

# Supplementary Information on “Quasiperiodic circuit quantum electrodynamics”

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## SUPPLEMENTARY NOTE 1

### Derivation of the Hamiltonians via the Lagrangian approach

#### Quasiperiodic nonlinear capacitor

The Hamiltonian in Eq. (3) of the main text characterizes the circuit in Fig. 1(a) of the main text displaying the quasiperiodic nonlinear capacitor (QPNC). To find this Hamiltonian, we first use the standard rules to write down the Lagrangian

$$\mathcal{L} = \frac{1}{2} \frac{C(\dot{\varphi}_T - \dot{\varphi})^2}{(2e)^2} + \frac{1}{2} \frac{C_{gT}(\dot{\varphi}_T - 2eV_{gT})^2}{(2e)^2} + \frac{1}{2} \frac{C_{JT}\dot{\varphi}_T^2}{(2e)^2} + E_{JT} \cos(\varphi_T) + \frac{1}{2} \sum_{\alpha} \frac{C_{\alpha}(\dot{\varphi} - \dot{\varphi}_{\alpha})^2}{(2e)^2} - V(\varphi), \quad (1)$$

where the last two terms correspond to additional elements connected to the right island (with phase  $\varphi$ ) not explicitly shown in the figure. We include these here since the circuit is meant to be a part of a larger circuit and to generalize the derivation and demonstrate what influence such elements have on the parameters of the final Hamiltonian. Here,  $V(\varphi)$  encapsulates any inductive contributions while the  $C_{\alpha}$  denote the capacities of any linear capacitance to an island or contact with superconducting phase  $\varphi_{\alpha}$ . We neglect any dynamics of these phases – that is, beyond external control like an applied voltage to ground – which also allows us to truncate any energy terms that do not depend on  $\varphi$  or  $\varphi_T$ . Treating the phases  $\varphi_{\alpha}$  as degrees of freedom within the Lagrange formalism would result in a much more complex calculation shifting the focus away from the QPNC. Part of this complexity is the appearance of capacitive cross-couplings between the auxiliary transmon island (with phase  $\varphi_T$ ) and the other islands whose strengths would have to be considered depending on the parameter regime. Here, we want to concentrate on the core behavior of the QPNC in circuits with few degrees of freedom. As an example of a more complex circuit we consider the multidimensional Aubry-André circuit (see Fig. 6 of the

main text); a derivation of the Hamiltonian for  $d = 2$  can be found below.

Now we continue with the Euler-Lagrange equations  $N = \partial\mathcal{L}/\partial\dot{\varphi}$  and  $N_T = \partial\mathcal{L}/\partial\dot{\varphi}_T$  to define the canonically conjugate charges,

$$N = \frac{C(\dot{\varphi} - \dot{\varphi}_T)}{(2e)^2} + \sum_{\alpha} \frac{C_{\alpha}(\dot{\varphi} - \dot{\varphi}_{\alpha})}{(2e)^2}, \quad (2)$$

$$N_T = \frac{C(\dot{\varphi}_T - \dot{\varphi})}{(2e)^2} + \frac{C_{gT}(\dot{\varphi}_T - 2eV_{gT})}{(2e)^2} + \frac{C_{JT}\dot{\varphi}_T}{(2e)^2}, \quad (3)$$

and define the offset charges  $N_g \equiv \sum_{\alpha} C_{\alpha}\dot{\varphi}_{\alpha}/(2e)^2$  and  $N_{gT} \equiv C_{gT}V_{gT}/2e$ . Therewith the Hamiltonian can be derived with a Legendre transformation  $H = N\dot{\varphi} + N_T\dot{\varphi}_T - \mathcal{L}$ . The kinetic part (i.e. the capacitive terms) of the Hamiltonian can also be written as  $T = \mathbf{Q}^T \underline{C}^{-1} \mathbf{Q}/2$  with the charge vector  $\mathbf{Q} = 2e(N + N_g, N_T + N_{gT})^T$  and the capacitance matrix  $\underline{C}$ . The latter can be constructed from the Euler-Lagrange equations such that  $\mathbf{Q} = \underline{C}\mathbf{V}$  with the voltage vector  $\mathbf{V} = (\dot{\varphi}, \dot{\varphi}_T)^T/2e$ . With Supplementary Equations (2) and (3) we identify

$$\underline{C} = \begin{pmatrix} C_{\text{tot}} & -C \\ -C & C_{T,\text{tot}} \end{pmatrix}, \quad (4)$$

introducing the total capacities of the islands  $C_{\text{tot}} \equiv C + \sum_{\alpha} C_{\alpha}$  and  $C_{T,\text{tot}} \equiv C + C_{gT} + C_{JT}$ , and the inverse

$$\underline{C}^{-1} = \frac{1}{C_{\text{tot}}C_{T,\text{tot}} - C^2} \begin{pmatrix} C_{T,\text{tot}} & C \\ C & C_{\text{tot}} \end{pmatrix}. \quad (5)$$

