

Supplementary Information

1. Performance Metrics Used to Calculate Error

MSE: Mean Square Error shown in Equation 1, where y^{Ph} is the physical observation, y^S is the NN model prediction and N_P is the total number of simulations.

$$MSE = \frac{\sum_{i=1}^{N_P} (y_i^{Ph} - y_i^S)^2}{N_P} \quad (1)$$

Root Mean Squared Error (RMSE) as defined in Equation 2 where y_i^E is the expected prediction from the analytical E-T model.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N_P} (y_i^E - y_i^P)^2}{N_P}} \quad (2)$$

Mean Absolute Error (MAE) as defined in Equation 3.

$$MAE = \frac{\sum_{i=1}^{N_P} (y_i^E - y_i^P)^2}{N_P} \quad (3)$$

Coefficient of Determination (R^2) as defined in Equation 4.

$$R^2 = 1 - \frac{\sum_{i=1}^{N_P} (y_i^E - y_i^P)^2}{\sum_{i=1}^{N_P} (y_i^E - y_i^{Ave})^2} \quad (4)$$

2. Details on the Surrogate Neural Network Model

2.1. Gaussian Process Framework

The calibration approach follows the canonical Kennedy and O'Hagan (KOH) framework [1], which accounts for three primary sources of uncertainty: parameter uncertainty, model discrepancy, and measurement error. This framework is expressed as:

$$y^E = \tau y^S(x, \theta^*) + \delta(x) + \epsilon \quad (5)$$

In this expression, y^E represents the experimental observation. The term τ is the scaling parameter, which is assumed to be 1 in this study. The prediction of the surrogate model is denoted by $y^S(x, \theta^*)$, where $\mathbf{x}$ refers to the control inputs and θ^* represents the optimal calibration parameters. The function $\delta(x)$ accounts for the model discrepancy and is modeled as a Gaussian Process (GP)

with a mean of zero and a covariance function $c(\cdot, \cdot)$. Finally, ϵ captures the measurement error when is model as Gaussian noise $\epsilon \sim N(0, \sigma_\epsilon^2)$.

In this framework, the surrogate model y^S and the discrepancy function $\delta(x)$ are both treated as Gaussian Processes (GPs). The calibration parameters θ and hyperparameters ϕ of the covariance function $c(\cdot, \cdot)$ are estimated using Bayesian modeling techniques. However, in this work, the surrogate model is the NN model, while the GP discrepancy term captures systematic errors not accounted for by the NN model.

2.2. Model Formulation with Deterministic NN and GP Discrepancy

The NN surrogate model serves as the deterministic mean function of the GP:

$$y^E|x, \theta^*, \Phi \sim GP(y^S(x, \theta^*), c(\cdot, \cdot)) + N(0, \sigma_\epsilon^2 I) \quad (6)$$

This combined model provides predictions while quantifying uncertainties through the GP framework.

2.3. Covariance Function Specification

The GP discrepancy term $\delta(x)$ uses a covariance function $c(x, x')$ (also referred to as a kernel) to quantify correlations between discrepancies at input points x and x' . For melt pool width the squared exponential (SE) kernel was used, as defined in Equation 7.

$$c(x, x') = \sigma_\delta^2 \exp\left(-\sum_{i=1}^2 \frac{(x_i - x'_i)^2}{2l_i^2}\right) \quad (7)$$

In this formulation, $\mathbf{x} = [P, v]$, where P represents the laser power (in watts) and v represents the velocity (in millimeters per second). The parameter σ_δ^2 is the variance, which controls the overall scale of the kernel and determines the level of variation that the Gaussian Process can capture. The length-scale parameters l_i , associated with each input dimension (P and v), define how quickly the correlation between data points decreases as their distance in input space increases. Larger values of l_i correspond to smoother variations in the respective dimension i , while smaller values allow the model to capture finer details. The exponential term ensures a smooth decay in correlation as the squared Euclidean distance $(x_i - x'_i)^2$ between points increases, resulting in a smooth covariance structure. Here, $\mathbf{x} = [P, v]$, where P is the laser power (W) and v is the velocity (mm/s).

For melt pool depth, the Matern kernel was used, as defined in Equation 8. This kernel is suitable for sharper variations, such as the transition of melt pool depth from conduction mode to keyholing mode:

$$c(x, x') = \sigma^2 \left(1 + \sqrt{5}r + \frac{5r^2}{3}\right) \exp(-\sqrt{5}r), \quad r = \frac{\|x - x'\|}{l} \quad (8)$$

In this formulation, σ^2 is the variance, controlling the overall scale of the kernel and determining the level of variation that the Gaussian Process can capture. The length scale of the kernel is

represented by ℓ , just as in the case of the SE kernel. The scaled Euclidean distance between input points $\mathbf{x}$ and $\mathbf{x}'$ is represented by r .

2.4. Calibration Workflow

The calibration process for the E-T emulator involves two main steps, namely the estimation of the calibration parameters and the hyperparameters using Markov Chain Monte Carlo (MCMC).

Estimation of Calibration Parameters θ^ .* The optimal calibration parameters θ^* minimize the sum of squared residuals between experimental observations y^E and the surrogate model predictions $y^S(x, \theta)$:

$$\theta^* = \arg \min_{\theta \in \Theta} \sum_{i=1}^{N_E} [(y_w^E - y_w^S(x, \theta))^2 + (y_d^E - y_d^S(x, \theta))^2] \quad (9)$$

This step, known as Maximum A Posteriori (MAP) estimation, assumes a Gaussian likelihood for the posterior distribution and uses deterministic optimization methods. Estimating calibration parameters by optimization is also discussed in [2].

Estimation of Hyperparameters ϕ Using MCMC. Markov Chain Monte Carlo (MCMC), specifically the No-U-Turn Sampler (NUTS) [3], is used to sample from the posterior distributions of the hyperparameters ϕ . NUTS is a gradient-based Hamiltonian Monte Carlo (HMC) method [4] that automatically tunes key parameters (e.g., step size and step count). It efficiently captures uncertainties in calibration by generating posterior distributions for ϕ .

The calibration process leads to the final posterior predictive equations. With the calibrated parameters θ^* and hyperparameters ϕ^* , posterior predictions for new input points x^P are expressed as [5]:

$$E[y^P | y^E, \theta^*, \phi^*] = y^S(x^P) + \Sigma^{PE} (\Sigma^{EE})^{-1} (y^E - y^S(x^E)) \quad (10)$$

$$\text{COV}[y^P | y^E, \theta^*, \phi^*] = \Sigma^{PP} - \Sigma^{PE} (\Sigma^{EE})^{-1} \Sigma^{EP} \quad (11)$$

where:

- $\Sigma^{EE} = c(x^E, x^E) + \sigma_\epsilon^2 I$
- $\Sigma^{PP} = c(x^P, x^P)$
- $\Sigma^{EP} = c(x^E, x^P), \Sigma^{PE} = c(x^P, x^E)$

2.5. Application to Ni-5Nb Alloy

The calibration framework was demonstrated on the Ni-5Nb alloy system to validate its efficacy. The NN surrogate model, trained on simulation data for melt pool width and depth, was integrated

with a GP discrepancy model. The SE kernel captured the smooth variations in melt pool width, while the Matern kernel modeled the rougher variations in melt pool depth.

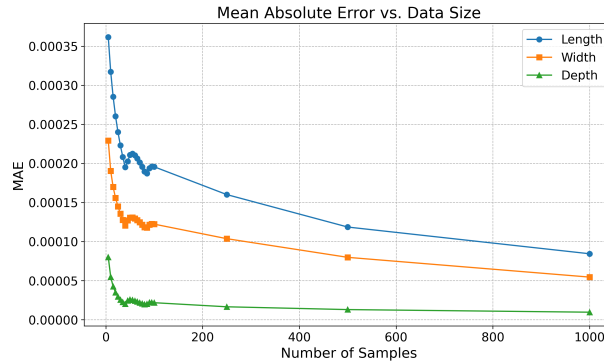
The calibration significantly improved performance metrics. For melt pool width, the Mean Absolute Error (MAE) decreased by 30% (from 15.2% to 10.6%), and for melt pool depth, the Root Mean Squared Error (RMSE) decreased by 25% (from 18.5% to 13.9%). These improvements underscore the ability of the calibration to align NN predictions with experimental observations while robustly accounting for uncertainties.

