Direct estimation of quantum coherence by collective measurements: Supplementary

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A. Estimating coherence of formation for qubits

We will now apply the collective measurement scheme to estimate the coherence of formation, which for single-qubit states takes the form [1]

$$C_f(\rho) = h\left(\frac{1+\sqrt{1-4|\rho_{01}|^2}}{2}\right), \tag{1}$$

where $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$ is the binary entropy. We perform numerical simulations and an optical experiment, following the same procedure as for estimating C_{ℓ_1}. In particular, we use $N = 1200$ copies of the state

$$|\Psi\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle \tag{2}$$

with $\theta \in [0, \pi/2]$, and average the data over 1000 repetitions. The results of the numerical simulation and experiment are shown in Fig. 1. Also in this case we compare the numerical simulation of CMS with three alternative schemes for coherence estimation, see main text for details.

B. Estimating C_{ℓ_1} for qutrit

In order to demonstrate the applicability of collective measurments to estimate C_{ℓ_1} for states of higher dimensions, we take qutrit as an example to theoretically show the performance of this method. First, we prepare two-copy qutrit state $\rho^{\otimes 2}$, where $\rho = |\Phi\rangle\langle\Phi|$ and

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(\sin\alpha|0\rangle + \cos\alpha|1\rangle + |2\rangle), \tag{3}$$

and then choose collective measurement to estimate the absolute value of the off-diagonal elements $|\rho_{01}|$, $|\rho_{02}|$, and $|\rho_{12}|$ according to

$$\left|\rho_{ij}\right| = \sqrt{\frac{1}{2}\left(\mathrm{Tr}\left[\rho^{\otimes 2}|\psi_{ij}^{+}\rangle\langle\psi_{ij}^{+}|\right] - \mathrm{Tr}\left[\rho^{\otimes 2}|\psi_{ij}^{-}\rangle\langle\psi_{ij}^{-}|\right]\right)}, \tag{4}$$

where $|\psi_{01}^{\pm}\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$, $|\psi_{02}^{\pm}\rangle = (|02\rangle \pm |20\rangle)/\sqrt{2}$, and $|\psi_{12}^{\pm}\rangle = (|12\rangle \pm |21\rangle)/\sqrt{2}$. Finally, according to the expression of C_{ℓ_1}, the ℓ_1-norm coherence of qutrit can be obtained by

$$C_{\ell_1}(\rho) = 2(\left|\rho_{01}\right| + \left|\rho_{02}\right| + \left|\rho_{12}\right|). \tag{5}$$

The results of the simulation are shown in Fig. 3 of the main text. As in the qubit case, we use $N = 1200$ copies of the state $|\Phi\rangle$ for both CMS and state tomography, and average over 1000 repetitions. The results show that CMS outperforms the tomography method for a large range of α.

C. Qutrit state tomography

The density matrix of an unknown qutrit state is reconstructed by performing projective measurements in 4 mutually unbiased bases. In the following, vectors $|\xi_{ij}\rangle$ form an orthonormal basis for a given i, i.e., $\langle\xi_{ij}|\xi_{ik}\rangle = \delta_{jk}$, and are mutually unbiased for different i: $|\langle\xi_{ij}|\xi_{kl}\rangle| = 1/\sqrt{3}$ for $i \neq k$. The vectors can be explicitly given as follows:

$$\begin{aligned}
|\xi_{00}\rangle &= |0\rangle, |\xi_{01}\rangle = |1\rangle, |\xi_{02}\rangle = |2\rangle,\\
|\xi_{10}\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle),\\
|\xi_{11}\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + e^{\frac{2i\pi}{3}}|1\rangle + e^{-\frac{2i\pi}{3}}|2\rangle),\\
|\xi_{12}\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + e^{-\frac{2i\pi}{3}}|1\rangle + e^{\frac{2i\pi}{3}}|2\rangle),\\
|\xi_{20}\rangle &= \frac{1}{\sqrt{3}}(e^{\frac{2i\pi}{3}}|0\rangle + |1\rangle + |2\rangle),\\
|\xi_{21}\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + e^{\frac{2i\pi}{3}}|1\rangle + |2\rangle),\\
|\xi_{22}\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + e^{\frac{2i\pi}{3}}|2\rangle),\\
|\xi_{30}\rangle &= \frac{1}{\sqrt{3}}(e^{-\frac{2i\pi}{3}}|0\rangle + |1\rangle + |2\rangle),\\
|\xi_{31}\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + e^{-\frac{2i\pi}{3}}|1\rangle + |2\rangle),\\
|\xi_{32}\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + e^{-\frac{2i\pi}{3}}|2\rangle).
\end{aligned} \tag{6}$$

For numerical simulation of coherence estimation via qutrit state tomography reported in the main text (see Fig. 3), we use the sample size $N = 1200$, and repeat the procedure 1000 times to infer the state from the results of the measurement. Here we use the method of maximum-likelihood reconstruction presented in Ref. [2].

