Supplementary Information

Thomas Corbin[1], Jason Livesay[1], Ishikawa Takehiko[2], Koyama Chihiro[2], Michael SanSoucie[3], Brandon Phillips[3], and Ranga Narayanan[1]

[1]University of Florida Department of Chemical Engineering, Gainesville, FL, 32611 USA
[2]Human Space Flight Technology Directive, Japan Aerospace Exploration Agency, Tsukuba, Ibaraki 305-8505, Japan
[3]NASA Marshall Space Flight Center, Huntsville, AL, 35812, USA
[1]ranga@ufl.edu tcorbin@ufl.edu

Supplementary Note 1: Conservation of Volume and Effective Droplet Radius

This work follows a similar procedure to that of Lord Rayleigh [1]. To ensure conservation of volume during droplet deformation, the effective radius of the droplet, $R(x)$, is defined such that the total volume remains constant. This condition is expressed as

$$\frac{4\pi}{3}R_0^3 = \frac{2\pi}{3}\int_{-1}^{1} R^3(x)\,dx, \tag{1}$$

where R_0 is the resting (undeformed) radius and $R(x)$ is the instantaneous droplet radius as a function of the cosine of the polar angle, i.e., $x = \cos\theta$.

The droplet surface is represented as a perturbation from a reference radius $\hat{R}$ in terms of Legendre polynomial modes:

$$R(x) = \hat{R} + \sum_{i=2}^{\infty} A_i P_i(x) \quad \approx \quad \hat{R} + A_2P_2(x) + A_3P_3(x) + A_4P_4(x), \tag{2}$$

where A_i are modal amplitudes and $P_i(x)$ are Legendre polynomials. The explicit forms of the first three deformation modes are

$$P_2(x) = \frac{1}{2}(3x^2 - 1), \tag{3}$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x), \tag{4}$$

$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3). \tag{5}$$

Normalizing Equation 1 by the resting volume and defining the normalized radius $\hat{r} = \hat{R}/R_0$ and normalized amplitudes $\epsilon_i = A_i/R_0$, we obtain

$$0 = \frac{2}{3}\int_{-1}^{1}\big(\hat{r} + \epsilon_2P_2(x) + \epsilon_3P_3(x) + \epsilon_4P_4(x)\big)^3 dx - \frac{4}{3}. \tag{6}$$

Substituting Equations 3-5 into Equation 6 and performing the integration yields

$$\begin{aligned} 0 = \frac{2}{3}\Bigg(&2\hat{r}^3 + \frac{6\hat{r}\epsilon_2^2}{5} + \frac{6\hat{r}\epsilon_3^2}{7} + \frac{2\hat{r}\epsilon_4^2}{3} + \frac{4\epsilon_2^3}{35} + \frac{12\epsilon_2^2\epsilon_4}{35} + \frac{8\epsilon_2\epsilon_3^2}{35} \\ &+ \frac{40\epsilon_2\epsilon_4^2}{231} + \frac{12\epsilon_3^2\epsilon_4}{77} + \frac{36\epsilon_4^3}{1001}\Bigg) - \frac{4}{3}. \end{aligned} \tag{7}$$

This expression is a cubic equation in $\hat{r}$, which can be solved to obtain the effective normalized radius as a function of modal amplitudes $\epsilon_2, \epsilon_3, \epsilon_4$.

For a single-mode oscillation with $\epsilon = 0.1$, the effective radius remains essentially unchanged from the resting radius. For example, when $\epsilon_4 = 0.1$ (with $\epsilon_2 = \epsilon_3 = 0$), the solution yields $\hat{r} = 0.999 \approx 1$. If all three modes are excited simultaneously with equal amplitude ($\epsilon_2 = \epsilon_3 = \epsilon_4 = 0.1$), the normalized radius decreases slightly to $\hat{r} = 0.995$. However, this deviation rapidly diminishes when the contributions of secondary modes are reduced, demonstrating that the effective radius remains close to the resting radius even under moderate deviations.

