

Supplementary Material for robust entanglement distribution via optical fibre assisted by an asynchronous counter-propagating laser light

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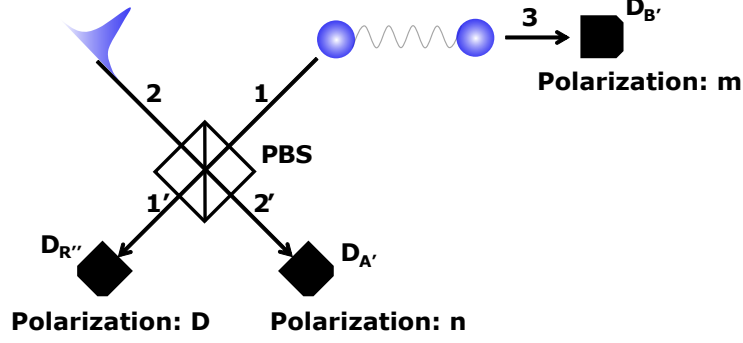
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In this supplemental material, we discuss the reason for the degradation of the fidelity of the final state based on the theoretical analysis in Ref. [1]. We define the coincidence probability among $D_{R''}$ with D-polarization, $D_{B'}$ with m -polarization and $D_{A'}$ with n -polarization by P_{mn} for $m, n = H, V, D, A, R, L$, where A, R, and L represent antidiagonal polarization, right circular polarization and left circular polarization, respectively. Assuming that the single detection probability at $D_{B'}$ does not depend on the measured polarization m , we define polarization correlation visibilities of the final state as: $V_Z := |P_{HH} + P_{VV} - P_{HV} - P_{VH}| / (P_{HH} + P_{VV} + P_{HV} + P_{VH})$, $V_X := |P_{DD} + P_{AA} - P_{DA} - P_{AD}| / (P_{DD} + P_{AA} + P_{DA} + P_{AD})$ and $V_Y := |P_{RR} + P_{LL} - P_{RL} - P_{LR}| / (P_{RR} + P_{LL} + P_{RL} + P_{LR})$. Then, the fidelity of the final state is given by

$$F = (1 + V_Z + V_X + V_Y)/4. \quad (S1)$$

In the following, we estimate the influence of the multiple photon generation at the SPDC photon source and the coherent laser pulse on the polarization correlation visibilities of the final state, while assuming the perfect indistinguishability. The model is shown in Fig 1. The states input to the quantum parity check from modes 1 and 2 are the state heralded by photon detection at $D_{B'}$ and D-polarized coherent state (average photon number: s_2 , intensity correlation function: 1). We assume that when an m -polarized photon in mode 3 is detected at $D_{B'}$, it heralds m^* -polarized signal photons (average photon number: s_1 , intensity correlation function: $g_s^{(2)}$) in mode 1, while multi-pair emission from the SPDC source produces m' -polarized noise photons (average photon number: n_1 , intensity correlation function: 2) which have no correlation or phase relations with signal photons. m^* is complex conjugate to the m -polarization and m' -polarized state is orthogonal to m^* -polarized state. Here, we assume that s_1 , n_1 and $g_s^{(2)}$ do not depend on heralding polarization m . As in our experiment, we assume that the overall transmittance of the system including the quantum efficiencies of $D_{A'}$, $D_{R''}$ and $D_{B'}$ are much less than 1 such that the detection probabilities are proportional to the photon number in the detected mode.



Supplementary Figure 1. The sketch of our theoretical model. The detection signal of a photon in the mode 3 is used to herald photon in the mode 1. The QPC is performed on the coherent light pulse in the mode 2 and photon in the mode 1.

Under the above assumptions and conditions, we first derive V_Z . The coincidence probability P_{mH} is given by

$$P_{mH} = \eta_{1'}\eta_{2'} \langle \hat{b}_{1'D}^\dagger \hat{b}_{2'H}^\dagger \hat{b}_{1'D} \hat{b}_{2'H} \rangle \quad (S2)$$

$$= \frac{1}{2} \eta_{1'}\eta_{2'} (\langle \hat{b}_{1'H}^\dagger \hat{b}_{2'H}^\dagger \hat{b}_{1'H} \hat{b}_{2'H} \rangle + \langle \hat{b}_{1'V}^\dagger \hat{b}_{2'H}^\dagger \hat{b}_{1'V} \hat{b}_{2'H} \rangle), \quad (S3)$$