This results in the Hamiltonian

$$H = 2E_{CT} [N_T + N_{gT} + \lambda(N + N_g)]^2 + 2E_C (N + N_g)^2 - E_{JT} \cos(\varphi_T) + V(\varphi), \quad (6)$$

with the parameter definitions  $E_{CT} = e^2 C_{\text{tot}} / (C_{\text{tot}} C_{T,\text{tot}} - C^2)$ ,  $E_C = e^2 / C_{\text{tot}}$  and  $\lambda = C / C_{\text{tot}}$ , analogously to the main text. Comparing this Hamiltonian (after quantizing) with Eq. (3) of the main text we find full agreement when removing the additional terms by setting  $V(\varphi) = 0$  and  $\sum_{\alpha} C_{\alpha} = 0$ , resulting in  $N_g = 0$  and  $C_{\text{tot}} = C$ . However, by including these terms we find out how every capacitive element connected to the right island contributes via  $C_{\text{tot}}$  to the value of  $\lambda$  and tunes it away from the trivial value 1.

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Now the two circuits in Figs. 1(c, d) of the main text are examples for complete circuits involving the QPNC that can be described by the general Hamiltonian in Supplementary Equation (6). For the Aubry-André circuit (Fig. 1(d) of the main text) we include a Josephson junction to ground (giving us the inductive term  $V(\varphi) = -E_J \cos(\varphi)$ ) and a capacitor connected to a gate voltage to ground (setting the total capacity  $C_{\text{tot}} = C + C_g$  and the offset charge  $N_g = C_g V_g / 2e$ ). The same elements are included in the Dirac circuit (Fig. 1(c) of the main text), however, an additional cotunneling junction connects to ground with a phase of  $\phi_\alpha = \delta$ . Here the full inductive potential reads  $V(\varphi) = -E_J \cos(\varphi) - E_{2J} \cos[2(\varphi - \delta)]$ . Note that the Josephson junctions and the cotunneling junction are also associated with capacities which are added to  $C_{\text{tot}}$ , too, but for simplicity we do not include them explicitly. After quantizing the Hamiltonians we just replace the terms containing  $\hat{N}_T$  and  $\hat{\varphi}_T$  by the low-energy effective treatment  $\sim E_S$  introduced and discussed in and around Eq. (5) of the main text to find the Hamiltonians shown in Eqs. (9) and (16) of the main text.

### 2D Aubry-André circuit

Here we derive the Hamiltonian given in Eq. (20) of the main text for the  $d = 2$  dimensional Aubry-André circuit; see Fig. 6 of the main text. The Lagrangian reads

$$\begin{aligned} \mathcal{L}_{\text{AA}}^{(2)} = & \frac{1}{2} \sum_{\mu \in \{x, y\}} \frac{C_{g, \mu} (\dot{\varphi}_\mu - 2eV_{g, \mu})^2}{(2e)^2} \\ & + \frac{1}{2} \sum_{j, \mu} \frac{C_{j\mu} (\dot{\varphi}_{T, j} - \dot{\varphi}_\mu)^2}{(2e)^2} + \frac{1}{2} \sum_j \frac{C_{gT, j} (\dot{\varphi}_{T, j} - 2eV_{gT, j})^2}{(2e)^2} \\ & + \sum_j E_{JT, j} \cos(\varphi_{T, j}) + \sum_\mu E_{J, \mu} \cos(\varphi_\mu), \quad (7) \end{aligned}$$

where each index takes two values  $j \in \{1, 2\}$  and  $\mu \in \{x, y\}$ . Thus we can derive the following four Euler-Lagrange equations

$$\begin{aligned} N_\mu &= \sum_j \frac{C_{j\mu} (\dot{\varphi}_\mu - \dot{\varphi}_{T, j})}{(2e)^2} + \frac{C_{g, \mu} (\dot{\varphi}_\mu - 2eV_{g, \mu})}{(2e)^2}, \quad (8) \\ N_{T, j} &= \sum_\mu \frac{C_{j\mu} (\dot{\varphi}_{T, j} - \dot{\varphi}_\mu)}{(2e)^2} + \frac{C_{gT, j} (\dot{\varphi}_{T, j} - 2eV_{gT, j})}{(2e)^2}, \quad (9) \end{aligned}$$

