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Supplementary Note 1. Band structure of acoustic guided mode resonance

To analyze the band structure of the acoustic guided mode resonance in the proposed acoustic topological metagrating model, we consider the complex wavevector of the guided waves, which must satisfy the condition:

$$(k_x + 2\pi n/\Lambda)^2 + k_y^2 = k_0^2 \quad (\text{S1})$$

where k_x and k_y represent the wavevector components in the x and y directions, respectively, and $k_0 = \omega/c_0$ depends on the sound speed in air. For the 1st-order mode (i.e., $n = 1$) and real k_x , the conditions for guided mode resonance can be expressed as follows:

$$|k_x + 2\pi/\Lambda| < k_0, \text{ when } k_y \text{ is a real number} \quad (\text{S2})$$

$$|k_x + 2\pi/\Lambda| > k_0, \text{ when } k_y \text{ is an imaginary number} \quad (\text{S3})$$

Eq. (S2) and Eq. (S3) indicate that the guided mode in the y -direction may exhibit two different propagation modes: a propagating wave mode and an evanescent wave mode. To achieve a propagating wave mode in the y -direction, the necessary condition can be represented as:

$$n_{\text{eff}} > n_0 \quad (\text{S4})$$

where n_{eff} is the effective refractive index of the acoustic grating, and n_0 is the refractive index of air. Meeting this condition requires the medium of the acoustic grating to have a larger effective refractive index (i.e., a smaller sound speed) than air, which poses significant experimental challenges. In fact, in our proposed acoustic topological metagrating model, the effective refractive index of the acoustic grating is much smaller than that of air, making it impossible to support a propagating wave mode in the y -direction. Consequently, only Eq. (S2) can be satisfied. In contrast, in general optical gratings, this refractive index condition is typically met, allowing for the emergence of two distinct guided mode resonance modes, corresponding to Eq. (S2) and Eq. (S3), respectively.

This implies that, for our proposed acoustic metagratings, the guided mode resonances occur below the sound cone. Due to the introduction of an impedance-discontinuous spacer in our acoustic topological metagrating model, the original single band splits into two bands, both of which lie below the sound cone, as shown in Supplementary Fig. 1a. In contrast, for optical gratings, the guided mode resonances can occur both below and above the light cone, naturally resulting in two bands that can open a bandgap, as shown in Supplementary Fig. 1b. These bands exhibit band

inversion and possess distinct topological characteristics (i.e., different Zak phases). Given these differences in band structure, we propose a mechanism of spontaneous symmetry breaking-induced real-space topological phase transition, which does not rely on traditional band topology. This approach overcomes the traditional constraints on the refractive index of the medium, offering a novel pathway for achieving topological effects in acoustic systems.

Supplementary Note 2. Derivation of the effective Hamiltonian for topological metagrating based on rigorous coupled-mode theory

The schematic of the acoustic metagrating is illustrated in [Supplementary Fig. 2](#). The metagrating consists of rectangular bars periodically arranged alongside a spacer in air. The air region is segmented into four parts, and the velocity potentials in these four regions I, II, III, IV are denoted by Φ_1, Φ_2, Φ_3 and Φ_4 , respectively. The acoustic impedance of the grating is significantly higher than that of air, thus the boundary of the slit, i.e., the region between the rectangular scatterers, can be considered as a hard boundary. Since the height of slit is substantially larger than the wavelength, we neglected higher-order decaying waves in the slit region to simplify the guided modes. Consequently, the acoustic field in the four regions can be expressed as follows:

$$\begin{cases} \Phi_1 = \sum_{n=-\infty}^{\infty} A_n \exp(j(k_x + qn)x + jk_y(y - d)), y > d \\ \Phi_2 = U(x)[B_1 \exp(jk_n y) + B_2 \exp(-jk_n y)], 0 < y < d \\ \Phi_3 = U(x)[C_1 \exp(jk_n y) + C_2 \exp(-jk_n y)], -d < y < 0 \\ \Phi_4 = \sum_{n=-\infty}^{\infty} D_n \exp(j(k_x + qn)x + jk_y(y + d)), y < -d, \end{cases} \quad (\text{S5})$$

