

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32

Supplementary Information:

Quantum Fredkin and Toffoli gates on a versatile programmable silicon photonic chip

Yuan Li¹, Lingxiao Wan¹, Hui Zhang¹, Huihui Zhu¹, Yuzhi Shi², Lip Ket Chin³
Xiaoqi Zhou^{4†}, Leong Chuan Kwek^{1,5,6†}, Ai Qun Liu^{1†}

¹ Quantum Science and Engineering Centre (QSec), Nanyang Technological
University, 50 Nanyang Ave, 639798, Singapore

² National Key Laboratory of Science and Technology on Micro/Nano Fabrication,
Department of Micro/Nano Electronics, Shanghai Jiao Tong University, Shanghai
200240, China

³ Department of Electrical Engineering, City University of Hong Kong, Kowloon,
Hong Kong SAR

⁴ State Key Laboratory of Optoelectronic Materials and Technologies and School of
Physics, Sun Yat-sen University, Guangzhou 510275, China

⁵ Centre for Quantum Technologies, National University of Singapore, Block S15, 3
Science Drive 2, 117543, Singapore

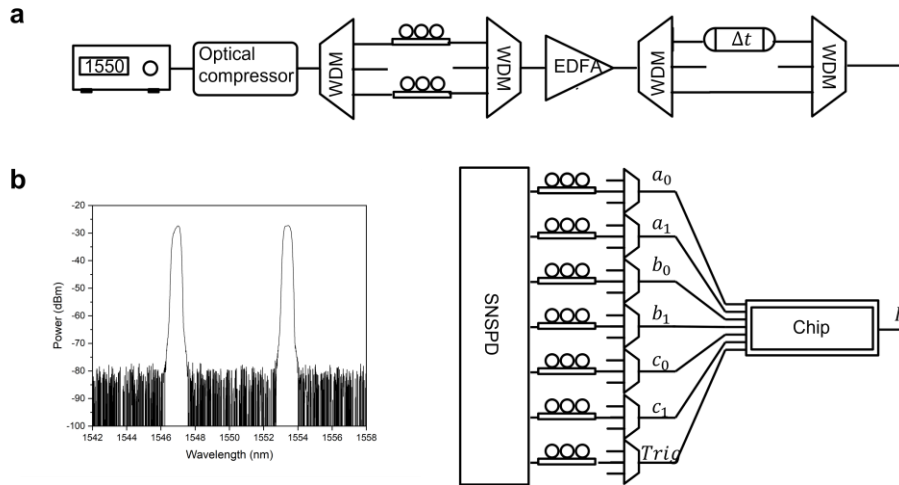
⁶ National Institute of Education, 1 Nanyang Walk, Singapore 637616

*e-mail: zhouxq8@mail.sysu.edu.cn (X.Z.); kwekleongchuan@nus.edu.sg (L.C.K.);
EAQLiu@ntu.edu.sg (A.Q.L.)

Supplementary Note 1: Experimental details

1.1 Experimental setup

The experimental setup is shown in Supplementary Figure 1. The laser is generated from the Ultrafast Optical Clock device (PriTel) with a repetition rate of 500 MHz, a central wavelength of 1550 nm and a 3dB bandwidth of 1.9 nm. The pulse laser is expanded to a bandwidth of about 10 nm by an optical compressor. A pair of wavelength-division multiplexing (WDM) are placed in order to select two wavelengths of 1546.8nm and 1553.2 nm with a 3-dB bandwidth of 0.5 nm. Two polarization controllers (PCs) are inserted to adjust the polarization of dual-wavelength beam separately which can ensure the maximum coupling efficiency to the chip. Erbium dope fibre amplifier (EDFA) is connected to amplify the beam power. However, the EDFA would also increase background light noise which is eliminated by another pair of WDM. A tunable optical delay line is inserted in one arm to assure the temporal indistinguishability of the dual-wavelength pump beam. The pump beam is coupled into the photonic chip and generate the 1550 nm photons by spontaneous four wavelength mixing (SFWM) processing. After passing through the optical circuit, the parametric photons are filtered by WDMs and detected by the superconducting nanowire single-photon detector (SNSPD).



