Supplementary information

Fourier-based estimators for the optical path difference between the two cores

Since we measure each interferogram as a function of the motor-position $p = \tau + p_0$ and not directly as a function of the optical path length difference τ between the two arms of the interferometer, we have to find p_0, the motor-position that corresponds to the optical-path equality with the highest precision possible. Therefore we use an estimator that is taking advantage of the property of the Fourier transform under translation. Let's suppose a function $u(\tau)$ and its Fourier transform with respect to τ is $\mathcal{FT}_\tau[u(\tau)] = \hat{u}(f)$, a function of the frequency f. The Fourier transform of the translated (time-shifted) function $v(\tau) = u(\tau - \tau_0)$ can then be written as $\hat{v}(f) = \hat{u}(f) \cdot \mathrm{e}^{-2\pi \mathrm{i} f \tau_0}$ where an extra phase term is appearing that is evolving linear with the frequency f with slope of $-2\pi\tau_0$. For an even function $u(\tau)$, $\hat{u}(f)$ is a real function and the only phase term of $\hat{v}(f) = \hat{u}(f) \cdot \mathrm{e}^{-2\pi \mathrm{i} f \tau_0}$ is $-2\pi f \tau_0$. Linear-fitting the phase of $\hat{v}(f)$, one thus can easily deduce τ_0.

Due to small third-order (and higher-odd order) dispersive effects, the interferograms (for quantum and classical measurement) are not perfectly symmetric [30, 31, 35]. The Fourier-transform of an unsymmetrical function $u(\tau)$ is complex $\hat{u}(f) = |\hat{u}(f)| \cdot \mathrm{e}^{\mathrm{i}\psi(f)}$, with a phase term $\psi(f)$ that is varying with the frequency f. The phase of the Fourier transform of the translated function $\hat{v}(f) = |\hat{u}(f)| \cdot \mathrm{e}^{\mathrm{i}\psi(f)} \cdot \mathrm{e}^{-2\pi \mathrm{i} f \tau_0}$ is thus a superposition of $\psi(f)$ and the linear term $-2\pi f \tau_0$. A simple linear-fit cannot determine τ_0.

Fortunately we are mainly interested in the difference $\Delta\tau$ of the optical path between the two cores and not the precise motor-position, that corresponds to the path equality for each core. Calculating the Fourier transform of the interferogram for each core, its phase for core 1 (core 2) takes the form $\Psi^{(1)} = \psi^{(1)}_{\mathrm{asym}}(f) - 2\pi f \tau_0^1$ ($\Psi^{(2)} = \psi^{(2)}_{\mathrm{asym}}(f) - 2\pi f \tau_0^2$), where $\psi^{(1)}_{\mathrm{asym}}(f)$ ($\psi^{(2)}_{\mathrm{asym}}(f)$) stands as the phase terms resulting form the unsymmetrical interferogram for core 1 (core 2). We suppose that the interferograms for the two cores are only translated in respect to each other, but the form of the dip/ envelope is the same, resulting in $\psi^{(1)}_{\mathrm{asym}}(f)) = \psi^{(2)}_{\mathrm{asym}}(f) = \psi_{\mathrm{asym}}(f))$. This is a valid approximation since we use the same spectra. Calculating the difference $\Psi^{(2)} - \Psi^{(1)} = \psi_{\mathrm{asym}}(f) - 2\pi f \tau_0^2 - (\psi_{\mathrm{asym}}(f) - 2\pi f \tau_0^1) = 2\pi f(\tau_1 - \tau_2)$, the phase terms resulting form the unsymmetrical interferograms cancels out and with a linear-fit we have directly access to $|\tau_1 - \tau_2| = \Delta\tau$.

Since the interferogram is a discrete set of data-points, that are non uniformly distributed due to the variance of $\sim 20\,\mathrm{nm}$ in the motor-position, we mathematically calculate its Fourier-transform with a nonuniform discrete Fourier transform (NDFT) algorithm (Supplementary Figure 1 and 2).