where k_x and k_y represent the wavevector components of delocalized waves in the x and y directions, respectively, and satisfy the condition $(k_x + 2\pi n/\Lambda)^2 + k_y^2 = k_0^2$. The term k_n denotes the imaginary wavevector component of the localized guided mode inside the slit along the y -direction. The Bragg wavevector is represented as $q = 2\pi/\Lambda$. Based on the rigorous coupled mode theory, we utilize the continuity conditions at the interface between these four regions to derive the coupling among guided modes. To support the SSB, it is essential to facilitate weak coupling between the guided modes on either side of the slit, which is enabled by the spacer composed of high-impedance material. Given that the thickness of the spacer is significantly smaller than that of the grating, it is appropriate to characterize it as a continuity condition described by an

effective transfer impedance calculated as $Z_{\text{eff}} = (Z_u - Z_l)/\rho_0 c_0$, where Z_u and Z_l represent the acoustic impedances of the upper and lower sides, respectively, and ρ_0 and c_0 are the density and sound speed of air. The characteristics of the spacer enable us to approximate that the velocities on both sides are equal, while the velocity potentials are different. Therefore, the continuity condition near the spacer can be represented as:

$$\begin{cases} \frac{jZ_{\text{eff}}}{k_n} \frac{\partial \Phi_2}{\partial y} = \Phi_2 - \Phi_3 \\ \frac{\partial \Phi_2}{\partial y} = \frac{\partial \Phi_3}{\partial y} \end{cases} \quad (\text{S6})$$

Combining all continuity conditions among the four regions, we obtain the resonance condition for guided modes $\psi = (B_1, C_2)^T$:

$$\begin{pmatrix} Z_{\text{eff}}(R_1 - 1)/2 - R_1 & 1 \\ 1 & Z_{\text{eff}}(R_2 - 1)/2 - R_2 \end{pmatrix} \psi = 0. \quad (\text{S7})$$

where $R_1 = B_2/B_1$, $R_2 = C_1/C_2$ depends on the impedance properties of the metagrating surface and represents the proportionality relationship between the guided modes propagating in opposite directions. Under this condition, the system can be described by a Hamiltonian that characterizes the band structures for two distinct symmetries, whereby Eq. (3) reduces to the form of a (1+1)-dimensional static Dirac equation:

$$H\psi = \left(\frac{Z_{\text{eff}}}{Z_{\text{eff}} - 2} - R \right) \psi = E(k_0)\psi, \quad (\text{S8})$$

where the Hamiltonian can be expressed as:

$$H = m\sigma_z + \frac{2}{Z_{\text{eff}} - 2} \sigma_x. \quad (\text{S9})$$

Supplementary Note 3. Experimental realization of topological metagrating in two-dimensional plane

To experimentally validate the numerical simulations presented in Fig. 2c and Fig. 2d, we constructed the metagrating within a planar acoustic waveguide with height equal to the metagrating thickness ($< 1/2\lambda$), ensuring strict two-dimensional confinement of the acoustic field. The metagrating sample is fabricated using ABS plastic, comprising two semi-infinite gratings joined at the interface: the left side has slit width 0.71λ with spacer transfer impedance $|Z_L| = 1200$, while the right side has slit width 0.67λ with $|Z_R| = 120$.

As shown in [Supplementary Fig. 3a](#), to precisely measure the confined wave distribution, the operating frequency is set to 3070 Hz, which is scaled relative to the simulation parameters to accommodate the experimental sample dimensions. The periodicity is $\Lambda = 0.1$ m with 5 periods on each side. The experimental setup employs a line array of speakers positioned beneath the waveguide to generate plane acoustic waves incident normally onto the metagrating. The acoustic pressure amplitude and phase at different positions are measured through small apertures drilled on the top surface of the waveguide. The entire setup is surrounded by absorbing material to eliminate unwanted boundary reflections.

