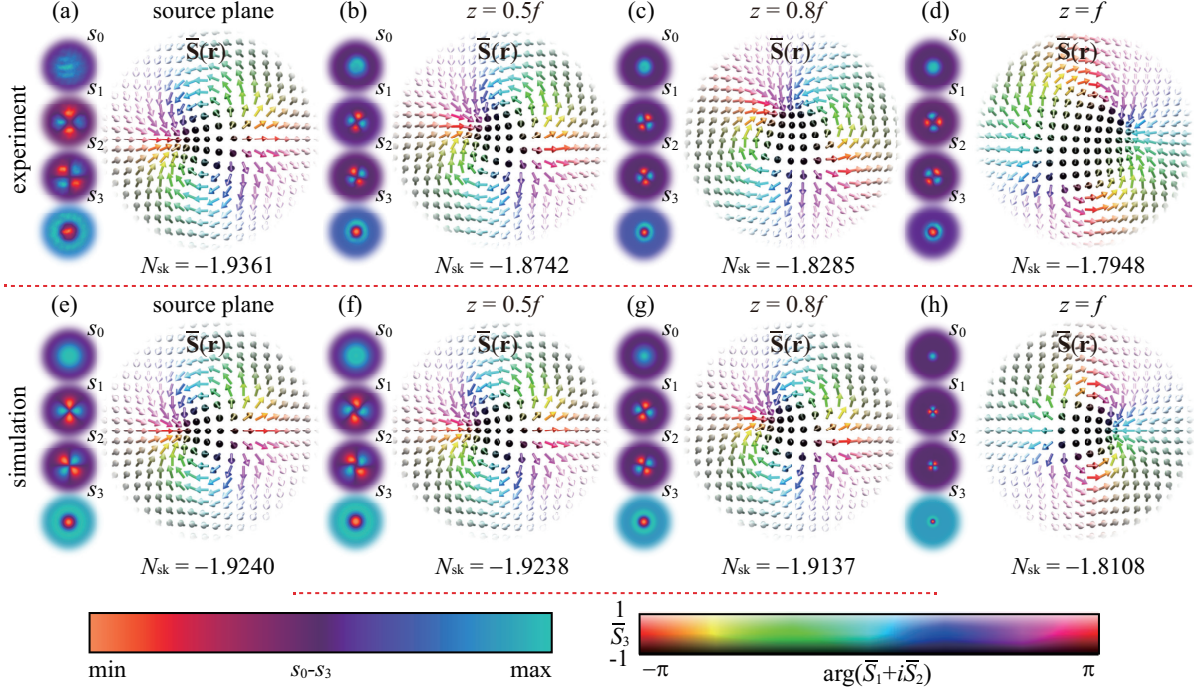
A. Effect of amplitude ratio

We first investigate how the amplitude ratio η affects the propagation of partially coherent skyrmion beams under focusing by a thin lens. In this case, the ABCD parameters are $A = 1 - z/f$, $B = z$, and $D = 1$, where f denotes the focal length of the lens. Supplementary Figure 1 shows simulation results for the evolution of the unnormalized Stokes parameters s_0, s_1, s_2, s_3 , and the normalized Stokes texture $\bar{\mathbf{S}}$ for $\eta = 1.00$ [Supplementary Figs. 1(a)–(f)], $\eta = 1.25$ [Supplementary Figs. 1(g)–(l)], and $\eta = 2.50$ [Supplementary Figs. 1(m)–



Supplementary Figure 3: Propagation dynamics of a partially coherent skyrmion beam whose left- and right-circular components are partially coherent LG_0^0 and LG_0^{-1} modes (Gaussian degree of coherence). (a)–(d) Experimental results and (e)–(h) simulation results. Parameters: $\gamma = 0$, $\lambda = 532$ nm, $\omega_0 = 1.0$ mm, $\delta_0 = 1.0$ mm, and $\eta = 1$.

