Supplementary Information for: A single atom noise probe operating beyond the Heisenberg limit

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(Dated: November 23, 2019)

PACS numbers: 32.70.Cs, 37.10.Ty, 06.30.Ft

Experimental setup

The experiment is based on a single barium ion trapped in a blade-like linear ion trap operated at 16 MHz radio-frequency with 3W power [S1, S2]. With this the radial frequency of motion of the ion is about 2 MHz and the axial frequency is 200 kHz for 100V dc applied to the endcaps. The ion is laser cooled to the Lamb-Dicke regime which ensures that the mean phonon number of the external motion is about five in the radial direction and slightly higher in the axial direction. The qubit is formed by the internal electronic state of the ion, the ground state $|S_{1/2}, m = -1/2\rangle$ and the excited state $|D_{5/2}, m = -1/2\rangle$ coupled by a narrow linewidth laser at 1760 nm. The qubit operations can be described by the Pauli spin operators σ_i, where $i = x, y, z$. The quantization axis is defined by an externally applied weak magnetic field about which the spin processes. There are several possible ways to implement a time-dependent Hamiltonian in the single ion qubit system, namely oscillating magnetic field, oscillating electric field gradient as well as oscillating electromagnetic (EM) field. In this experiment the magnetic field has been kept constant as inherent magnetic field fluctuations at the line frequency (50 Hz) limits the coherence time of the qubit. Instead, we employed a time-dependent non-resonant EM field (de-tuned by -100 kHz from the carrier transition) to generate a periodically oscillating light-atom interaction in a controlled manner as given by the Hamiltonian in eq.(10) of the main article. In order to generate such an EM field we modulate the acusto-optic modulator (AOM) with an oscillating amplitude such that the intensity varies as $I(t) = I_0 + I_d \sin(\omega t)$ (eq.(12) in the main article). The constant AC Stark shift has been measured to be about 25 kHz with a modulation depth of intensity modulation $I_d/I_0 \approx 0.6$. The frequency sensitivity has been measured by scanning a frequency range of $\pm 5\%$ while in the case of amplitude sensitivity measurement it is $\pm 50\%$ for uncontrolled measurement. In the case of controlled one the range is smaller. The amplitude modulation pulse has been generated using an arbitrary waveform generator with a resolution of 400 ps. The measurements have been performed with a Rabi $\pi-$ time of 3 μs and a total measurement

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time of 3 ms, out of which 2 ms is state detection time. We repeated 500 independent measurements to estimate the phase which limits the phase uncertainty to $\leq 6 \times 10^{-4}$ rad. The qubit can be driven to at most 500 kHz frequency mainly limited by the available power of the laser while the laser linewidth is estimated to be < 100 Hz.

Experimental method

As mentioned earlier, the experiment starts by preparing the probe in an optimal state for the best estimation of the estimator, in this case the quantum fisher information (QFI). The state we prepare is

$$|\phi(t=0)\rangle = 1/\sqrt{2}(|0\rangle + |1\rangle), \tag{S1}$$

by application of a $\sigma_x^{\pi/2}$ pulse. After a certain wait time if the final state is again probed by another $\sigma_x^{\pi/2}$ pulse one would expect to observe $|1\rangle$ with 100% probability. This measurement technique known as the Ramsey technique, then probes the change in the relative phase of $|\psi(t)\rangle$ if the state or the laser has evolved at different rates with respect to each other. The uncertainty in the phase estimation is measured by repeating the Ramsey sequence with the phase of the second $\sigma_\phi^{\pi/2}$ pulse adjusted such that the mean probability of finding the qubit in state $|0,1\rangle$ is 0.5. One such measurement is shown in Figure (S1) where each phase measurement has an uncertainty of 0.06 while the slope which is the inverse frequency sensitivity has been measured with an uncertainty of 3.9×10^{-4}. The standard deviation of this measurement is limited by the projection shot noise to $1/\sqrt{n}$ as is observed in Figure (S2), where we have plotted the variance of the final probability to be projected in $|1\rangle$ for identical and independent n measurements. In our experiment, we ensure that the phase between the laser and the atom is maintained over a long time, determined by de-coherence effects associated with the experiment such as a magnetic field noise, laser frequency noise etc..

