

Supplementary information to "Deterministically separable polarization-entangled photons from battery-powered diode lasers"

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SUPPLEMENTARY NOTE 1: POLARIZATION ENTANGLEMENT

The reconstruction of the density matrix, ρ , was calculated via maximum likelihood quantum state tomography [1, 2]. The results presented in the Subsec. Entanglement Characterization of the main text shows a plot of the den-

sity matrix with the reconstructed from the data without subtracted accidental, ρ_{raw} . Here we present the matrix representation of ρ_{raw} along with, ρ_{net} , whose density matrix was reconstructed from the same data with accidentals subtracted from coincidence counts (note: ρ_{net} was not plotted in the main text). Here the exact density matrices for both the net and raw data is presented:

$$\rho_{net} = \frac{1}{100} \begin{pmatrix} 0.4 & 1.6 - i1.1 & 2.3 - i0.8 & 0.2 + i0.0 \\ 1.7 + i1.1 & 52.8 & 48.3 - i0.1 & -2.0 + i1.3 \\ 2.3 + i0.9 & 48.3 + i0.1 & 46.5 & -1.3 + i0.9 \\ 0.1 - i0.0 & -2.0 - i1.3 & -1.3 - i0.9 & 0.2 \end{pmatrix} \quad (1)$$

$$\rho_{raw} = \frac{1}{100} \begin{pmatrix} 4.7 & 0.1 - i0.5 & 4.2 - i1.7 & 0.6 - i2.2 \\ -0.1 + i0.5 & 48.0 & 41.4 - i0.4 & -2.1 + i0.4 \\ 4.2 + i1.7 & 41.4 + i0.4 & 41.9 & -1.5 + i1.4 \\ 0.6 + i2.2 & -2.1 - i0.4 & -1.5 - i1.4 & 5.4 \end{pmatrix}. \quad (2)$$

Furthermore, the data used in the maximum likelihood quantum state tomography is plotted in Fig. S1; in these figures, a bin width and coincidence window of 150 ps were used. In Fig. S1, the "coincidence bin" is highlighted in red and the "accidental bins" are highlighted in blue. When analyzing the data for maximum likelihood quantum state tomography of ρ_{net} the accidentals are calculated by averaging the coincidences in the accidental bins and are then subtracted from the peak bin, this procedure is done for each coincidence plot individually.

To calculate the error in our maximum likelihood tomography calculations different data files of the same measurements are used to reconstruct different density matrices. The average of the magnitude of the difference between different reconstructed density matrices then provides us with the average magnitude error. From this procedure we obtain that the average magnitude error of ± 0.0024 for ρ_{raw} and ± 0.0016 for ρ_{net} .

SUPPLEMENTARY NOTE 2: FORMALISM EXTENSION

To reproduce the phase convention of the results presented in section Methods of the main text, one can adopt the notation of phase-free variable beam-splitter (BS) with transmission $\eta \in [0, 1]$. A BS between the modes i and j is given by the operator

$$\hat{B}_{ij}(\eta) = \exp \left[\left(\arcsin(\sqrt{\eta}) - \frac{\pi}{2} \right) (\hat{a}_i \hat{a}_j^\dagger - \hat{a}_i^\dagger \hat{a}_j) \right], \quad (3)$$

where $\hat{a}_x$ and $\hat{a}_x^\dagger$, represent the bosonic mode annihilation and creation operators for a mode $x \in \{i, j\}$ respectively. Then by denoting

$$\begin{pmatrix} \hat{b}_i \\ \hat{b}_j \end{pmatrix} = \hat{B}_{ij}(\eta)^\dagger \begin{pmatrix} \hat{a}_i \\ \hat{a}_j \end{pmatrix} \hat{B}_{ij}(\eta), \quad (4)$$

where $\hat{b}_x$ and $\hat{b}_x^\dagger$, represents the annihilation and creation operators at the output of the beam splitter in mode $x \in \{i, j\}$ respectively. Alternatively, the phase-free BS operator can be equivalently represented by the Bogoliubov transformation

$$\begin{pmatrix} \hat{b}_i \\ \hat{b}_j \end{pmatrix} = \begin{pmatrix} \sqrt{\eta} & \sqrt{1-\eta} \\ \sqrt{1-\eta} & -\sqrt{\eta} \end{pmatrix} \begin{pmatrix} \hat{a}_i \\ \hat{a}_j \end{pmatrix}. \quad (5)$$