Supplementary Figure 1: Experimental setup. **a**, The experimental setup for performing the quantum logic gate. WDM: wavelength-division multiplexer; EDFA: Erbium-doped fiber amplifier; SNSPD: superconducting nanowire single-photon detector. **b**, The spectra

54 of dual-wavelength pump beam with an attenuated power measured by the spectrometer at
 55 the input point (I) of chip.

56 1.2 Details of constructing $[Y_{ij}]$

57 We take the two-qubit gate as an example to explain how to construct the nonunitary matrix
 58 $[Y_{ij}]$. The two-qubit gate can be described as a 4×4 matrix in the computation basis
 59 $|00\rangle, |01\rangle, |10\rangle, |11\rangle$, denoted as $\llbracket X_{ij} \rrbracket$.

$$60 \quad \llbracket X_{ij} \rrbracket = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} \\ X_{21} & X_{22} & X_{23} & X_{24} \\ X_{31} & X_{32} & X_{33} & X_{34} \\ X_{41} & X_{42} & X_{43} & X_{44} \end{bmatrix}$$

61 Thus, the input-output relation of the gate for the four basis is given as:

$$62 \quad |00\rangle_{in} \xrightarrow{\text{gate}} X_{11}|00\rangle_{out} + X_{12}|01\rangle_{out} + X_{13}|10\rangle_{out} + X_{14}|11\rangle_{out}$$

$$63 \quad |01\rangle_{in} \xrightarrow{\text{gate}} X_{21}|00\rangle_{out} + X_{22}|01\rangle_{out} + X_{23}|10\rangle_{out} + X_{24}|11\rangle_{out}$$

$$64 \quad |10\rangle_{in} \xrightarrow{\text{gate}} X_{31}|00\rangle_{out} + X_{32}|01\rangle_{out} + X_{33}|10\rangle_{out} + X_{34}|11\rangle_{out}$$

$$65 \quad |11\rangle_{in} \xrightarrow{\text{gate}} X_{41}|00\rangle_{out} + X_{42}|01\rangle_{out} + X_{43}|10\rangle_{out} + X_{44}|11\rangle_{out}$$

66 On the other hand, we consider a 4×4 matrix $[Y_{ij}]$ acting at the four waveguides denoted
 67 as: a_0, a_1, b_0 and b_1 . The evolution for each input mode can be described as:

$$68 \quad \begin{bmatrix} a_{0in}^\dagger \\ a_{1in}^\dagger \\ b_{0in}^\dagger \\ b_{1in}^\dagger \end{bmatrix} \xrightarrow{[Y_{ij}]} \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \cdot \begin{bmatrix} a_{0out}^\dagger \\ a_{1out}^\dagger \\ b_{0out}^\dagger \\ b_{1out}^\dagger \end{bmatrix}$$

69 Then we can also consider the evolutions for four computation bases input for two-qubit
 70 states:

$$71 \quad |00\rangle_{in} = a_{0in}^\dagger b_{0in}^\dagger |\emptyset\emptyset\rangle$$

$$72 \quad \xrightarrow{U} (Y_{11}a_{0out}^\dagger + Y_{12}a_{1out}^\dagger + Y_{13}b_{0out}^\dagger + Y_{14}b_{1out}^\dagger)(Y_{31}a_{0out}^\dagger + Y_{32}a_{1out}^\dagger + Y_{33}b_{0out}^\dagger + Y_{34}b_{1out}^\dagger)|\emptyset\emptyset\rangle$$

$$73 \quad \xrightarrow{\text{Postselection}} \text{Per} \begin{bmatrix} Y_{11} & Y_{13} \\ Y_{31} & Y_{33} \end{bmatrix} |00\rangle_{out} + \text{Per} \begin{bmatrix} Y_{11} & Y_{14} \\ Y_{31} & Y_{34} \end{bmatrix} |01\rangle_{out} + \text{Per} \begin{bmatrix} Y_{12} & Y_{13} \\ Y_{32} & Y_{33} \end{bmatrix} |10\rangle_{out} + \text{Per} \begin{bmatrix} Y_{12} & Y_{14} \\ Y_{32} & Y_{34} \end{bmatrix} |11\rangle_{out}$$

$$74 \quad |01\rangle_{in} = a_{0in}^\dagger b_{1in}^\dagger |\emptyset\emptyset\rangle$$