This detailed explanation is included here to provide supplementary context for readers interested in the calibration process and its impact on performance.

2.6. Performance and Runtime Trends of Surrogate Model

To demonstrate the practicality and efficiency of the NN surrogate model, we evaluate its predictive performance and runtime scalability as the number of test samples increases. The model was pre-trained, and predictions were compared against ground truth values obtained from the analytical E-T model.

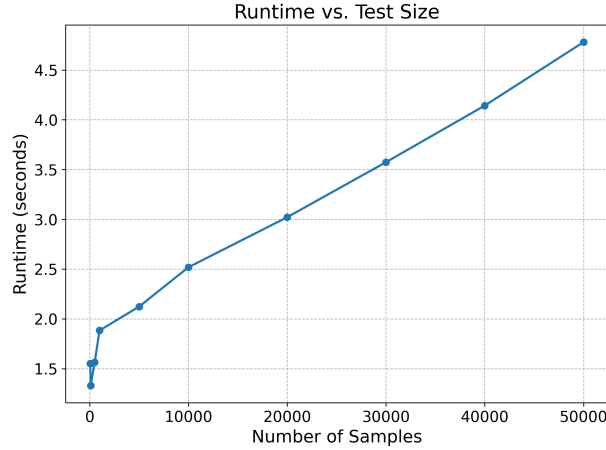
Figure 1 shows the MAE for melt pool length, width, and depth as a function of test dataset size. The model maintains low MAE values across all three dimensions, even when evaluated on relatively small test sets. Notably, the error for depth stabilizes quickly, while length and width continue to improve as more samples are tested. These results indicate that the model generalizes well across the design space and remains consistent in predictive performance, regardless of the number of test points.



Supplementary Figure 1: MAE of the NN surrogate model predictions for melt pool length, width, and depth as a function of training dataset size are visualized. The results demonstrate that the model achieves low error even with small sample sizes. This ensures that a reasonable accuracy can be obtained with significantly fewer points, enabling faster model performance without sacrificing predictive performance.

To show the efficiency of the NN model, Figure 2 visualizes the simulation time (in seconds) to run for various data set sizes up to 50,000 samples. The simulation time ranges from 1 to 5 seconds, depending on the number of test points. While the full dataset took 45 days to generate using the analytical E-T model, the surrogate model—once trained—can simulate tens of thousands of

samples in seconds. The linear trend and consistently low runtime confirm the model’s scalability and suitability for HTP applications, especially when repeated inference is required.



Supplementary Figure 2: The computational run time for various data sizes is visualized. The linear scalability and high-throughput performance of the model, with computational time increasing modestly as the number of test samples increases. The total runtime remains under 5 seconds, highlighting the efficiency of the surrogate model for large-scale predictions.

3. Printability Maps

The 12 printability maps were constructed using the boundary conditions in the Supplementary Table 1. In addition, precision, recall, and accuracy were used to evaluate the performance of the printability maps as described in the Methods section of the main manuscript. In the context of the printability map, precision measures the accuracy with which the prediction was made, where 100% precision means that every point predicted to belong to a defect class truly belongs to that defect class. On the other hand, an undefined precision value corresponds to all positive points being misclassified and all negative points being correctly classified (TP and FP are 0). Recall measures the ability of the combination of criteria to capture the entire region of the processing space where a specific defect occurs. A 100% recall indicates that the combination of criteria could fully map the region of the defect. Finally, accuracy measures how effectively the printability maps made correct predictions for all the defect modes.

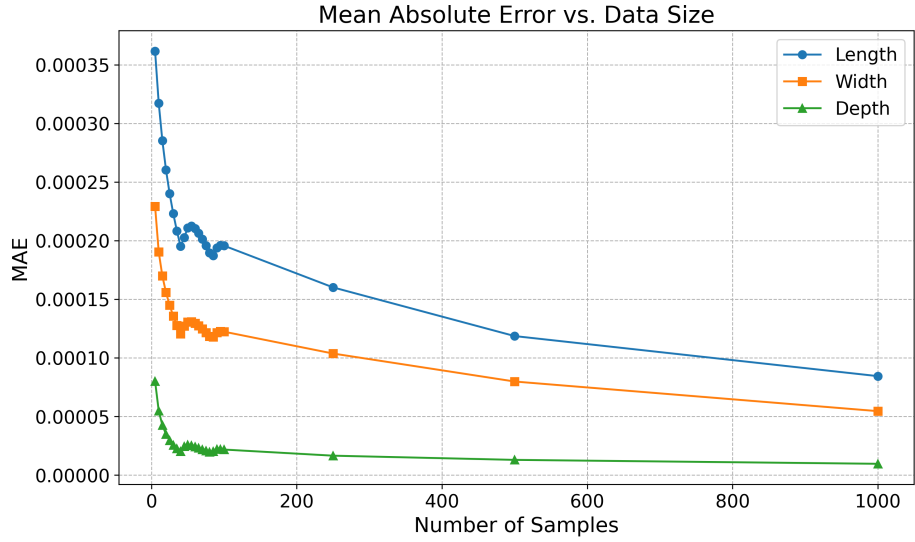
3.1. Ensemble Printability Maps for the Analytical and Dimensionless E-T Thermal Model

These profiles helped in constructing the ensemble probability printability maps, as illustrated in Supplementary Figure 6. The alloy that had a maximum printability index was Alloy E (5.62%), while Alloy A was the least printable alloy in the Pareto front alloys with a printable index of 4.13%. The maximum printability index reached a value of 5.62%, exceeding the printability index of 3.32% that the analytical E-T thermal model determined for the equiatomic CoCrFeMnNi alloy.

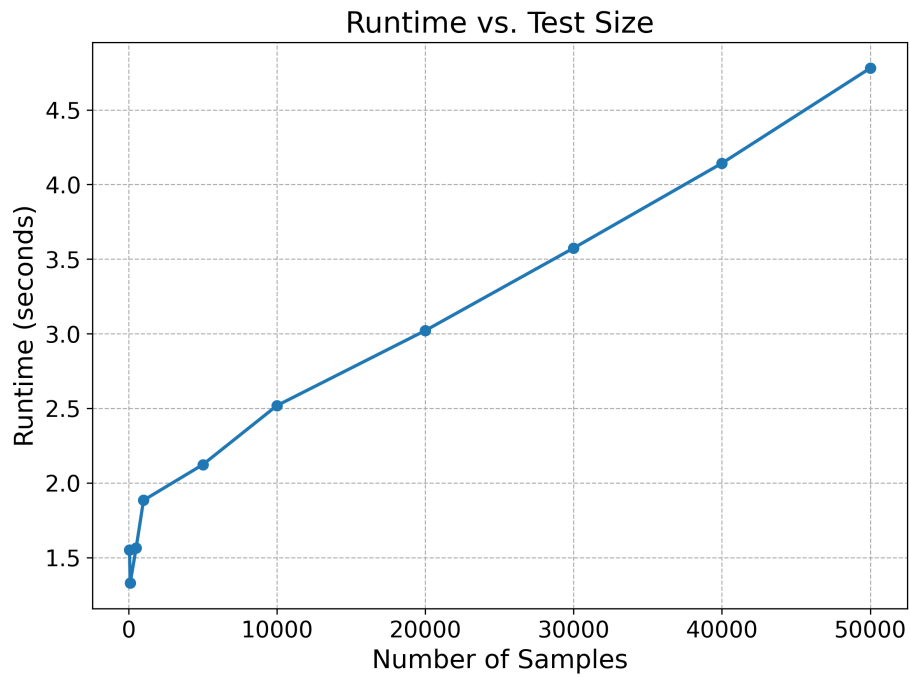
Supplementary Figure 7 visualizes the ranking of each alloy based on objectives: susceptibility to hot cracking, solidification range, composition-based balling criteria and printability index. The