(r)]. For $\eta = 1.00$, the source plane exhibits a Néel-type skyrmion texture with $N_{\text{sk}} = -0.9954$ (evaluated within the same finite measurement region used throughout). During propagation, N_{sk} remains nearly unchanged, while the transverse patterns of s_1 and s_2 rotate clockwise by approximately $\pi/2$ from the source plane to the focal plane, transforming the texture from Néel-type to Bloch-type. This Gouy-phase-driven Néel–Bloch conversion is the same as in the fully coherent counterpart and is essentially insensitive to spatial coherence. By contrast, for larger amplitude ratios the topology can break down: for $\eta = 1.25$, the skyrmion number at $z = f$ approaches zero, and for $\eta = 2.50$ the breakdown occurs already around $z \simeq 0.9f$. In both cases, the corruption is associated with a central sign reversal of $\bar{S}_3$ at $r = 0$ at a critical propagation distance z_c , which changes the effective polarity of the texture and prevents the Stokes mapping from covering the full parameter sphere.

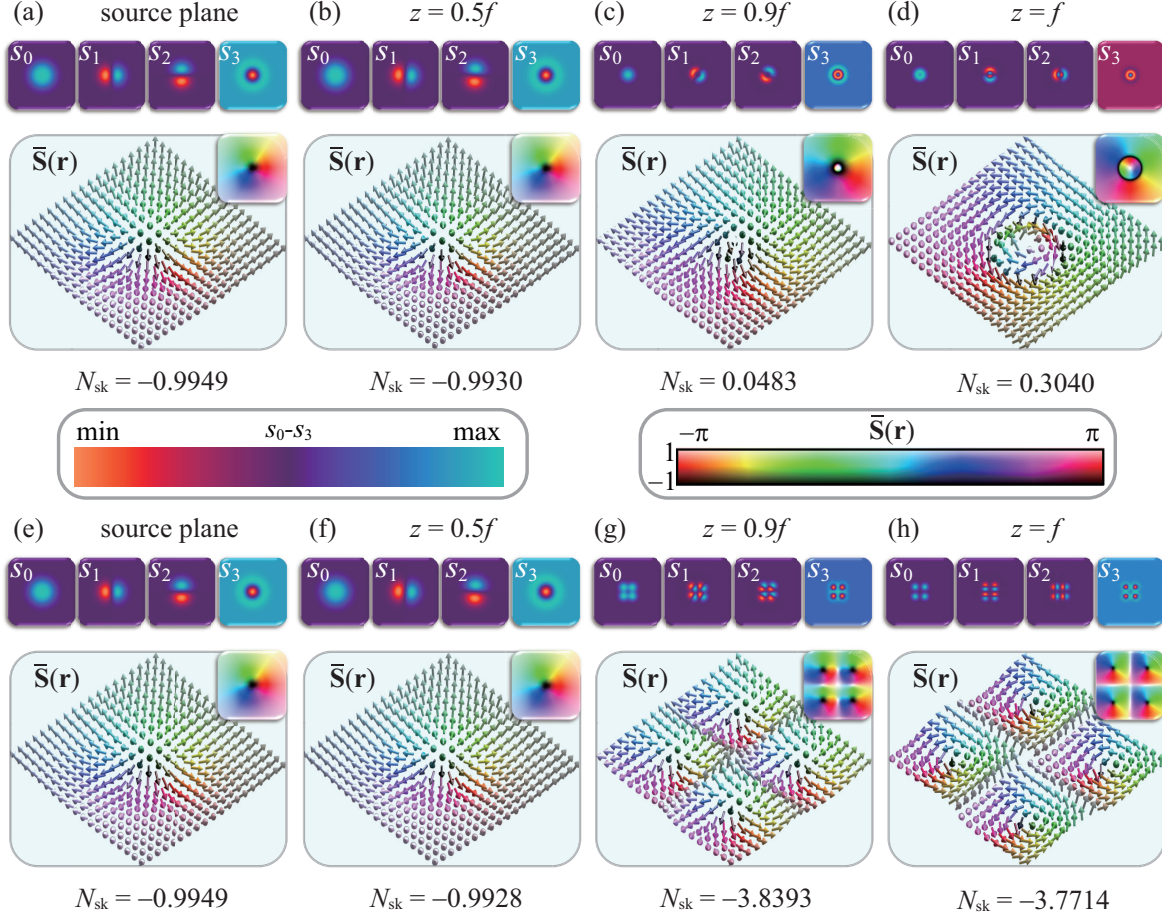


Supplementary Figure 4: Propagation dynamics of a partially coherent skyrmion beam whose left- and right-circular components are partially coherent LG_0^0 and LG_0^2 modes (Gaussian degree of coherence). (a)–(d) Experimental results and (e)–(h) simulation results. Parameters: $\gamma = 0$, $\lambda = 532$ nm, $\omega_0 = 1.0$ mm, $\delta_0 = 1.0$ mm, and $\eta = 1$.