To assess the accuracy of simply taking $\hat{r} = 1$, we compute the deviation from exact volume conservation for an experimental case with a large measured deformation ($99Nb_2O_5$-$1Fe_2O_3$). Substituting the experimental amplitudes $\epsilon_2 = 0.0187$, $\epsilon_3 = 0.0021$, and $\epsilon_4 = 0.001$, in Equation 8 with $\hat{r} = 1$ and δV representing the dimensionless volume deviation from a resting sphere,

$$\delta V = \frac{2}{3}\left(2\hat{r}^3 + \frac{6\hat{r}\epsilon_2^2}{5} + \frac{6\hat{r}\epsilon_3^2}{7} + \frac{2\hat{r}\epsilon_4^2}{3} + \frac{4\epsilon_2^3}{35} + \frac{12\epsilon_2^2\epsilon_4}{35} + \frac{8\epsilon_2\epsilon_3^2}{35} + \frac{40\epsilon_2\epsilon_4^2}{231} + \frac{12\epsilon_3^2\epsilon_4}{77} + \frac{36\epsilon_4^3}{1001}\right) - \frac{4}{3}, \tag{8}$$

which, when dimensionalized, corresponds to a volume deviation of

$$2.8 \times 10^{-4}\, R_0^3,$$

a negligible amount for all practical purposes. This confirms that using the resting radius as the effective radius introduces only a vanishingly small error.

Supplementary Note 2: Uncertainty Analysis

The uncertainty in the measured surface tension is evaluated by computing the relative error associated with both the mass and frequency determinations [2,3]. Solving Rayleigh Equation for the surface tension yields

$$\gamma = \frac{f_n^2\, 3\pi m}{n(n-1)(n+2)}. \tag{9}$$

Treating m and f_n as independent variables, the uncertainty in γ is given by

$$u(\gamma) = \sqrt{\left(\frac{\partial\gamma}{\partial f_n}u(f_n)\right)^2 + \left(\frac{\partial\gamma}{\partial m}u(m)\right)^2}, \tag{10}$$

where

$$\frac{\partial\gamma}{\partial f_n} = \frac{2f_n\, 3\pi m}{n(n-1)(n+2)}, \qquad \frac{\partial\gamma}{\partial m} = \frac{f_n^2\, 3\pi}{n(n-1)(n+2)}.$$

Dividing Equation 10 by Equation 9 and substituting the expressions for the partial derivatives gives

$$\frac{u(\gamma)}{\gamma} = \sqrt{\left(2\,\frac{u(f_n)}{f_n}\right)^2 + \left(\frac{u(m)}{m}\right)^2}. \tag{11}$$

The relative error in the frequency measurement is estimated from the sampling rate and number of samples used during the experiment, which determine the frequency resolution Δf, where

$$\Delta f = \frac{\text{sampling rate}}{\text{number of samples}}.$$

Given the camera parameters used, the largest Δf was 1 Hz. Assuming a negligible mass loss and a representative resonance frequency of $f_n = 100$ Hz, the relative uncertainty in the surface tension becomes

$$\frac{u(\gamma)}{\gamma} = \sqrt{\left(2\frac{1\,\mathrm{Hz}}{100\,\mathrm{Hz}}\right)^2} = 2\%.$$

Supplementary Table 1: List of Materials and Conditions

Material	Mass (g)	Atmosphere
Au	0.06904	Ar
Au	0.09358	Ar
Au	0.08133	Ar
Au	0.1075	Air
Au	0.08075	Air
Au	0.08076	Air
Au	0.1078	Air
FeO	0.02252	Ar
FeO	0.02251	Ar
FeO	0.03057	Ar
FeO	0.02731	Ar
Pt	0.07374	Ar
Pt	0.07421	Ar
Pt	0.07421	Ar
Pt	0.08814	Ar
99Nb2O5-1Fe2O3	0.02113	Air
99Nb2O5-1Fe2O3	0.01701	Air
99Nb2O5-1Fe2O3	0.01809	Air
99Nb2O5-1Fe2O3	0.02015	Air
99Nb2O5-1Fe2O3	0.02082	Air
99Nb2O5-1Fe2O3	0.01902	Air

References

[1] Lord Rayleigh, "On the Capillary Phenomena of Jets," *Royal Society*, vol. 29, pp. 71–97, 1879.

[2] T. Ishikawa, C. Koyama, H. Oda, H. Saruwatari, and P.-F. Paradis, "Status of the Electrostatic Levitation Furnace in the ISS-Surface Tension and Viscosity Measurements," *Int. J. Microgravity Sci. Appl*, vol. 39, no. 1, p. 390101, 2022.

[3] J. Nawer, T. Ishikawa, H. Oda, H. Saruwatari, C. Koyama, X. Xiao, S. Schneider, M. Kolbe, and D. M. Matson, "Uncertainty analysis and performance evaluation of thermophysical property measurement of liquid au in microgravity," *npj Microgravity*, vol. 9, no. 1, p. 38, 2023.