

Supplementary Material: A phononic interface between a superconducting quantum processor and quantum networked spin memories

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Supplementary Note 1. TRANSDUCTION FROM A SC QUBIT TO A SPIN QUBIT VIA A WAVEGUIDE

The main text discussed a scheme where the SC qubit is directly electro-mechanically coupled to a phononic cavity. Alternatively, the mechanical mode of the phononic cavity can be coupled to the microwave circuit via a microwave or phononic waveguide. For example, this waveguide may serve as an interconnect between a SC qubit of a quantum computer which is physically separated from the phononic cavity across large distance, or it might represent a guided phonon wave connecting a piezoelectric coupler (interdigital coupler - IDT) with a discrete high- Q mechanical mode of a phononic cavity surrounded by a phononic crystal.

We break down the transduction of the qubit stored in the SC device to the spin and describe in this subsection the ‘pitch-and-catch’ state transfer of the SC state to the mechanical resonator via the waveguide. The transduction of the quantum state stored in the phonon into the electron spin via an effective Jaynes-Cummings interaction can be performed as described in the main text.

A. State transfer from the SC qubit to the phononic cavity via a waveguide

As schematically shown in [Supplementary Figure 1\(a\)](#), we assume that the SC qubit is coupled to a waveguide which is electro-mechanically coupled to the phononic cavity (or, alternatively, to a phononic waveguide mechanically coupled to a discrete mechanical mode of a cavity). Such a system can be described by the following

Hamiltonian [1–3]:

$$\begin{aligned} H_{\text{sc-m-p}} = & \hbar\omega_{\text{sc}}\sigma_{\text{sc}}^{\dagger}\sigma_{\text{sc}} + \hbar\omega_{\text{p}}b^{\dagger}b + \sum_k \hbar\omega_k a_k^{\dagger}a_k \\ & + \sum_k \hbar g_{\text{sc-m}}(t)(\sigma_{\text{sc}}^{\dagger}a_k + \sigma_{\text{sc}}a_k^{\dagger}) \\ & + \sum_k \hbar g_{\text{m-p}}(t)(b^{\dagger}a_k + ba_k^{\dagger}). \end{aligned} \quad (1)$$

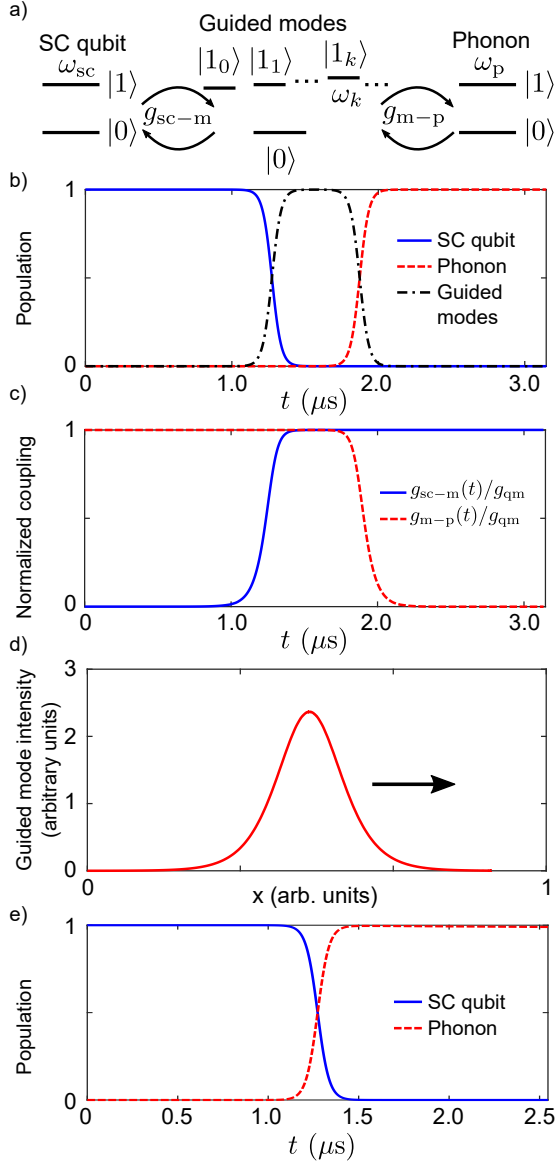
where a_k ($a_k^{\dagger}$) are the annihilation (creation) operators of a waveguide mode k of frequency ω_k . The SC qubit and the mechanical mode are coupled to the waveguide via a controllable time-dependent coupling $g_{\text{sc-m}}(t)$ [4–6] and $g_{\text{m-p}}(t)$, respectively. The coupling $g_{\text{m-p}}$ can be either realized as a tunable IDT coupler, or as a tunable mechanical interconnect that could be e.g. based on interferometric modulation of coupling in analogy with optical implementations [7, 8], although an implementation of such a controllable phononic coupler is yet to be demonstrated.

The quantum state stored in the SC device can be released into the waveguide and subsequently absorbed by the phononic cavity. To accomplish the pitch-and-catch state transfer with high fidelity we need to ensure that the processes of phonon emission by the SC qubit and phonon absorption by the phononic cavity are mutually time-reversed. To that end the pulse emitted by the SC qubit has to be time-symmetrical and the couplings have to fulfill $g_{\text{sc-m}}(t) = g_{\text{m-p}}(-[t - \tau])$ [9], where τ is the delay time due to the finite length of the waveguide.