and define the offset charges  $N_{g, \mu} \equiv C_{g, \mu} V_{g, \mu} / 2e$  and  $N_{gT, j} \equiv C_{gT, j} V_{gT, j} / 2e$  and the shifted charges  $\tilde{N}_\mu \equiv N_\mu + N_{g, \mu}$  and  $\tilde{N}_{T, j} \equiv N_{T, j} + N_{gT, j}$ . Analogously to above, we construct a capacitance matrix  $\underline{C}$  satisfying the equation  $\mathbf{Q} = \underline{C}\mathbf{V}$  with the charge vector  $\mathbf{Q} = 2e(\tilde{N}_x, \tilde{N}_y, \tilde{N}_{T, 1}, \tilde{N}_{T, 2})^\top$  and the voltage vector  $\mathbf{V} = (\dot{\varphi}_x, \dot{\varphi}_y, \dot{\varphi}_{T, 1}, \dot{\varphi}_{T, 2})^\top / 2e$ . Then we invert the matrix

to find the kinetic part of the Hamiltonian via Legendre transformation  $T = \mathbf{Q}^T \underline{C}^{-1} \mathbf{Q} / 2$ . With Supplementary Equations (8) and (9) we thus find the capacitance matrix

$$\underline{C} = \begin{pmatrix} \underline{A} & \underline{B} \\ \underline{B} & \underline{D} \end{pmatrix}, \quad (10)$$

with

$$\underline{A} = \begin{pmatrix} C_{x, \text{tot}} & 0 \\ 0 & C_{y, \text{tot}} \end{pmatrix}, \quad (11)$$

$$\underline{B} = \begin{pmatrix} -C_{1x} & -C_{2x} \\ -C_{1y} & -C_{2y} \end{pmatrix}, \quad (12)$$

and

$$\underline{D} = \begin{pmatrix} C_{T1, \text{tot}} & 0 \\ 0 & C_{T2, \text{tot}} \end{pmatrix}, \quad (13)$$

where we defined the total capacities  $C_{\mu, \text{tot}} \equiv \sum_j C_{j\mu} + C_{g, \mu}$  and  $C_{T, j, \text{tot}} \equiv C_{gT, j} + \sum_\mu C_{j\mu}$ . We denote the inverse of this matrix as

$$\underline{C}^{-1} = \begin{pmatrix} \underline{U} & \underline{V} \\ \underline{V}^\top & \underline{W} \end{pmatrix}, \quad (14)$$

which can be derived to be

$$\underline{U} = \frac{1}{A - B \frac{1}{D} B^T}, \quad (15)$$

$$\underline{V} = -\frac{1}{A - B \frac{1}{D} B^T} B \frac{1}{D}, \quad (16)$$

and

$$\underline{W} = \frac{1}{D - B^T \frac{1}{A} B}. \quad (17)$$

Defining the vectors  $\mathbf{E}_J \equiv (E_{J, x}, E_{J, y})^\top$ ,  $\boldsymbol{\varphi} \equiv (\varphi_x, \varphi_y)^\top$ , and  $\tilde{\mathbf{N}} \equiv (\tilde{N}_x, \tilde{N}_y)^\top$ , we find for the full Hamiltonian

$$\begin{aligned} H_{\text{AA}}^{(2)} = & -\sum_{j=1}^2 E_{JT, j} \cos(\varphi_{T, j}) + V_{\text{CT}}^{(2)} \\ & - \mathbf{E}_J \cdot \cos(\boldsymbol{\varphi}) + V_C^{(2)}, \quad (18) \end{aligned}$$

where we define the cosine of a vector component-wise  $[\cos(\boldsymbol{\varphi})]_\mu := \cos(\varphi_\mu)$ . The islands forming the charge lattice contribute capacitively as

$$V_C^{(2)} = 2\tilde{\mathbf{N}}^\top \underline{E}_C \tilde{\mathbf{N}}, \quad (19)$$

with the charging energy matrix  $\underline{E}_C$ . Furthermore, the capacitive energy contribution of the transmons is given by