Defect Type	Label	Equation
Lack-of-Fusion	LOF1	$D \leq t$
	LOF2	$(\frac{h}{W})^2 + \frac{th}{th+D} \geq 1$
Balling	Ball1	$\frac{L}{W} \geq 2.3$
	Ball2	$\frac{\pi W}{L} < \sqrt{\frac{2}{3}}$
Keyholing	KH1	$\frac{W}{D} \leq 2.5$
	KH2	$\frac{\Delta H}{h_s} = \frac{AP}{\pi h_s \sqrt{\alpha \nu a^3}} > \frac{\pi T_{boiling}}{T_{liquidus}}$
	KH3	$\frac{Ke}{\eta P} = \frac{1}{(T_{liquidus} - T_0) \pi \rho C_p \sqrt{\alpha \nu r_0^3}} > 6$

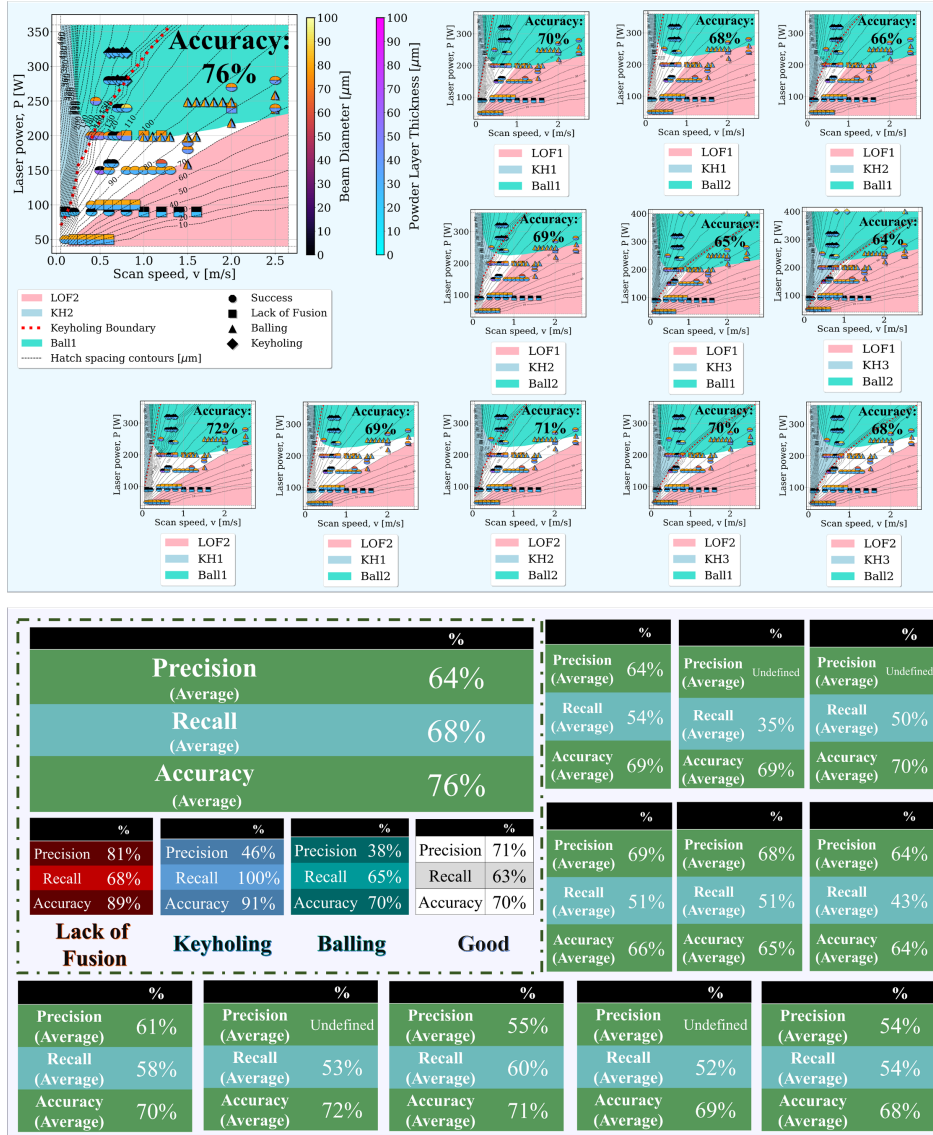
Supplementary Table 1: The equation and labels for each respective defect used to graph the printability maps



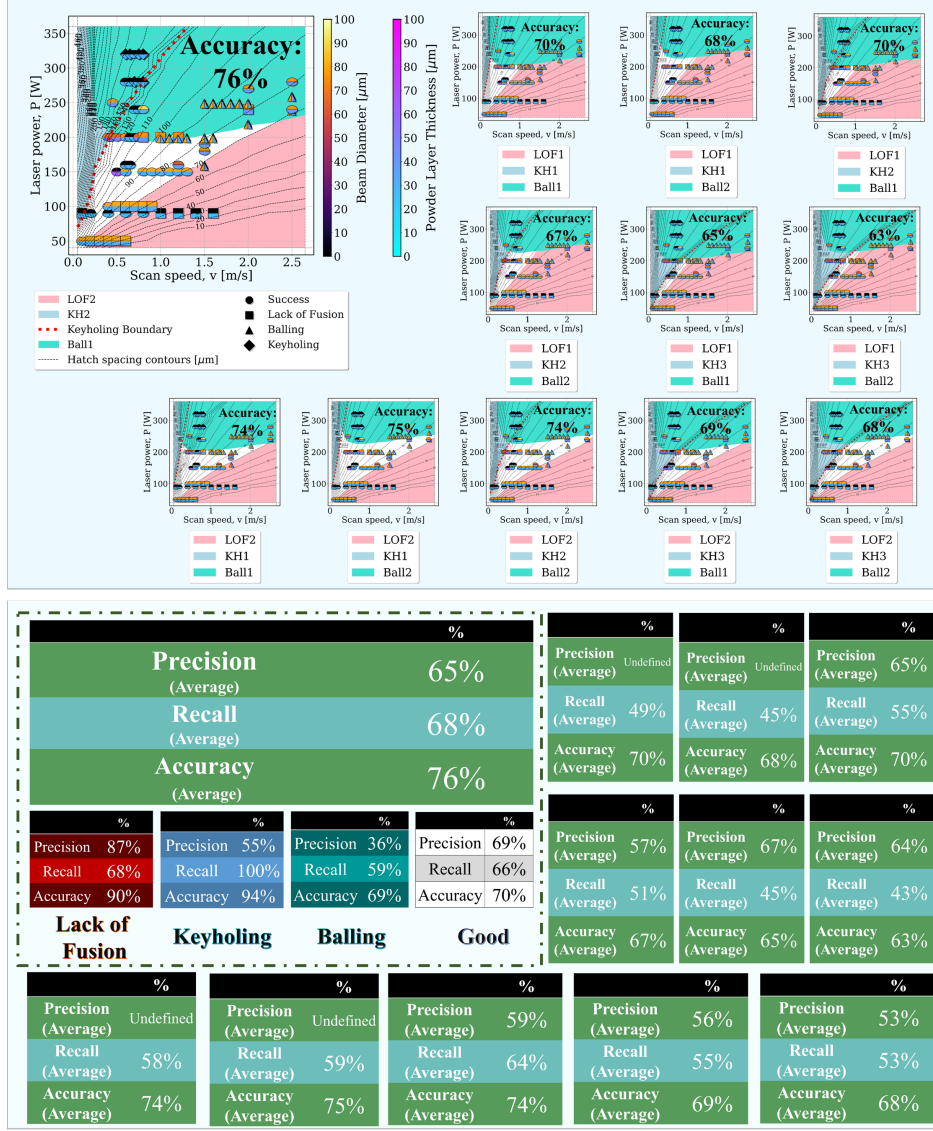
Supplementary Figure 3: Full Performance Evaluation of Printability Maps Constructed Using Analytical E-T Model. By obtaining the melt pool profile using the analytical E-T model, the highest accuracy of 76% was obtained by using a hatch distance of $70\mu\text{m}$ and beam diameter of $80\mu\text{m}$ with criteria set of **LOF2**, **KH2**, and **Ball1**. In addition, for the best map, the precision and recall values were 64% and 68%, respectively. The printable index is 17.1%. LOF: Lack-of-fusion. KH: Keyholing. Ball: Balling. Circle: Experimentally observed defect-free single tracks. Square: Experimentally observed lack-of-fusion defect. Triangle: Experimentally observed balling defect. Diamond: Experimentally observed Keyholing defect.