The measurement results substantiate the presence of localized standing waves near the interface along the surface of the metagrating, with the acoustic intensity and phase distributions shown in [Supplementary Fig. 3b](#) and [Supplementary Fig. 3c](#), respectively. The acoustic intensity reaches maximum at the interface and decays exponentially toward both sides, with decay rate consistent with the theoretical prediction. Due to the real-space topological transition, a phase jump of π for the reflected wave is observed near the interface. This phase discontinuity validates the proposed real-space topology where the topological invariant is defined by the sign of the Dirac mass. As shown in [Supplementary Fig. 3d](#), the measured reflected wave phase distribution shows good agreement with simulation results.

Supplementary Note 4. Robustness analysis of acoustic Jackiw-Rebbi states

Due to topological protection, acoustic Jackiw-Rebbi states exhibit inherent robustness. To analyze this robustness, we provide numerical simulations illustrating the relationship between transmittance, structural defects, and material properties. Specifically, we investigate the robustness of Jackiw-Rebbi states against deviations in scatter size, spacer thickness, and the acoustic impedance of the materials.

(a) Robustness against deviations in scatter size

Considering random structural defects that lead to size inconsistencies among the scatters (i.e., the rectangular bars in the metagrating), we assume, without loss of generality, that the lengths L and widths W of the bars follow a normal distribution, given by:

$$L \sim N(L_0, \sigma_1 L_0), W \sim N(W_0, \sigma_1 W_0), \quad (\text{S10})$$

where L and W correspond to the dimensions that enable the Jackiw-Rebbi states in the absence of deviations. The relative standard deviation σ_1 determines the magnitude of

the structural defects. Variations in the scatter size disrupt the periodicity and mirror symmetry of the structure and hinder the guided mode resonance to some extent. As a result, the resonance strength of the acoustic Jackiw-Rebbi state is expected to decrease, which can be observed in the transmission spectrum. As shown in [Supplementary Fig. 4a](#), as σ_1 increases, the central frequency experiences random shifts, and the Q-factor decreases accordingly. Simulation results indicate that a high transmission peak can still be maintained under moderate deviations (with $\sigma_1 < 2\%$), as illustrated in [Supplementary Fig. 4b](#), which demonstrates the robustness of the Jackiw-Rebbi states against deviations in scatter size.

(b) *Robustness against deviations in spacer thickness*

Given that the spacer in the metagrating is very thin, manufacturing errors can lead to uneven acoustic impedance of the spacer. Without loss of generality, we assume this unevenness is caused by deviations in the spacer thickness D , which follows the normal distribution:

$$D \sim N(D_0, \sigma_2 D_0), \quad (\text{S11})$$

where D_0 is the spacer thickness that enables the Jackiw-Rebbi states in the absence of deviations. The relative standard deviation σ_2 determines the degree of unevenness. Variations in the thickness of the spacers disrupt the periodicity of the metagrating, causing shifts in the resonance frequencies of both the even and odd modes, and consequently altering the spectral splitting. However, since the curvature difference between the two bands is sufficiently large, the band crossing is almost unaffected by changes in the spectral splitting, ensuring the persistence of the Jackiw-Rebbi states. To verify this, as shown in [Supplementary Fig. 4c](#), the transmission spectrum shows minimal changes as σ_2 increases. Simulation results, depicted in [Supplementary Fig. 4d](#), indicate that the transmission maximum remains above 0.89 even when σ_2 reaches 12%, confirming the robustness of the Jackiw-Rebbi states against deviations in spacer thickness.