$$75 \quad \xrightarrow{U} (Y_{11}a_{0out}^\dagger + Y_{12}a_{1out}^\dagger + Y_{13}b_{0out}^\dagger + Y_{14}b_{1out}^\dagger)(Y_{41}a_{0out}^\dagger + Y_{42}a_{1out}^\dagger + Y_{43}b_{0out}^\dagger + Y_{44}b_{1out}^\dagger)|\emptyset\emptyset\rangle$$

$$76 \quad \xrightarrow{\text{Postselection}} \text{Per} \begin{bmatrix} Y_{11} & Y_{13} \\ Y_{41} & Y_{43} \end{bmatrix} |00\rangle_{out} + \text{Per} \begin{bmatrix} Y_{11} & Y_{14} \\ Y_{41} & Y_{44} \end{bmatrix} |01\rangle_{out} + \text{Per} \begin{bmatrix} Y_{12} & Y_{13} \\ Y_{42} & Y_{43} \end{bmatrix} |10\rangle_{out} + \text{Per} \begin{bmatrix} Y_{12} & Y_{14} \\ Y_{42} & Y_{44} \end{bmatrix} |11\rangle_{out}$$

$$77 \quad |10\rangle_{in} = a_{1in}^\dagger b_{0in}^\dagger |\emptyset\emptyset\rangle$$

$$78 \quad \xrightarrow{U} (Y_{21}a_{0out}^\dagger + Y_{22}a_{1out}^\dagger + Y_{23}b_{0out}^\dagger + Y_{24}b_{1out}^\dagger)(Y_{31}a_{0out}^\dagger + Y_{32}a_{1out}^\dagger + Y_{33}b_{0out}^\dagger + Y_{34}b_{1out}^\dagger)|\emptyset\emptyset\rangle$$

$$79 \quad \xrightarrow{\text{Postselection}} \text{Per} \begin{bmatrix} Y_{21} & Y_{23} \\ Y_{31} & Y_{33} \end{bmatrix} |00\rangle_{out} + \text{Per} \begin{bmatrix} Y_{21} & Y_{24} \\ Y_{31} & Y_{34} \end{bmatrix} |01\rangle_{out} + \text{Per} \begin{bmatrix} Y_{22} & Y_{23} \\ Y_{32} & Y_{33} \end{bmatrix} |10\rangle_{out} + \text{Per} \begin{bmatrix} Y_{22} & Y_{24} \\ Y_{32} & Y_{34} \end{bmatrix} |11\rangle_{out}$$

$$80 \quad |11\rangle_{in} = a_{1in}^\dagger b_{1in}^\dagger |\emptyset\emptyset\rangle$$

$$81 \quad \xrightarrow{U} (Y_{21}a_{0out}^\dagger + Y_{22}a_{1out}^\dagger + Y_{23}b_{0out}^\dagger + Y_{24}b_{1out}^\dagger)(Y_{41}a_{0out}^\dagger + Y_{42}a_{1out}^\dagger + Y_{43}b_{0out}^\dagger + Y_{44}b_{1out}^\dagger)|\emptyset\emptyset\rangle$$

$$82 \quad \xrightarrow{\text{Postselection}} \text{Per} \begin{bmatrix} Y_{21} & Y_{23} \\ Y_{41} & Y_{43} \end{bmatrix} |00\rangle_{out} + \text{Per} \begin{bmatrix} Y_{21} & Y_{24} \\ Y_{41} & Y_{44} \end{bmatrix} |01\rangle_{out} + \text{Per} \begin{bmatrix} Y_{22} & Y_{23} \\ Y_{42} & Y_{43} \end{bmatrix} |10\rangle_{out} + \text{Per} \begin{bmatrix} Y_{22} & Y_{24} \\ Y_{42} & Y_{44} \end{bmatrix} |11\rangle_{out}$$

83 Where $|\emptyset\emptyset\rangle$ is the vacuum state and $\text{Per} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad + bc$ is the permanent of the
84 matrix.