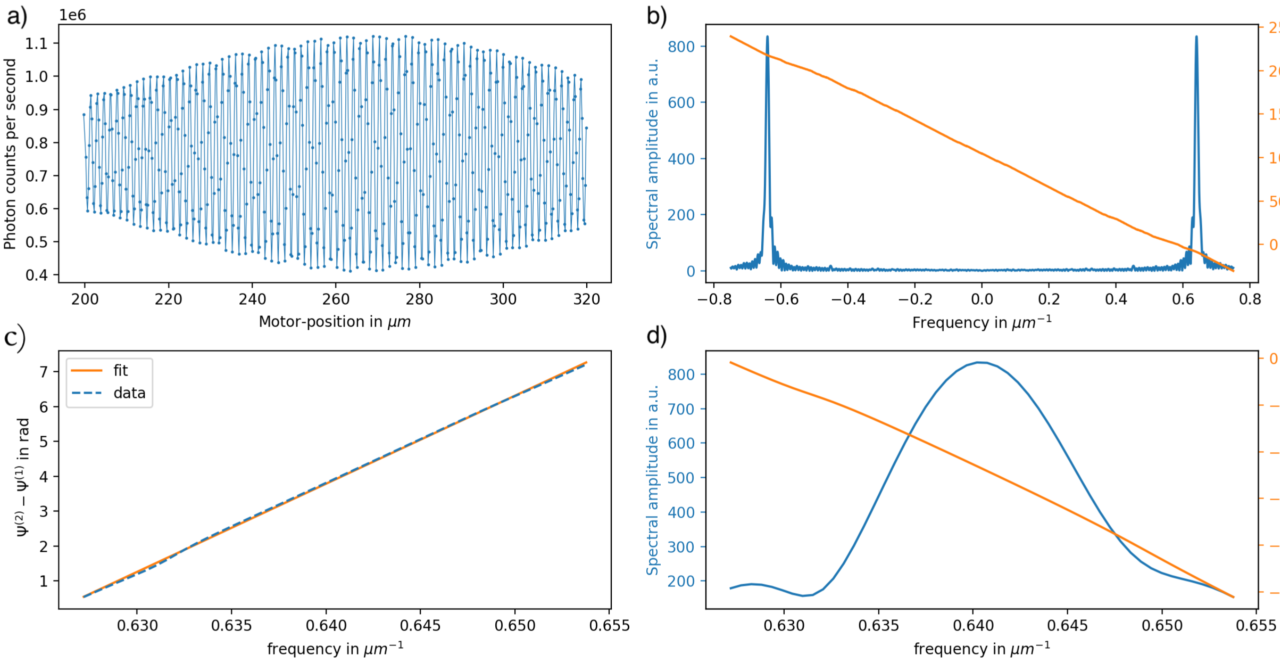
We perform prior simulations of the expected interferogram according to (2), supposing Gaussian distributed fluctuations within the motor-position and a Poisson distribution for the photon counts/coincidences in order to find the optimal stepsize and total range of the HOM-dip scan. This leads to a trade-off between the total number of points and the resulting precision of the estimator, since the faster we can perform the scan the less we are sensible to temperature fluctuations during consecutive measurements. We can reduce the total number of points to 500, before the precision degrades. This gives us a total range of $120\,\mu\mathrm{m}$, centered around the prior estimated optical path equality, with a step-size of $0.24\,\mu\mathrm{m}$. With an integration time of $0.5\,\mathrm{s}$ per point this results in a measurement time of ~ 5 minutes per core.