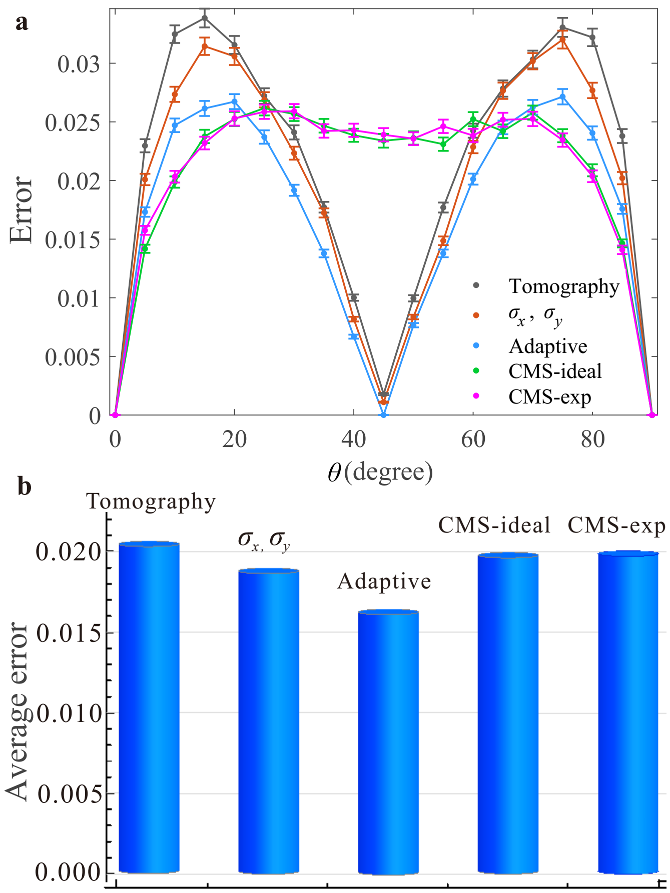

Figure 1. **a** Mean errors for estimating coherence of formation for a family of qubit states with different measurement schemes. The states have the form $|\Psi\rangle = \sin\theta|0\rangle + \cos\theta|1\rangle$ with θ ranging from 0 to $\pi/2$. In **a**, the performances of CMS (numerical simulation and experiment); σ_x, σ_y measurement (simulation); two-step adaptive measurements (simulation); and tomography (simulation) are shown for comparison. The sample size is $N = 1200$. Each data point is the average of 1000 repetitions, and the error bar denotes the standard deviation. **b** Average results of the mean error for all input states shown in **a**. The corresponding average values of these methods are from left to right: 0.0211, 0.0194, 0.0168, 0.0204, 0.0205.

D. Angles of wave plates used to prepare the states

In the state preparation module, H_1, H_2, H_3, Q_1, Q_2 are the rotation angles of the HWPs and QWPs shown in Fig. 4 of the main text. In the preparation of states $|\psi^+\rangle$, $|\psi^-\rangle$, $|\varphi^+\rangle$, $|\varphi^-\rangle$, the QWP corresponding to Q_2 is removed. Details of the angles of wave plates used to prepare all states are given in Table I.

Table I. The angles of wave plates used to prepare the states.

State	H_1	Q_1	H_3	H_2	Q_2
$\|\psi^+\rangle$	67.5°	45°	0°	45°	–
$\|\psi^-\rangle$	22.5°	45°	0°	45°	–
$\|\varphi^+\rangle$	67.5°	45°	0°	–	–
$\|\varphi^-\rangle$	22.5°	45°	0°	–	–
$\|\Psi(\theta)\rangle$	$(90-\frac{\theta}{2})°$	$-\theta°$	45°	$(90-\frac{\theta}{2})°$	$-\theta°$

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