(c) *Robustness against deviations in acoustic impedance of the materials*

The materials used in the metagrating must have sufficiently high acoustic impedance relative to air to approximate an acoustic hard boundary condition. When material properties of the metagrating change, the ratio of acoustic impedance between

the metagrating and air is altered, denoted as:

$$\xi = \rho_1 c_1 / \rho_0 c_0, \quad (\text{S12})$$

where ρ_1 and ρ_0 represent the mass densities of the metagrating material and air, respectively, while c_1 and c_0 denote their respective sound speeds. In the theoretical model, the hard boundary condition assumption means the Jackiw-Rebbi state conditions are independent of this impedance ratio. However, in practical scenarios, variations in ξ can cause slight shifts in the guided mode resonance frequency, which can be observed in numerical simulations. As illustrated in [Supplementary Fig. 4e](#), the overall profile remains largely unchanged, although the central frequency of the Jackiw-Rebbi states decreases with increasing ξ . The simulation results in [Supplementary Fig. 4f](#) show that the transmission maximum remains above 0.92 within the acoustic impedance range ratio of 1600 to 2000, exhibiting the robustness of the Jackiw-Rebbi states against variations in the acoustic impedance of the materials.

(d) *Robustness against deviations in vertical position of the spacer*

The emergence of the Jackiw-Rebbi state requires paired exceptional points with opposite masses across the interface. When the spacer is centered at the symmetry plane ($m_0 = 0$), the non-Hermitian system resides in the PT-symmetry-broken phase that supports the topologically protected zero mode. When the spacer undergoes vertical displacement δy from the centered position, an effective mass bias m_0 is introduced, where m_0 is proportional to the displacement. For small displacements satisfying $|m_0| < \kappa$, the mass still changes sign across the interface and the system remains in the PT-broken phase, preserving the Jackiw-Rebbi state. However, when $|m_0| > \kappa$, both effective masses acquire the same sign, the exceptional points annihilate each other, the system transitions to the PT-unbroken phase, and the Jackiw-Rebbi state disappears. Therefore, the Jackiw-Rebbi state is robust against vertical displacements below the critical threshold. To verify this theoretical prediction, we performed numerical simulations of the transmission spectrum as a function of spacer displacement, where the relative displacement is defined as:

$$\Delta = \delta y / 2d, \quad (\text{S13})$$

where $2d$ represents the total height of the double-well potential. The simulation results demonstrate that when $\Delta < 1\%$, which corresponds to the PT-broken phase, the system maintains high transmittance with the Jackiw-Rebbi state preserved, as shown in

Supplementary Fig. 4g and Supplementary Fig. 4h, confirming the robustness of the Jackiw-Rebbi states against vertical spacer displacements.

Supplementary Note 5. Realization of Jackiw-Rebbi states in three-dimensional space

To realize Jackiw-Rebbi states in a three-dimensional experimental setup, we extend the theoretical model to a planar metagrating structure based on coupled-mode theory. In this generalized three-dimensional configuration, the acoustic field above the metagrating surface supports evanescent waves that decay along the out-of-plane (z) direction, expressed as:

$$\Phi_1 = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} A_n \exp(j(k_x + qn)x + j(k_y + qm)y + jk_z(y - d)), z > d \quad (\text{S14})$$

In this three-dimensional space, the acoustic field distribution in the xy -plane does not affect the propagation characteristics of the guided modes in the z -direction. Therefore, similar to the two-dimensional case, the acoustic field inside the slit is described as a superposition of two guided modes propagating in opposite directions, given by:

$$\Phi_2 = U(x, y)[B_1 \exp(jk_n z) + B_2 \exp(-jk_n z)], 0 < z < d \quad (\text{S15})$$

Here, k_x , k_y and k_z represent the wavevector components outside the metagrating in the x , y , and z directions, respectively, and satisfy the condition $(k_x + 2\pi n/\Lambda)^2 + (k_y + 2\pi m/\Lambda)^2 + k_z^2 = k_0^2$. The term k_n denotes the wavevector component of the guided mode inside the slit along the z -direction. Compared to the two-dimensional case, the metagrating introduces an additional wavevector component k_y . However, the form of the guided mode resonance condition remains unchanged:

$$\begin{cases} \frac{jZ_{\text{eff}}}{k_n} \frac{\partial \Phi_2}{\partial z} = \Phi_2 - \Phi_3 \\ \frac{\partial \Phi_2}{\partial z} = \frac{\partial \Phi_3}{\partial z} \end{cases} \quad (\text{S16})$$