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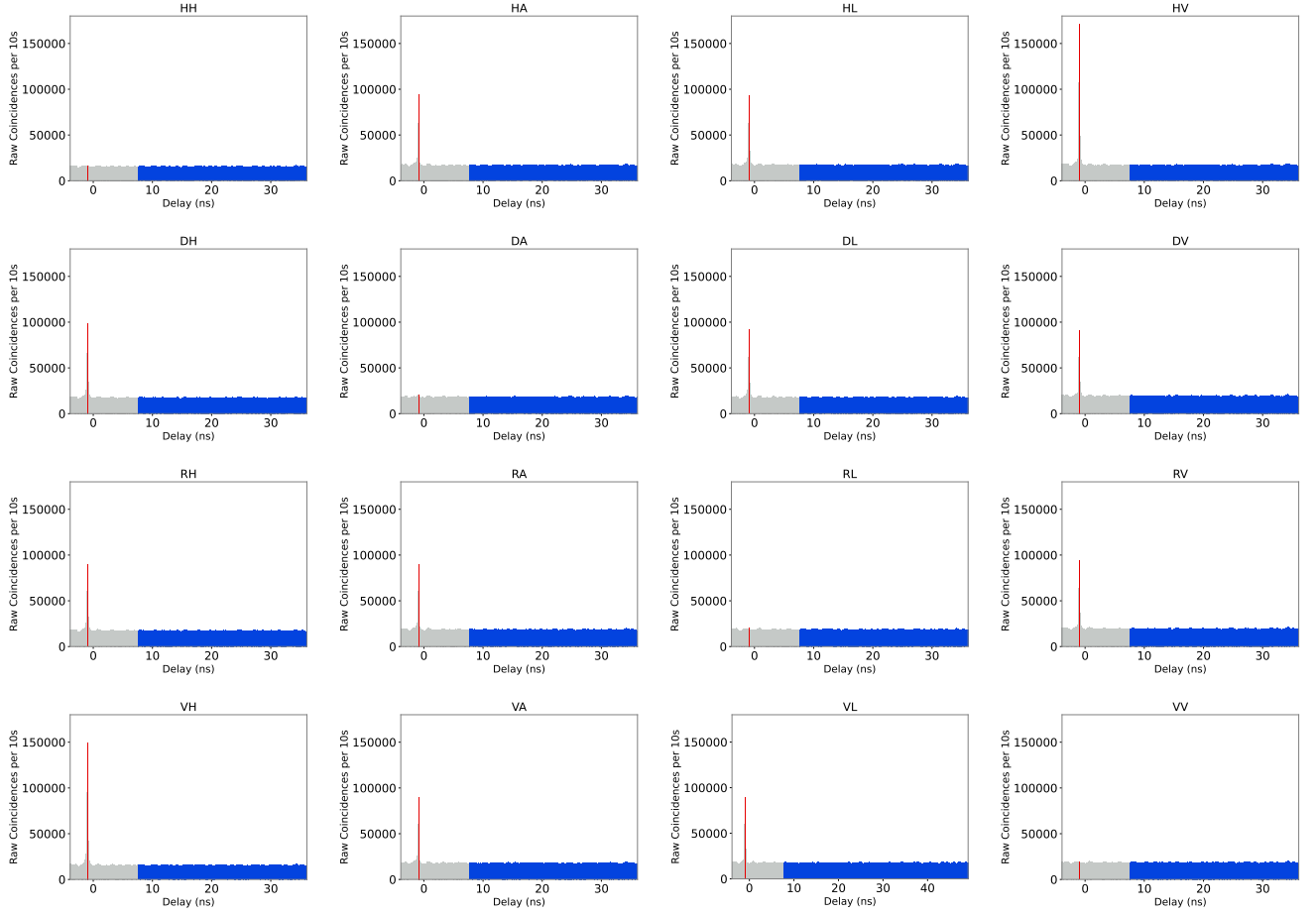


FIG. S1: Raw coincidence data from the quantum state tomography. Plots XY $X \in \{H, D, R, V\}$ and $Y \in \{H, A, L, V\}$ represent polarization measurements in path 1 and 2 respectively, and H, V, D, A, R, L refers to the horizontal, vertical, diagonal, anti-diagonal, right-hand circular, and left-hand circular polarizations respectively. The coincidence peak is plotted in red and the section used to calculate accidentals is plotted in blue.