For concreteness, we consider a waveguide of length L supporting phononic (or electromagnetic) modes of the form $\propto \cos(k_j x)$, with $k_j = (N_0 + j)\pi/L$ and x being a position along the waveguide, where N_0 is a mode number that in connection with the waveguide length L and the mode velocity c (assuming a linear dispersion) determines a central frequency of the selected set of modes. This function may represent a vector potential in a MW transmission line or e.g. a mechanical displacement of a phononic wave. The free spectral range of this finite waveguide is $\delta = c\pi/L$ and the spontaneous decay rate of each qubit into the waveguide modes occurs with the

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Supplementary Figure 1. Pitch-and-catch scheme for state transfer between the SC qubit and the phononic cavity. (a) The general scheme describing a SC qubit (of frequency ω_{sc}) coupled to a continuum of waveguide modes k of their respective frequencies ω_k via time-dependent coupling rate g_{sc-m} that mediates the state transfer to the phononic cavity of frequency ω_p via a time-dependent coupling rate g_{m-p} . (b) Populations of the SC qubit (full blue), the phonon (red dashed), and the propagating photon (black dash-dotted) as a function of time. (c) The time-dependent coupling rates g_{sc-m} and g_{m-p} applied to control the state transfer. (d) The SC qubit releases a symmetrical photon wave packet (photon intensity shown as a function of distance along the waveguide) propagating towards the receiving phononic cavity (shown by the arrow). (e) The state transfer as in (b) but described using the master-equation approach [Eq. (5)]. Only the SC-qubit population (blue line) and the phonon population (dashed red line) are shown as the photons are eliminated from the dynamics. We disregard the time delay caused by the photon propagation along the finite-length waveguide for simplicity.

rate (assuming time-independent $g_{scm} = g_{mp} \equiv g_{qm}$):

$$\kappa_{sc} = \frac{2\pi g_{qm}^2}{\delta}. \quad (2)$$

The objective of releasing a perfectly symmetrical microwave pulse in a form proportional to $\text{sech}(\kappa_{sc}t/2)$ can be achieved if we modulate the coupling constant in time via an electromechanical coupler [5]:

$$g_{sc-m}(t) = g_{qm} \sqrt{\frac{e^{\kappa_{sc}t}}{1 + e^{\kappa_{sc}t}}}, \quad (3)$$

The wave packet released by the superconducting qubit can then be fully absorbed by the phononic cavity if the time-reversed delayed coupling is:

$$g_{m-p}(t) = g_{qm} \sqrt{\frac{e^{-\kappa_{sc}(t-\tau)}}{1 + e^{-\kappa_{sc}(t-\tau)}}}. \quad (4)$$

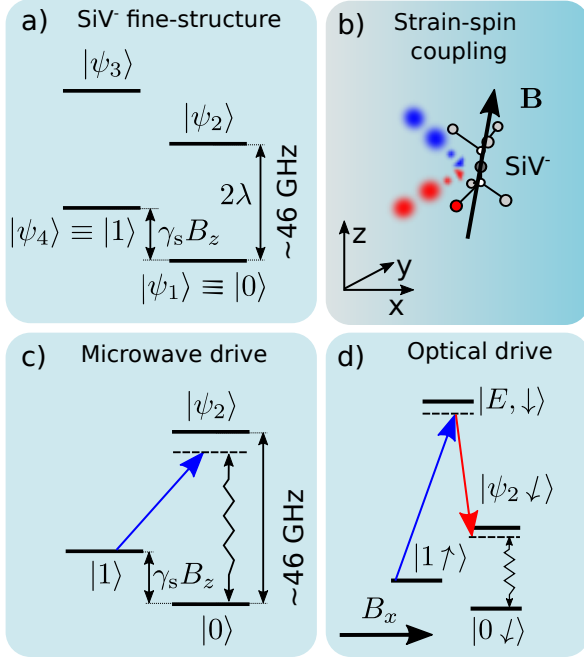
We demonstrate the pitch-and-catch scheme in [Supplementary Figure 1\(b-d\)](#). [Supplementary Figure 1\(b\)](#) shows the time-dependence of the populations of the SC qubit (full blue), the phonon (red dashed), and the MW photon in the waveguide (black dash-dotted). As shown, the pitch-and-catch scheme leads to an almost perfect transfer of population from the SC qubit to the phonon [final phonon population $\langle \sigma_p^\dagger(t_{\text{end}}) \sigma_p(t_{\text{end}}) \rangle \approx 1$]. The sequence of time-dependent couplings shown in [Supplementary Figure 1\(c\)](#) first releases a fully symmetrical propagating wave packet [a snapshot of the photon intensity is shown in [Supplementary Figure 1\(d\)](#) as a function of position along the waveguide] and is subsequently perfectly absorbed by the receiving qubit. We cast the model outlined above into the form of a master equation [10] for the density matrix ρ_{scp} describing the SC qubit and the phononic cavity in the single-excitation basis, but only effectively accounting for the modes of the MW waveguide:

$$\begin{aligned} \frac{\partial \rho_{scp}}{\partial t} = & -\frac{i}{\hbar} [H_{sc} + H_p, \rho_{scp}] \\ & + \kappa_{sc}(t) \mathcal{L}_{\sigma_{sc}}(\rho_{scp}) + \gamma_{sc} \mathcal{L}_{\sigma_{sc}}(\rho_{scp}) \\ & + \kappa_p(t) \mathcal{L}_b(\rho_{scp}) + \gamma_p \mathcal{L}_b(\rho_{scp}) \\ & + \sqrt{\kappa_p(t) \kappa_{sc}(t)} \\ & \times (e^{i\phi} [\sigma_{sc} \rho_{scp}, b^\dagger] + e^{-i\phi} [b, \rho_{scp} \sigma_{sc}^\dagger]), \end{aligned} \quad (5)$$

with $H_{sc} = \hbar \omega_{sc} \sigma_{sc}^\dagger \sigma_{sc}$, and $H_p = \hbar \omega_p b^\dagger b$. It is understood that the density matrix of the phonon is evaluated at a later time $t + \tau$ (in the following we always set $\tau = 0$ s for simplicity) and the phase accumulated due to the propagation of the photon wave packet is absorbed in the definition of ϕ . The respective time-dependent decay rates are given by:

$$\kappa_{sc} = \frac{2\pi g_{sc-m}^2(t - \tau_{pc})}{\delta}, \quad (6)$$

$$\kappa_p = \frac{2\pi g_{m-p}^2(t - \tau_{pc})}{\delta}, \quad (7)$$



Supplementary Figure 2. Coupling between fine-structure states of a SiV^- defect and strain. (a) Fine structure states described in [Supplementary Note 2](#). (b) External magnetic field $\mathbf{B}$ and optical drive is applied to the defect to enable the strain-spin coupling. (c) A microwave-based Raman scheme: states $|1\rangle$ and $|\psi_2\rangle$ can couple via magnetic field whereas states $|0\rangle$ and $|\psi_2\rangle$ are coupled via strain. (d) An optical Raman scheme exploiting a virtual excitation via an optically-active excited state can be used to induce an effective strain-mediated coupling between $|0\rangle$ and $|1\rangle$.

in accordance with Eqs.(2)-(4). We consider that the pulses are applied at a later time τ_{pc} to ensure smooth dynamics. The resulting time-dependent populations shown in [Supplementary Figure 1\(e\)](#) perfectly capture the pitch-and-catch scheme described previously in the framework of Schrödinger equation [cf. populations in [Supplementary Figure 1\(b\)](#)]. Last we note that by effectively eliminating the waveguide we neglect the waveguide propagation losses that could further decrease the state-transfer fidelity. Nevertheless, we estimate that for phonon decay rates $\sim 1 \text{ Hz}$ achieved in state-of-the-art acoustical systems, speed of sound $c \sim 10^3 \text{ ms}^{-1}$, and waveguide length $L \sim 1 \text{ mm}$, the propagation losses are so small to result in near-unity transmission $\sim e^{-10^{-6}} \approx 0.999999$. We integrate the master-equation description of the pitch-and-catch scheme [Eq. (5)] to describe the full dynamics of the state transfer from the SC qubit to the electron spin and show the result of the state-transfer protocol in [Supplementary Figure 1\(e\)](#).

Supplementary Note 2. EFFECTS OF STRAIN ON SiV^- NEGATIVE CENTER

The effects of strain on a SiV^- center have been considered in the literature [11, 12] theoretically and experimentally. The theory predicts that the strain effects can be divided into three categories according to the transformation properties of the strain field under symmetry operation of the D_{3d} symmetry group. based on symmetry, the strain can be classified as $\epsilon_{A_{1g}}$, $\epsilon_{E_{gx}}$ and $\epsilon_{E_{gy}}$. These strain components then give rise to the longitudinal, α , and transverse, β and γ , strain coupling to the spin-orbit states of the color center:

$$\alpha = t_{\perp}(\epsilon_{xx} + \epsilon_{yy}) + t_{\parallel}\epsilon_{zz} \sim \epsilon_{A_{1g}}, \quad (8)$$

$$\beta = d(\epsilon_{xx} - \epsilon_{yy}) + f\epsilon_{zx} \sim \epsilon_{E_{gx}}, \quad (9)$$

$$\gamma = -2d(\epsilon_{xy}) + f\epsilon_{yz} \sim \epsilon_{E_{gy}}, \quad (10)$$

where z is oriented along the high-symmetry axis of the defect $[111]$, x is oriented along $[\bar{1}\bar{1}2]$, and y is defined by $[\bar{1}10]$. The respective values of the constants $t_{\parallel}$, $t_{\perp}$, d , and f have been estimated to be in the range of 1 PHz to 2 PHz (we transform the relevant tensor components into the coordinate system defined by $[100]$, $[010]$, and $[001]$ in [Supplementary Note 3](#)). We will use these values to estimate all necessary constants to design a potential transducer. As has been shown [11–15], the A_{1g} strain uniformly shifts all the fine-structure-state energies and we thus disregard its effects in the following discussion.

We further consider that the Hamiltonian of the fine-structure states of a SiV^- in a longitudinal magnetic field is (neglecting the Jahn-Teller effect and the Orbital Zeeman effect [14], for simplicity):

$$H_{\text{tot}} = \begin{pmatrix} B_z\gamma_S & 0 & -i\lambda & 0 \\ 0 & -B_z\gamma_S & 0 & i\lambda \\ i\lambda & 0 & B_z\gamma_S & 0 \\ 0 & -i\lambda & 0 & -B_z\gamma_S \end{pmatrix}. \quad (11)$$

The Hamiltonian is expressed in the basis of spin-orbit states $\{|e_y \uparrow\rangle, |e_y \downarrow\rangle, |e_x \uparrow\rangle, |e_x \downarrow\rangle\}$ [11, 12]. Here λ is the spin-orbit coupling strength ($\lambda/(2\pi\hbar) \approx 23 \text{ GHz}$), B_z is the magnetic field applied along the high-symmetry axis of the defect, and $\gamma_S/(2\pi) \approx 28 \text{ GHz/T}$ is the spin gyromagnetic ratio. The Hamiltonian in Eq. (11) can be diagonalized to obtain the eigenfrequencies:

$$\nu_1 = -B_z\gamma_S - \lambda, \quad (12)$$

$$\nu_2 = -B_z\gamma_S + \lambda, \quad (13)$$

$$\nu_3 = B_z\gamma_S + \lambda, \quad (14)$$

$$\nu_4 = B_z\gamma_S - \lambda. \quad (15)$$

and the corresponding eigenstates:

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(-i|e_y \downarrow\rangle + |e_x \downarrow\rangle), \quad (16)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(i|e_y \downarrow\rangle + |e_x \downarrow\rangle), \quad (17)$$