B. Effect of polarization state

We next examine the propagation of three additional partially coherent skyrmion beams (Gaussian degree of coherence) corresponding to different input polarization textures. Experimental results and the corresponding numerical simulations are compared in Supplementary Figs. 2–4.

In Supplementary Fig. 2, the field is synthesized from a right-circular LG_0^1 component and a left-circular LG_0^0 component with $\eta = 1$ and relative phase $\gamma = \pi/2$. In Supplementary Fig. 3, the right-circular component is LG_0^{-1} with $\gamma = 0$ and $\eta = 1$, while in Supplementary Fig. 4 the right-circular component is LG_0^2 with $\gamma = 0$ and $\eta = 1$. In all cases, the simulations agree well with the experimental observations. The results further confirm that Gouy-phase accumulation drives systematic texture rotations: for example, the Bloch-type texture in Supplementary Fig. 2 evolves into a Néel-type texture in the far field (focal plane), whereas the anti-Néel-type texture in Supplementary Fig. 3 evolves into an anti-Bloch-type texture.

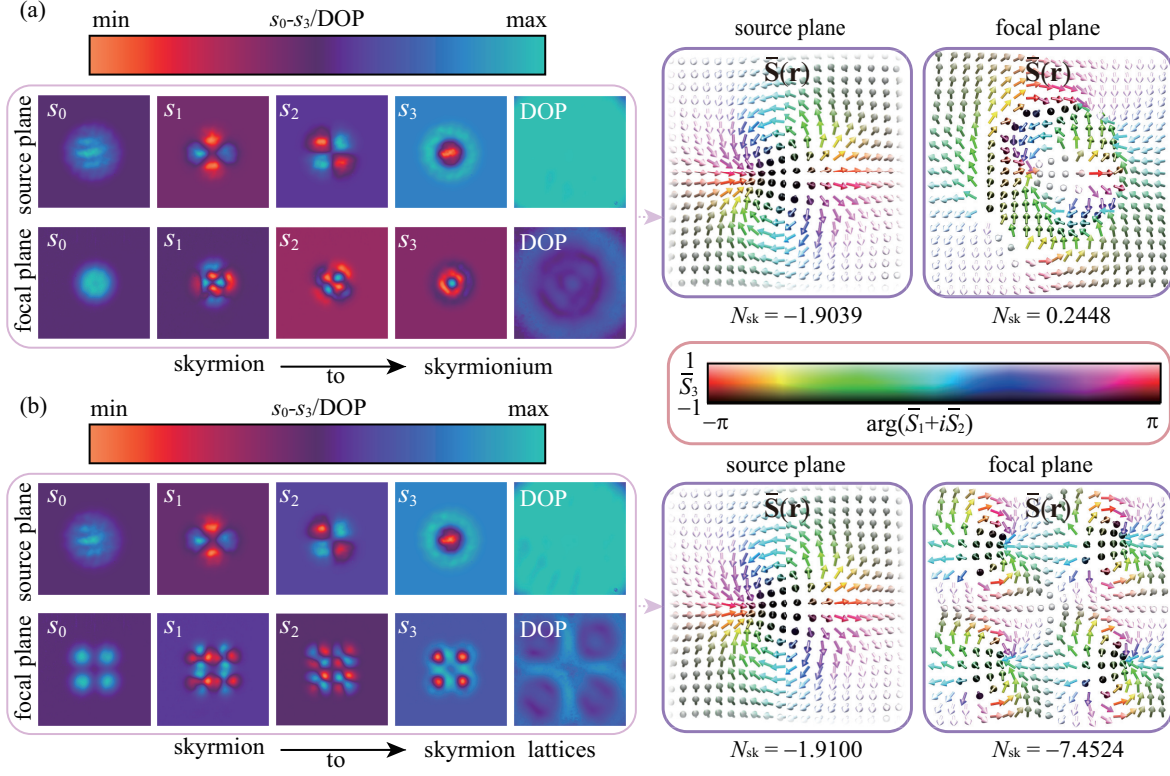


Supplementary Figure 5: Simulation results for coherence-driven topological transitions of partially coherent optical skyrmions enabled by non-Gaussian spatial correlations. (a)–(d) Unnormalized Stokes parameters and the corresponding normalized Stokes texture during propagation for partially coherent skyrmion beams with an LG-type coherence function. (e)–(h) Same as (a)–(d) but for an HG-type coherence function. Beam parameters are the same as in Fig. 6 of the main text.