SUPPLEMENTARY NOTE 3: PAIR SEPARATION PROBABILITIES

We confirmed that the theory of time-reversed Hong-Ou-Mandel (TRHOM) extends itself to the deterministic separation of photons even if the two photons in a single pair are themselves distinguishable as seen in Eq. (7) of the main manuscript [5].

The maximum and minimum probabilities of observing a coincidence (anti-bunched and bunched output states) during a TRHOM interference experiment assuming a perfect modal overlap at the BS given in Fig. S2. We note that the maximum probability of finding a coincidence monotonically increases with the transmission of both channels. However, the minimum probability of finding a coincidence can always reach zero, allowing for unit visibility. Hence, as observed in the subsection Time-Reversed Hong-Ou-Mandel of the main text, increasing the loss can achieve a higher visibility even though it can lead to a lower probability of achieving a deterministic pair separation.

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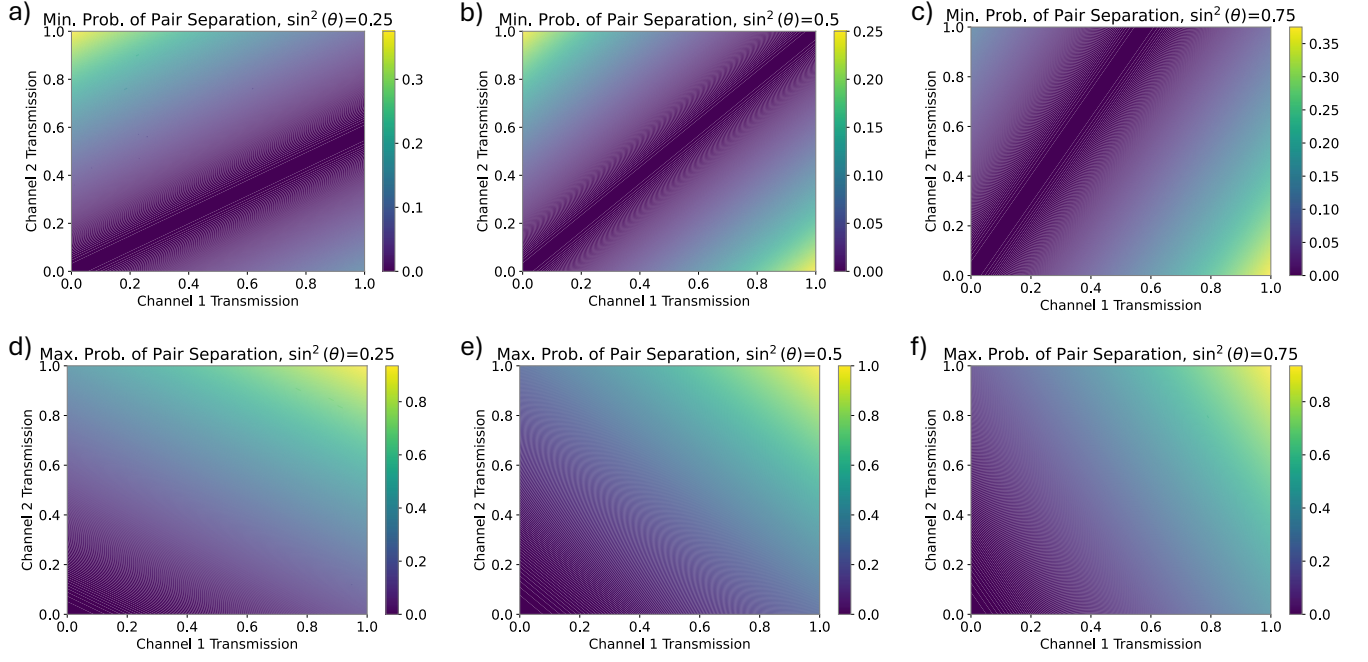


FIG. S2: **a, b, c** The minimum probability of seeing a coincidence when the ratio of pump power in channel 1 and channel 2 given by $\cos^2 \theta : \sin^2 \theta$ is set to $\sin^2(\theta) = 0.25, 0.5, 0.75$ respectively. **d, e, f** The maximum probability of seeing pair separation when the ratio is set to $\sin^2(\theta) = 0.25, 0.5, 0.75$ respectively. The plots are generated using QuTiP [3, 4].