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}(-i|e_y \uparrow\rangle + |e_x \uparrow\rangle), \quad (18)$$

$$|\psi_4\rangle = \frac{1}{\sqrt{2}}(i|e_y \uparrow\rangle + |e_x \uparrow\rangle). \quad (19)$$

The structure of the fine-structure states is schematically depicted in [Supplementary Figure 2\(a\)](#). The two lowest-energy states $|\psi_4\rangle$ and $|\psi_1\rangle$ can be conveniently used as the spin-qubit states. In this basis the transverse-strain Hamiltonian becomes:

$$H_\beta = \begin{pmatrix} 0 & \beta & 0 & 0 \\ \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta \\ 0 & 0 & \beta & 0 \end{pmatrix} \quad (20)$$

and

$$H_\gamma = \begin{pmatrix} 0 & i\gamma & 0 & 0 \\ -i\gamma & 0 & 0 & 0 \\ 0 & 0 & 0 & i\gamma \\ 0 & 0 & -i\gamma & 0 \end{pmatrix}, \quad (21)$$

where β and γ are the strain components shown in Eq. (9) and Eq. (10). The spin degree of freedom thus cannot be flipped by the sole application of a transverse strain. Considering that the transition $|\psi_1\rangle \leftrightarrow |\psi_4\rangle$ is spin-forbidden and the states $|\psi_1\rangle$ and $|\psi_4\rangle$ have distinct orbital character, it is necessary to apply a combination of a transverse magnetic field and strain to couple to the spin qubit. We next consider possible scenarios that allow the transition $|\psi_1\rangle \leftrightarrow |\psi_4\rangle$ including (i) the application of a quasi-static magnetic field, (ii) a microwave drive, and (iii) an optical Raman scheme.

A. Quasi-static magnetic field

To allow the spin-qubit states to couple to strain we add to the system a perturbation in the form of an x -polarized magnetic field:

$$H_{B_x} = \begin{pmatrix} 0 & 0 & B_x\gamma_S & 0 \\ 0 & 0 & 0 & B_x\gamma_S \\ B_x\gamma_S & 0 & 0 & 0 \\ 0 & B_x\gamma_S & 0 & 0 \end{pmatrix}. \quad (22)$$

In the lowest order of perturbation theory, this Hamiltonian causes the following modification to the system

eigenstates:

$$|\psi'_1\rangle \approx |\psi_1\rangle + \frac{B_x\gamma_S}{\nu_1 - \nu_3}|\psi_3\rangle, \quad (23)$$

$$|\psi'_2\rangle \approx |\psi_2\rangle + \frac{B_x\gamma_S}{\nu_2 - \nu_4}|\psi_4\rangle, \quad (24)$$

$$|\psi'_3\rangle \approx |\psi_3\rangle + \frac{B_x\gamma_S}{\nu_3 - \nu_1}|\psi_1\rangle, \quad (25)$$

$$|\psi'_4\rangle \approx |\psi_4\rangle + \frac{B_x\gamma_S}{\nu_4 - \nu_2}|\psi_2\rangle. \quad (26)$$

The two lowest-lying spin states $|\psi_4\rangle$ and $|\psi_1\rangle$ are therefore modified to $|\psi'_4\rangle$ and $|\psi'_1\rangle$ which can be coupled via strain. In particular, in the lowest order of perturbation theory, this coupling can be estimated as:

$$\begin{aligned} \langle\psi'_4|H_\beta|\psi'_1\rangle &\approx \beta \left(\frac{B_x\gamma_S}{\nu_1 - \nu_3} + \frac{B_x\gamma_S}{\nu_4 - \nu_2} \right) \\ &= \beta \left(\frac{B_x\gamma_S}{-2B_z\gamma_S - 2\lambda} + \frac{B_x\gamma_S}{2B_z\gamma_S - 2\lambda} \right). \end{aligned} \quad (27)$$

Similarly for the γ component of strain we obtain:

$$\begin{aligned} \langle\psi'_4|H_\gamma|\psi'_1\rangle &\approx -i\gamma \left(\frac{B_x\gamma_S}{\nu_1 - \nu_3} + \frac{B_x\gamma_S}{\nu_4 - \nu_2} \right) \\ &= -i\gamma \left(\frac{B_x\gamma_S}{-2B_z\gamma_S - 2\lambda} + \frac{B_x\gamma_S}{2B_z\gamma_S - 2\lambda} \right). \end{aligned} \quad (28)$$

Based on our simulations we further consider $\beta/(2\pi) \sim 10$ MHz ($\gamma/(2\pi) \sim 10$ MHz) we obtain that the direct coupling of states $|\psi'_1\rangle$ and $|\psi'_4\rangle$, $g_{pe} = \Gamma_{pe}B_x$ is going to be of the order $\Gamma_{pe}/(2\pi) \sim 5$ MHz/T. A moderate magnetic bias field of 0.2 T would therefore be required to achieve the coupling rate $g_{pe}/(2\pi) \sim 1$ MHz used in the state-transfer analysis. The frequency of the spin transition $|\psi'_1\rangle \leftrightarrow |\psi'_4\rangle$ can be tuned by an external field B_z to achieve resonant spin-phonon interaction. The pulsed modulation of the coupling could be realized by modulating the value of the magnetic field $B_x(t)$.