$$V_{CT}^{(2)} = \sum_{j=1}^2 E_{CT,j} \left( \tilde{N}_{T,j} + \mathbf{\Lambda}_j \cdot \tilde{\mathbf{N}} \right)^2 + E_{CT,cross} \left( \tilde{N}_{T,1} + \mathbf{\Lambda}_1 \cdot \tilde{\mathbf{N}} \right) \left( \tilde{N}_{T,2} + \mathbf{\Lambda}_2 \cdot \tilde{\mathbf{N}} \right), \quad (20)$$

with the transmon charging energies

$$E_{CT,j} = 2e^2 \frac{C_{x,tot} C_{y,tot} C_{T\bar{j},tot} - C_{\bar{j}x}^2 C_{y,tot} - C_{\bar{j}y}^2 C_{x,tot}}{\det[\mathbf{C}]}, \quad (21)$$

( $\bar{j} \neq j$ ), the cross coupling energy

$$E_{CT,cross} = 4e^2 \frac{C_{x,tot} C_{1y} C_{2y} + C_{y,tot} C_{1x} C_{2x}}{\det[\mathbf{C}]}, \quad (22)$$

and the quasiperiodicity parameters

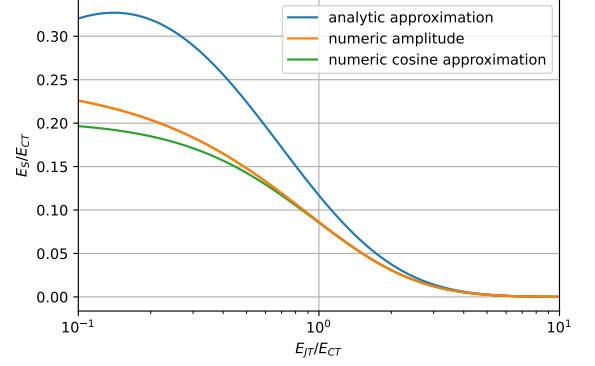
$$\lambda_{j\mu} = \frac{C_{j\mu}}{C_{\mu,tot}}, \quad (23)$$

forming the vectors  $\mathbf{\Lambda}_j \equiv (\lambda_{jx}, \lambda_{jy})^\top$ . After quantizing the Hamiltonian, we assume tuning into an appropriate parameter regime where the transmon charges  $\hat{N}_{T,j}$  have much faster dynamics than the island charges  $\hat{N}_\mu$  (the extensive discussion on this regime for the 1D case of the quasiperiodic capacitor in the main text can easily be extended to this multidimensional case). Consequently, we can derive an effective version of this Hamiltonian given by Eq. (20) of the main text, characterized by a quasiperiodic capacitive behavior. The cross-coupling between the transmons  $\sim E_{CT,cross}$  is suppressed in the transmon limit, that is  $E_{CT,j}/E_{JT,j} \ll 1$ .

## SUPPLEMENTARY NOTE 2

### Numerical evaluation of $E_S$

The essence of the quasiperiodic nonlinear capacitor is to make use of the offset-charge dependence of the transmon ground state while requantizing this offset-charge living on a finite superconducting island. In this process we used the cosine approximation  $-E_S \cos(2\pi N_g)$  which is perfectly valid in the transmon regime ( $E_{JT} \gg E_{CT}$ ) where the amplitude  $E_S$  is exponentially suppressed; see Eq. (6) of the main text. However, since we are interested in a measurable effect of this term, we suggest to go beyond the transmon regime, where  $E_{JT} \sim E_{CT}$  or even  $E_{JT} < E_{CT}$ . This is outside of the regime where the above approximation is justified. This motivates us to do a numerical evaluation of the exact amplitude  $E_S$  as well as the harmonicity of the term; see Supplementary Figure 1. To test the latter, we compute the Fourier coefficient of the numeric eigenvalue with the same frequency as the analytic term and compare it to the full numeric amplitude. We find that, while we overestimate  $E_S$  a bit outside the transmon regime, the analytic result still captures roughly the correct magnitude. Furthermore, the anharmonicity stays rather small, which yields only a small contribution of phase cotunneling events.

