

Supplementary information for  
**Topological Diversity and Scintillating Dynamics of Dipolar Surface Twists in a  
van der Waals Magnet**

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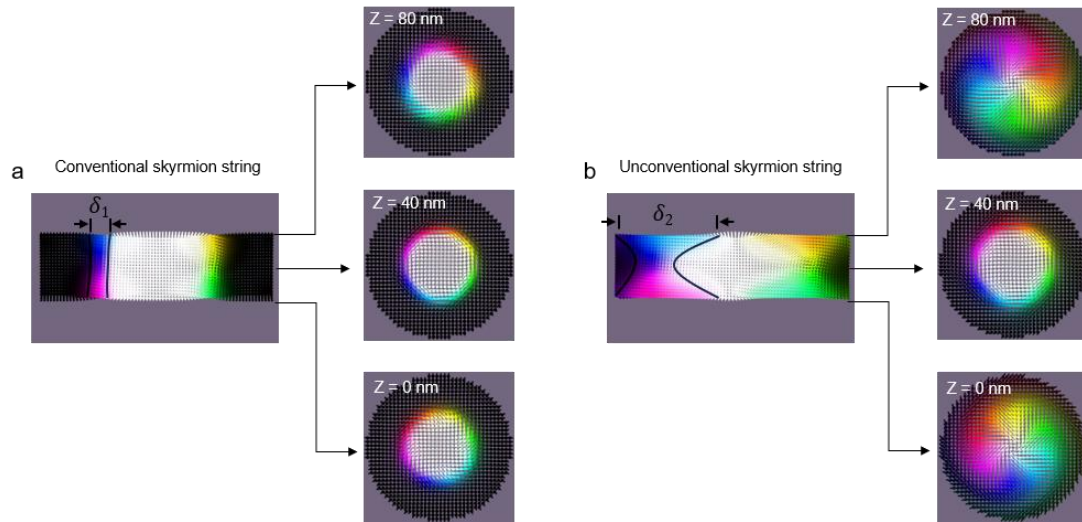
**Supplementary Fig. 26** | Simulated current-induced dynamics of DSTs with  $K_{\text{pinning}} = 7.5K_u$  at  $j = 8 \times 10^{11} \text{ A/m}^2$

**Supplementary Fig. 27** | Simulated current-induced dynamics of DSTs with  $K_{\text{pinning}} = 10K_u$  at  $j = 8 \times 10^{11} \text{ A/m}^2$

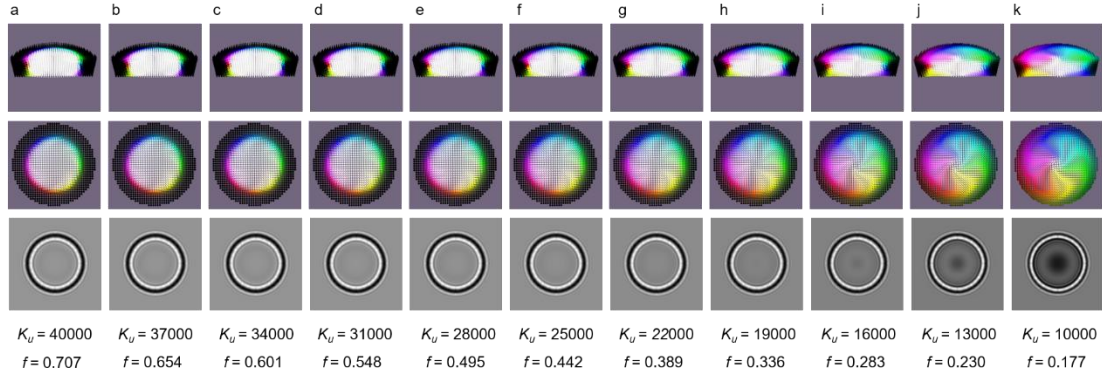
**Supplementary Fig. 28** | Simulated current-induced dynamics of DSTs with  $K_{\text{pinning}} = 10K_u$  at  $j = 5.6 \times 10^{11} \text{ A/m}^2$

**Supplementary Fig. 29** | Temperature-dependent stability of DSTs.

**Supplementary Note 1** | Explanation of small skyrmion Hall angle

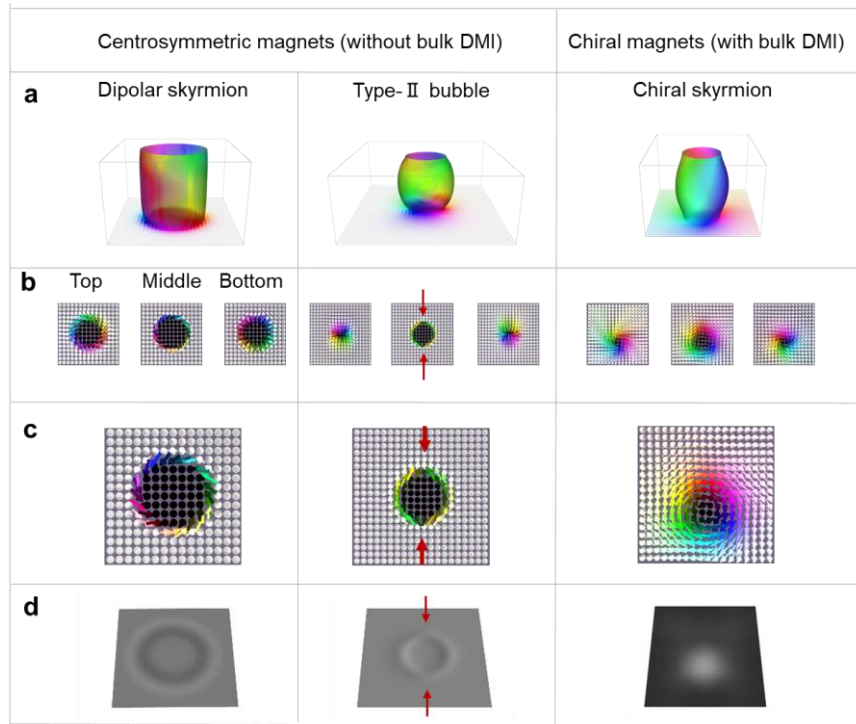


**Supplementary Fig. 1** | Simulated 3D spin configurations. Conventional **a** and unconventional **b** skyrmion strings in a disk at zero magnetic field. The simulated disk has a diameter of 300 nm and a thickness of 80 nm. The cross-sectional spin configurations of these two strings and magnetizations in the top surface layer ( $Z = 80$  nm), middle layer ( $Z = 40$  nm) and bottom surface layer ( $Z = 0$  nm) are presented. The domain walls ( $\delta_1$  and  $\delta_2$ ) along the cross section are outlined in black.

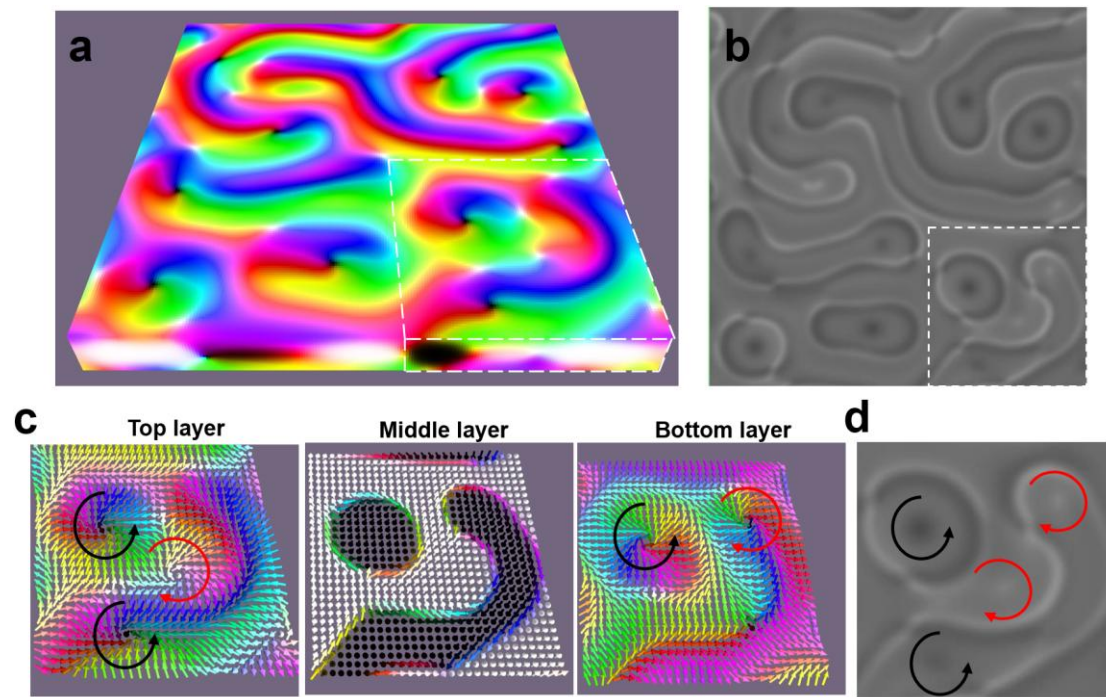


**Supplementary Fig. 2** | Simulated string structures and corresponding L-TEM images as functions of  $K_u$  and  $f$ . **a-k** The first, second, and third rows show cross-section, top layer, and L-TEM contrasts of the 3D string, with the respective  $K_u$  and  $f$  values provided beneath each panel. The values of  $M_s$  and  $A$  are fixed as  $300 \text{ kA.m}^{-1}$  and  $2 \text{ pJ.m}^{-1}$ , respectively.