Likewise, the second-order Néel-type texture in Supplementary Fig. 4 evolves into a second-order Bloch-type texture.

C. Effect of spatial coherence structure

Supplementary Figure 5 presents numerical results for coherence-driven topological transitions of partially coherent skyrmion beams engineered with non-Gaussian spatial correlations. The corresponding experimental results are shown in Fig. 5 of the main text. Supple-



Supplementary Figure 6: Experimental results for coherence-driven topological transitions of partially coherent optical skyrmions enabled by non-Gaussian spatial correlations. The initial left- and right-circular component modes are partially coherent LG_0^0 and LG_0^2 , respectively. (a) LG-type coherence induces a transition from a second-order skyrmion at the source to a skyrmionium at the focal plane. (b) HG-type coherence induces controlled splitting into an ordered lattice of four second-order skyrmions at the focal plane.

Parameters: $\gamma = 0$, $\lambda = 532$ nm, $\omega_0 = 1.0$ mm, $\delta_0 = 0.5$ mm, and $\eta = 1$.

mentary Figures 5(a)–(d) show the evolution under an LG-type coherence function, where an initial Néel-type skyrmion transforms into a skyrmionium upon propagation. In the focal plane, the skyrmionium manifests as two nested Bloch-type textures. Supplementary Figures 5(e)–(h) show the corresponding evolution under an HG-type coherence function, where the skyrmion texture controllably splits into an ordered skyrmion lattice. Both results consistent with the experimental observations in the main text.

Supplementary Figure 6 shows experimental measurements of the Stokes parameters, degree of polarization (DOP), and normalized Stokes texture for partially coherent skyrmion beams with LG- and HG-type coherence functions at the source and focal planes. Here the

left- and right-circular components are encoded by partially coherent LG_0^0 and LG_0^2 modes. The LG coherence drives a topological transition from a second-order skyrmion at the source to a skyrmionium at the focal plane [Supplementary Fig. 6(a)]. In contrast, the HG coherence causes the initial second-order Néel-type skyrmion to split into an ordered lattice of four identical second-order Bloch-type skyrmions at the focal plane [Supplementary Fig. 6(b)]. These results highlight the versatility of structured spatial coherence for engineering complex topological polarization textures.

S4. SKYRMION BEAMS IN THE TURBULENT ATMOSPHERE

The multi-phase-screen method is a widely used wave-optics approach for simulating light propagation through atmospheric turbulence [1, 2]. A schematic of the computational model is shown in Supplementary Fig. 7(a), where a continuous turbulent path is approximated by a sequence of thin random phase screens separated by free-space propagation segments. Let N be the number of phase screens and z the total propagation distance between the transmitter and receiver; the spacing between adjacent screens is then $\Delta z = z/N$.