where η_i is the transmittance of the system in modes $i = 1', 2'$. Here, we introduced the annihilation and creation operators $\hat{b}_{i,m}$ and $\hat{b}_{i,m}^\dagger$ of the m -polarized photons in the output modes of PBS for $i = 1', 2'$. They satisfy the commutation relation as $[\hat{b}_{ik}, \hat{b}_{i'k'}^\dagger] = \delta_{ii'}\delta_{kk'}$. Similarly, we define the annihilation and creation operators $\hat{a}_{i,m}$ and $\hat{a}_{i,m}^\dagger$ of the m -polarized input modes, which satisfy the commutation relation as $[\hat{a}_{jk}, \hat{a}_{j'k'}^\dagger] = \delta_{jj'}\delta_{kk'}$. Using a unitary operator $\hat{U}$ of the PBS satisfying $\hat{U}\hat{b}_{1'H(1'V)}^\dagger\hat{U}^\dagger = \hat{a}_{1H(2V)}^\dagger$ and $\hat{U}\hat{b}_{2'H(2'V)}^\dagger\hat{U}^\dagger = \hat{a}_{2H(1V)}^\dagger$, Eq. (S3) is transformed into

$$P_{mH} = \frac{1}{2} \eta_{1'}\eta_{2'} \left(\langle \hat{a}_{1H}^\dagger \hat{a}_{2H}^\dagger \hat{a}_{1H} \hat{a}_{2H} \rangle + \langle \hat{a}_{2V}^\dagger \hat{a}_{2H}^\dagger \hat{a}_{2V} \hat{a}_{2H} \rangle \right). \quad (S4)$$

Rewriting the operators in mode 2 of Eq. (S4) in terms of the polarizations D and A, we obtain

$$P_{mH} = \frac{1}{4} \eta_{1'}\eta_{2'} \left\{ \langle \hat{a}_{1H}^\dagger \hat{a}_{2D}^\dagger \hat{a}_{1H} \hat{a}_{2D} \rangle + \langle \hat{a}_{1H}^\dagger \hat{a}_{2A}^\dagger \hat{a}_{1H} \hat{a}_{2A} \rangle + \frac{1}{2} (\langle \hat{a}_{2D}^\dagger \hat{a}_{2D}^\dagger \hat{a}_{2D} \hat{a}_{2D} \rangle + \langle \hat{a}_{2A}^\dagger \hat{a}_{2A}^\dagger \hat{a}_{2A} \hat{a}_{2A} \rangle) \right\}. \quad (S5)$$

For calculating P_{HH} , we substitute $\langle \hat{a}_{1H}^\dagger \hat{a}_{1H} \rangle = s_1$, $\langle \hat{a}_{2D}^\dagger \hat{a}_{2D} \rangle = s_2$, $\langle \hat{a}_{2A}^\dagger \hat{a}_{2A} \rangle = 0$, $\langle \hat{a}_{2D}^\dagger \hat{a}_{2D}^\dagger \hat{a}_{2D} \hat{a}_{2D} \rangle = s_2^2$ and $\langle \hat{a}_{2A}^\dagger \hat{a}_{2A}^\dagger \hat{a}_{2A} \hat{a}_{2A} \rangle = 0$ leading to

$$P_{HH} = \frac{1}{4} \eta_{1'}\eta_{2'} \left(s_1 s_2 + \frac{1}{2} s_2^2 \right). \quad (S6)$$

Similarly, P_{VH} is calculated by substituting $\langle \hat{a}_{1H}^\dagger \hat{a}_{1H} \rangle = n_1$, $\langle \hat{a}_{2D}^\dagger \hat{a}_{2D} \rangle = s_2$, $\langle \hat{a}_{2A}^\dagger \hat{a}_{2A} \rangle = 0$, $\langle \hat{a}_{2D}^\dagger \hat{a}_{2D}^\dagger \hat{a}_{2D} \hat{a}_{2D} \rangle = s_2^2$ and $\langle \hat{a}_{2A}^\dagger \hat{a}_{2A}^\dagger \hat{a}_{2A} \hat{a}_{2A} \rangle = 0$ as

$$P_{VH} = \frac{1}{4} \eta_{1'}\eta_{2'} \left(n_1 s_2 + \frac{1}{2} s_2^2 \right). \quad (S7)$$