Supplementary Figure 4: Full Performance Evaluation of Printability Maps Constructed Using NN E-T Model. Using the NN (ML) E-T model, the highest accuracy that was obtained was 76% for the set of criteria of **LOF2**, **KH2**, and **Ball1**. For the best map, the precision and recall values are 65% and 68%, respectively. It can be noted that the results are similar, if not the same, as those obtained from the analytical E-T model. However, the NN E-T model is computationally less expensive than the analytical E-T thermal model. The printable index is 18.5%. LOF: Lack-of-fusion. KH: Keyholing. Ball: Balling. Circle: Experimentally observed defect-free single tracks. Square: Experimentally observed lack-of-fusion defect. Triangle: Experimentally observed balling defect. Diamond: Experimentally observed Keyholing defect.



Supplementary Figure 5: Full Performance Evaluation of Printability Maps Constructed Using Dimensionless Form of the E-T Model. By obtaining the melt pool profile with the dimensionless E-T model, the highest accuracy obtained was 72% with the optimal combination criteria of **LOF1, Key1, and Ball1**. The boundary for each region is smooth, which is a more realistic description of the boundary behavior than that of the analytical E-T and NN (ML) E-T models. The precision value for the best map is defined as *undefined* because experimental points labeled as keyholing points were all misclassified on the predicted printability map. On the other hand, the recall value for the best map is 50%. The printable index when using the dimensionless E-T model is 22.2%. LOF: Lack-of-fusion. KH: Keyholing. Ball: Balling. Circle: Experimentally observed defect-free single tracks. Square: Experimentally observed lack-of-fusion defect. Triangle: Experimentally observed balling defect. Diamond: Experimentally observed Keyholing defect.



Supplementary Figure 6: Ensemble Probability Printability Maps for Pareto Front Alloys Using Analytical E-T Model. Using the analytical E-T thermal model, we generate the ensemble probability printability maps for the five sampled alloys with their respective printability index. The alloy, Alloy E ($\text{Co}_{20}\text{Cr}_{40}\text{Fe}_5\text{Mn}_5\text{Ni}_{30}$) achieves the highest printability index of 5.62%, while Alloy A ($\text{Co}_{15}\text{Cr}_{15}\text{Fe}_{60}\text{Mn}_5\text{Ni}_5$) shows a minimum printability index of 4.13%. However, both values exceed 3.32%, the printability index of the equiatomic CoCrFeMnNi alloy's ensemble probability printability map generated using the analytical E-T model. Salmon-shaded region: Area that represents the predicted lack-of-fusion region. Blue-shaded region: Area that represents the predicted keyholing region. Teal-shaded region: Area that represents the predicted balling region.

ranking maximizes the printability index and composition-based balling, while minimizing the susceptibility to hot cracking and the solidification range. In the Supplementary Figure 7, the visualization maximizes each objective and inverts the axes for the susceptibility to hot cracking and the solidification range to represent their minimized values. The difference between the printable indices for the Pareto front alloys is between 0.07% and 1.49%.

Using the dimensionless E-T thermal model, the maximum printability index obtained was 10.60% for Alloy E, as shown in Supplementary Figure 8e, followed by Alloy D (Supplementary Figure 8d), Alloy B (Supplementary Figure 8b), and Alloy C (Supplementary Figure 8c). Alloy A (Supplementary Figure 8a) showed the lowest printability index, as summarized in Supplementary Table 2.

Alloy	Composition	Analytical E-T Model Ranking	NN (ML) E-T Model Ranking	Dimensionless E-T Model Ranking
Alloy A	Co ₁₅ Cr ₁₅ Fe ₆₀ Mn ₅ Ni ₅	5	5	5
Alloy B	Co ₄₅ Cr ₂₀ Fe ₂₅ Mn ₅ Ni ₅	3	3	3
Alloy C	Co ₃₀ Cr ₃₀ Fe ₄₀ Mn ₅ Ni ₅	4	4	4
Alloy D	Co ₄₀ Cr ₃₀ Fe ₂₀ Mn ₅ Ni ₅	2	2	2
Alloy E	Co ₂₀ Cr ₄₀ Fe ₅ Mn ₅ Ni ₃₀	1	1	1

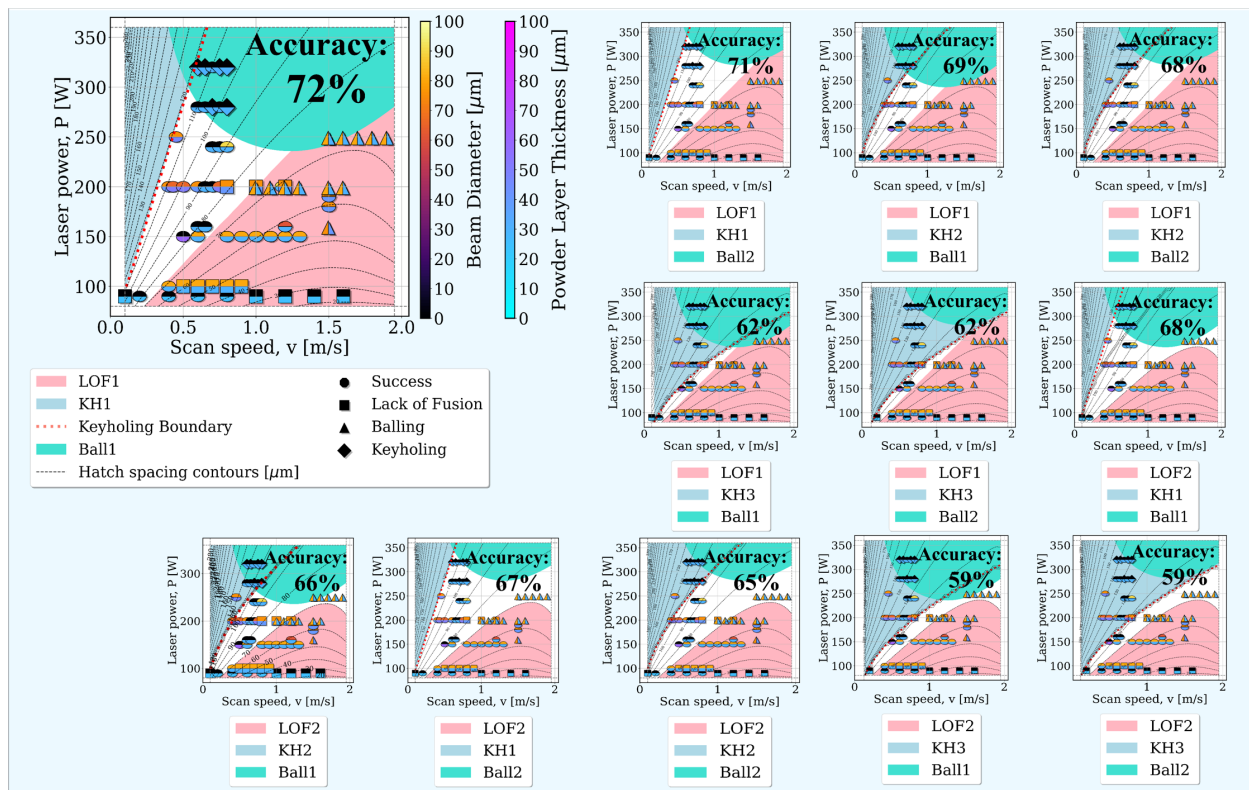
Supplementary Table 2: We constructed the printability maps for the five sampled alloys using the optimal combination criteria (**LOF2-KH2-Ball1**) and the three E-T thermal models. We calculated the printability index for each map and ranked the alloys from the highest printability index (1) to the lowest (5). Alloy E consistently achieved the highest printability index, even when ranked using the alternate criteria set from the dimensionless E-T thermal model.