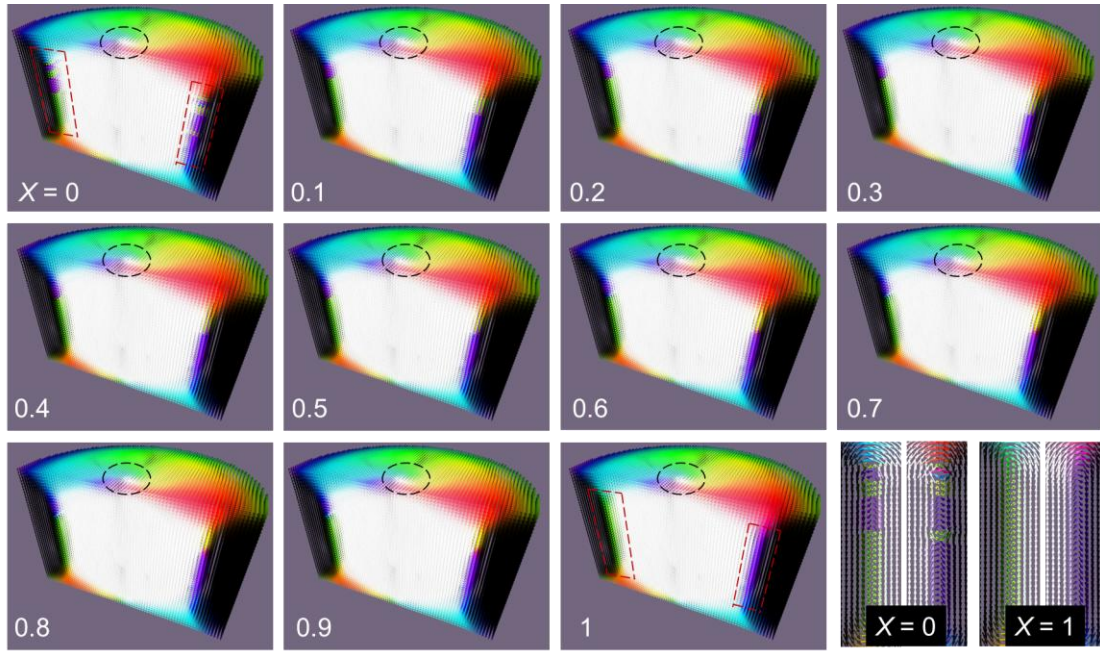
Micromagnetic simulations show that the formation of DSTs is governed primarily by the intrinsic magnetic properties of the material, particularly its low magnetic quality factor. Although Ga implantation and FIB-induced surface damage are unlikely to play a dominant role in their nucleation, such defects may locally influence the current-driven dynamics and stability of DSTs through additional pinning effects.



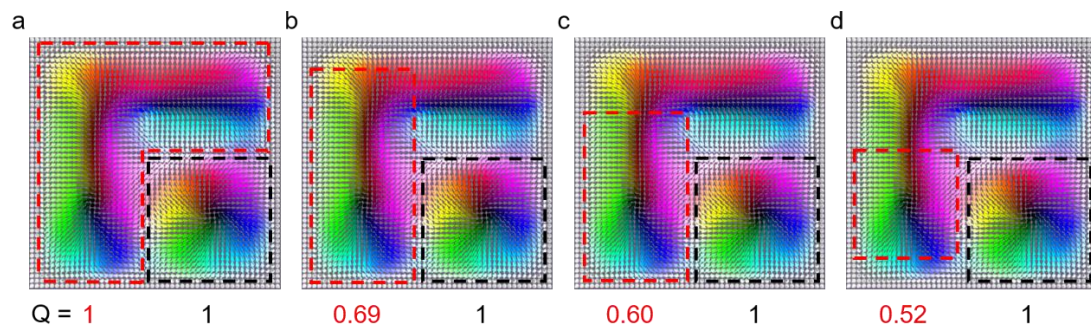
**Supplementary Fig. 3** | Simulated L-TEM contrast for different magnetic structures. **a** Schematic three-dimensional spin configuration of representative magnetic textures, including dipolar skyrmions, type-II bubbles, and chiral skyrmions. **b** Corresponding magnetic configurations at the top, middle, and bottom layers. **c** Enlarged views of the magnetic configurations at the middle layer, highlighting the characteristic in-plane spin arrangements. **d** Simulated L-TEM contrast of the corresponding magnetic structures.



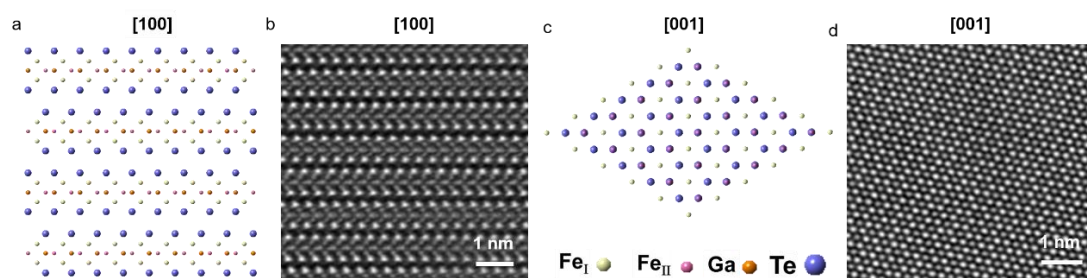
**Supplementary Fig. 4** | Depth-resolved separation of surface DSTs and bulk stripe domains. **a** Simulated three-dimensional spin configurations of DSTs on the surfaces of stripe domains. **b** Corresponding simulated L-TEM image. **c,d** Enlarged views of the marked regions in **a,b** showing the spin configurations at the top surface, middle layer, and bottom surface, together with the corresponding simulated L-TEM contrasts. The surface layers contain localized DSTs and give rise to the solid dot-like contrast, whereas the middle layer mainly represents the bulk stripe-domain background. Black and red arrows denote DSTs with anticlockwise and clockwise helicities, respectively.



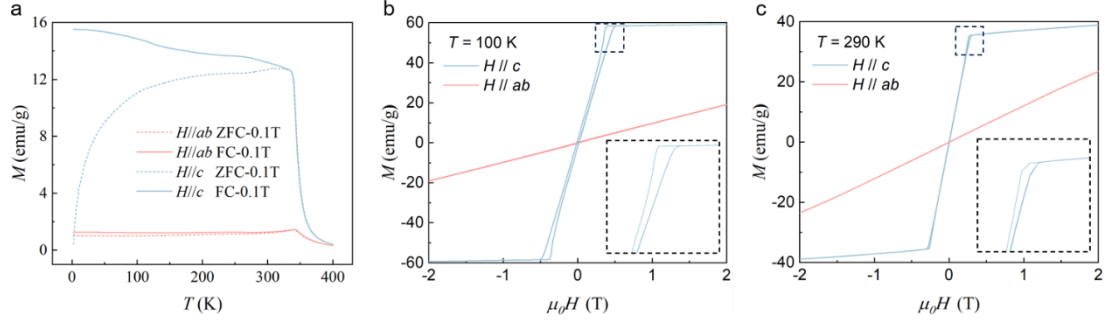
**Supplementary Fig. 5** | Micromagnetic simulations of three-dimensional magnetic structures under varying interlayer exchange coupling ( $xA$ , with  $0 \leq x \leq 1$ ), where  $A$  denotes the intralayer exchange stiffness fixed at  $2 \times 10^{-12}$  J/m. The bottom-right panels show cross-sectional views of the domain walls, highlighting the variation of Bloch lines with different  $x$  values.



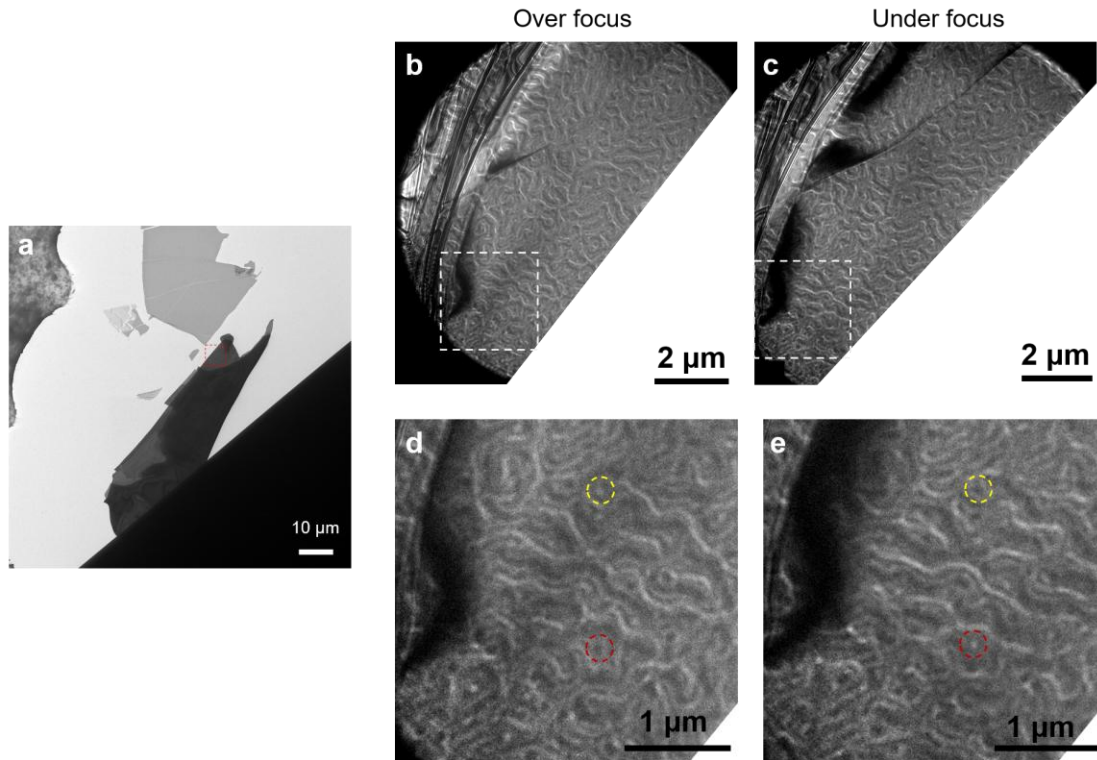
**Supplementary Fig. 6** | Calculation of topological charge  $Q$ . **a–d** Spin configurations used to calculate  $Q$ . Red and black numbers indicate the  $Q$  values in the corresponding boxed regions, respectively.