B. Microwave drive

Another way to induce the resonant interaction of the lowest lying spin states (states $|\psi_1\rangle$ and $|\psi_4\rangle$) considering that λ is the dominant scale is to drive the spin transition between states $|\psi_4\rangle$ and $|\psi_1\rangle$ that are orbitally allowed via a microwave drive at the correct frequency ω_d (that we determine later) as shown in [14]. This scheme is schematically depicted in [Supplementary Figure 2\(c\)](#). The orbital transitions $|\psi_1\rangle \rightarrow |\psi_2\rangle$ and $|\psi_4\rangle \rightarrow |\psi_3\rangle$ are coupled to the acoustic phonon via the strain susceptibility with a rate $g_{orb} \approx 2\pi \times 10$ MHz. We further introduce the shorthand notation: $\sigma_{ij} = |\psi_i\rangle\langle\psi_j|$ and write the effective Hamiltonian of the system under consideration:

$$\begin{aligned} H_{sys} &= \Delta\sigma_{22} + \omega_B\sigma_{44} + \Omega(t) \left(e^{i[\theta(t) + \omega_d t]} \sigma_{42} + \text{H.c.} \right) \\ &+ g_{orb} (b^\dagger \sigma_{12} + \text{H.c.}) + \omega_p b^\dagger b. \end{aligned} \quad (29)$$

Here we neglected any influence of the off-resonant state $|\psi_3\rangle$, $\Delta = E_2 - E_1 = E_4 - E_1$, ω_B is the phonon frequency, b ($b^\dagger$) is the phonon annihilation (creation) operator, and $\Omega(t)$ and $\theta(t)$ are the amplitude- and phase-envelope of the external microwave drive, respectively. The Hamiltonian in Eq. (29) can be used to find an approximation in such a way that the Raman-mediated coupling of the two lowest spin states with the phonon can be made explicit. To that end we first introduce the interaction picture given by the Hamiltonian:

$$H_{ip} = \omega_B \sigma_{44} + \Delta \sigma_{22} + \omega_p b^\dagger b. \quad (30)$$

This leads to the following rotating-frame Hamiltonian:

$$H_{rot} = \Omega(t) \left[\sigma_{42} e^{i[\theta(t) + (\omega_d + \omega_B - \Delta)t]} + \text{H.c.} \right] + g_{orb} \left[b^\dagger \sigma_{12} e^{i(\omega_p - \Delta)t} + \text{H.c.} \right]. \quad (31)$$

We further set $\omega_d = \omega_p - \omega_B$ to ensure resonant drive.

Adiabatic elimination can be applied to the Hamiltonian in Eq. (31) to obtain the effective coupling constant between the phonon and the electron spin:

$$g_{p-e} \approx g_{eff}^{mw} = g_{orb} \frac{\Omega(t) e^{i\theta(t)}}{\delta}, \quad (32)$$

with $\delta = \omega_p - \Delta$. To ensure the validity of the adiabatic approximation we further require that $|\Omega| < |\delta|$ and we therefore estimate $g_{pe}/(2\pi) = g_{eff}^{mw}/(2\pi) \approx 0.1 g_{orb}/(2\pi) \approx 1$ MHz. The microwave drive employed in this scheme ensures the resonant character of the phonon-spin coupling and eliminates the necessity to tune the magnitude of the magnetic field B_z (i.e. of ω_B).

C. Optical Raman drive

Finally, an optical Raman drive has been proposed [14] to enable resonant coupling between the transition connecting the lowest-energy spin-orbit states and the cavity phonon, as shown in [Supplementary Figure 2\(d\)](#). The Hamiltonian describing this Raman scheme, expressed in the basis of states perturbed by the magnetic field [Eqs. (23) to (26)] and an optically accessible excited state, $|E \uparrow\rangle$, can be written as:

$$H_{Raman} = \Delta \sigma'_{22} + \omega_B \sigma'_{44} + \omega_p b^\dagger b + \omega_E \sigma'_{EE} + \Omega_A (\sigma'_{2E} e^{i[\theta_A(t) + \omega_A t]} + \text{H.c.}) + \Omega_C (\sigma'_{4E} e^{i[\theta_C(t) + \omega_C t]} + \text{H.c.}) + g_{orb} (\sigma'_{12} b^\dagger + \sigma_{21} b), \quad (33)$$

where $\sigma'_{ij} = |\psi'_i\rangle\langle\psi'_j|$ and $|\psi'_E\rangle$ is an electronic excited state of SiV^- . Here Ω_A and Ω_B are related to the amplitude of the two pumping lasers and are proportional to the dipole coupling elements between the respective states, and θ_A (θ_C) are slowly varying phases. The respective laser-drive frequencies ω_A and ω_C are adjusted

so that $\omega_p = \omega_B + \omega_A - \omega_C$. Under such conditions, it is possible to obtain the following effective phonon-electron-spin coupling:

$$g_{p-e} \approx g_{eff}^{Raman} = \frac{\Omega_A e^{i\theta_A(t)} \Omega_C e^{-i\theta_C(t)} g_{orb}}{(\omega_p - \Delta)(\omega_C - \omega_E + \omega_p)}. \quad (34)$$

Since g_{eff}^{Raman} has been obtained perturbatively, it is necessary that $\Omega_A \Omega_C / [(\omega_p - \Delta)(\omega_C - \omega_E + \omega_p)] \ll 1$, and the effective phonon-electron-spin coupling is thus substantially reduced. The advantage of this scheme is in the tunability of the externally applied lasers that can be used to rapidly adjust the condition for the resonant phonon-spin coupling or modulate the magnitude of the coupling strength. Notice also that in order for this scheme to be efficient, the phonon frequency must be close to the transition frequency Δ .

Last we mention that the different strain susceptibility of the ground and excited electronic-state manifolds could also be used to induce the ground-state spin-strain coupling. This scheme has, for example, been described in Ref. [16] for a nitrogen-vacancy color center.