This implies that, while the acoustic field distribution in the plane parallel to the metagrating surface differs between two-dimensional and three-dimensional configuration, it is identical in the direction perpendicular to the metagrating surface. This consistency allows for the resonance mode to undergo spectral splitting by introducing an acoustic potential barrier, which results in the formation of paired even- and odd-parity resonant modes, as shown in Supplementary Fig. 5a and Supplementary

Fig. 5b. When the intensity and phase distributions of the incident plane wave are as shown in Supplementary Fig. 5c and Supplementary Fig. 5d, it can be observed that the acoustic field distribution of the guided modes in the yz-plane for the metagrating closely resembles that of the two-dimensional case. Notably, the spontaneous symmetry breaking mechanism we propose is unrelated to the topological properties of the bands, which implies that the specific distribution of the eigenmodes is not crucial for the emergence of interface states. When these guided modes with different parity undergo crossing, an acoustic Jackiw-Rebbi state emerges, which aligns with the mechanism of the two-dimensional case.

However, it is important to note two key differences between the Jackiw-Rebbi states in two-dimensional and three-dimensional configuration, despite the similarities. First, due to the higher dimensionality, the interface state transitions from a point-like state in two-dimensional case to a line-like state in three-dimensional case. Second, the presence of a wavevector component in the y-direction reduces the group velocity $\partial k_0 / \partial k_x$ in the x-direction, leading to an increase in the bandwidth and a reduction in the Q-factor of the acoustic Jackiw-Rebbi state.

Supplementary Note 6. Simulation setup of acoustic metagratings

(a) *Simulation setup of the metagrating in two-dimensional plane*

The metagrating shown in Fig. 2(c) consists of two finite-length metagratings joined together. For the right-side metagrating, the rectangular bars have a length of $L = 4.1$ mm, width $W_1 = 2.4$ mm, and a periodicity of $\Lambda = 7.3$ mm, with 25 periods in total. The spacer between the bars has a thickness of $D = 0.073$ mm. For the left-side metagrating, the rectangular bars maintain the same length $L = 4.1$ mm, but have a reduced width of $W_2 = 2.1$ mm, while the periodicity remains $\Lambda = 7.3$ mm, also with 25 periods. The spacer thickness here is $D = 0.73$ mm. The surrounding medium is air, with a density of 1.29 kg/m^3 , and a sound speed of 343 m/s . The spacer has a density of 1000 kg/m^3 and a sound speed of 1000 m/s .

The incident plane acoustic wave propagates perpendicularly to the surface of the metagrating, with a perfectly matched layer (PML) applied around the simulation domain to eliminate reflections. The reflected pressure is obtained from COMSOL Multiphysics simulations by calculating the scattered acoustic field under an incident background field. The simulation domain is discretized with a maximum mesh element

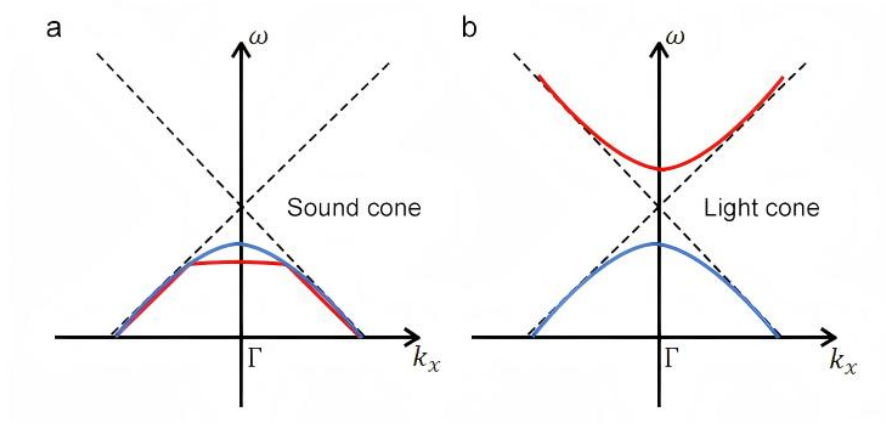
size of $\lambda/6$, where λ is the wavelength in air at the operating frequency. Acoustic field distributions are simulated at different frequencies ranging from 41,000 Hz to 43,000 Hz, and the normal transmission coefficient is calculated as the ratio of the averaged sound intensity of the transmitted wave to that of the incident wave across the metagrating.