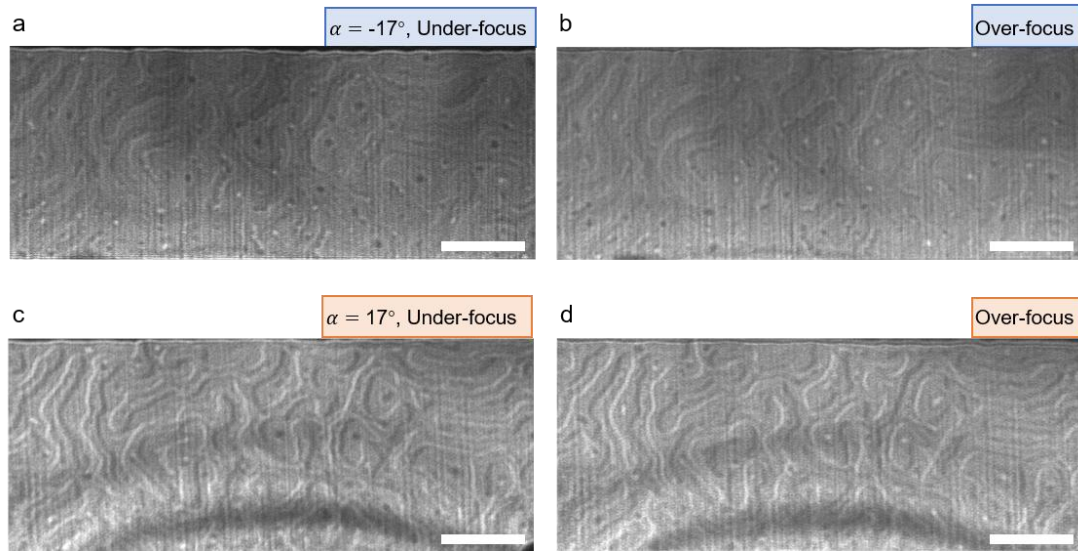
**Supplementary Fig. 7** | Crystal structure of FGT. Schematic **a, c** and high-angle annular dark-field scanning transmission electron microscopy (HAADF-STEM) images **b, d** of FGT crystal structure along [100] and [001] orientations.



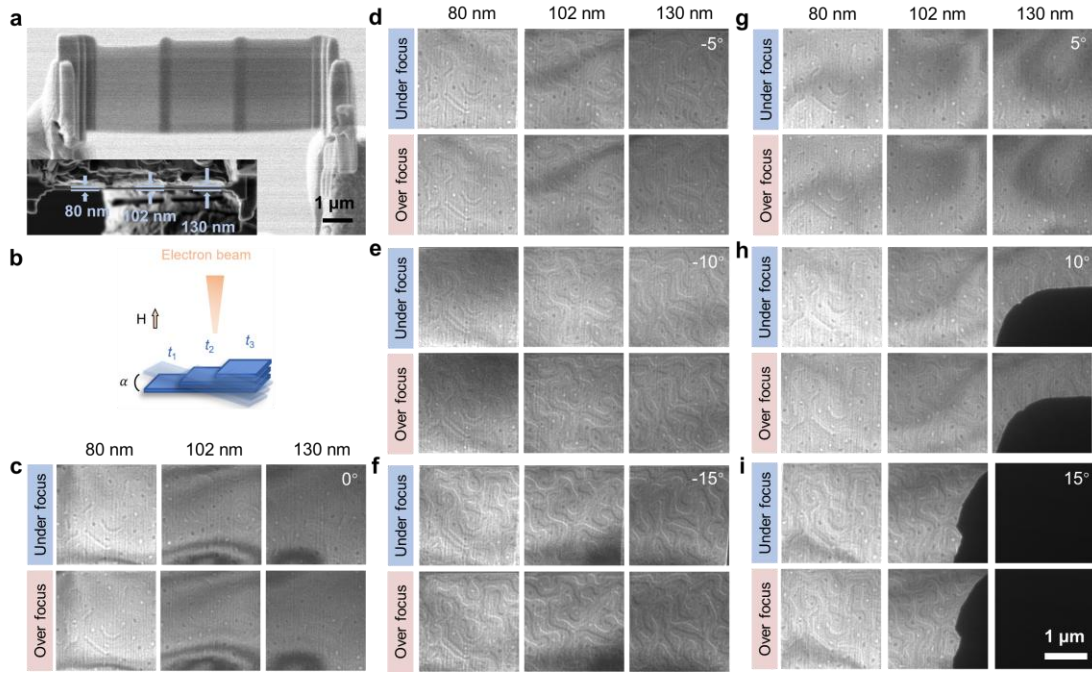
**Supplementary Fig. 8** | Magnetic property characterization of FGT. **a** Temperature-dependent zero field cooling (ZFC) and field cooling (FC) curves along  $c$ -axis and  $ab$ -plane under a magnetic field of 0.1 T, respectively. **b-c** Magnetic field-dependent magnetization curves along  $c$ -axis and  $ab$ -plane measured at 100 K **b** and 290 K **c**. The insets display magnified views of the corresponding regions.



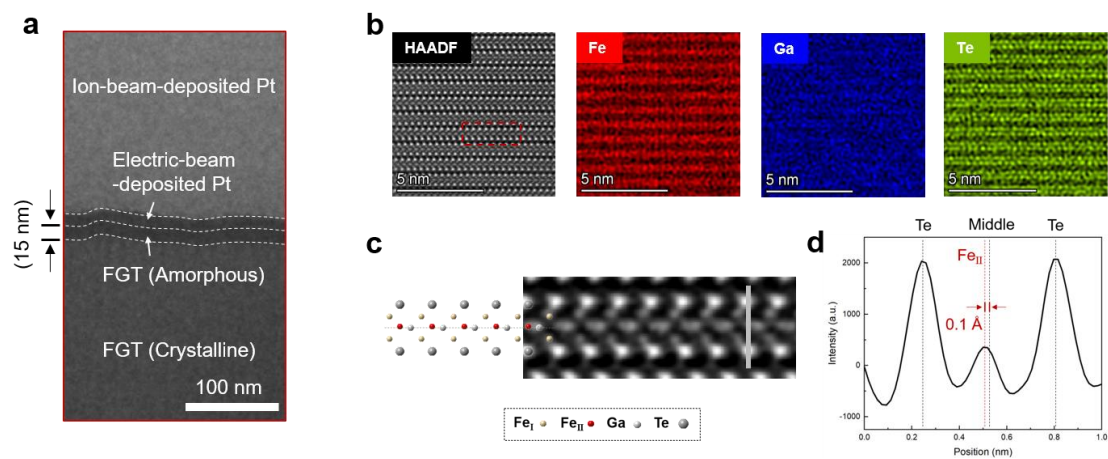
**Supplementary Fig. 9** | Observation of DSTs in mechanically exfoliated FGT flakes. **a** Low-magnification TEM image of mechanically exfoliated FGT flakes transferred onto a SiN membrane. The red box indicates the region selected for L-TEM imaging. **b,c** Over-focused **b** and under-focused **c** L-TEM images acquired from the selected region at zero magnetic field and room temperature. **d-e** Enlarged views of the white-boxed regions in **b,c**, showing high-density dot-like DST contrast. Representative dark and bright dots marked by the red and yellow boxes in **d** reverse to bright and dark dots in **e**, respectively. The observation of DST contrast in mechanically exfoliated flakes prepared without Ga-ion irradiation or FIB milling demonstrates that DST formation does not require FIB processing.