85 In order to assure that all the evolution coefficients are exactly the same, the following
86 16 equations should be satisfied simultaneously.

$$87 \quad \begin{cases} \text{Per} \begin{bmatrix} Y_{11} & Y_{13} \\ Y_{31} & Y_{33} \end{bmatrix} = X_{11} & \text{Per} \begin{bmatrix} Y_{11} & Y_{14} \\ Y_{31} & Y_{34} \end{bmatrix} = X_{12} & \text{Per} \begin{bmatrix} Y_{12} & Y_{13} \\ Y_{32} & Y_{33} \end{bmatrix} = X_{13} & \text{Per} \begin{bmatrix} Y_{12} & Y_{14} \\ Y_{32} & Y_{34} \end{bmatrix} = X_{14} \\ \text{Per} \begin{bmatrix} Y_{11} & Y_{13} \\ Y_{41} & Y_{43} \end{bmatrix} = X_{21} & \text{Per} \begin{bmatrix} Y_{11} & Y_{14} \\ Y_{41} & Y_{44} \end{bmatrix} = X_{22} & \text{Per} \begin{bmatrix} Y_{12} & Y_{13} \\ Y_{42} & Y_{43} \end{bmatrix} = X_{23} & \text{Per} \begin{bmatrix} Y_{12} & Y_{14} \\ Y_{42} & Y_{44} \end{bmatrix} = X_{24} \\ \text{Per} \begin{bmatrix} Y_{21} & Y_{23} \\ Y_{31} & Y_{33} \end{bmatrix} = X_{31} & \text{Per} \begin{bmatrix} Y_{21} & Y_{24} \\ Y_{31} & Y_{34} \end{bmatrix} = X_{32} & \text{Per} \begin{bmatrix} Y_{22} & Y_{23} \\ Y_{32} & Y_{33} \end{bmatrix} = X_{33} & \text{Per} \begin{bmatrix} Y_{22} & Y_{24} \\ Y_{32} & Y_{34} \end{bmatrix} = X_{34} \\ \text{Per} \begin{bmatrix} Y_{21} & Y_{23} \\ Y_{41} & Y_{43} \end{bmatrix} = X_{41} & \text{Per} \begin{bmatrix} Y_{21} & Y_{24} \\ Y_{41} & Y_{44} \end{bmatrix} = X_{42} & \text{Per} \begin{bmatrix} Y_{22} & Y_{23} \\ Y_{42} & Y_{43} \end{bmatrix} = X_{43} & \text{Per} \begin{bmatrix} Y_{22} & Y_{24} \\ Y_{42} & Y_{44} \end{bmatrix} = X_{44} \end{cases}$$

88 We then perform the numerical solution to solve $[Y_{ij}]$ for the desired gates, which can also
89 be applied for solving the three-qubit Fredkin gate and Toffoli gate with 64 equations.

90 1.3 Construction of the matrices $V'^\dagger$ and U'

91 To realize the three-qubit gate, theoretically a 6×6 nonunitary matrix $[Y_{ij}]$ should be
92 constructed. For a three-qubit input state $|a_{in}, b_{in}, c_{in}\rangle$ which can be expressed as:
93 $|a_{in}, b_{in}, c_{in}\rangle = (P_a \oplus P_b \oplus P_c)|a_0, b_0, c_0\rangle$ after the state preparation, where P_a, P_b, P_c
94 are 2×2 unitary matrices and a_0, b_0, c_0 are the $|0\rangle$ input state. Then the state of output
95 photons $|a_{out}, b_{out}, c_{out}\rangle$ after passing through the photonic circuit $[Y_{ij}]$ and the
96 tomography part is:

$$97 \quad |a_{out}, b_{out}, c_{out}\rangle = (M_a \oplus M_b \oplus M_c)[Y_{ij}](P_a \oplus P_b \oplus P_c)|a_0, b_0, c_0\rangle$$

Where M_a, M_b, M_c are 2×2 unitary matrices determined by the measurement basis. Then we define the nonunitary matrix: $[\tilde{Y}] = (M_a \oplus M_b \oplus M_c)[Y_{ij}] (P_a \oplus P_b \oplus P_c)$. As can be seen that the output state of a three-qubit gate is determined by the columns 1, 3 and 5 of the nonunitary matrix $[\tilde{Y}]$ which is just a 6×3 nonunitary matrix denoted as $[\tilde{Y}]_{6 \times 3}$.

By performing the SVD decomposition of $[\tilde{Y}]_{6 \times 3}$ for each input state and measurement basis, we obtain:

$$[\tilde{Y}]_{6 \times 3} = U'_{6 \times 6} \Sigma V'^{\dagger}_{3 \times 3} \quad (1)$$

which is experimentally constructed on our photonic chip with the corresponding attenuations.