Supplementary Figure 9 summarizes the ranking of the alloys based on the Pareto front objectives and printability index when the melt pool profile is predicted using the dimensionless form of the E-T model.

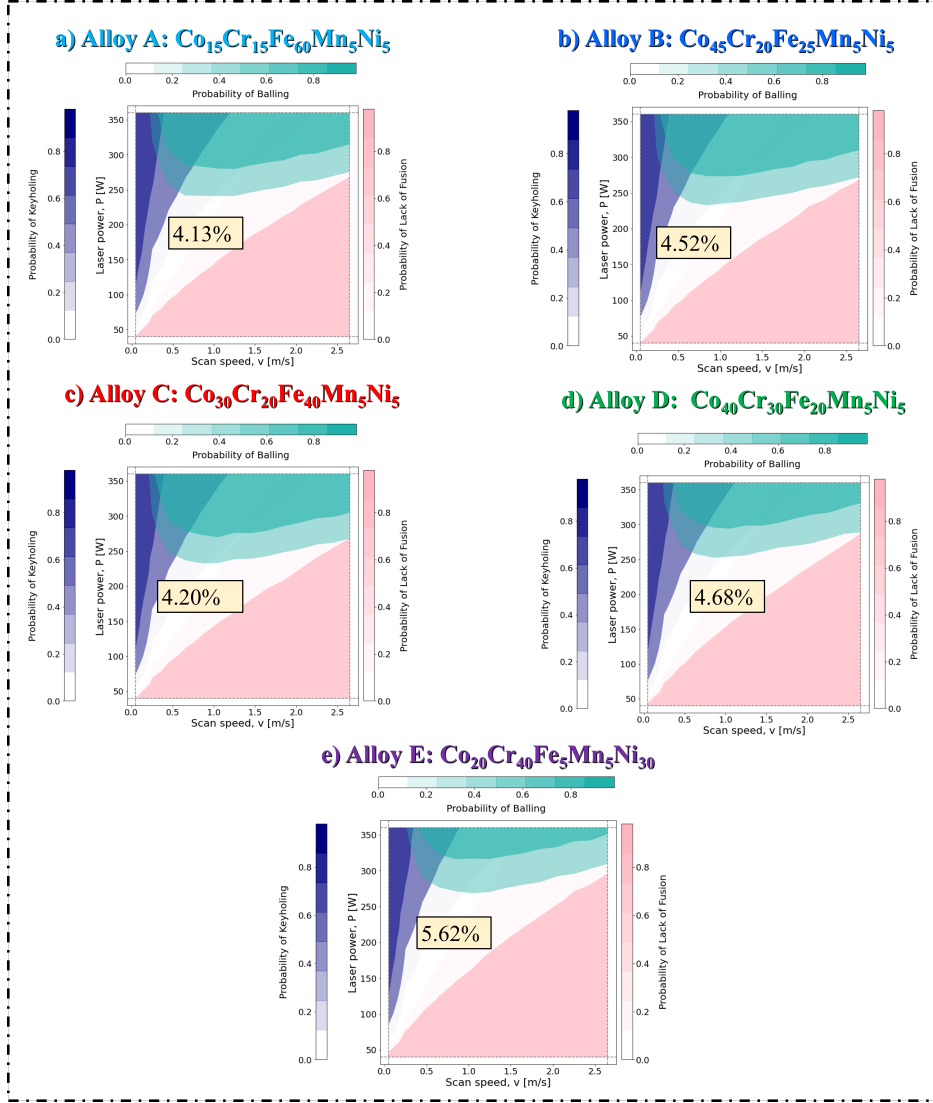
As shown in Supplementary Table 2, all three thermal models agree on the rankings of the alloys with the highest printability for Alloy E. Comparing the printability index of the equiatomic CoCrFeMnNi to the five sampled alloys demonstrates that our proposed framework effectively designs HEAs containing elements Co, Cr, Fe, Mn, and Ni, which can be printed across a large power and velocity processing space.

3.2. Analysis of Using the Optimal Criteria Set to Rank the Pareto Front Alloys

The printability analysis used a 17-by-14 design of experiments for power and velocity while maintaining the hatch spacing, the laser beam diameter, and the powder layer thickness constant at 70 μm , 80 μm , and 30 μm , respectively, consistent with the equiatomic CoCrFeMnNi alloy. Using the analytical thermal E-T model, the melt pool profile for each of the sampled alloys was obtained, and the process constructed printability maps, as shown in Supplementary Figure 10 using the optimal defect criteria of **LOF2-KH2-Ball1**. The printability index rankings for the sampled alloys are