(b) *Simulation setup of the metagrating in three-dimensional space*

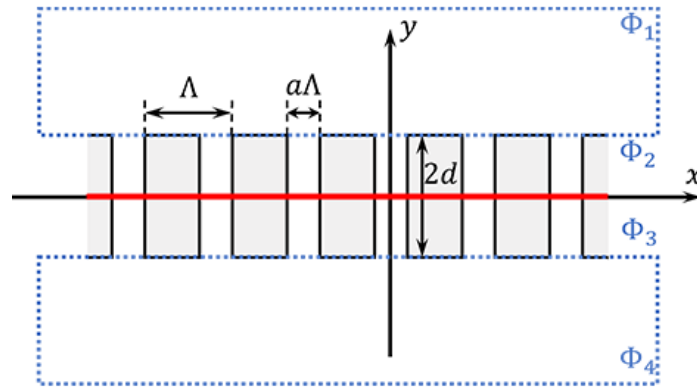
The metagrating shown in Fig. 4(a) has overall dimensions of $0.2\text{ m} \times 0.2\text{ m}$ and consists of two equal-area rectangular metagratings joined together. One of the metagratings (on the positive x-axis side) contains periodically arranged apertures, each of which is a square (viewed from above) with a side length of $W_1 = 5.5\text{ mm}$, depth $L = 4\text{ mm}$, and a periodicity of $\Lambda = 7.5\text{ mm}$. The other metagrating (on the negative x-axis side) also has periodically arranged apertures, with a slightly smaller square side length of $W_2 = 5.2\text{ mm}$, depth $L = 4\text{ mm}$, and the same periodicity of $\Lambda = 7.5\text{ mm}$. The spacer thickness in this case is $D = 0.01\text{ mm}$. The surrounding medium is air, with a density of 1.29 kg/m^3 and a sound speed of 343 m/s . The material of the metagrating is steel, with a density of 7850 kg/m^3 and a sound speed of 5300 m/s . The spacer is made of aluminum, with a density of 2700 kg/m^3 and a sound speed of 6300 m/s .

The incident plane wave propagates along the z-axis, with periodic boundary conditions applied to the lateral boundaries of the unit cell to simulate an infinite periodic structure, and PMLs placed at the top and bottom of the computational domain to absorb outgoing waves. The simulation domain is discretized with a maximum mesh element size of $\lambda/6$ in the air regions and $\lambda/10$ in the solid regions to ensure accurate resolution of the acoustic field. Acoustic field distributions are simulated at different frequencies ranging from 41,000 Hz to 43,000 Hz, and the normal transmission coefficient is determined by the ratio of the averaged sound intensity across the top and bottom surfaces of the metagrating.