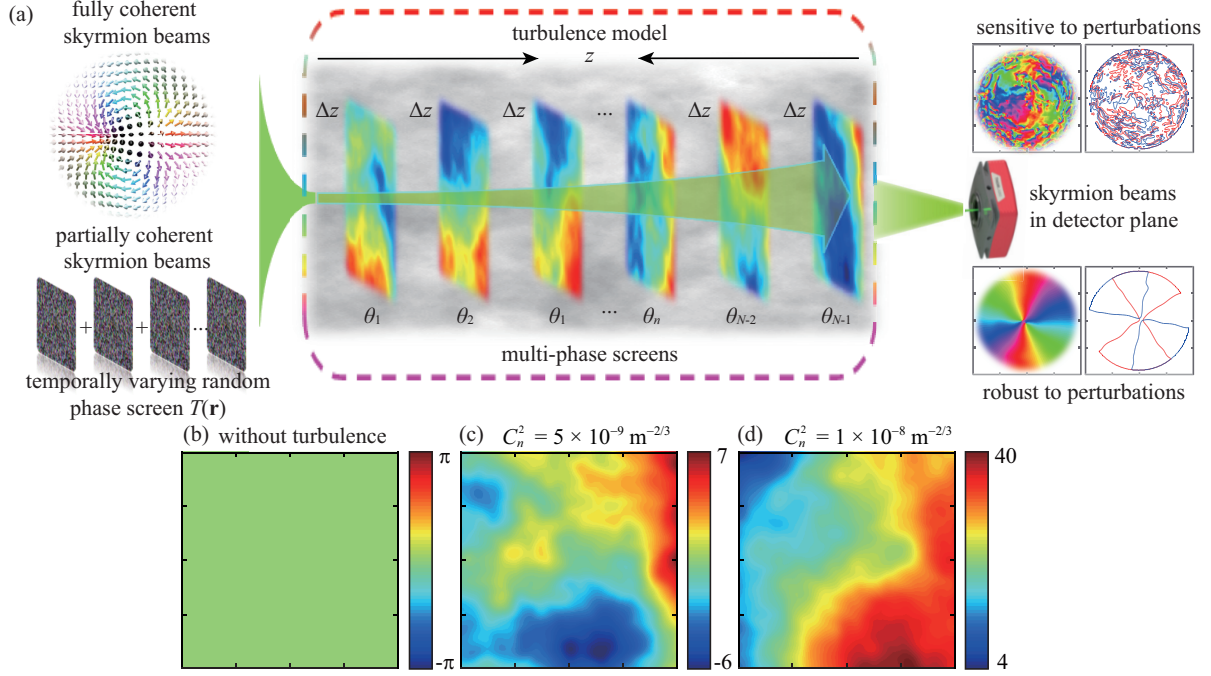
For fully coherent (deterministic) Stokes skyrmion beams, the Stokes parameters at the receiver plane can be directly computed from the propagated vector field using standard Fourier-based Fresnel propagation. For partially coherent skyrmion beams, however, three characteristic timescales become relevant: the fluctuation time of the source realizations τ_s , the turbulence evolution time τ_t , and the detector integration time τ_d . In our simulations we assume

$$\tau_s \ll \tau_d \ll \tau_t, \quad (\text{S31})$$

so that the detector averages over many source realizations while the turbulence can be treated as quasi-static during a single measurement. In other words, the detector does not resolve the rapid field fluctuations introduced by the partially coherent source, but it does capture changes induced by different turbulence realizations.

To implement the propagation of a partially coherent skyrmion beam, we represent the source CSD matrix as an incoherent superposition of random vector-field realizations [3, 4],

$$W_{ab}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{m=1}^M \Phi_{am}^*(\mathbf{r}_1) \Phi_{bm}(\mathbf{r}_2), \quad (\text{S32})$$



Supplementary Figure 7: Computational model for fully coherent and partially coherent skyrmion beams propagating through atmospheric turbulence. (a) Schematic of the multi-phase-screen propagation model. (b–d) Representative random phase screens for three turbulence strengths.

where $a, b \in \{r, l\}$ denote the circular polarization components and M is the number of realizations used in the ensemble. Each realization is written as

$$\Phi_{am}(\mathbf{r}) = E_a(\mathbf{r}) T_m(\mathbf{r}), \quad a \in \{r, l\}, \quad (\text{S33})$$

with $\mathbf{E}(\mathbf{r}) = [E_r(\mathbf{r}), E_l(\mathbf{r})]^T$ the deterministic vector field and $T_m(\mathbf{r})$ a complex random screen obeying Gaussian statistics. For Schell-model sources, $T_m(\mathbf{r})$ can be generated via spectral filtering,

$$T_m(\mathbf{r}) = \mathcal{F}^{-1} \left[\sqrt{p(\mathbf{f})} R_m(\mathbf{f}) \right], \quad (\text{S34})$$

where $p(\mathbf{f})$ is the power spectrum [Fourier transform of $\mu(\mathbf{r}_1, \mathbf{r}_2)$ with respect to $\mathbf{r}_1 - \mathbf{r}_2$], $R_m(\mathbf{f})$ is complex Gaussian white noise with zero mean and unit variance, and $\mathcal{F}^{-1}$ denotes the inverse Fourier transform.