Experimental Note 2: Two-photon logic gates

Here, some typical two-qubit gates we have experimentally realized including the CNOT gate et, al. are listed in Supplementary Table 1.

Supplementary Table 1: The matrix forms $[[X_{ij}]]$ and the optimal nonunitary matrices $[Y_{ij}]$ for different two-qubit logic gates.

Logic gate	$[[X_{ij}]]$	Optimal $[Y_{ij}]$	Success probability
CNOT	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$	1/9
CH	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0.54 & -1.31 \\ 0 & 1 & 0 & 0 \\ 0.54 & 0 & 0.71 & 0.71 \\ -1.31 & 0 & 0.71 & -0.71 \end{bmatrix}$	1/9
$\sqrt{i}$ SWAP	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2}i & 0 \\ 0 & 1/\sqrt{2}i & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1.30 & 0 & 0.84i & 0 \\ 0 & 0.54 & 0 & 0.84 \\ 0.84i & 0 & 1.31 & 0 \\ 0 & 0.84 & 0 & 0.54 \end{bmatrix}$	0.17
QFT	$\frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$	$\begin{bmatrix} -0.6 & 0.6 & -0.7 & -0.7 \\ -0.6 & 0.6 & -0.7 & 0.7 \\ -0.71 & -0.71 & 0 & 0 \\ -0.71i & 0.71i & -0.83 + 0.83i & 0 \end{bmatrix}$	0.09

Supplementary Note 3: n -qubit Toffoli gate

For a n -qubit Toffoli gate, the n^{th} qubit is flipped only when all the other controlled $n - 1$ qubits are $|1\rangle$. In this section, we list the calculated nonunitary matrices $[Y_{ij}]$ for 4-7 qubits Toffoli gates which provides the method to realize the large-qubit Toffoli gates with a larger-scale photonic network. The unknown numbers in the following matrices such as $A_0, B_0, \dots$ can be arbitrary values which are determined by optimizing the successful rate of the gate. It can be extended to obtain n -qubit Toffoli gate's form instead of calculating the permanent of a large size matrix due to its very regular form as listed as following for 4-7 qubit Toffoli gate (lots of zero elements and infinite nonzero elements in specific positions of the matrix).

4-qubit Toffoli gate

$$\begin{bmatrix} A_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_0 & 0 & 0 & 0 & D_0 & 0 & 0 \\ 0 & 0 & B_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_0 & 0 & 0 & -E_0 & E_0 \\ 0 & 0 & 0 & 0 & C_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & F_0 & 0 & C_0 & 0 & 0 \\ 0 & \frac{1}{D_0 E_0 F_0} & 0 & 0 & 0 & 0 & \frac{1}{A_0 B_0 C_0} & 0 \\ 0 & -\frac{1}{D_0 E_0 F_0} & 0 & 0 & 0 & 0 & 0 & \frac{1}{A_0 B_0 C_0} \end{bmatrix}$$

5-qubit Toffoli gate

$$\begin{bmatrix} A_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_0 & 0 & 0 & 0 & 0 & 0 & E_0 & 0 & 0 \\ 0 & 0 & B_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_0 & 0 & 0 & 0 & 0 & -F_0 & F_0 \\ 0 & 0 & 0 & 0 & C_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_0 & 0 & C_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & D_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & H_0 & 0 & D_0 & 0 & 0 \\ 0 & \frac{1}{E_0 F_0 G_0 H_0} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{A_0 B_0 C_0 D_0} & 0 \\ 0 & -\frac{1}{E_0 F_0 G_0 H_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{A_0 B_0 C_0 D_0} \end{bmatrix}$$

6-qubit Toffoli gate

129

$$\begin{bmatrix}
A_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & A_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & F_0 & 0 & 0 \\
0 & 0 & B_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & B_0 & 0 & 0 & 0 & 0 & 0 & 0 & -G_0 & G_0 \\
0 & 0 & 0 & 0 & C_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & C_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & H_0 & 0 & 0 & D_0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & D_0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E_0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & J_0 & 0 & E_0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & \frac{F_0 G_0 H_0 I_0 J_0}{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{A_0 B_0 C_0 D_0 E_0}{1} & 0 \\
0 & -\frac{1}{F_0 G_0 H_0 I_0 J_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{A_0 B_0 C_0 D_0 E_0}
\end{bmatrix}$$