In the same manner, P_{mV} is calculated as

$$\begin{aligned}
P_{mV} &= \eta_{1'}\eta_{2'}\langle\hat{b}_{1'D}^\dagger\hat{b}_{2'V}^\dagger\hat{b}_{1'D}\hat{b}_{2'V}\rangle \\
&= \frac{1}{2}\eta_{1'}\eta_{2'}\langle(\hat{b}_{1'H}^\dagger + \hat{b}_{1'V}^\dagger)\hat{b}_{2'V}^\dagger(\hat{b}_{1'H} + \hat{b}_{1'V})\hat{b}_{2'V}\rangle \\
&= \frac{1}{2}\eta_{1'}\eta_{2'}\langle(\hat{a}_{1H}^\dagger + \hat{a}_{2V}^\dagger)\hat{a}_{1V}^\dagger(\hat{a}_{1H} + \hat{a}_{2V})\hat{a}_{1V}\rangle \\
&= \frac{1}{2}\eta_{1'}\eta_{2'}(\langle\hat{a}_{1H}^\dagger\hat{a}_{1V}^\dagger\hat{a}_{1H}\hat{a}_{1V}\rangle + \langle\hat{a}_{2V}^\dagger\hat{a}_{1V}^\dagger\hat{a}_{2V}\hat{a}_{1V}\rangle) \\
&= \frac{1}{2}\eta_{1'}\eta_{2'}\left\{\langle\hat{a}_{1H}^\dagger\hat{a}_{1V}^\dagger\hat{a}_{1H}\hat{a}_{1V}\rangle + \frac{1}{2}(\langle\hat{a}_{2D}^\dagger\hat{a}_{1V}^\dagger\hat{a}_{2D}\hat{a}_{1V}\rangle + \langle\hat{a}_{2A}^\dagger\hat{a}_{1V}^\dagger\hat{a}_{2A}\hat{a}_{1V}\rangle)\right\}. \tag{S8}
\end{aligned}$$

For calculating P_{VV} , we substitute $\langle\hat{a}_{1V}^\dagger\hat{a}_{1V}\rangle = s_1$, $\langle\hat{a}_{1H}^\dagger\hat{a}_{1H}\rangle = n_1$, $\langle\hat{a}_{2D}^\dagger\hat{a}_{2D}\rangle = s_2$ and $\langle\hat{a}_{2A}^\dagger\hat{a}_{2A}\rangle = 0$ leading to

$$P_{VV} = \frac{1}{2}\eta_{1'}\eta_{2'}\left(n_1s_1 + \frac{1}{2}s_1s_2\right). \tag{S9}$$

P_{HV} is calculated by substituting $\langle\hat{a}_{1V}^\dagger\hat{a}_{1V}\rangle = n_1$, $\langle\hat{a}_{1H}^\dagger\hat{a}_{1H}\rangle = s_1$, $\langle\hat{a}_{2D}^\dagger\hat{a}_{2D}\rangle = s_2$ and $\langle\hat{a}_{2A}^\dagger\hat{a}_{2A}\rangle = 0$ as

$$P_{HV} = \frac{1}{2}\eta_{1'}\eta_{2'}\left(n_1s_1 + \frac{1}{2}n_1s_2\right). \tag{S10}$$

From Eq. (S6), (S7), (S9) and (S10), the visibility V_z is calculated to be

$$V_z = \frac{2\chi_s(1 - \chi_n)}{4\chi_n + 2\chi_s(\chi_n + 1) + \chi_s^2}, \tag{S11}$$

where $\chi_s = s_2/s_1$ and $\chi_n = n_1/s_1$.