The simulation procedure for a given turbulence realization is as follows: (1) Generate in advance a set of N turbulence phase screens $\{\theta_1, \theta_2, \dots, \theta_N\}$ corresponding to successive segments of length Δz . (2) For a given source realization m , propagate the vector field

$\Phi_m(\mathbf{r}) = \mathbf{E}(\mathbf{r})T_m(\mathbf{r})$ through free space over Δz to the first screen, and then apply the phase modulation:

$$\Phi_m^{(1)}(\mathbf{r}, \Delta z) = \tilde{\Phi}_m^{(1)}(\mathbf{r}, \Delta z) \exp[i\theta_1(\mathbf{r})], \quad (\text{S35})$$

where $\tilde{\Phi}_m^{(1)}$ denotes the field immediately before the screen. (3) Repeat the free-space propagation over Δz followed by multiplication by $\exp[i\theta_j(\mathbf{r})]$ for $j = 2, \dots, N$, until the field reaches the receiver plane. (4) After propagating all M source realizations through the same turbulence realization, form the output CSD matrix at the receiver plane as

$$W_{ab}^{(\text{out})}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{m=1}^M \Phi_{am}^{(N)*}(\mathbf{r}_1, z) \Phi_{bm}^{(N)}(\mathbf{r}_2, z), \quad (\text{S36})$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are transverse coordinates in the receiver plane and the superscript (N) indicates the field after the last phase screen.

The turbulence phase screens $\theta_n(\mathbf{r})$ are generated using the von Kármán refractive-index power spectrum

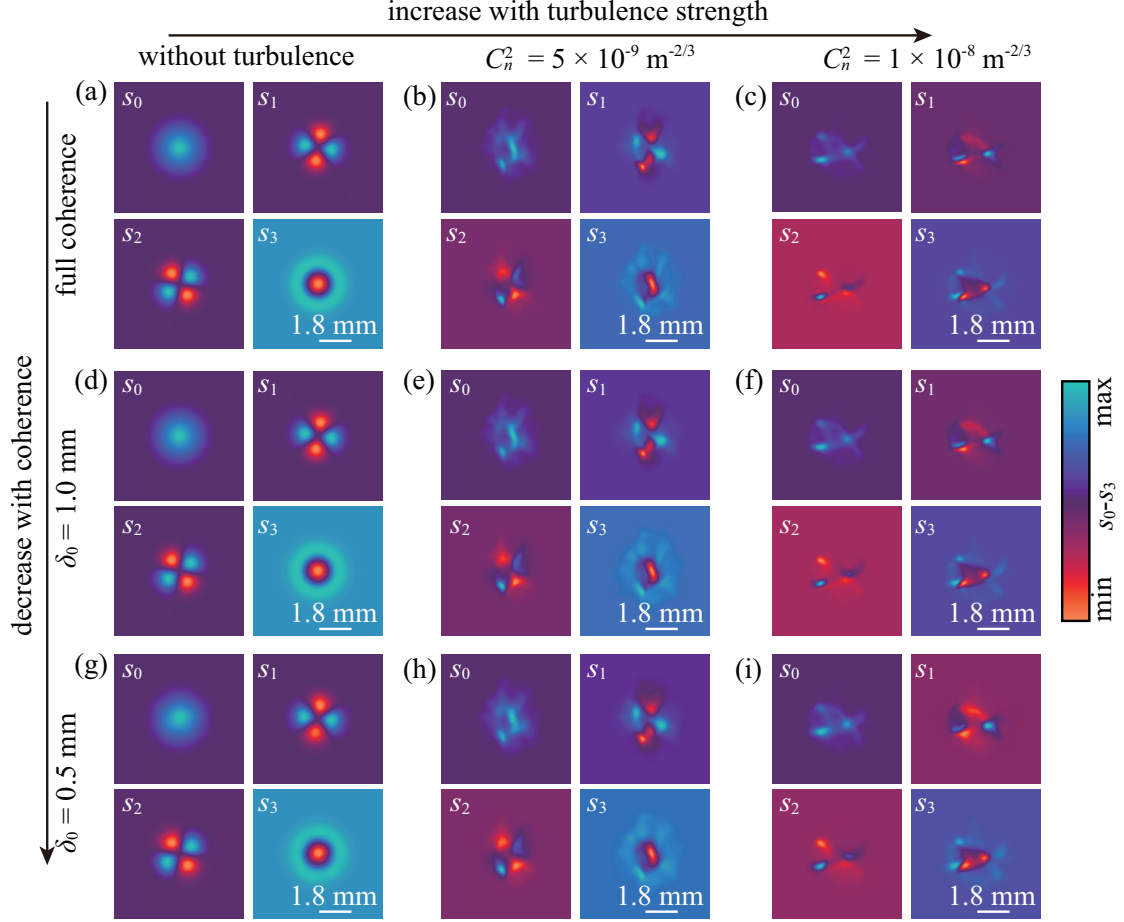
$$\Phi_{\text{RI}}(\kappa) = 0.033 C_n^2 (\kappa^2 + \kappa_0^2)^{-11/6} \exp(-\kappa^2/\kappa_m^2), \quad (\text{S37})$$