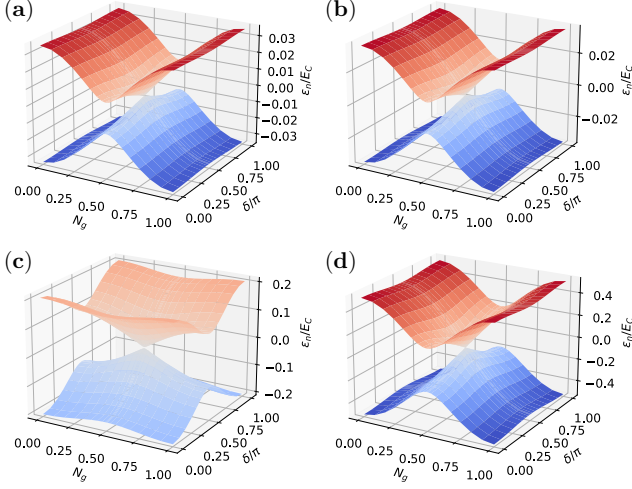


Supplementary Figure 1. **Numeric evaluation of  $E_S$ .** We compare the analytic approximation from Eq. (6) of the main text (blue line) with a numerically calculated amplitude (orange line) and the cosine component of the numerical result (green line). This is done as a function of  $E_{JT}/E_{CT}$  in the transmon regime ( $E_{JT} \gg E_{CT}$ ) and beyond.

## SUPPLEMENTARY NOTE 3

### Numerical evaluation of the Dirac spectrum

To derive the Dirac Hamiltonian (see Eq. (10) of the main text) the Hilbert space was reduced to only a single pseudo-spin degree of freedom and two externally controlled parameters. First we got rid of the degrees of freedom of the auxiliary transmon of the QPNC and afterwards we used Bloch's theorem to find the final 2D low-energy basis with Dirac-like dynamics. To affirm the validity of this process and the resulting energy profile, we numerically evaluate the Hamiltonian before and after analytically solving the dynamics of the auxiliary transmon (see Supplementary Figure 2) and compare it to the expected Dirac spectrum. We perform this comparison for two sets of parameters. First, we choose a parameter regime where our approximations in the main text are expected to hold; see Supplementary Figures 2(a, b). Here we see indeed a very good qualitative and quantitative agreement with the expected Dirac cone spectrum, i.e., the eigenvalues of the Hamiltonian in Eq. (10) of the main text. Then, in Supplementary Figures 2(c, d), we plug in values for which the approximations cannot be expected to hold. Surprisingly, we see that nonetheless, the Dirac point feature is still preserved in this regime. But the agreement is now only qualitative, and no longer quantitative between Supplementary Figures 2(c) and (d). In particular, the amplitude of the energy spectrum with respect to  $N_g$  is reduced in the full calculation of Supplementary Figure 2(c). This is due to the fact that here, the energy of the Cooper-pair cotunneling junction,  $E_{2J} = 20E_C$ , is chosen very large compared to the excitation gap of the auxiliary transmon, which, for the chosen parameters, scales as  $\sim 2\sqrt{E_{CT}E_{JT}} = 8E_C$ .



**Supplementary Figure 2. Tests of the approximation procedure arriving at the Dirac cones.** We compare the eigenspectrum  $\epsilon_n$  of the Dirac circuit (see Fig. 1(c) of the main text) including explicitly the auxiliary transmon dynamics [shown in panels (a, c)] and with the effective treatment of the quasiperiodic nonlinear capacitor, i.e., where we assume that the auxiliary transmon remains in the ground state (b, d). In (a, b) we used the parameters  $E_J = 0.01E_C$ ,  $E_{2J} = 20E_C$ , and in (a)  $E_{JT} = 4E_{CT} = 16E_C$ , and in (b) the equivalent  $E_S \approx 0.024E_C$ . We find a very good agreement of the spectra of the two Hamiltonians. In (c) we choose a lower  $E_{JT} = E_{CT}$ , and respectively  $E_S \approx 0.47E_C$  in (d), while tuning up  $E_J = 0.2E_C$  accordingly in both (c, d). Here, neglecting the auxiliary transmon dynamics was not justified (since (c) and (d) give quantitatively different spectra). However, the discrepancy can be easily understood by means of excitations in the auxiliary transmon, see also main text.

As pointed out in the main text, this leads to excitations in the auxiliary transmon state, effectively reducing  $E_S$ . This fully explains the difference between the two figures.