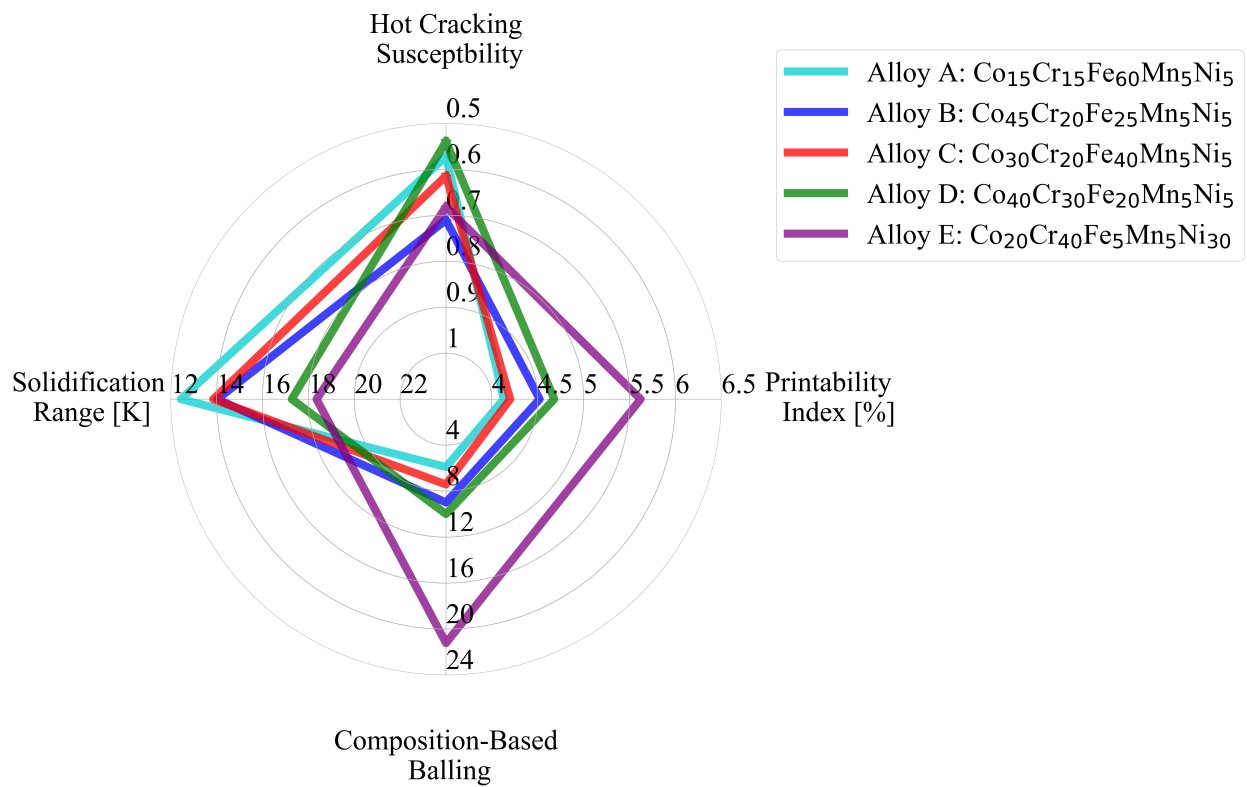


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Supplementary Figure 8: Ensemble Probability Printability Maps for Pareto Front Alloys Using Dimensionless E-T Model. Using the dimensionless E-T model, we generated the ensemble probability printability maps for the five sampled alloys. Alloy E ($\text{Co}_{20}\text{Cr}_{40}\text{Mn}_5\text{Fe}_5\text{Ni}_{30}$), achieved the maximum printability index of 10.60%, while the minimum value for the printability index was 7.41%. Both values exceed the printability index for the printability maps for the equiatomic CoCrFeMnNi using the dimensionless E-T model of 5.32%. Salmon-shaded region: Area that represents the predicted lack-of-fusion region. Blue-shaded region: Area that represents the predicted keyholing region. Teal-shaded region: Area that represents the predicted balling region. Red dashed line: Keyholing boundary. Black dashed lines: Maximum hatch spacing required for sufficient overlap in a fully built part.

Analytical E-T Model Ranking



Supplementary Figure 9: Ranking Optimized Alloys for High-Temperature Application with Dimensionless E-T Model. The radar chart graphically displays the ranking of the alloys based on the Pareto front objectives and printability index when using the dimensionless E-T model, as well as the optimal combination criteria set. The chart inverts the axes for solidification range and hot cracking susceptibility to show the rankings where all objectives are maximized.

summarized in Supplementary Table 2, where Alloy E showed the highest printability index value when using the optimal defect criteria.

Supplementary Figure 11 visualizes the ranking of each alloy based on the objectives: hot cracking susceptibility, solidification range, composition-based balling criteria, and the printability index. The ranking maximizes the printability index and composition-based balling while minimizing hot cracking susceptibility and the solidification range. In Supplementary Figure 11, the visualization maximizes each objective and inverts the axes for hot cracking susceptibility and solidification range to represent their minimized values.

The maximum printability index reached a value of 26.3%, exceeding the printability index of 17.1% that the analytical E-T thermal model determined for the equiatomic CoCrFeMnNi alloy. The NN (ML) E-T thermal model achieved a higher maximum printability index of 27.2% when compared to the analytical E-T thermal model, as shown in Supplementary Figure 12. In addition, the current printability index surpasses 18.5%, a value obtained for the printability maps constructed using the NN (ML) E-T thermal model for the equiatomic CoCrFeMnNi alloy. Ranking the alloys based on the printability index produces results similar to the rankings generated by the analytical E-T thermal model. As summarized in Supplementary Table 2, Alloy E showed the highest printability index, followed by the remaining alloys: Alloy B, Alloy D, Alloy C, and Alloy A.

The ranking of the alloys using the neural network E-T model is summarized graphically in Supplementary Figure 13. Similar to Supplementary Figure 11, the radar chart inverts the axes for hot cracking susceptibility and solidification range to the problems as one of maximization. We want to note that the ranking between printability maps ranked at 3 and 4 differs by 0.1% in printable indices for both thermal models.

Using the dimensionless E-T thermal model, combined with the optimal criteria set of Equation 12 for lack-of-fusion, Equation 17 for keyholing, and Equation 14 from the main text (**LOF2-KH2-Ball1**), the maximum printability index obtained was 31.9% for Alloy E, as shown in Supplementary Figure 14, followed by Alloy D, Alloy B, and Alloy A. Alloy C showed the lowest printability index, as summarized in Supplementary Table 2. Supplementary Figure 15 summarizes the ranking of the alloys based on the Pareto front objectives and printability index when the melt pool profile is predicted using the dimensionless form of the E-T model.

As shown in Supplementary Table 1, all three thermal models agree that the alloy with the highest printability is Alloy E. Comparing the printability index of the equiatomic CoCrFeMnNi to the five sampled alloys demonstrates that our proposed framework effectively designs HEAs containing elements Co, Cr, Fe, Mn, and Ni, which can be printed across a large power and velocity processing space.

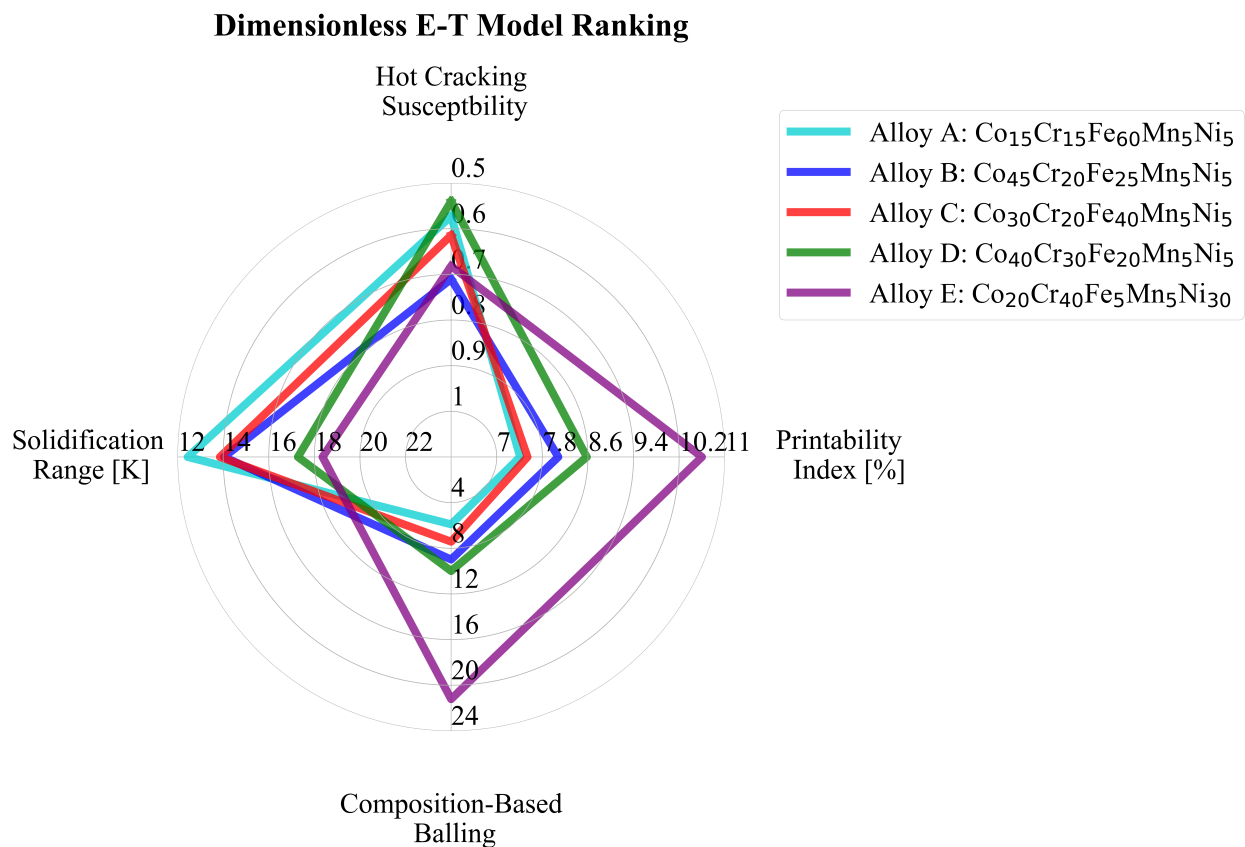
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Supplementary Figure 8 .png

Supplementary Figure 10: Printability Maps for the Optimized Alloys Constructed Using Analytical E-T Model.