where C_n^2 is the refractive-index structure constant (units $\text{m}^{-2/3}$), $\kappa_0 = 2\pi/L_0$, and $\kappa_m = 5.92/l_0$, with L_0 and l_0 being the outer and inner scales, respectively. The relation between the refractive-index power spectrum $\Phi_{\text{RI}}(\kappa)$ and the spectrum $\Phi_\theta(\kappa)$ of the random phase screen is given by $\Phi_\theta(\kappa) = 2\pi\Delta z k^2 \Phi_{\text{RI}}(\kappa)$. The random phase screens are obtained as

$$\theta_n(\mathbf{r}) = \text{Re}\{\mathcal{F}[\sqrt{\Phi_\theta(\kappa)} R_n(\kappa)]\}, \quad (\text{S38})$$

where $R_n(\kappa)$ is complex Gaussian white noise with zero mean and unit variance. Representative phase screens for different C_n^2 are shown in Supplementary Fig. 7(b)–(d), illustrating that the imposed phase varies continuously across the transverse plane.

Supplementary Fig. 8 shows the unnormalized Stokes parameters of fully coherent and partially coherent skyrmion beams at the receiver plane under three different turbulence strengths. It can be observed that, as the turbulence strength increases, the Stokes parameters of both fully coherent and partially coherent skyrmion beams become significantly distorted after propagation through turbulence.



Supplementary Figure 8: Computational results for the unnormalized Stokes parameters of the fully coherent and partially coherent skyrmion beams after propagation through atmospheric turbulence.

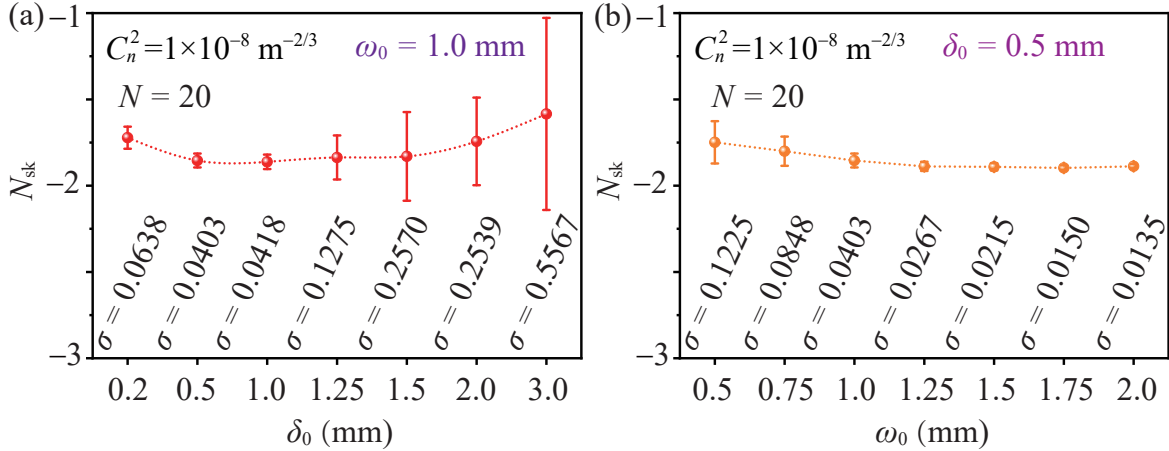
S5. EFFECT OF SPATIAL COHERENCE WIDTH AND BEAM RADIUS ON THE ROBUSTNESS OF THE SKYRMION NUMBER