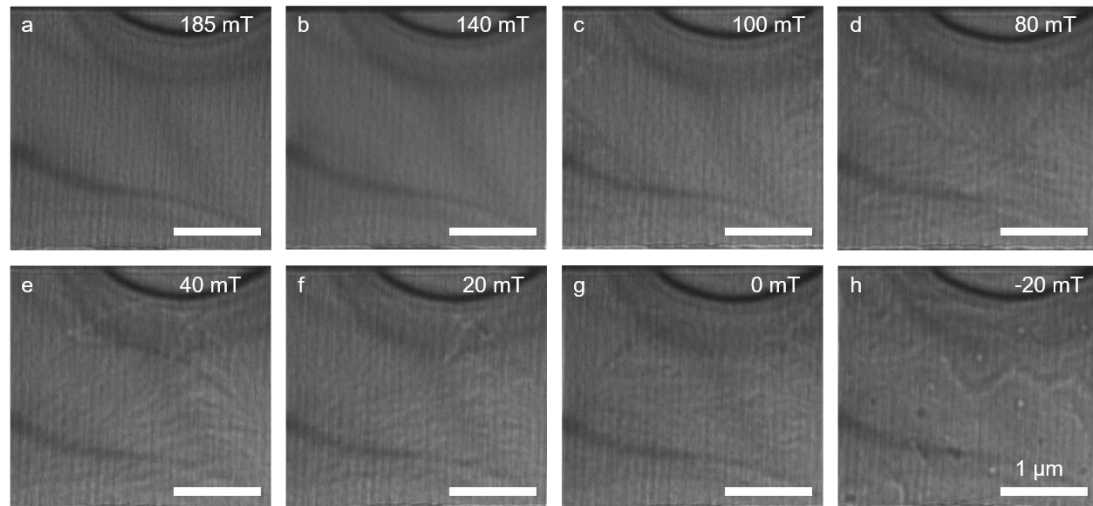
**Supplementary Fig. 10** | Observation of magnetic domains at different tilted conditions. **a-b** Under-focused **a** and over-focused **b** L-TEM images obtained at  $-17^\circ$ . **c-d** Under-focused **c** and over-focused **d** L-TEM images obtained at  $17^\circ$ . The defocus value is set as  $\pm 1$  mm. The scale bars are  $1\ \mu\text{m}$ .



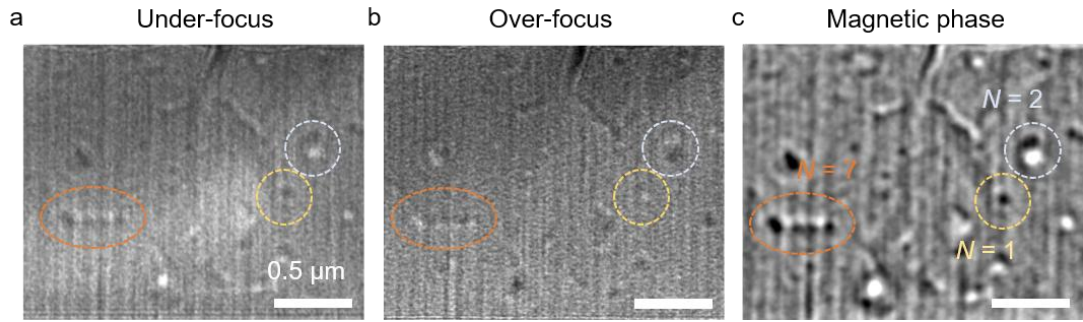
**Supplementary Fig. 11** | Thickness- and tilt-dependent L-TEM contrast of magnetic domains. **a** Cross-sectional SEM image with an inset (bottom left) showing the corresponding top-view image, highlighting three regions with thicknesses of 80, 102 and 130 nm. **b** Schematic illustration of sample tilting for L-TEM imaging. **c–i** L-TEM images of samples with different thicknesses acquired at various tilt angles; upper and lower panels correspond to under-focused and over-focused conditions, respectively. The dark regions in **h–i** arise from Pt shadowing during sample tilting.



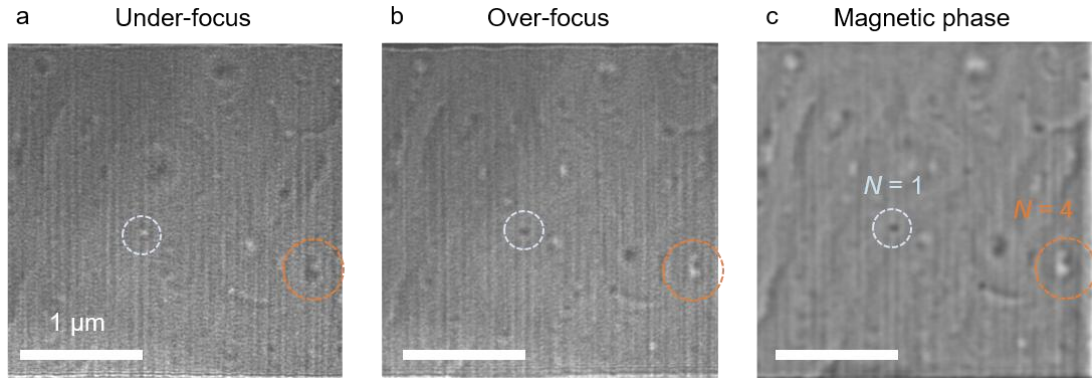
**Supplementary Fig. 12** | Structural characterizations for FIB-induced surface disorder and intrinsic atomic displacement in FGT. **a** Cross-sectional HAADF-STEM image of the FIB-prepared FGT lamella, showing an amorphous FGT surface layer above the crystalline FGT region. The non-flat interface indicates local structural disorder introduced by FIB processing. **b** Atomic-resolution HAADF-STEM image and corresponding elemental maps of Fe, Ga and Te acquired from the crystalline FGT region. **c** Enlarged HAADF-STEM image of the red-boxed region in **b**, together with the structural model. **d** Intensity profile showing an Fe<sub>II</sub> displacement of approximately 0.1 Å along the *c*-axis.



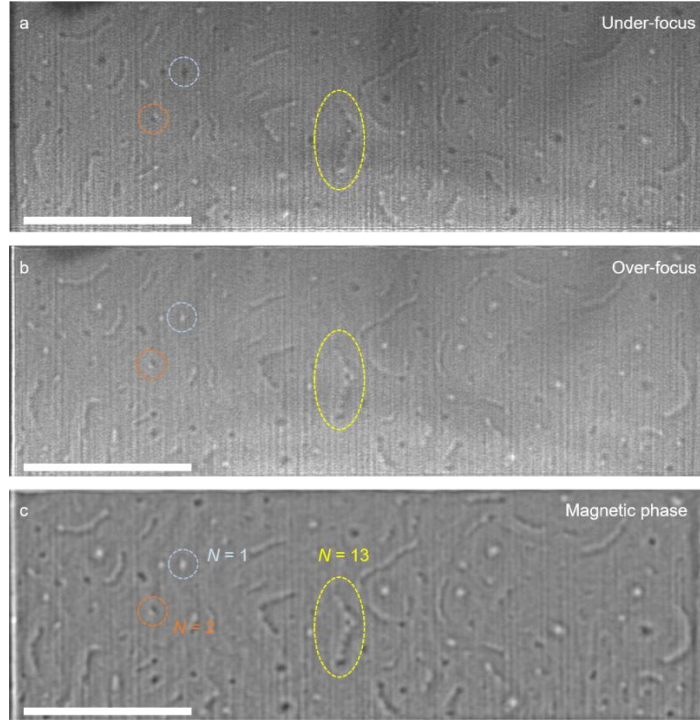
**Supplementary Fig. 13** | Evolution of magnetic domains at 290 K. **a–h** Under-focused L-TEM images acquired during the demagnetization process.



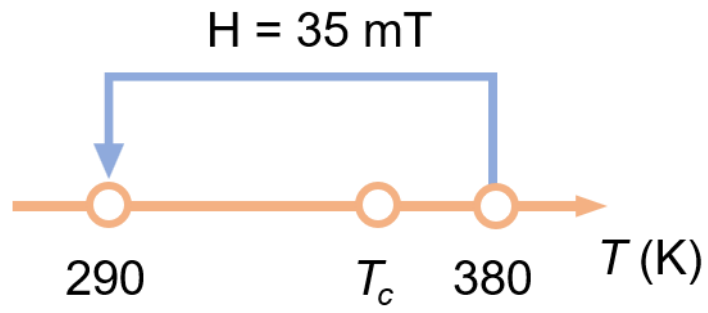
**Supplementary Fig. 14** | DSTs with  $N = 7, 2$  and  $1$ . **a–c** Under-focused and over-focused L-TEM images **a** and **b**, respectively, acquired at room temperature, and the corresponding magnetic phase image **c**.  $N$  denotes the number of fractional topological charges associated with each DST.