Estimator for the HOM and classical white-light interferometry

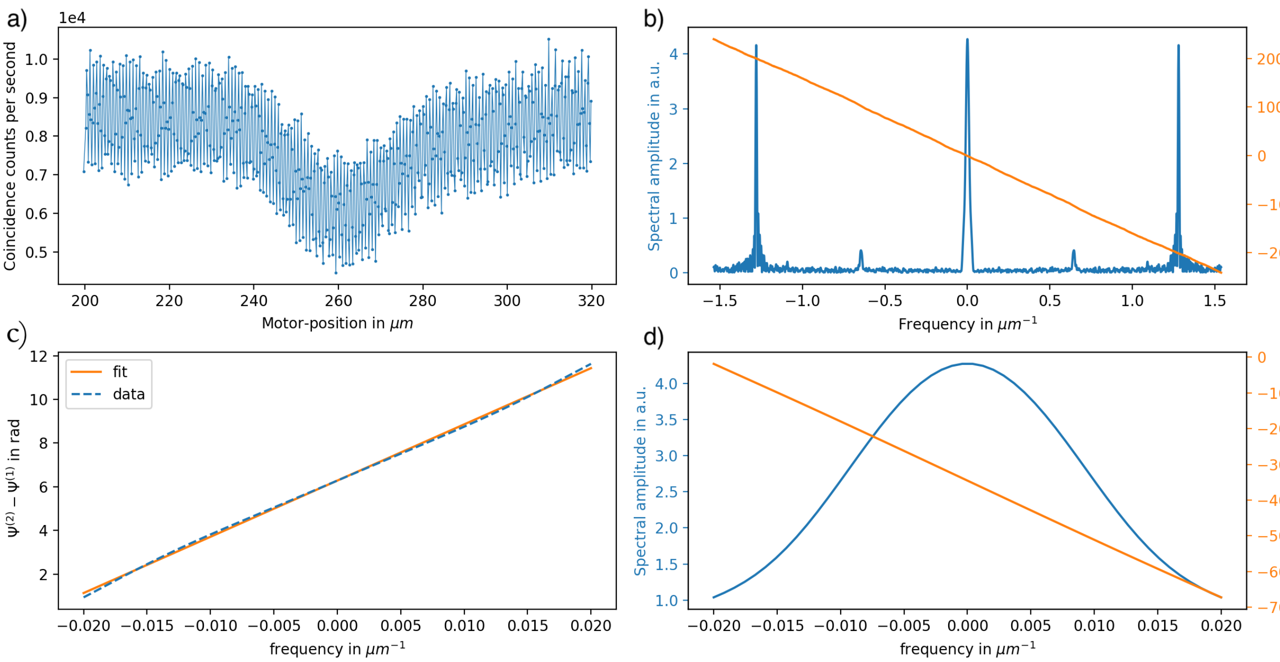
A measured interferogram and its calculated Fourier transform can be seen in Supplementary Figure 1 and 2, corresponding to the quantum and classical regime respectively. Before calculating the Fourier transform, we subtract the average outside the dip [36]. In the quantum approach, we therefore filter the Fourier transform around the low-frequencies that correspond to the HOM-dip.

On the other hand, in the classical regime, the envelop function is modulated by the single-photon frequency, we thus filter the Fourier transform around the side peak at $\frac{1}{1560\,\mathrm{nm}}$ in order to estimate $\Delta\tau$. Since the interferogram is a real function, its Fourier transform is symmetric, we therefore do not gain any more information by using the two side-peaks at $\frac{1}{1560\,\mathrm{nm}}$ and $-\frac{1}{1560\,\mathrm{nm}}$.

In both cases, we find a good agreement between the differential phase $\Psi^{(2)} - \Psi^{(1)}$ and the linear fit (Supplementary Figure 1(c) and 2(c)), which is as well promising for the precision of the classical estimator.



Supplementary Figure 1: a) Measured classical interferogram with a step-size of $0.24\,\mu$m and an integration time of 0.5 s. b) amplitude and phase of the Fourier transform of the classical interferogram. c) amplitude and phase of the Fourier transform, zoomed-in around side peak at $\frac{1}{1560\,\text{nm}}$. d) Differential phase $\Psi^{(2)} - \Psi^{(1)}$ of the Fourier transform for two different measurements (each in a different core) with a linear fit to find $\Delta\tau = 40.9(4)\,\mu$m.



Supplementary Figure 2: a) Measured quantum interferogram with a step-size of $0.24\,\mu$m and an integration time of 0.5 s. b) amplitude and phase of the Fourier transform of the quantum interferogram. c) amplitude and phase of the Fourier transform, zoomed-in around central peak, corresponding to the HOM-dip. d) Differential phase $\Psi^{(2)} - \Psi^{(1)}$ of the Fourier transform for two different measurements (each in a different core) with a linear fit to find $\Delta\tau = 41.3(3)\,\mu$m.