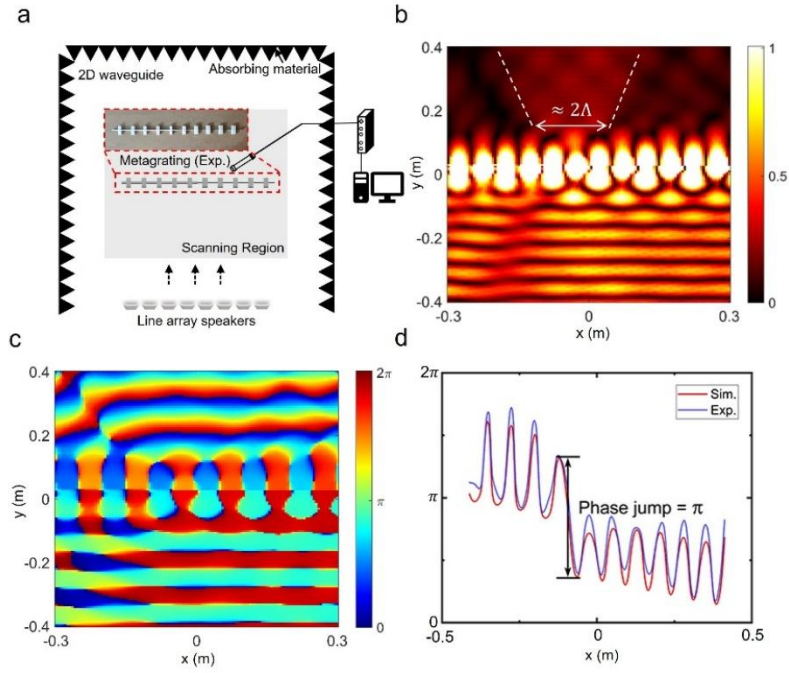
Supplementary Figures



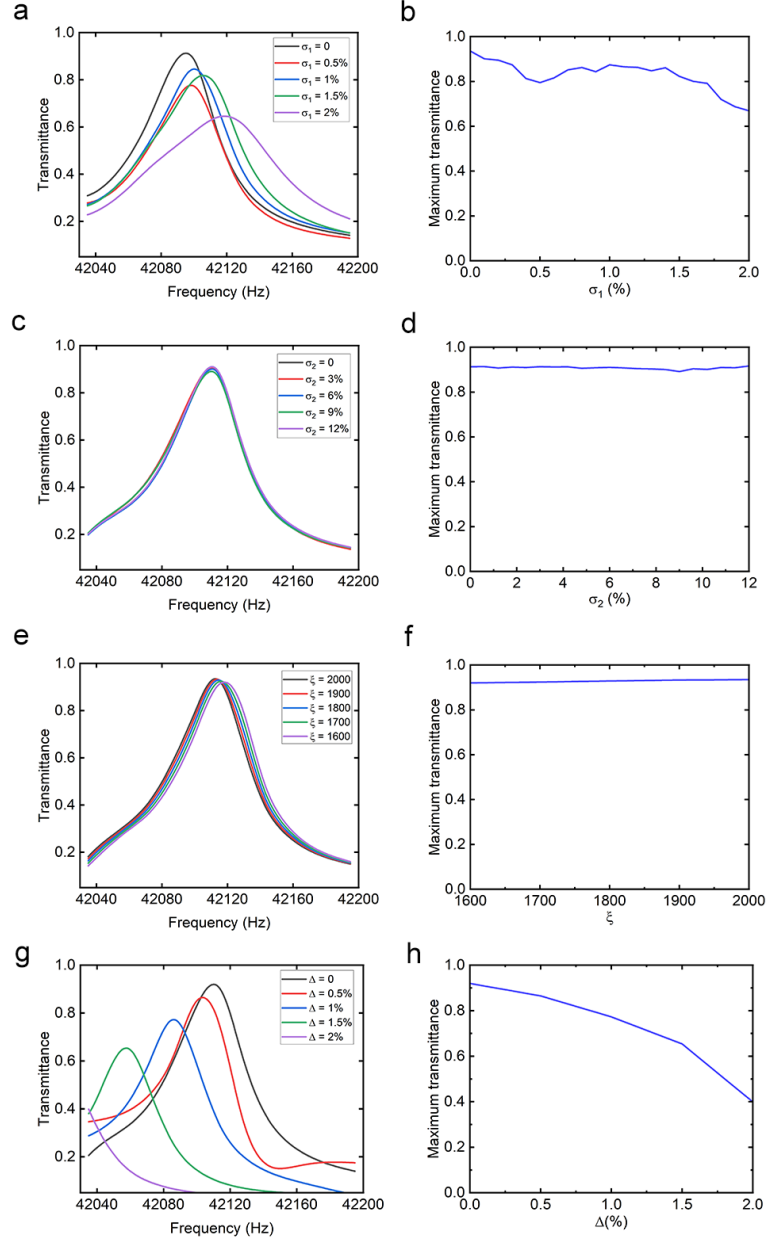
Supplementary Fig. 1 | Band structure comparison between acoustic and optical guided mode resonances. (a) The red and blue lines represent two distinct bands, both of which lie below the sound cone. (b) The red and blue lines represent two bands that can occur both below and above the light cone.