The turbulence robustness of the skyrmion number is governed by both the initial spatial coherence width δ_0 and the beam radius ω_0 . To clarify this dependence, we present additional numerical results in Supplementary Fig. 9. First, we fix the beam radius at $\omega_0 = 1$ mm and vary the spatial coherence width δ_0 . As shown in Supplementary Fig. 9(a), when δ_0 is smaller than or comparable to the beam radius, the skyrmion number remains highly stable under strong turbulence. As δ_0 becomes larger than ω_0 , the beam contains fewer effectively incoherent spatial modes, and the skyrmion number begins to show larger realization-to-realization fluctuations. When δ_0 is further increased to 3 mm, the behavior approaches

that of a fully coherent beam.

Conversely, we fix the coherence width at $\delta_0 = 0.5$ mm and vary the beam radius ω_0 . As shown in Supplementary Fig. 9(b), the skyrmion number remains stable for larger beam radii, where the relative coherence width δ_0/ω_0 is small. When the beam radius is reduced and becomes comparable to or smaller than δ_0 , the effective modal averaging is weakened and fluctuations of the skyrmion number reappear.

Therefore, coherence-enhanced turbulence robustness is not determined by the absolute value of δ_0 alone. Rather, it is governed by the relative coherence width δ_0/ω_0 . Robust stabilization is obtained when the spatial coherence width is comparable to or smaller than the beam radius, approximately $\delta_0 \lesssim \omega_0$. Under this condition, the partially coherent beam contains a sufficient number of mutually incoherent spatial modes to suppress turbulence-induced singularity proliferation and stabilize the skyrmion number within a single turbulence realization.



Supplementary Figure 9: Effect of coherence width and beam radius on skyrmion-number stability in turbulence. (a) Dependence of the skyrmion number on the spatial coherence width δ_0 under atmospheric turbulence. (b) Dependence of the skyrmion number on the beam radius ω_0 . Other beam and turbulence parameters are the same as those used in Fig. 5 of the main text. The quantity σ denotes the standard deviation of the skyrmion number calculated over $N = 20$ independent turbulence realizations.

S6. DEFINITIONS OF THE SCINTILLATION INDEX AND THE TOTAL ORBITAL ANGULAR MOMENTUM PER PHOTON

To quantitatively characterize turbulence-induced intensity fluctuations, we adopt the scintillation index, defined as

$$\text{SI}(x_i, y_j) = \frac{\langle I^2(x_i, y_j) \rangle}{\langle I(x_i, y_j) \rangle^2} - 1 = \frac{\sum_{n=1}^N I_n^2(x_i, y_j)}{N [\bar{I}(x_i, y_j)]^2} - 1, \quad (\text{S39})$$

where N is the number of turbulence realizations, $i, j = 1, 2, 3, \dots, M$ are the pixel indices of the intensity distribution, and the angle brackets denote ensemble averaging over turbulence-induced intensity fluctuations. Here

$$\bar{I}(x_i, y_j) = \frac{1}{N} \sum_{n=1}^N I_n(x_i, y_j) \quad (\text{S40})$$

is the mean intensity. As indicated by Eq. (S39), the scintillation index is generally spatially dependent. In this work, we evaluate the scintillation index at the centroid of the average intensity distribution in the detector plane, namely $\text{SI} = \text{SI}(0, 0)$.

For a vector field $\mathbf{E}_n(\mathbf{r}, z) = E_{x,n}(\mathbf{r}, z)\hat{\mathbf{e}}_x + E_{y,n}(\mathbf{r}, z)\hat{\mathbf{e}}_y$, the ensemble-averaged total OAM per photon, in units of $\hbar$, is defined as

$$h = \frac{\sum_{n=1}^N \int \text{Im} \left[E_{x,n}^*(\mathbf{r}, z) \frac{\partial E_{x,n}(\mathbf{r}, z)}{\partial \varphi} + E_{y,n}^*(\mathbf{r}, z) \frac{\partial E_{y,n}(\mathbf{r}, z)}{\partial \varphi} \right] d^2\mathbf{r}}{\sum_{n=1}^N \int [|E_{x,n}(\mathbf{r}, z)|^2 + |E_{y,n}(\mathbf{r}, z)|^2] d^2\mathbf{r}}. \quad (\text{S41})$$

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