130 **7-qubit Toffoli gate**

131

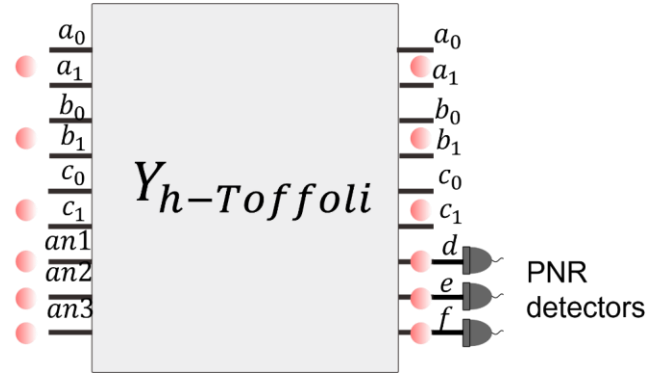
$$\begin{bmatrix}
A_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & A_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & G_0 & 0 \\
0 & 0 & B_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & B_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -H_0 \\
0 & 0 & 0 & 0 & C_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_0 \\
0 & 0 & 0 & 0 & 0 & C_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & I_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & D_0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E_0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E_0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & F_0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_0 & 0 & F_0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & \frac{G_0 H_0 I_0 J_0 K_0 L_0}{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{A_0 B_0 C_0 D_0 E_0 F_0}{1} & 0 \\
0 & -\frac{1}{G_0 H_0 I_0 J_0 K_0 L_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{A_0 B_0 C_0 D_0 E_0 F_0}
\end{bmatrix}$$

132 **Supplementary Note 4: Scheme of constructing heralded Toffoli gate**

133 Our scheme can also be used to construct a heralded three-qubit Toffoli gate with three
134 ancillary independent photons as shown in Supplementary Figure 2. Here, we construct a
135 9×9 nonunitary matrix $Y_{h-Toffoli}$ with six photons injected into this optical circuit to
136 implement a heralded three-qubit Toffoli gate, which is expressed as:

$$137 \quad Y_{h-Toffoli} = \begin{bmatrix}
0.82 & 0 & 0 & 0 & 0 & 0 & -0.86 & 0 & 0.81 \\
0 & 1.6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.84 & 0 & 0 & 0 & 0 & -0.35 & 0.54 \\
0 & 0 & 0 & 1.7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.3 & -0.43 & -0.5 & -0.6 & 0 \\
0 & 0 & 0 & 0 & -0.43 & 1.3 & -0.5 & -0.6 & 0 \\
0 & 0 & -1.1 & 0 & 0 & 0 & 0 & 0.47 & 0.73 \\
0 & 0 & 0 & 0 & 0.58 & 0.58 & -0.66 & 0.8 & 0 \\
-0.69 & 0 & 0 & 0 & 0 & 0 & -0.72 & 0 & -0.68
\end{bmatrix}.$$

The control qubits C_1 , C_2 and target qubit T are encoded in the first six modes (a_0, a_1), (b_0, b_1) and (c_0, c_1), and the bottom three modes are for the ancillary inputs. At the output of the circuit, three photon number resolving (PNR) detectors are placed at last three modes d, e and f to assure a deterministic three-photon coincidence (one and only one photon arrive at d, e and f respectively) at the last three modes. It is guaranteed that there is one and only one photon in every two modes of the upper six modes (a_0, a_1), (b_0, b_1) and (c_0, c_1) at the output, thus realizing the desired heralded three-qubit Toffoli gate. The success probability of this heralded Toffoli gate is 9.4×10^{-4} . Combined with the KLM scheme¹, it is possible to boost the success probability of the heralded Toffoli gate to near-deterministic level. Such heralded three-qubit gates can be potentially applied to future linear optical quantum circuits.



Supplementary Figure 2: Scheme for generating heralded Toffoli gate. The scheme for realizing a heralded Toffoli gate with three independent photons acting as the gate qubits and three ancillary qubits injected in the $an1$, $an2$ and $an3$ mode of the 9×9 nonunitary circuits $Y_{h-Toffoli}$. A detection of a three-photon coincidence at modes d, e and f heralds a success generation of the Toffoli gate for photons a, b and c .

Supplementary References

1. Knill, E., Laflamme, R. & Milburn, G. J. A scheme for efficient quantum computation with linear optics. *Nature* **409**, 46–52 (2001).