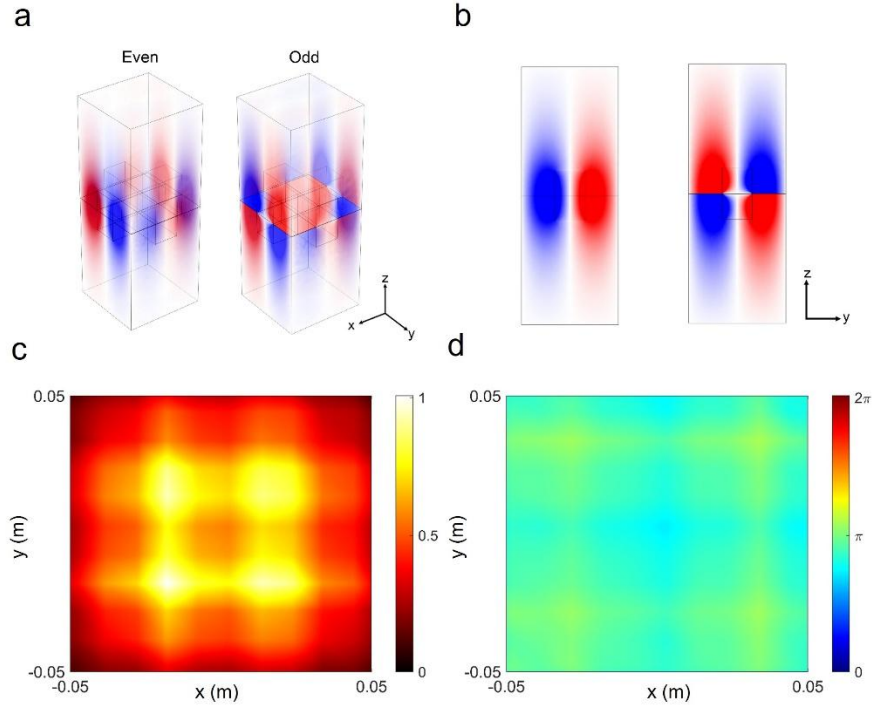
Supplementary Fig. 2 | Schematic of the acoustic topological metagrating model. The spacer of negligible thickness (colored in red) and the one-dimensional binary lattice (colored in grey) are aligned with the x-axis. The period of the metagrating is designated as Λ , where the rectangular scattering unit has a width of $(1 - a)\Lambda$, with $0 < a < 1$, and a height of $2d$.



Supplementary Fig. 3 | Experimental observation of Jackiw-Rebbi states in acoustic metagrating within a two-dimensional planar acoustic waveguide. (a) Experimental setup. (b) Measured confined wave intensity distribution. (c) Measured confined wave phase distribution. (d) Comparison between measured and simulated reflected wave phase distributions.



Supplementary Fig. 4 | Robustness of the JR State against four types of deviations. (a) Transmission spectra under deviations in scatter size. (b) Maximum transmittance as a function of deviations in scatter size. (c) Transmission spectra under deviations in spacer thickness. (d) Maximum transmittance as a function of deviations in spacer thickness. (e) Transmission spectra under deviations in acoustic impedance of the materials. (f) Maximum transmittance as a function of deviations in acoustic impedance of the materials. (g) Transmission spectra under vertical displacement of the spacer. (h) Maximum transmittance as a function of relative displacement Δ .



Supplementary Fig. 5 | Acoustic field distributions of metagrating in three-dimensional space.

(a) The normalized acoustic pressure distributions of the even and odd eigenmodes within one metagrating period, shown in a 3D Cartesian coordinate system, obtained through simulations. (b) The normalized acoustic pressure distributions of the even and odd eigenmodes in the yz-plane, averaged along the x-direction. (c) Acoustic intensity and (d) phase distributions of the measured incident sound wave generated by the 8×8 ultrasonic transducer array.