Supplementary Note 3. COORDINATE TRANSFORMATION OF THE STRAIN TENSOR COMPONENTS

In [Supplementary Note 2](#) we discuss the effects of strain on the fine-structure states of a SiV^- color center and express the strain tensor in the internal system of coordinates of the color center defined with respect to the diamond crystallographic directions as: z along $[111]$, x along $[\bar{1}\bar{1}2]$, and y along $[\bar{1}10]$. However, in applications it is more natural to consider the strain tensor in the set of coordinates defined by the basis vectors of the diamond cubic lattice. For convenience, we therefore transform the relevant tensor components that yield electron-spin-phonon coupling into this natural coordinate system defined by the basis vectors of the diamond cubic lattice and use the numbered indexes 1, 2, and 3 to denote the coordinates $[100]$, $[010]$, and $[001]$, respectively:

$$\epsilon_{xx} - \epsilon_{yy} = (-\epsilon_{11} - \epsilon_{22} + 2\epsilon_{33} + 2[\epsilon_{12} + \epsilon_{21}] - [\epsilon_{13} + \epsilon_{31}] - [\epsilon_{23} + \epsilon_{32}])/3 \quad (35)$$

$$\epsilon_{zx} = -(\epsilon_{11} + \epsilon_{22} - 2\epsilon_{33} - 2\epsilon_{13} - 2\epsilon_{23} + \epsilon_{12} + \epsilon_{21} + \epsilon_{31} + \epsilon_{32})/(3\sqrt{2}) \quad (36)$$

$$\epsilon_{xy} = \frac{\epsilon_{11} - \epsilon_{12} + \epsilon_{21} - \epsilon_{22} - 2\epsilon_{31} + 2\epsilon_{32}}{2\sqrt{3}} \quad (37)$$

$$\epsilon_{yz} = \frac{-\epsilon_{11} - \epsilon_{12} - \epsilon_{13} + \epsilon_{21} + \epsilon_{22} + \epsilon_{23}}{\sqrt{6}}. \quad (38)$$

This form is convenient to express the effect of strained diamond slab etched along the (100) crystallographic plane of diamond, which we consider in the design of the phononic cavity.

Supplementary Note 4. STATE TRANSFER FROM THE ELECTRON SPIN TO THE NUCLEAR SPIN

To complete the chain of state-transfer steps leading to the transduction of a state stored in an SC qubit to a nuclear-spin qubit, following reference [17] we discuss an example of a state-transfer protocol than can be applied to perform the step connecting the electronic and nuclear-spin qubits.

We assume that the nuclear spin described by the Hamiltonian

$$H_{nn} = \frac{\hbar\omega_L}{2}\sigma_z^n \quad (39)$$

is coupled to the electron spin via a longitudinal interaction:

$$H_{e-n} = \frac{A_{\parallel}}{4}\sigma_z^e\sigma_z^n, \quad (40)$$

where σ_z^e and σ_z^n are the electron-spin and nuclear-spin Pauli z operators, respectively. This interaction Hamiltonian is a result of a hyperfine interaction between the electronic and the nuclear spin. The nuclear spin is furthermore driven by a microwave field of frequency $\omega_{mw} = \omega_L + \frac{A_{\parallel}}{2}$, amplitude Ω_{mw} , and adjustable phase θ_{mw} :

$$H_{mw} = \Omega_{mw} \left[\sigma_n^e e^{i(\theta_{mw} + \omega_{mw}t)} + \sigma_n^{\dagger} e^{-i(\theta_{mw} + \omega_{mw}t)} \right]. \quad (41)$$

This drive is conditionally resonant when the electron spin is in state $|1_e\rangle$ and is off-resonant when the electron is in $|0_e\rangle$. After transforming the total Hamiltonian $H_{nn} + H_{e-n} + H_{mw}$ into an interaction picture and considering the conditional character of the drive, we obtain the effective Hamiltonian H_{en} :

$$H_{en} = -\hbar A_{\parallel} \sigma_n^{\dagger} \sigma_n |0_e\rangle\langle 0_e| + \hbar \Omega_{mw} [\cos(\theta_{mw})\sigma_x^n + \sin(\theta_{mw})\sigma_y^n] |1_e\rangle\langle 1_e|. \quad (42)$$

Importantly, H_{en} describes the time evolution of the system accurately only if $\Omega_{mw} \ll A_{\parallel}$. When the electron spin is in $|0_e\rangle$ the nuclear spin undergoes a free precession with an angular velocity $-A_{\parallel}$, and when the electron spin is in $|1_e\rangle$ the nuclear spin rotates around an axis $\mathbf{e}_{\theta_{mw}} = \cos(\theta_{mw})\mathbf{e}_x + \sin(\theta_{mw})\mathbf{e}_y$ ($\mathbf{e}_x, \mathbf{e}_y$ being unit vectors along x and y , respectively) with angular velocity $2\Omega_{mw}$.

We next consider that the electron spin is periodically flipped via a dynamical decoupling sequence of the form $(\tau - \pi - 2\tau - \pi - \tau)^{N/2}$, where N is an (even) number of pulses applied to the system. The total duration of the pulse sequence is $T_N = 2N\tau$ and we consider that the gate applied to the nuclear spin is completed at $t = T_N$. The phase θ_{mw} of the microwave drive must be adjusted after each pulse k as:

$$\begin{aligned} \theta_{mw} &= (k-1)\phi_k + \phi_c + \phi_0 & \text{for } k \text{ odd,} \\ \theta_{mw} &= (k-1)\phi_k + \phi_0 & \text{for } k \text{ even,} \end{aligned} \quad (43)$$

where $\phi_k = -(2 - \delta_{1k})\tau A_{\parallel}$, and $\phi_c = 0$ for unconditional rotations of the nuclear spin ($\phi_c = \pi$ for conditional rotations of the nuclear spin). The angle of rotation φ of the nuclear spin about the axis determined by $\cos(\phi_0)\mathbf{e}_x + \sin(\phi_0)\mathbf{e}_y$ is $\varphi = 2\Omega_{mw}\tau N$. The Rabi frequency Ω_{mw} must therefore be appropriately adjusted in order to achieve the desired rotation angle φ . We denote the unconditional gate implemented by the above described protocol as $R_{\phi_0, \varphi}^n$ and the conditional gate as $C_{\phi_0, \varphi}^n$. The conditional gate rotates the nuclear spin by an angle $-\varphi$ if the electron spin is initially in $|1_e\rangle$. The following sequence of controlled and uncontrolled rotations produces a SWAP gate exchanging the states of the electron and the nuclear spin:

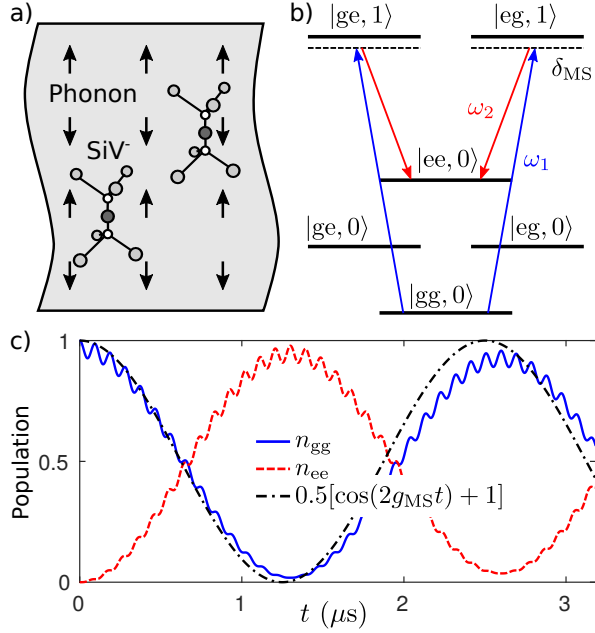
$$|\psi_f\rangle = CX^n \cdot H^e \cdot H^n \cdot CX^n \cdot H^e \cdot H^n \cdot CX^n |\psi_i\rangle, \quad (44)$$

where CX^n is the controlled not gate conditionally flipping the nuclear spin, H^s is the single-qubit Hadamard gate acting on the electron qubit, $s = e$, or the nuclear qubit, $s = n$. The single and two-qubit gates outlined above can be constructed from the conditional rotation of the nuclear spin and local qubit operations. In particular, the Hadamard gate acting on the nuclear spin can be constructed as $H^n = R_{0, \pi}^n \cdot R_{\frac{\pi}{2}, \frac{\pi}{2}}^n$. Similarly, $CX^n = S_{\frac{\pi}{2}} \cdot R_{0, \frac{\pi}{2}}^n \cdot C_{0, \frac{\pi}{2}}^n$, with $S_{\frac{\pi}{2}} = \sigma_e^{\dagger} \sigma_e + i\sigma_e^{\dagger} \sigma_e$ (a rotation around the z axis).

Note that the time-duration of the single-qubit rotations applied to the electron spin is mainly dependent on the intensity of the applied pulses and we treat it as practically instantaneous. On the other hand, the gates applied to the nuclear spin rely on a free time evolution of the system limited by $\Omega_{mw} \ll A_{\parallel}$. This sets the limit to the achievable state-transfer fidelity $\mathcal{F}_{en}$ when spin dephasing is taken into account. We phenomenologically account for pure dephasing of both the electron and the nuclear spin via the Lindblad superoperators $\gamma_e \mathcal{L}_{\sigma_e^{\dagger} \sigma_e}(\rho)$ and $\gamma_n \mathcal{L}_{\sigma_n^{\dagger} \sigma_n}(\rho)$ [see Eq. (??)] that together with H_{en} [Eq. (42)] describe the dynamics of the system. We estimate the fidelity of the state transfer performed by the SWAP gate for a moderate value of the longitudinal spin-spin coupling $A_{\parallel}/(2\pi) = 500$ kHz and we set the drive frequency to $\Omega_{mw}/(2\pi) \approx 3.9$ kHz. We further consider $\gamma_e/(2\pi) = 10$ kHz and $\gamma_n/(2\pi) = 1$ Hz. With these values we estimate $\mathcal{F}_{en} \approx 0.9975$, as given in the main text.

Supplementary Note 5. TWO-QUBIT GATES APPLICABLE TO THE ELECTRON SPINS

One of the advantages of the architecture proposed in this paper is that the color-center electron spins can be used to prepare non-classical many-body quantum-mechanical states that can be further utilized for processing of quantum information, quantum teleportation, or speedup of quantum-state transduction. In this section we provide a suggestion of a gate that could be



Supplementary Figure 3. Mølmer-Sørensen gate can be used to entangle electron spins. (a) Two electron spins of diamond color centers interact with a shared mechanical mode of the phononic cavity. (b) A two-photon drive can be used to perform a transition from the ground state of the system $|gg, 0\rangle$ to the doubly excited state $|ee, 0\rangle$ without changing the number of phonons in the system or populating an intermediate state. (c) Dynamics of the ground-state ($|gg, 0\rangle$) population, n_{gg} , and the doubly excited-state ($|ee, 0\rangle$) population n_{ee} compared with the ideal time dependence $0.5[\cos(2g_{MS}t) + 1]$ following from the effective model implementing the Mølmer-Sørensen gate.