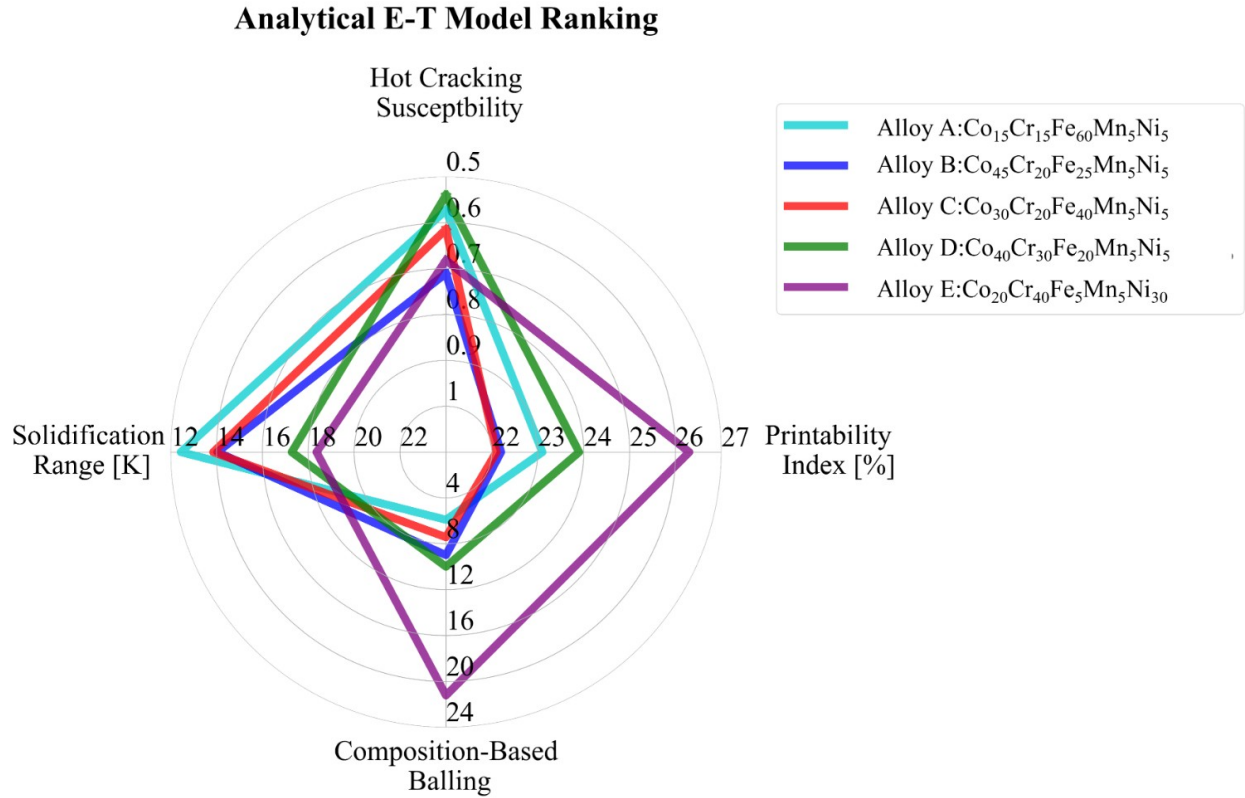
Using the analytical E-T thermal model and the optimal criteria set, **LOF2-KH2-Ball1**, we generate the printability maps for the five sampled alloys with their respective printability index. The alloy, Alloy E ($\text{Co}_{20}\text{Cr}_{40}\text{Fe}_5\text{Mn}_5\text{Ni}_{30}$), achieves the highest printability index of 26.3% while the minimum printability index of 22.1%. However, both values exceed the printability index of the equiatomic CoCrFeMnNi alloy's printability map generated using the analytical E-T model. LOF: lack-of-fusion. W: melt pool width. h: hatch spacing. t: powder layer thickness. D: melt pool depth. ΔH : specific enthalpy. T_{boiling} : boiling temperature. T_{melting} : melting temperature. h_s : specific enthalpy at melting. Salmon-shaded region: Area that represents the predicted lack-of-fusion region. Blue-shaded region: area that represents the predicted keyholing region. Teal-shaded region: Area that represents the predicted balling region. Red dashed line: keyholing boundary. Black dashed lines: Maximum hatch spacing required for sufficient overlap in a fully built part.



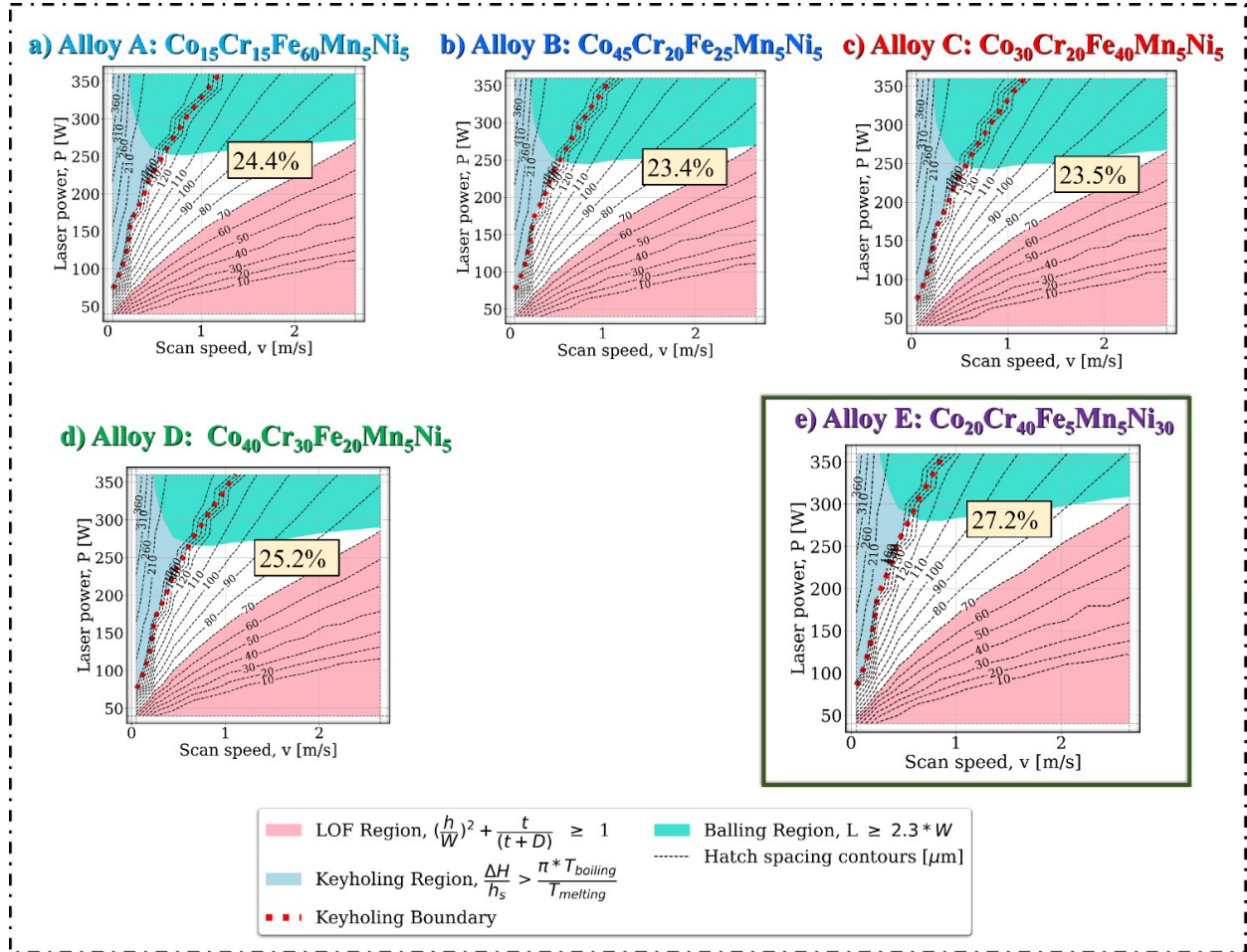
Supplementary Figure 11: Ranking Optimized Alloys for High-Temperature Application with Analytical E-T Model using LOF2-KH2-Ball1. Using the analytical E-T model to calculate the melt pool geometry, the radar chart summarizes the objectives used to rank the alloys for printability. The chart maximizes the printability index and the composition-based balling criterion, while minimizing the solidification range and the hot cracking susceptibility. To account for the two optimization problems, the chart inverts the solidification range and hot cracking susceptibility axis, so the radar chart maximizes all objectives.