Next, we calculate V_x . The coincidence probability P_{mD} is given by

$$\begin{aligned}
P_{mD} &= \eta_{1'}\eta_{2'}\langle\hat{b}_{1'D}^\dagger\hat{b}_{2'D}^\dagger\hat{b}_{1'D}\hat{b}_{2'D}\rangle \\
&= \frac{1}{4}\eta_{1'}\eta_{2'}\langle(\hat{b}_{1'H}^\dagger + \hat{b}_{1'V}^\dagger)(\hat{b}_{2'H}^\dagger + \hat{b}_{2'V}^\dagger)(\hat{b}_{1'H} + \hat{b}_{1'V})(\hat{b}_{2'H} + \hat{b}_{2'V})\rangle \\
&= \frac{1}{4}\eta_{1'}\eta_{2'}\langle(\hat{a}_{1H}^\dagger + \hat{a}_{2V}^\dagger)(\hat{a}_{2H}^\dagger + \hat{a}_{1V}^\dagger)(\hat{a}_{1H} + \hat{a}_{2V})(\hat{a}_{2H} + \hat{a}_{1V})\rangle \\
&= \frac{1}{16}\eta_{1'}\eta_{2'}(\langle\hat{a}_{1D}^\dagger\hat{a}_{1D}^\dagger\hat{a}_{1D}\hat{a}_{1D}\rangle + \langle\hat{a}_{1A}^\dagger\hat{a}_{1A}^\dagger\hat{a}_{1A}\hat{a}_{1A}\rangle + \langle\hat{a}_{2D}^\dagger\hat{a}_{2D}^\dagger\hat{a}_{2A}\hat{a}_{2D}\rangle + \langle\hat{a}_{2A}^\dagger\hat{a}_{2A}^\dagger\hat{a}_{2A}\hat{a}_{2A}\rangle) \\
&\quad + 4\langle\hat{a}_{1D}^\dagger\hat{a}_{2D}^\dagger\hat{a}_{1D}\hat{a}_{2D}\rangle + 4\langle\hat{a}_{1A}^\dagger\hat{a}_{2A}^\dagger\hat{a}_{1A}\hat{a}_{2A}\rangle. \tag{S12}
\end{aligned}$$

By substituting $\langle\hat{a}_{1D}^\dagger\hat{a}_{1D}\rangle = s_1$, $\langle\hat{a}_{1A}^\dagger\hat{a}_{1A}\rangle = n_1$, $\langle\hat{a}_{2D}^\dagger\hat{a}_{2D}\rangle = s_2$, $\langle\hat{a}_{2A}^\dagger\hat{a}_{2A}\rangle = 0$, $\langle\hat{a}_{1D}^\dagger\hat{a}_{1D}^\dagger\hat{a}_{1D}\hat{a}_{1D}\rangle = g_s^{(2)}s_1^2$, $\langle\hat{a}_{1A}^\dagger\hat{a}_{1A}^\dagger\hat{a}_{1A}\hat{a}_{1A}\rangle = 2n_1^2$, $\langle\hat{a}_{2D}^\dagger\hat{a}_{2D}^\dagger\hat{a}_{2D}\hat{a}_{2D}\rangle = s_2^2$,

$$P_{DD} = \frac{1}{16}\eta_{1'}\eta_{2'}\left\{s_2^2 + s_1^2g_s^{(2)} + 2n_1^2 + 4s_1s_2\right\}. \tag{S13}$$

Similarly, P_{AD} is calculated by substituting $\langle\hat{a}_{1D}^\dagger\hat{a}_{1D}\rangle = n_1$, $\langle\hat{a}_{1A}^\dagger\hat{a}_{1A}\rangle = s_1$, $\langle\hat{a}_{2D}^\dagger\hat{a}_{2D}\rangle = s_2$, $\langle\hat{a}_{2A}^\dagger\hat{a}_{2A}\rangle = 0$, $\langle\hat{a}_{1D}^\dagger\hat{a}_{1D}^\dagger\hat{a}_{1D}\hat{a}_{1D}\rangle = 2n_1^2$, $\langle\hat{a}_{1A}^\dagger\hat{a}_{1A}^\dagger\hat{a}_{1A}\hat{a}_{1A}\rangle = g_s^{(2)}s_1^2$, and $\langle\hat{a}_{2D}^\dagger\hat{a}_{2D}^\dagger\hat{a}_{2D}\hat{a}_{2D}\rangle = s_2^2$ as

$$P_{AD} = \frac{1}{16}\eta_{1'}\eta_{2'}\left\{s_2^2 + s_1^2g_s^{(2)} + 2n_1^2 + 4n_1s_2\right\}. \tag{S14}$$