## SUPPLEMENTARY NOTE 4

### Effect of a $\delta\lambda$ -detuning on the Dirac spectrum

In the Dirac circuit, the large  $E_{2J}$  element stabilizes phase slips of exactly  $\pi$ , which enter in the effective Dirac Hamiltonian via the  $-E_S \cos(\pi\hat{N})$  term. This allows us to suppress the linear capacitance with a  $\cos(2\hat{\varphi})$ -term without suppressing the nonlinear capacitance. As we already discussed in the main text, a detuning  $\delta\lambda$  from the theoretically optimal value,  $\lambda = 1/2 + \delta\lambda$ , does not influence the spectrum within the 2D low-energy basis. In addition, a numerical analysis of the full Hamiltonian shows that, to the contrary, a small detuning from  $\lambda = 1/2$  can surprisingly be beneficial. Namely, a specific finite detuning  $\delta\lambda$  will actually exactly close the gap and thus counteract the perturbing effects of other imperfections

like the influence of higher-energy states.

## SUPPLEMENTARY NOTE 5

### Classical current-noise spectrum in more details

We consider an adiabatic current response (see Eq. (12) of the main text) due to classical offset charge noise, as described in the main text. As the two contributions to the response we find the Josephson current  $I_0(t) = 2e\partial_\delta\epsilon_n$  and a linear correction term  $I_1(t) = -2e\dot{N}_g\mathcal{B}_0[\delta, N_g(t)]$  proportional to the Berry curvature  $\mathcal{B}_0$ . Exploiting the periodicity of the current in  $N_g$ , we now cast the different contributions into a discrete Fourier series,  $I_0 = \sum_{n \in \mathbb{Z}} i_n \exp(i\pi n N_g)$  and  $2e\mathcal{B}_0 = \sum_{n \in \mathbb{Z}} b_n \exp(i\pi n N_g)$  such that  $I_1 = \partial_t \sum_{n \in \mathbb{Z}} i b_n \exp(i\pi n N_g)/(\pi n)$ . Thus we can straightforwardly calculate the noise spectrum and find two contributions, one from the Josephson current (with subscript J) and a Berry curvature contribution (with subscript B)

$$S(\omega) = S_J(\omega) + S_B(\omega), \quad (24)$$

with

$$S_J(\omega) = 4T \sum_{n>0} \frac{(\pi n)^2 |i_n|^2}{(\pi n)^4 + T^2 \omega^2}, \quad (25)$$

and

$$S_B(\omega) = 4T \sum_{n>0} \frac{\omega^2 |b_n|^2}{(\pi n)^4 + T^2 \omega^2}. \quad (26)$$

The total current noise  $S$  and its two contributions  $S_J$  and  $S_B$  are shown for the Dirac system and its gapped version in Fig. 3 of the main text.

When considering the phase  $\delta$  at a value away from the degeneracies at  $\pi/2$  and  $3\pi/2$ , we can identify the dominant Fourier coefficients  $i_{m_i}$  and  $b_{m_b}$ . We thus find an approximate Lorenz curve of height  $S_0 = 4T|i_{m_i}|^2/(\pi m_i)^2$  for  $S_J$ , and with an asymptotic behavior of  $S_J(\omega) \rightarrow \omega^{-2} (2\pi m_i)^2 |i_{m_i}|^2 / T$ . For  $S_B$  we find an inverted Lorenz curve, that is, a constant  $S_\infty$  minus a Lorenz curve of the same height  $S_\infty = 4|b_{m_b}|^2/T$ , which gives us the asymptotic value of the spectrum for high frequencies  $|\omega| \rightarrow \infty$ . Importantly, the asymptotic value vanishes  $S_\infty = 0$  in the Dirac system since the Berry curvature (and thus every  $b_n$ ) is zero, in contrast to a gapped system; see Fig. 3 of the main text. Therefore, we can probe whether or not the system is gapped by measuring this offset value  $S_\infty$ . With this in mind let us briefly discuss the behavior of the current noise contributions for a system that is possibly gapped.

To compare the two contributions,  $S_J$  and  $S_B$ , it is useful to consider the parameter  $\beta \equiv S_\infty/S_0 = (m_i\pi)^2 |b_1|^2 / |i_2|^2 T^2$ . However, note that the two Lorenz curves in general have different widths due to different

dominant Fourier coefficients of the current contributions contributing at different frequency values  $m_i$  and  $m_b$ , as discussed above. Hence we find a non-monotonous noise spectrum  $S(\omega)$  for values of  $\beta$  close to 1; see Fig. 3(a)

of the main text. Considering the symmetric case of  $E_S = E_J \equiv E$  and a small gap  $\Delta^2 \ll E^2$ , we can estimate the scaling of the noise spectrum contributions as  $S_0 \sim TE^2$  and  $S_\infty \sim \Delta^2/TE^2$  such that  $\beta \sim \Delta^2/T^2E^4$ .