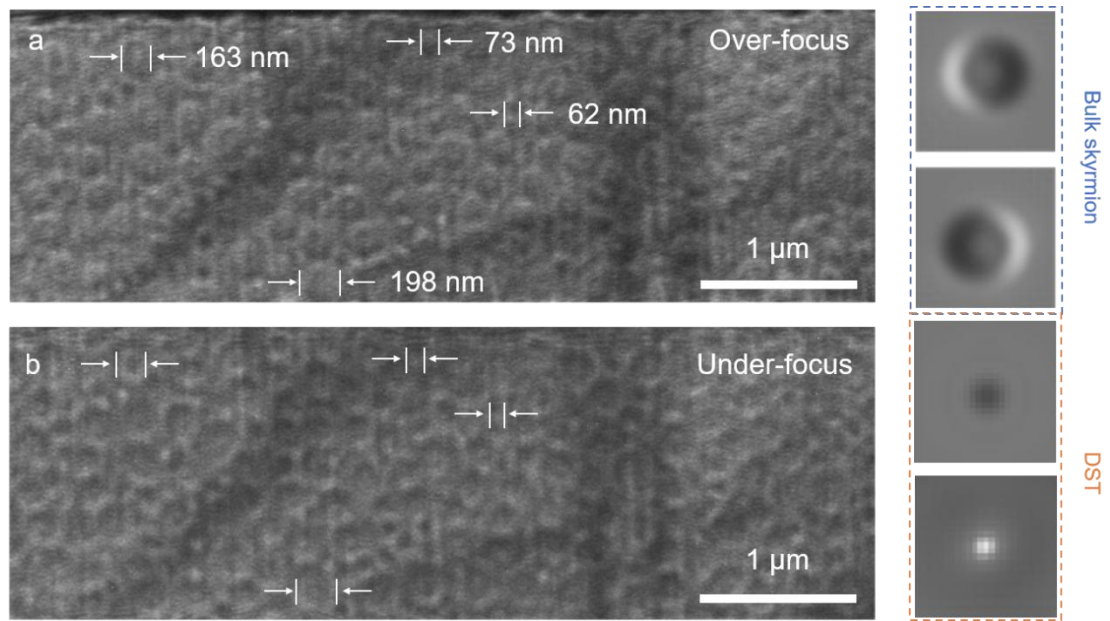
**Supplementary Fig. 15** | DSTs with  $N = 4$  and 1. **a–c** Under-focused and over-focused L-TEM images **a** and **b**, respectively, acquired at room temperature, and the corresponding magnetic phase image **c**.  $N$  denotes the number of fractional topological charges associated with each DST.



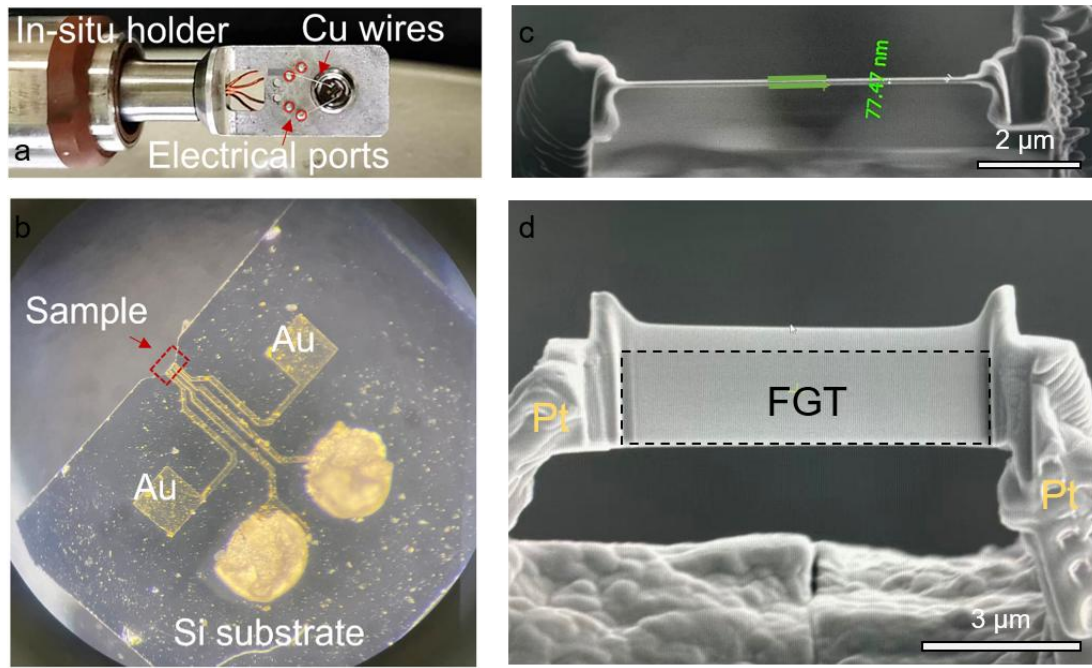
**Supplementary Fig. 16** | DSTs with  $N = 13, 2$  and  $1$ . **a–c** Under-focused and over-focused L-TEM images **a** and **b**, respectively, acquired at room temperature, and the corresponding magnetic phase image **c**.  $N$  denotes the number of fractional topological charges associated with each DST.



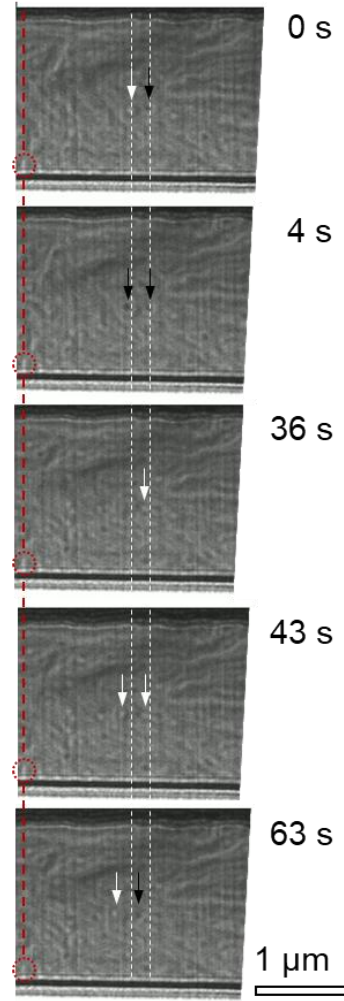
**Supplementary Fig. 17** | Schematic of the FC process. The FGT lamella was cooled from 380 K (well above  $T_c$ ) to 290 K in an out-of-plane magnetic field of 35 mT.



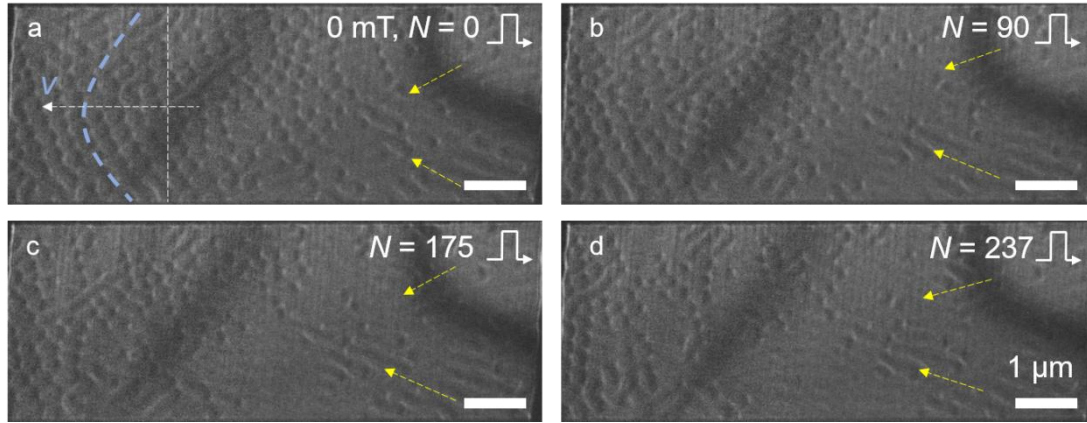
**Supplementary Fig. 18** | Coexistence of bulk skyrmions and DSTs after a FC procedure. **a-b** Over-focused **a** and under-focused **b** L-TEM images obtained at 290 K and a tilt sample of  $10^\circ$  with  $\pm 1$  mm defocus value. The magnetic field was set as 35 mT during the FC process and tuned off before imaging.



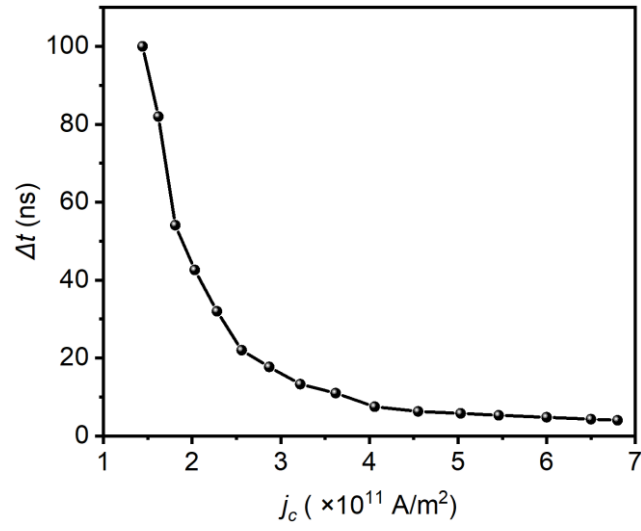
**Supplementary Fig. 19** | In-situ electrical experimental setup of FGT micro-device. **a** Image of a Gatan sample holder, where the electrodes in FGT microdevice are connected to the external electrical ports using the Cu wires and silver colloids. **b** Image of the electrical chip, which is comprised of a Si substrate, Au pads and a FGT sample. **c-d** FIB-SEM images of the top **c** and side **d** view of FGT microdevice. The sample was protected by a carbon layer during the fabrication process, and the sample thickness was measured at  $\sim 77$  nm.