Supplementary Figure 10.png

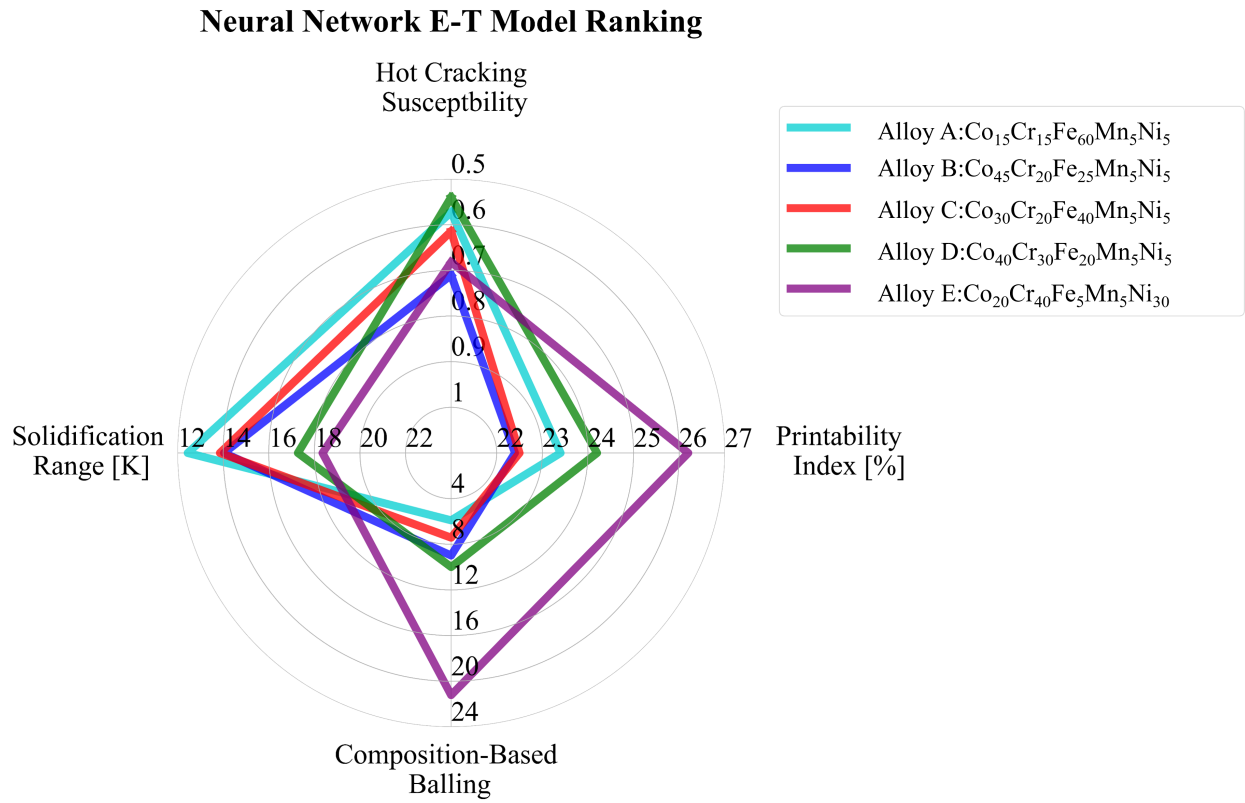
Supplementary Figure 12: Printability Maps for the Optimized Alloys Constructed Using NN E-T Model. Using the NN (ML) E-T model to calculate the melt pool profile for the five sampled alloys, Alloy E ($\text{Co}_{20}\text{Cr}_{40}\text{Mn}_5\text{Fe}_5\text{Ni}_{30}$) achieved the highest printability index of 27.2%. Alloy A ($\text{Co}_{15}\text{Cr}_{15}\text{Mn}_5\text{Fe}_{60}\text{Ni}_5$) has the lowest printability index at 24.4%. Both printability index values exceed the printability index of 18.5%, calculated from the printability map of equiatomic CoCrFeMnNi using the NN (ML) E-T model. Salmon-shaded region: Area that represents the predicted lack-of-fusion region. Blue-shaded region: Area that represents the predicted keyholing region. Teal-shaded region: Area that represents the predicted balling region. Red dashed line: Keyholing boundary. Black dashed lines: Maximum hatch spacing required for sufficient overlap in a fully built part.



Supplementary Figure 13: Ranking Optimized Alloys for High-Temperature Application with NN E-T Model using LOF2-KH2-Ball1. The radar chart summarizes the results for the alloys predicted using the Pareto front objectives and further analyzes the performance and printability of the alloys based on the four objectives: hot cracking susceptibility, solidification range, composition-based balling, and the printability index when using the neural network E-T model. The chart inverts the axis for hot cracking susceptibility and solidification range to maximize all objectives.



Supplementary Figure 14: Printability Maps of Optimized Alloys Using Dimensionless E-T Model and LOF2-KH2-Ball1 Criteria. Using the dimensionless E-T model, we generated the printability maps with the optimal combination criteria set for the five sampled alloys. The alloy, **Alloy E** ($\text{Co}_{20}\text{Cr}_{40}\text{Mn}_5\text{Fe}_5\text{Ni}_{30}$), achieved the maximum printability index of 31.9%, while the minimum value for the printability index was 25.6%. Both values exceeded the printability index for the printability maps for the equiatomic CoCrFeMnNi using the dimensionless E-T model of 22.2%. W: Melt pool width. h: Hatch spacing. t: Powder layer thickness. D: Melt pool depth. ΔH : Specific enthalpy. T_{boiling} : Boiling temperature. T_{melting} : Melting temperature. h_s : Specific enthalpy at melting. Salmon-shaded region: Area that represents the predicted lack-of-fusion region. Blue-shaded region: Area that represents the predicted keyholing region. Teal-shaded region: Area that represents the predicted balling region. Red dashed line: Keyholing boundary. Black dashed lines: Maximum hatch spacing required for sufficient overlap in a fully built part.



Supplementary Figure 15: Ranking Optimized Alloys for High-Temperature Application with Dimensionless E-T Model and LOF2-KH2-Ball1 Criteria. The radar chart graphically displays the ranking of the alloys based on the Pareto front objectives and printability index when using the dimensionless E-T model, as well as the optimal combination criteria set. The chart inverts the axes for solidification range and hot cracking susceptibility to show the rankings where all objectives are maximized.

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