The coherence time can be as large as minutes, however in our present experiment we are mainly limited by the ambient magnetic field amplitude noise since the qubit states are susceptible to magnetic field strength variations. We create a time-dependent Hamiltonian for the frequency and amplitude estimation by modulating an external field which couples off-resonantly to the atomic qubit. Therefore, the relative phase between the eigenkets of the state $|\psi(t)\rangle$ will also modulate. Thus one would expect to observe a change in the relative phase acquired during the evolution time t. The final phase is observed by suitably projecting the state to the σ_z axis. The rate of change of phase as a function of the applied frequency provides the QFI for the frequency estimation. As shown in [S3], a control R_x^π pulse at the right frequency and phase with respect to the external oscillating field saturates the QFI as in eq.(6) in the main article.

References

[S1] T. Dutta, Thesis *Precision measurement to study strongly correlated systems-single ion to phonons in an ion chain.*, https://scholarbank.nus.edu.sg/handle/10635/134407 (2016).

[S2] D. De Munshi, Thesis *Precision measurements to explore underlying geometries and interactions in a trapped Ba^+ ion.*, https://scholarbank.nus.edu.sg/handle/10635/134946(2016).

[S3] Jing Yang, Shengshi Pang & Andrew N. Jordan, *Quantum parameter estimation with the Landau-Zener transition.* Phys. Rev. A. **96**, 020301 (2017).

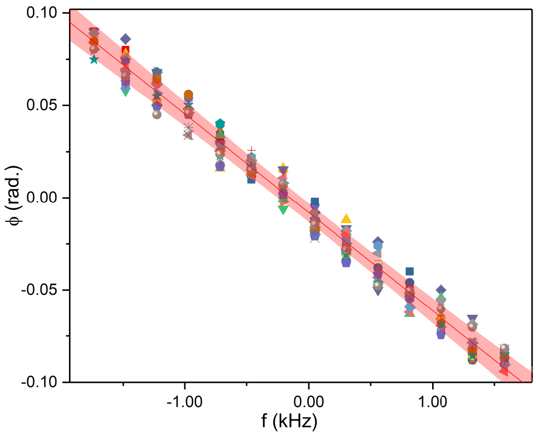

FIG. S1: The variation of the acquired phase as a function of the frequency of the external field. The slope of the curve determines the sensitivity. The dots correspond to 22 independent measurements per frequency point while the 95% confidence limit band for a linear fit is shown by the shaded region. This slope corresponds to 55μs observation time.

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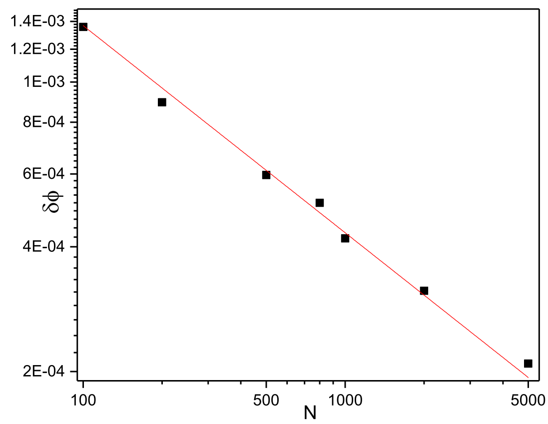

FIG. S2: The variance of the measured phase as a function of the number of repetition. The fitted curve matches well with the standard limit of $1/\sqrt{n}$. In this measurement the phases of the Ramsey pulses are adjusted such that the final state is an equal superposition of the eigenkets.

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