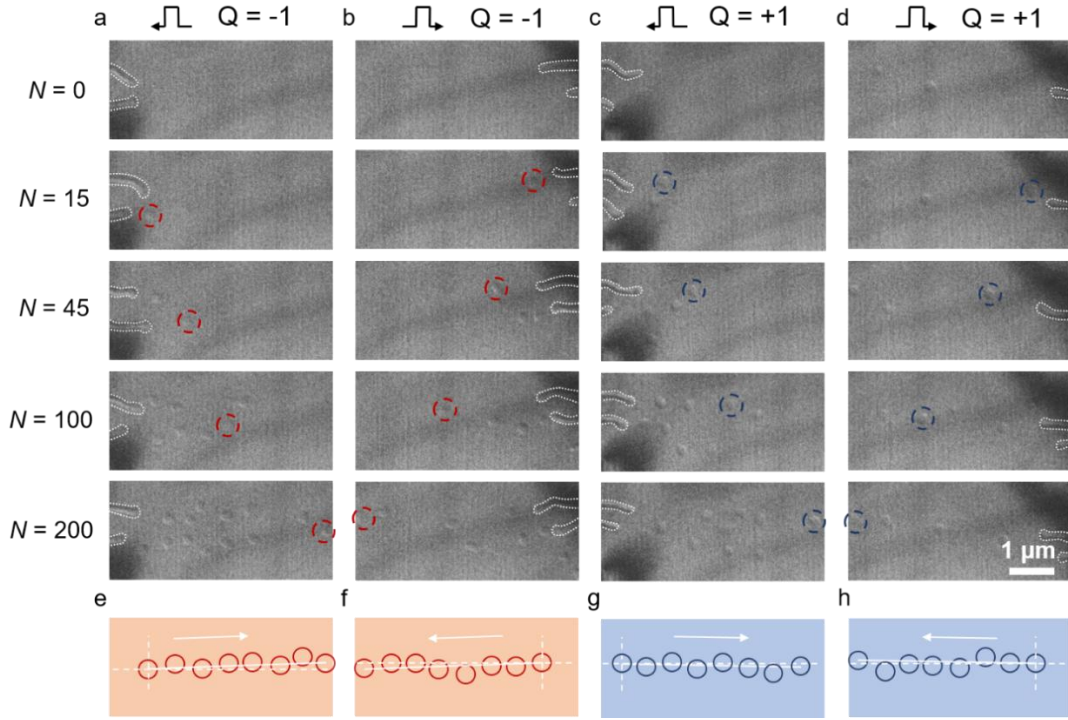
**Supplementary Fig. 20** | Current-driven dynamics of DSTs at zero magnetic field and room temperature. The applied current density is  $j = 2.20 \times 10^{11} \text{ A.m}^{-2}$  with a pulse width of 40 ns. Red circles mark sample defects serving as fixed reference points. Clockwise and anticlockwise DSTs are indicated by black and white arrows, respectively.



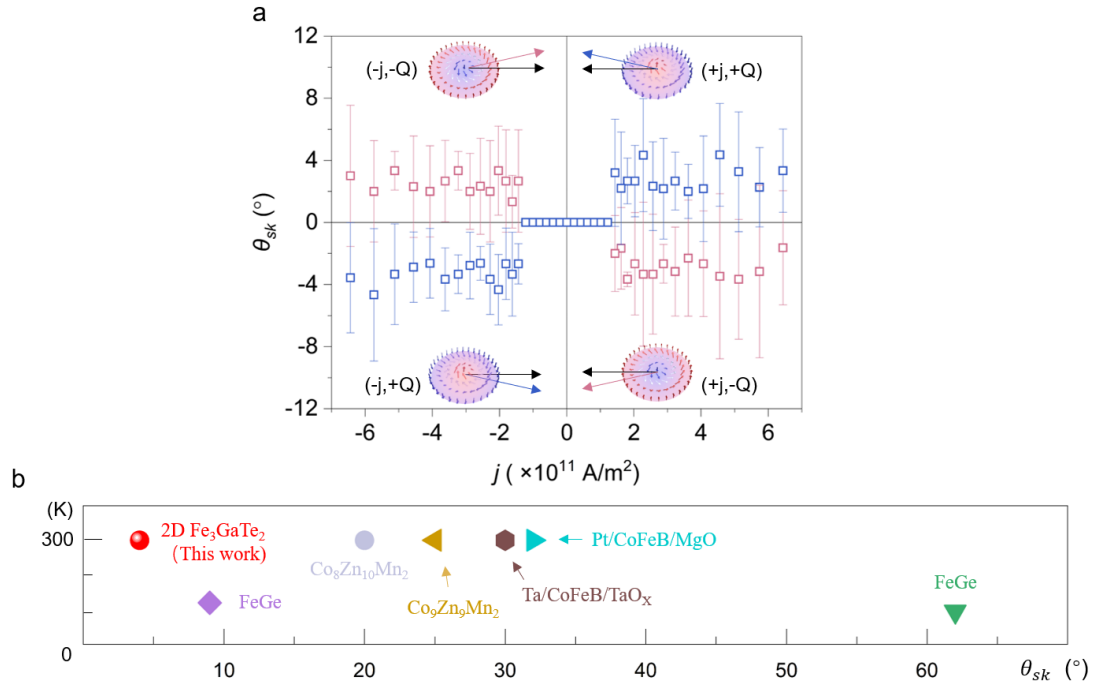
**Supplementary Fig. 21** | Ordered movement of mixed skyrmions and stripe driven by current at 0 mT and room temperature. **a–d** Under-focused L-TEM images acquired after  $N = 0, 90, 175$ , and  $237$  current pulses, respectively. The nonuniform velocity of magnetic domains in the middle and near the edges of the sample demonstrates the pinning effect arising from the edges. The yellow arrows indicate the orientations of stripes domain.  $N$  represents the pulse number in a frequency of 10 Hz.



**Supplementary Fig. 22** | Dependence of pulse width on the critical current density ( $j_c$ ) for driving skyrmions movement along the device.

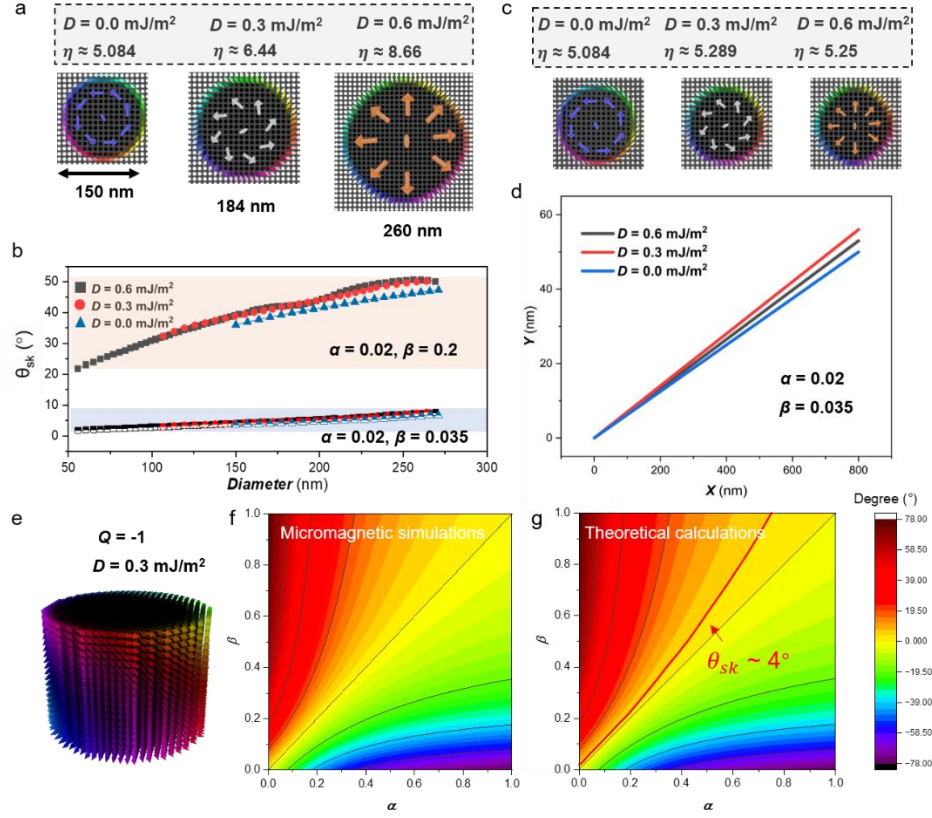


**Supplementary Fig. 23** | Motion tracking of isolated bulk skyrmions driven by sequential current pulses at room temperature. **a-b** Isolated bulk skyrmions with  $Q = -1$  move forwards under a series of negative ( $j = -2.18 \times 10^{11} \text{ A.m}^{-2}$ ) **a** and positive ( $j = 2.25 \times 10^{11} \text{ A.m}^{-2}$ ) **b** current pulses with a pulse width of 40 ns at +80 mT and room temperature. **c-d** Isolated skyrmions with  $Q = +1$  move forwards under a series of negative ( $j = -2.18 \times 10^{11} \text{ A.m}^{-2}$ ) **c** and positive ( $j = 2.25 \times 10^{11} \text{ A.m}^{-2}$ ) **d** current pulses with a pulse width of 40 ns at -80 mT and room temperature. The skyrmions with opposite  $Q$  were marked in blue and red dashed circles, and the stripe domains near the device edge were marked in white dotted lines. **e-h** Summary of the skyrmions trajectories. The circles show the changes of skyrmions positions as injecting current pulses, with the arrows and fitted lines shown to guide the eyes. The presence of small longitudinal displacements indicates the negligible skyrmion Hall effect.