In the same way, P_{mA} is calculated as

$$\begin{aligned}
P_{mA} &= \eta_{1'}\eta_{1'}\langle \hat{b}_{1'D}^\dagger \hat{b}_{2'A}^\dagger \hat{b}_{1'D} \hat{b}_{2'A} \rangle \\
&= \frac{1}{4}\eta_{1'}\eta_{2'}\langle (\hat{b}_{1'H}^\dagger + \hat{b}_{1'V}^\dagger)(\hat{b}_{2'H}^\dagger - \hat{b}_{2'V}^\dagger)(\hat{b}_{1'H} + \hat{b}_{1'V})(\hat{b}_{2'H} - \hat{b}_{2'V}) \rangle \\
&= \frac{1}{4}\eta_{1'}\eta_{2'}\langle (\hat{a}_{1H}^\dagger + \hat{a}_{2V}^\dagger)(\hat{a}_{2H}^\dagger - \hat{a}_{1V}^\dagger)(\hat{a}_{1H} + \hat{a}_{2V})(\hat{a}_{2H} - \hat{a}_{1V}) \rangle \\
&= \frac{1}{16}\eta_{1'}\eta_{2'}\langle (\hat{a}_{1D}^\dagger \hat{a}_{1D}^\dagger \hat{a}_{1D} \hat{a}_{1D}) + \langle \hat{a}_{1A}^\dagger \hat{a}_{1A}^\dagger \hat{a}_{1A} \hat{a}_{1A} \rangle + \langle \hat{a}_{2D}^\dagger \hat{a}_{2D}^\dagger \hat{a}_{2D} \hat{a}_{2D} \rangle + \langle \hat{a}_{2A}^\dagger \hat{a}_{2A}^\dagger \hat{a}_{2A} \hat{a}_{2A} \rangle \\
&\quad + 4\langle \hat{a}_{2D}^\dagger \hat{a}_{1A}^\dagger \hat{a}_{2D} \hat{a}_{1A} \rangle + 4\langle \hat{a}_{1D}^\dagger \hat{a}_{2A}^\dagger \hat{a}_{1D} \hat{a}_{2A} \rangle \rangle.
\end{aligned} \tag{S15}$$

For calculating P_{AA} , we substitute $\langle \hat{a}_{1D}^\dagger \hat{a}_{1D} \rangle = s_1$, $\langle \hat{a}_{1A}^\dagger \hat{a}_{1A} \rangle = n_1$, $\langle \hat{a}_{2D}^\dagger \hat{a}_{2D} \rangle = s_2$, $\langle \hat{a}_{2A}^\dagger \hat{a}_{2A} \rangle = 0$, $\langle \hat{a}_{1D}^\dagger \hat{a}_{1D}^\dagger \hat{a}_{1D} \hat{a}_{1D} \rangle = g_1^{(2)} s_1^2$, $\langle \hat{a}_{1A}^\dagger \hat{a}_{1A}^\dagger \hat{a}_{1A} \hat{a}_{1A} \rangle = 2n_1^2$, $\langle \hat{a}_{2D}^\dagger \hat{a}_{2D}^\dagger \hat{a}_{2D} \hat{a}_{2D} \rangle = s_2^2$, leading to

$$P_{AA} = \frac{1}{16}\eta_{1'}\eta_{2'} \left\{ s_2^2 + s_1^2 g_s^{(2)} + 2n_1^2 + 4s_1 s_2 \right\}. \tag{S16}$$

Similarly, P_{DA} is calculated by substituting $\langle \hat{a}_{1D}^\dagger \hat{a}_{1D} \rangle = n_1$, $\langle \hat{a}_{1A}^\dagger \hat{a}_{1A} \rangle = s_1$, $\langle \hat{a}_{2D}^\dagger \hat{a}_{2D} \rangle = s_2$, $\langle \hat{a}_{2A}^\dagger \hat{a}_{2A} \rangle = 0$, $\langle \hat{a}_{1D}^\dagger \hat{a}_{1D}^\dagger \hat{a}_{1D} \hat{a}_{1D} \rangle = 2n_1^2$, $\langle \hat{a}_{1A}^\dagger \hat{a}_{1A}^\dagger \hat{a}_{1A} \hat{a}_{1A} \rangle = g_s^{(2)} s_1^2$, $\langle \hat{a}_{2D}^\dagger \hat{a}_{2D}^\dagger \hat{a}_{2D} \hat{a}_{2D} \rangle = s_2^2$, as

$$P_{DA} = \frac{1}{16}\eta_{1'}\eta_{2'} \left\{ s_2^2 + s_1^2 g_s^{(2)} + 2n_1^2 + 4n_1 s_2 \right\}. \tag{S17}$$