used to generate a GHZ state (i.e. an entangled Bell state) of a pair of electron-spin qubits coupled to a common vibrational mode. In [Supplementary Note 2](#) we have shown that the electron-spin states can be coupled to a strain field via effective controllable coupling schemes. This leads to the effective interaction between a mode of an acoustical cavity coupled to two electron spins:

$$H_{\text{eff}} = \hbar\omega_{e1}\sigma_{e1}^\dagger\sigma_{e1} + \hbar\omega_{e2}\sigma_{e2}^\dagger\sigma_{e2} + \hbar\omega_p b^\dagger b + \hbar g_{\text{eff}}(\sigma_{e1} + \sigma_{e1}^\dagger)(b + b^\dagger) + \hbar g_{\text{eff}}(\sigma_{e2} + \sigma_{e2}^\dagger)(b + b^\dagger). \quad (45)$$

Here $\sigma_{e1} = |0_1\rangle\langle 1_1|$ ($\sigma_{e2} = |0_2\rangle\langle 1_2|$) are the lowering operators of the respective two-level spin systems, b ($b^\dagger$) is the annihilation (creation) operator of the shared phonon mode, ω_{e1} and ω_{e2} are the frequencies of the respective spins, and ω_p is the frequency of the phonon mode. The effective coupling g_{eff} can be realized as described in [Supplementary Note 2](#). It is more convenient to transform the Hamiltonian in Eq. (45) into the interaction picture:

$$H_{\text{eff}}^I = g_{\text{eff}}(\sigma_{e1}e^{-i\omega_{e1}t} + \sigma_{e1}^\dagger e^{i\omega_{e1}t})(be^{-i\omega_p t} + b^\dagger e^{i\omega_p t}) + g_{\text{eff}}(\sigma_{e2}e^{-i\omega_{e2}t} + \sigma_{e2}^\dagger e^{i\omega_{e2}t})(be^{-i\omega_p t} + b^\dagger e^{i\omega_p t}). \quad (46)$$

Next, we assume that the coupling g_{eff} can be modulated in time as $g_{\text{eff}}(t) = \frac{g_{\text{eff}}^0}{4}(e^{i\omega_1 t} + e^{i\omega_2 t} + \text{H.c.})$ (H.c. stands for the Hermitian conjugate). We assume a situation where $\omega_p > \omega_e = \omega_{e1} = \omega_{e2}$ and therefore select the two drive frequencies as $\omega_1 = \omega_e + \omega_p - \delta_{MS}$, and $\omega_2 = \omega_p - \omega_e - \delta_{MS}$, with δ_{MS} a small detuning. We further simplify the Hamiltonian by assuming $\omega_{e1} = \omega_{e2} = \omega_e$. The interaction-picture Hamiltonian [Eq. (46)] then becomes (considering only slowly oscillating terms in the RWA):

$$H_{\text{eff}}^{\text{RWA}} \approx g_{\text{eff}}^0[(\sigma_{e1} + \sigma_{e2})b^\dagger e^{-i(\omega_e - \omega_p + \omega_2)t} + (\sigma_{e1}^\dagger + \sigma_{e2}^\dagger)b e^{i(\omega_e + \omega_p - \omega_1)t} + \text{H.c.}]. \quad (47)$$

From this Hamiltonian we can obtain the effective coupling g_{M-S} between the state $|0_1\rangle \otimes |0_2\rangle \otimes |0\rangle \equiv |gg, 0\rangle$ and the doubly excited state $|1_1\rangle \otimes |1_2\rangle \otimes |0\rangle \equiv |ee, 0\rangle$ (more generally $|gg, n\rangle$ and $|ee, n\rangle$, with n the number of phonons):

$$g_{M-S} \approx \frac{(g_{\text{eff}}^0)^2}{8\delta_{MS}}. \quad (48)$$

We plot the resulting dynamics of the populations of the two excited states in [Supplementary Figure 3](#). The population of the state $|gg, 0\rangle$ (blue line) coherently transfers into the population of $|ee, 0\rangle$ (orange line). For comparison we plot the expression $0.5[\cos(2g_{MS}t) + 1]$ as the yellow line in [Supplementary Figure 3](#). We consider that both electron spins and the phonon are subject to decoherence as described in the main text. If we stop the time evolution at $t \approx 0.675 \mu\text{s}$ we obtain a highly entangled Bell state, the two-qubit GHZ state (up to a phase factor).

Supplementary Note 6. DETERMINISTIC STATE TELEPORTATION

The use of an optically-active spin enables the teleportation of states between two superconducting qubits without transduction of their states to the optical domain as described below. First, nuclear-spin entanglement-at-a-distance is established using previously demonstrated methods[18, 19]. These methods first produce entanglement between two electron-photon pairs separated at a distance, route the photons to a beamsplitter, and perform a Bell-state measurement (BSM) in the optical domain that destroys the which-path information on the measured photons. Upon feedback of the BSM result to the initial nodes and application of appropriate conditional gates, entanglement is produced between the two electron spins and finally transferred to the nuclear spin following [Supplementary Note 4](#). Crucially, the success of this probabilistic entanglement protocol is heralded by the detection of a photon in the BSM apparatus, resulting in a fidelity that is immune to photon loss in the routing network and, with sufficient generation rate, on-demand entanglement within a given time[20].

Once nuclear-spin entanglement is produced, the electron spins are optically reset, and the SC qubit of interest is transferred to its local electron spin using the phononic bus as described in the main text. Finally, quantum teleportation is performed using the prepared entangled nuclear spins as a resource in an analogous fashion to Pfaff *et al.*[19]. A BSM is performed between the local nuclear spin and the electron spin containing the quantum state of the target SC qubit. Upon feed-forward of the BSM results to the distant target node and subsequent single-qubit operations, the SC state is teleported to the distant nuclear spin. The state can be transferred to distant SC qubit via the phononic bus as needed, or maintained in the nuclear-spin quantum memory. In this way the quantum emitter enables SC quantum state transfer without direct transduction of the relevant quantum information into lossy, long-distance links.

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