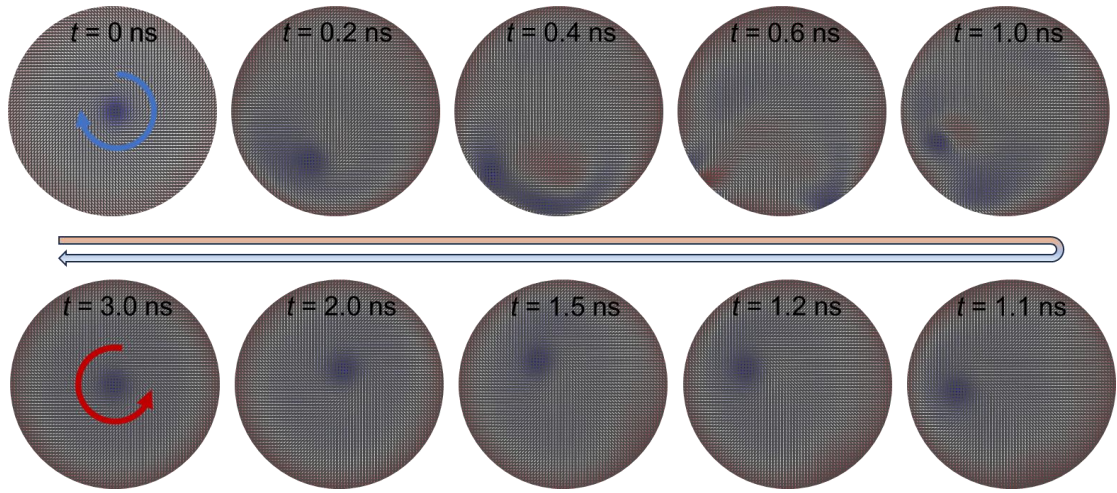


**Supplementary Fig. 24** | Dependence of current density on the skyrmions Hall angle.

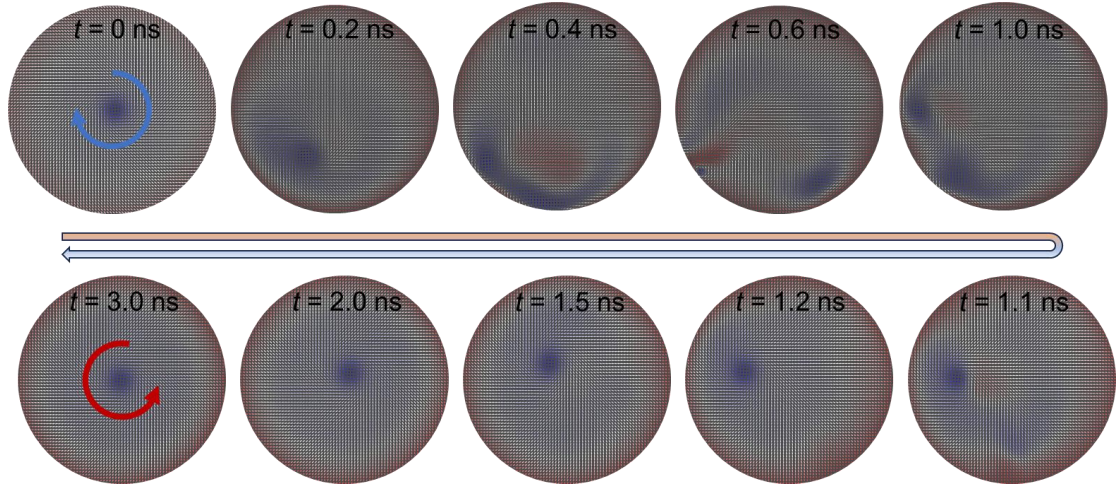
**a** Summarized skyrmion Hall angles  $\theta_{sk}$  measured at different current density and opposite magnetic field of  $\sim \pm 80$  mT. Each data point is an average result of the statistics of five isolated skyrmions, and the error bars show the standard deviation. The data points with  $\theta_{sk} \sim 0^\circ$  means no skyrmions were blown from the stripes under a low current density. The insets show the spin configurations of skyrmions with different topological charges ( $Q$ ), accompanied with the direction of electrons (black arrows) and skyrmions movement (blue or red arrows). **b** Comparison of  $\theta_{sk}$  and service temperature in FGT and other ferromagnetic systems<sup>1-6</sup>.



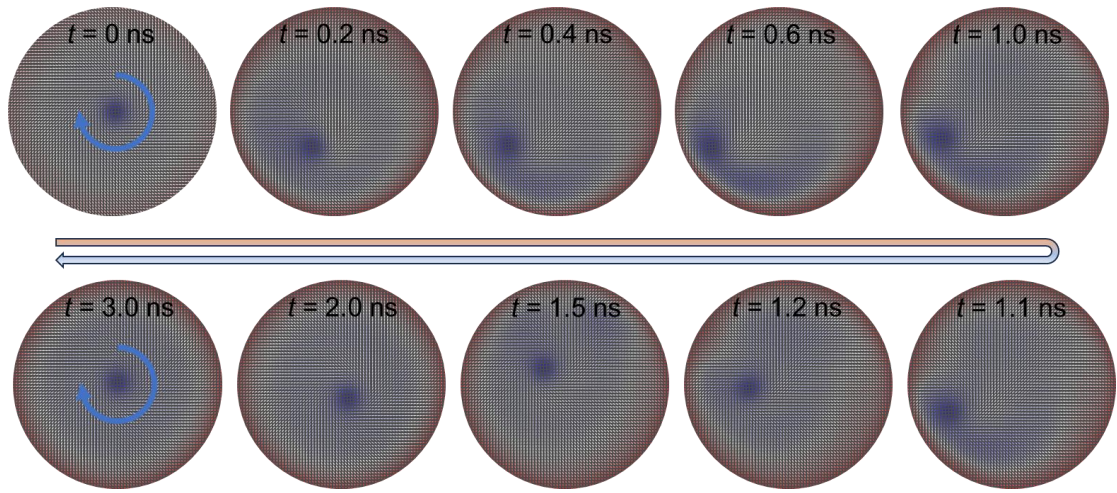
**Supplementary Fig. 25 | Micromagnetic simulations and theoretical calculations. a** Spin configurations and corresponding schematic drawings of bulk skyrmion with different DMI and shape factor ( $\eta$ ) at 80 mT. **b** Dependence of Skyrmion diameter on the skyrmion Hall angle ( $\theta_{sk}$ ), The black, red, and blue data points represent magnetic field-modulated sizes of skyrmions with DMI strength of 0.6, 0.3, and 0  $\text{mJ/m}^2$ , respectively. The damping constant ( $\alpha$ ) and non-adiabatic coefficient ( $\beta$ ) used here is  $\alpha = 0.02, \beta = 0.035$  (lower blue area) and  $\alpha = 0.02, \beta = 0.2$  (upper orange area). **c-d** Shape factor ( $\eta$ ) **c** and trajectory **d** of skyrmions as a function of DMI with coincident sizes of 150 nm by controlling the magnetic field. The parameters used here are:  $\alpha = 0.02, \beta = 0.035$ . **e** 3D structure of a bulk skyrmion with  $Q = -1$  and  $D = 0.3 \text{ mJ/m}^2$  that used in (f) and (g). **f-g** Simulated **f** and theoretically calculated **g** results of the influence of  $\alpha$  and  $\beta$  values on the  $\theta_{sk}$ .



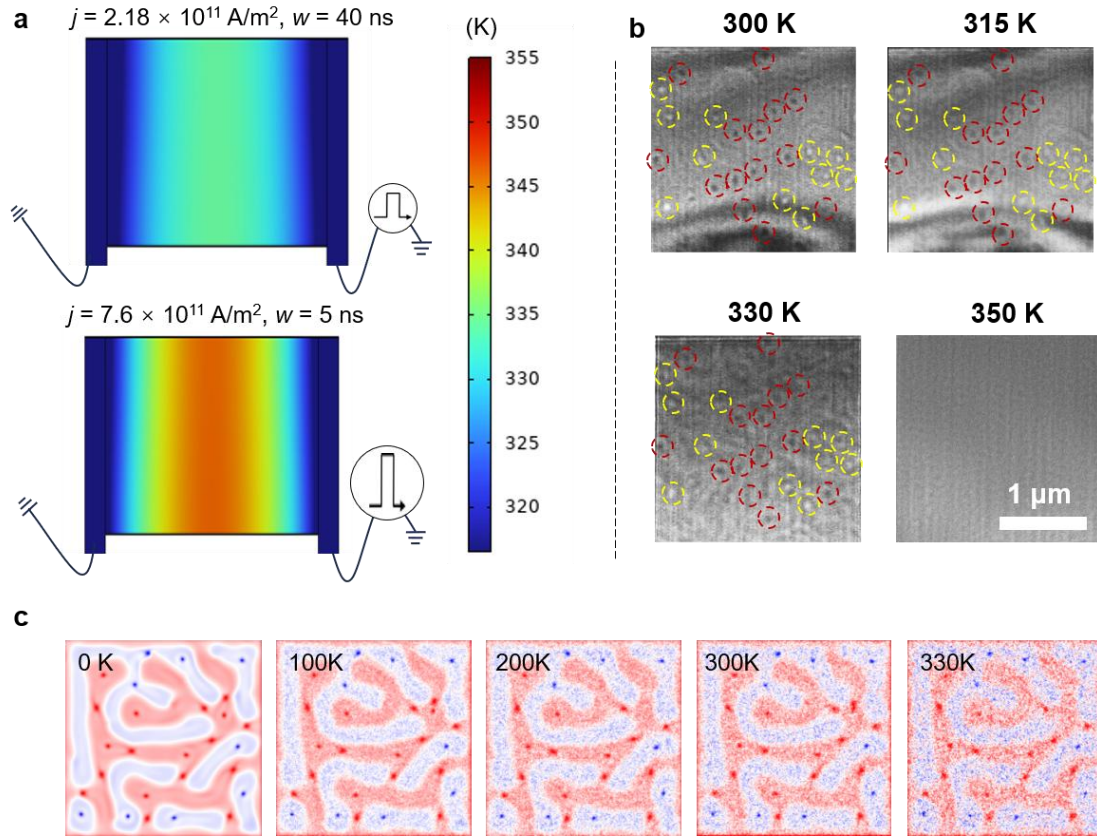
**Supplementary Fig. 26** | Simulated current-induced dynamics of DSTs with  $K_{\text{pinning}} = 7.5K_u$  at  $j = 8 \times 10^{11} \text{ A/m}^2$ . A current pulse was applied for 1 ns and subsequently removed to allow relaxation into a stable state.