From Eq. (S13), (S14), (S16) and (S17), the visibility V_x is calculated to be

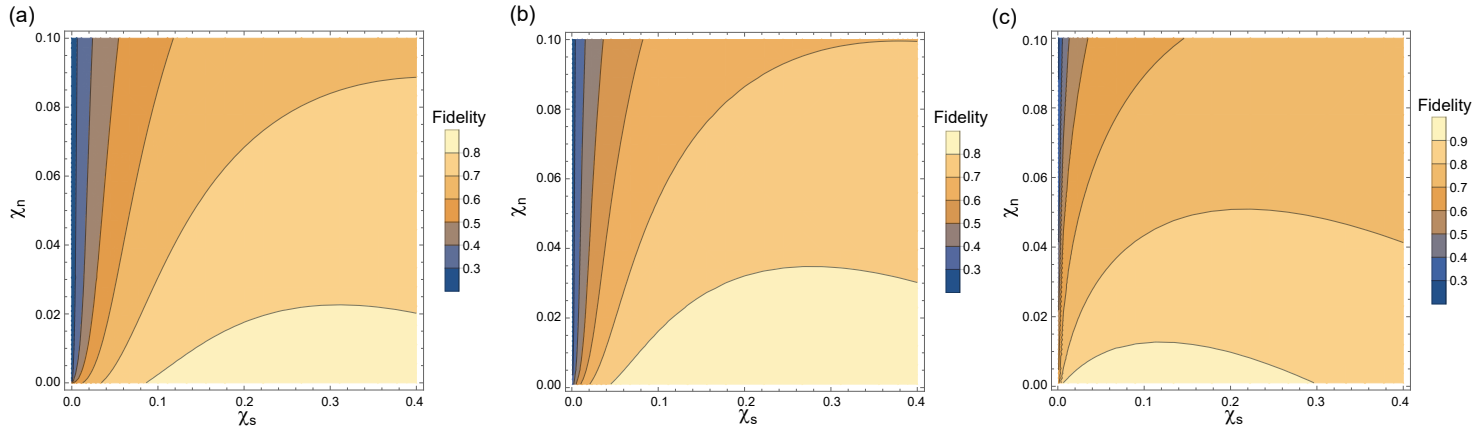
$$V_x = \frac{2\chi_s(1 - \chi_n)}{\chi_s^2 + 2\chi_n^2 + g_s^{(2)} + 2(\chi_n + 1)\chi_s}. \tag{S18}$$

The coincidence probabilities P_{RR}, P_{RL}, P_{LR} and P_{LL} are calculated in a similar manner, which leads to $V_x = V_y$. As a result, we obtain

$$F = \frac{1 + V_z + 2V_x}{4}. \tag{S19}$$

In our experiment, the parameters $g_s^{(2)}$, χ_s , and χ_n are obtained as follows. The intensity correlation function $g_s^{(2)} = 0.098$ is obtained by running another experiment. The average number of the m^* -polarized heralded photon s'_1 is estimated from the experimental data. The detection probability of H-polarized photon at $D_{R''}$ conditioned on the H-polarized photon-detection at $D_{B'}$ is given by $s'_1 := C_{HH}(D_{R''} \cap D_{B'})/S_H(D_{B'}) = 6.0 \times 10^{-3}$, where $C_{HH}(D_{R''} \cap D_{B'})$ is the two-fold coincidence count between $D_{R''}$ with H-polarization and $D_{B'}$ with H-polarization, and $S_H(D_{B'})$ is the single detection count at $D_{B'}$ with H-polarization. The relation between s_1 and s'_1 is given by $s_1 = s'_1/\eta_d$, where η_d is the system transmittance after the PBS including the quantum efficiency of the SSPD. Similarly, n_1 is given by $n_1 = n'_1/\eta_d$, where $n'_1 := C_{HV}(D_{R''} \cap D_{B'})/S_H(D_{B'}) = 1.1 \times 10^{-4}$. Then, χ_n is calculated to be $\chi_n = n_1/s_1 = n'_1/s'_1 = 0.018$. Finally, we estimate the average photon number of the coherent light pulse s_2 . This is given by $s_2 = s'_2/\eta_d$, where $s'_2 = 2S_H(D_{R''})/f = 1.7 \times 10^{-3}$. Here f is the repetition rate of the mode lock laser. The factor 2 arises from the fact that D-polarized coherent light pulse is divided into two at the PBS. Then, χ_s is given by $\chi_s = s_2/s_1 = s'_2/s'_1 = 0.28$.

Using those parameters, Eq. (S11), (S18) and (S19), we obtain $V_z = 0.76$, $V_x = 0.74$ and $F = 0.81$. In addition, it is possible to estimate how much fidelity will be expected improving the SPDC source. In Fig. 2, we show the dependence of the fidelity on $g_s^{(2)}$, χ_s , and χ_n .



Supplementary Figure 2. The relation between the fidelity and the parameters $g_s^{(2)}$, $\chi\beta n$ and χ_s . (a) $g_s^{(2)} = 0.098$. (b) $g_s^{(2)} = 0.05$. (c) $g_s^{(2)} = 0.01$.