**Supplementary Fig. 27** | Simulated current-induced dynamics of DSTs with  $K_{\text{pinning}} = 10K_{\text{u}}$  at  $j = 8 \times 10^{11} \text{ A/m}^2$ . A current pulse was applied for 1 ns and subsequently removed to allow relaxation into a stable state.



**Supplementary Fig. 28** | Simulated current-induced dynamics of DSTs with  $K_{\text{pinning}} = 10K_u$  at  $j = 5.6 \times 10^{11} \text{ A/m}^2$ . A current pulse was applied for 1 ns and subsequently removed to allow relaxation into a stable state.



**Supplementary Fig. 29** | Temperature-dependent stability of DSTs. **a** The temperature distributions in the FGT microdevice under different current density. **b** Under-focused L-TEM images showing the temperature-dependent evolution of DSTs. DSTs with clockwise (black contrast) and anticlockwise (white contrast) helicities are marked by red and yellow circles, respectively. **c** Micromagnetic simulations of temperature-dependent spin textures.

## Supplementary Note 1 | Explanation of small skyrmion Hall angle

The current-driven dynamic behaviors of bulk skyrmions can typically be understood by the Landau-Lifshitz-Gilbert (LLG) equation incorporating the STT effect, which can be expressed in the following form<sup>7,8</sup>:

$$\frac{d\mathbf{m}}{dt} = -\gamma_0(\mathbf{m} \times \mathbf{H}_{\text{eff}}) + \alpha(\mathbf{m} \times \frac{d\mathbf{m}}{dt}) + T_{\text{ad}} + T_{\text{na}} \quad (1)$$

Where  $\mathbf{m}$  is the normalized spin vector,  $\gamma_0$  is the gyromagnetic ratio,  $\mathbf{H}_{\text{eff}}$  is the total effective field,  $\alpha$  is the Gilbert damping constant, the sum of the adiabatic torque term  $T_{\text{ad}}$  and the non-adiabatic torque term  $T_{\text{na}}$  form the STT effect. If the current flows along the x-axis direction,  $T_{\text{ad}}$  and  $T_{\text{na}}$  are respectively expressed as follows:

$$\mathbf{T}_{\text{ad}} = +v_s \mathbf{m} \times \left( \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial x} \right) = -v_s \frac{\partial \mathbf{m}}{\partial x} \quad (2)$$

$$\mathbf{T}_{\text{na}} = +\beta v_s \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial x} \quad (3)$$

where, the effective polarized electron velocity  $v_s = \frac{\mu_B P j}{(e M_s)}$ ,  $P$  is the current polarization factor,  $j$  is the current density, and  $\beta$  is a dimensionless parameter named the non-adiabatic coefficient. Simplifying the LLG equation yields the Thiele equation as follows<sup>9-11</sup>:

$$\mathbf{G} \times (\mathbf{v}_s - \mathbf{v}_d) + \mathbf{D}(\beta \mathbf{v}_s - \alpha \mathbf{v}_d) = 0 \quad (4)$$

Where the first term denotes the contributions of Magnus force with  $\mathbf{G} = 4\pi Q \mathbf{e}_z$ ,  $Q$  and  $\mathbf{e}_z$  are the topological charge and unit vector along the  $z$  axis, the velocity of the driving electrons and skyrmions drift are represented by  $\mathbf{v}_d$  and  $\mathbf{v}_s$ , respectively. It is noted that the  $\mathbf{v}_s$  term is comprised of the translational skyrmions velocity  $v_x$  and the transverse velocity  $v_y$ . The second term represents the dissipative tensor  $\mathbf{D}$ -related dissipative force, for isolated skyrmions

$$\mathbf{D} = 4\pi \begin{pmatrix} \eta & 0 \\ 0 & \eta \end{pmatrix} \quad (5)$$

The shape factor  $\eta = -\frac{1}{4\pi} \iint \frac{\partial \mathbf{m}}{\partial x} \cdot \frac{\partial \mathbf{m}}{\partial y} dx dy$ , related to the diameter  $L$  and domain wall width of skyrmions. Consequently, the solution of the Thiele equation can be expressed as below:

$$\begin{aligned} v_x &= \left( \frac{\beta}{\alpha} + \frac{Q^2(\alpha-\beta)}{\alpha(Q^2+\alpha^2\eta^2)} \right) v_s \\ v_y &= \frac{Q\eta(\alpha-\beta)}{Q^2+\alpha^2\eta^2} v_s \end{aligned} \quad (6)$$

Note that the  $v_x$  is an even function of  $Q$ , hence the translational drift direction of skyrmions ( $\Delta x$ ) will not change with  $Q$ . On the contrary, the transverse skyrmions movement ( $\Delta y$ ) varies with  $Q$ , since the odd function between  $v_y$  and  $Q$ , as evidenced by our experimental observations in Figure 2.

Using the four-quadrant angle calculation formula:

$$\theta = \text{sign}(v_y) \cdot \arccos\left(\frac{v_x}{\sqrt{v_x^2+v_y^2}}\right) \quad (7)$$

we get the skyrmion Hall angle:

$$\theta_{sk} = \arctan\left(\frac{Q\eta(\alpha-\beta)}{Q^2+\alpha\beta\eta^2}\right) \approx \arctan(\alpha-\beta)\frac{\eta}{Q} \quad (8)$$

under the limitation of  $\alpha \ll 1$  and  $\beta \ll 1$ . Note that for a given skyrmion in magnetic systems, its topological charge is determined to be  $\pm 1$ . Consequently, the magnitude of  $\theta_{sk}$  is determined by two factors: the difference value between the Gilbert damping parameter and the non-adiabatic coefficient ( $\alpha-\beta$ ), and the value of the shape factor ( $\eta$ ), the latter of which can be numerically calculated based on the spin configuration of skyrmions. Supplementary Fig. 25a shows the DMI-dependent  $\eta$ , skyrmion diameter ( $L$ ), and spin configurations. It is evident that appropriately increasing the DMI strength changes  $\eta$  from 5.084 to 8.66,  $L$  from 150 to 260 nm, and spin configurations from Bloch- to Néel-type. The specific value of  $D$  depends on factors such as the saturated magnetization and the sample thickness of the magnet. Injecting current pulses into these three systems shows a minor variation within  $\theta_{sk} < 6^\circ$  in their respective skyrmion Hall angles when the coefficient  $\alpha-\beta$  is -0.015, and it becomes pronounced from  $\theta_{sk} \sim 22^\circ$  to  $47^\circ$  at  $\alpha-\beta = -0.18$ , as shown in Supplementary Fig. 25b. This suggests that the skyrmion Hall angle depends on both the DMI and the coefficient  $\alpha-\beta$ . Subsequently, we use the size  $L = 150$  nm and the coefficient  $\alpha-\beta = 0.015$  as control variables to check the spin configuration- and  $\eta$  -dependent  $\theta_{sk}$ , as shown in Supplementary Fig. 25c-d. It is worth mentioning that the coincident sizes of skyrmions with  $D$  of 0.0, 0.3, and 0.6 mJ/m<sup>2</sup> were achieved by applying various magnetic field of

90, 97, and 120 mT, respectively. The negligible variation in their trajectories confirmed the irrelevance between the  $\theta_{sk}$  and the spin configurations of skyrmions. Thus, the DMI-induced difference in  $\theta_{sk}$  can be attributed to the shape factor  $\eta$  and skyrmion sizes. This is inconsistent with the behaviors of skyrmions driven by spin-orbit torque (SOT) in multilayer films, where  $\theta_{sk}$  depends on the spin configurations of skyrmions.<sup>1,12</sup> Therefore, it is concluded that the small  $\theta_{sk}$  represented in this study can be primarily attributed to the small difference value of  $\alpha-\beta$ , which was further verified by both the simulated and theoretical results, as shown in Supplementary Fig. 25e-g. The skyrmion with  $Q = -1$  was used to extract the  $\theta_{sk}$ , demonstrating the correspondence between  $\theta_{sk} \approx 4^\circ$  and  $62.5(\alpha - \beta) = 20.34 \alpha\beta + 1$ , as shown in the red fitting curve. In the range of  $\alpha \ll 1$ , the above relationship can be simplified as  $\alpha - \beta = -0.015$ .

## Supplementary References

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